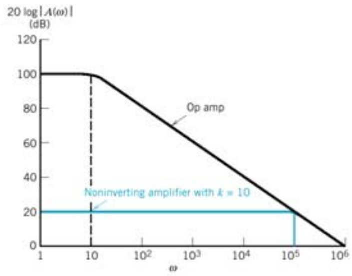
Frequency response

- Static analysis
  - Low frequency response
  - Response without time dependent elements
- Real circuit response
  - Strongly dependent on frequency
  - Small signal behavior with reactive passives
    - capacitances
      - Planar technology have parasitic capacitors all over the place
    - Inductors
    - Reactances are also constructive

Typical operational amplifier response

•Gain strongly dependent on frequency



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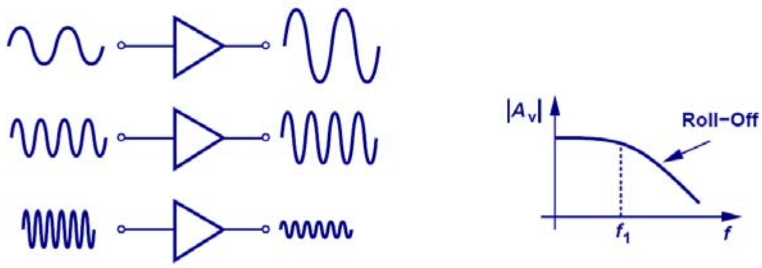
Frequency response

- Signal out changing with input signal frequency



(a) (b)

- Magnitude (amplitude) may change
- Phase (delay) may change

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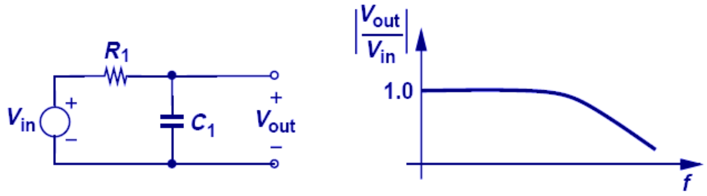
Passive low-pass filter example

- Passive filter



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    - high impedance a low frequency
    - Reduced impedance with increasing frequency
  - Giving gain of 1 in passband
  - Higher frequencies → reduced gain (transition band)
  - Phase?
- Circuit behavior with time-dependent elements

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Laplace transform

- Solving differential equations
  - Differentials → polynomials
- Reactive circuit elements time dependent
  - Capacitor: $i(t) = C\frac{\partial V}{\partial t}$  Open at low frequency / Short at high frequency
  - Inductor: $v(t) = L\frac{\partial i}{\partial t}$  Short at low frequency / Open at high frequency
- Laplace enable frequency analysis directly in frequency domain
  - Analysis for circuit performances
  - Analysis for circuit stability

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Transfer function

- In frequency domain (Laplace transformed from time-domain)

$$X_{out}(s) = H(s)\,X_{in}(s)$$

- Limited types of transfer functions
  – Stick to real values (avoid complex voltages and currents)
  – Limited to lumped circuit
- Transfer function form

$$H(s) = \frac{a_0 + a_1 s + \ldots + a_m s^m}{1 + b_1 s + b_2 s^2 + \ldots + b_n s^n}$$

  - a and b are reals and normally m≤n
  - All b positive for stability

  – Alternative forms

$$H(s) = K\frac{(s+z_1)(s+z_2)\ldots(s+z_m)}{(s+\omega_1)(s+\omega_2)\ldots(s+\omega_n)} = a\frac{\left(1+\frac{s}{z_1}\right)\left(1+\frac{s}{z_2}\right)\ldots\left(1+\frac{s}{z_m}\right)}{\left(1+\frac{s}{\omega_1}\right)\left(1+\frac{s}{\omega_2}\right)\ldots\left(1+\frac{s}{\omega_n}\right)}$$

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Poles and zeros

$$H(s) = K\frac{(s+z_1)(s+z_2)\ldots(s+z_m)}{(s+\omega_1)(s+\omega_2)\ldots(s+\omega_n)} = a\frac{\left(1+\frac{s}{z_1}\right)\left(1+\frac{s}{z_2}\right)\ldots\left(1+\frac{s}{z_m}\right)}{\left(1+\frac{s}{\omega_1}\right)\left(1+\frac{s}{\omega_2}\right)\ldots\left(1+\frac{s}{\omega_n}\right)}$$

  – Poles and zeros are real or complex conjugates
  – The actual roots are the *negatives* of the poles and zeros
  – Referred to as a positive frequency
- Magnitude and phase

Euler's formula

  – Sinusoidal: $x_{in}(t) = A_{in}\cos(\omega_{in}t) = A_{in}\dfrac{e^{j\omega_{in}t} + e^{-j\omega_{in}t}}{2}$

  – Frequency domain two solutions $s = j\omega_m$ and $s = -j\omega_m$
  – May find for particular $s = j\omega_m$:

$$x_{out}(t) = \frac{A_{in}}{2}\left|H(j\omega_{in})\right|\left(e^{j(\omega_{in}t+\phi)} + e^{-j(\omega_{in}t+\phi)}\right) = A_{in}\left|H(j\omega_{in})\right|\cos(\omega_{in}t+\phi)$$

  – Where magnitude is $\left|H(j\omega_{in})\right|$ and phase $\phi = \angle H(j\omega_{in})$

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Decibels

- Linear electric circuit excited by sinusoidal
  - Always give an output with the same frequency
  - Magnitude modified by $\left|H\left(j\omega_{in}\right)\right|$
  - And phase modified by $\phi = \angle H\left(j\omega_{in}\right)$
- Decibel
  - Magnitude response often logarithmic

$$20\log_{10}\left|H\left(j\omega_{in}\right)\right| dB$$

  - Two linear systems $H_1\left(j\omega_{in}\right)$ and $H_2\left(j\omega_{in}\right)$ in series
  - Have magnitude response

$$20\log_{10}\left(\left|H_1\left(j\omega_{in}\right)\right|\left|H_2\left(j\omega_{in}\right)\right|\right) dB = 20\log_{10}\left|H_1\left(j\omega_{in}\right)\right| dB + 20\log_{10}\left|H_2\left(j\omega_{in}\right)\right| dB$$

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First-order circuit

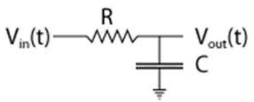
- Like passive 1. order low-pass filter

$$H(s) = \frac{A_0}{1+\frac{s}{\omega_0}} = \frac{A_0}{1+j\frac{\omega}{\omega_0}} = \frac{A_0}{\sqrt{1+\left(\frac{\omega}{\omega_0}\right)^2}} e^{j\phi} \qquad \phi = -\tan^{-1}\frac{\omega}{\omega_0}$$

Rads/s or Hz will do here
Units cancel.

- Example: passive low-pass filter

$$H(s) = \frac{1}{1+sRC} = \frac{1}{\sqrt{1+(\omega RC)^2}} e^{j\phi}$$



$$\omega_0 = \frac{1}{RC}$$

  - Assume $R = 100\Omega$ and $C = 0.00001F \rightarrow RC = 0.001$ $f_c = \frac{1}{2\pi RC} = 159Hz$
  - For $f = 50Hz \rightarrow \omega = 2\pi 50 = 314.5$ we get:

$$|H(s)| = \frac{1}{\sqrt{1+(\omega RC)^2}} = \frac{1}{\sqrt{1+(314.5\cdot 0.001)^2}} \approx 0.9530 = -0{,}42dB \qquad \phi = -\tan^{-1}\frac{\omega}{\omega_0} = -\tan^{-1}\frac{50}{159} \approx -17.5^\circ$$

  - For $f = 159Hz \rightarrow \omega = 2\pi 159 = 999$ we get: $|H(s)| \approx 0.7074 = -3dB$
    - Signal reduced to half in magnitude

$$\phi = -\tan^{-1}\frac{159}{159} \approx -45^\circ$$

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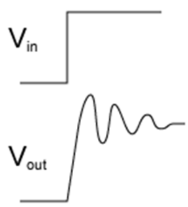
Impulse response

- Large signal step response
  - Slur-rate
    - Sharp transition →less steep
    - Problem in digital and analog
  - Ringing
    - More or less damped oscillations
    - Signal distortion
- Step response function
  - Laplace

$$u(t)=\begin{cases}0 \text{ for } t\le 0\\ 1 \text{ for } t>0\end{cases} \qquad U(t)=\frac{1}{s}$$

  - Output of linear system and step response

$$X_{out}(t)=A_{in}\frac{H(s)}{s}$$



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Slur rate

- Frequency → time
  - 1. order LP filter

$$H(s)=\frac{A_0}{1+\frac{s}{\omega_0}}$$

  - Applied step response

$$X_{out}(s)=A_{in}\frac{1}{s}\frac{A_0}{1+\frac{s}{\omega_0}}$$

  - Residue method

$$X_{out}(s)=A_{in}A_0\left[\frac{1}{s}-\frac{1}{s+\omega_0}\right]$$

  - Inverse Laplace give time domain

$$x_{out}(t)=u(t)A_{in}A_0\left(1-e^{-\frac{t}{\tau}}\right) \qquad \tau=\frac{1}{\omega_0}$$

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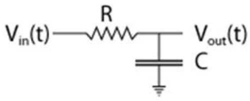
Slur rate with zero

- May be extended $$H(s) = A_0 \frac{1 + \frac{s}{\omega_Z}}{1 + \frac{s}{\omega_0}}$$
- And solved $$x_{out}(t) = u(t) A_{in} A_0 \left(1 - \left[1 - \frac{\omega_0}{\omega_Z} \right] e^{-\frac{t}{\tau}} \right)$$
- Example: 1. order passive LP filter

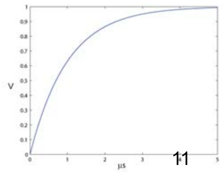


$$H(s) = \frac{1}{1 + sRC} \qquad \omega_0 = \frac{1}{RC}$$

$$R = 10\Omega \text{ and } C = 0.1\mu F \rightarrow RC = 0.000001 \quad f_{-3dB} = \frac{1}{2\pi RC} = 1.59kHz$$

  – Time constant:
  – Voltage at 2μs assuming 1V supply: $RC = 10\mu \text{s}$

$$x_{out}(t + 2\mu s) = 1\left(1 - e^{-\frac{2\cdot 10^{-6}}{10^{-6}}} \right) = 1 - e^{-2} = 0.8647V$$



2. order LP transfer function

- Real roots

$$H(s) = \frac{K}{(1 + s\tau_1)(1 + s\tau_2)} = \frac{K}{\left(1 + \frac{s}{\omega_{p1}}\right)\left(1 + \frac{s}{\omega_{p2}}\right)} = \frac{K\omega_{p1}\omega_{p2}}{(s + \omega_{p1})(s + \omega_{p2})}$$

- Coefficients real and positive or conjugate pairs
  – Popular form

$$H(s) = \frac{K\omega_{p1}\omega_{p2}}{\omega_{p1}\omega_{p2} + s(\omega_{p1} + \omega_{p2}) + s^2} = \frac{K\omega_0^2}{\omega_0^2 + s\frac{\omega_0}{Q} + s^2}$$

  – ω_0 – resonant frequency, Q – Q (quality) factor and K DC gain
  – Avoiding complex numbers (almost)

2. order transfer

- Analyze some cases
  - May equate denominators

$$H(s) = \frac{K\omega_0^2}{\omega_0^2 + s\dfrac{\omega_0}{Q} + s^2}$$

$$D(s) = \omega_0^2 + s\frac{\omega_0}{Q} + s^2 = (s + \omega_{p1})(s + \omega_{p2}) = \omega_{p1}\omega_{p2} + s(\omega_{p1} + \omega_{p2}) + s^2$$

  - Equating coefficients: $\omega_0^2 = \omega_{p1}\omega_{p2}$ $\quad \dfrac{\omega_0}{Q} = (\omega_{p1} + \omega_{p2})$
  - Solving yields

$$\omega_{p1}, \omega_{p2} = \frac{\omega_0}{2Q}\left(1 \pm \sqrt{1 - 4Q^2}\right)$$

  - Assuming roots $\omega_{p1} \ll \omega_{p2}$ giving $Q \ll 1$ and $\sqrt{1-4Q^2} \cong 1 - 2Q^2$

$$\omega_{p1} \approx \omega_0 Q \qquad \omega_{p2} \approx \frac{\omega_0}{Q}$$

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Simplifications for 2. order LP

- $\omega \ll \omega_{p1}$

$$|H(\omega)| = K$$

- $\omega = \omega_{p1}$

$$|H(\omega)| = \frac{K}{\sqrt{2}} \qquad \angle H(\omega) = -45^\circ$$

- $\omega_{p1} < \omega < \omega_{p2}$

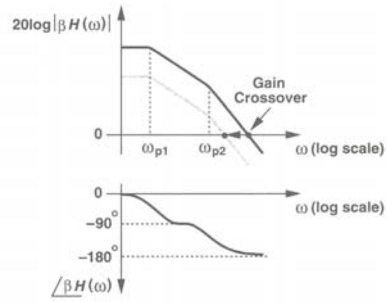
$$|H(\omega)| = \frac{K\omega_{p1}}{\omega} \qquad \angle H(\omega) \cong -90^\circ$$

- $\omega = \omega_{p2}$

$$|H(\omega)| = \frac{K\omega_{p1}}{\sqrt{2}\omega_{p2}} \qquad \angle H(\omega) \cong -135^\circ$$

- $\omega > \omega_{p2}$

$$|H(\omega)| = \frac{K\omega_{p1}\omega_{p2}}{\omega^2} \qquad \angle H(\omega) \cong -180^\circ$$



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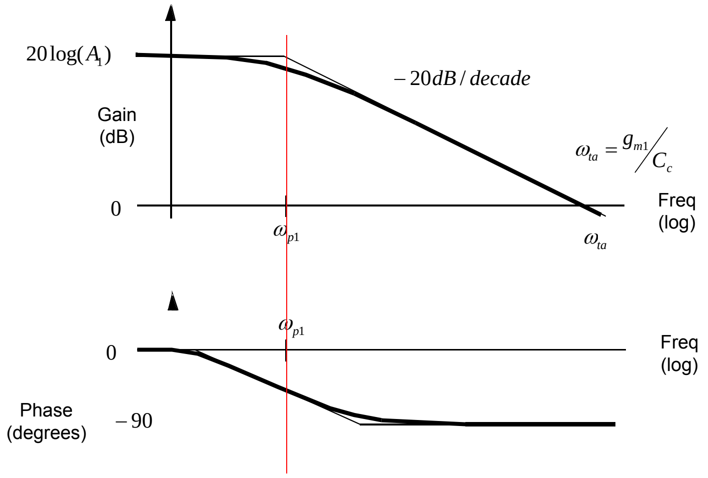
Bode plot

- 1. order system



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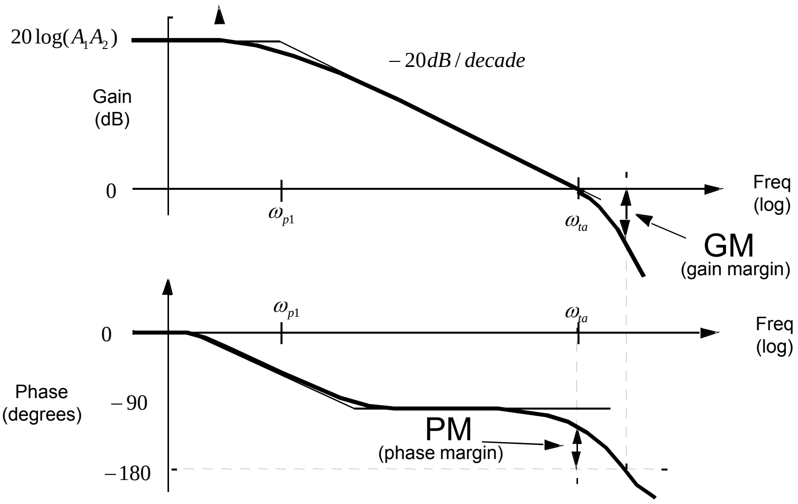
Bode plot

- 2. order system



High frequency pole may give instability

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Analytical stability analysis

Determine stability
- ω_0 -- pole frequency
- Q – quality factor

$$A(s) = A(0)\frac{N(s)}{1 + \frac{s}{\omega_0 Q} + \frac{s^2}{\omega_0}}$$

$Q < 0.5$
  - No overshoot (only real poles)
  - Unconditionally stable
  - Max magnitude at DC

$0.5 < Q < \sqrt{1/2} = 0.707$
  - Ringing overshoot

$$\%overshoot = 100e^{-\pi\sqrt{4Q^2-1}}$$

$Q = \sqrt{1/2}$
  - Critically damped system
  - -3db frequency is ω_0

$Q > \sqrt{1/2}$
  - Unstable and oscillations may occur
  - May be carefully used for improved performance

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MOS caps in active region

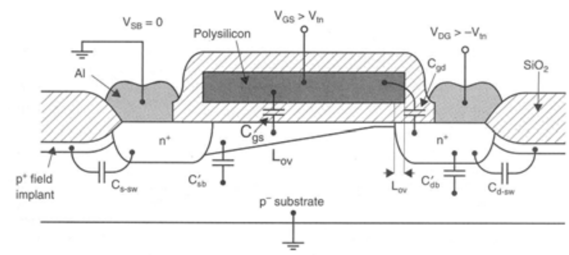
- Gate capacitance

$$C_{gs} = \frac{2}{3}WLC_{ox}$$

- Fringing capacitances
  - Overlap

$$C_{ov} = WL_{ov}C_{ox} \Rightarrow$$

$$C_{gs} = WC_{ox}\left(\frac{2}{3}L + L_{ov}\right)$$



- Source-bulk capacitance (+channel cap when present)

$$C_{sb}^{'} = \left(A_s + A_{ch}\right)\frac{C_{j0}}{\sqrt{1 + V_{SB}/\Phi_0}}$$

A_S – source area

A_{ch} – channel area

C_{j0} – unit depletion capacitance at 0V

Φ_0 – build in junction potential

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MOS model active region

- Drain-bulk capacitance

$$C_{sb}' = A_D \frac{C_{j0}}{\sqrt{1 + V_{DB}/\Phi_0}}$$

- Gate-drain overlap
  - Miller capacitance

$$C_{gd} = C_{ox} W L_{ov}$$

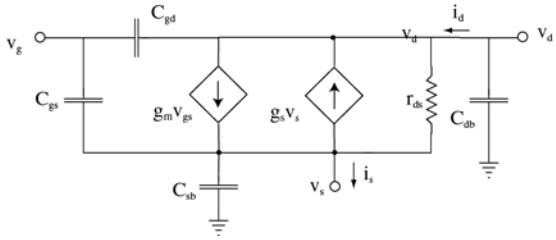
- Sidewall capacitances

$$C_{s-sw} = P_S \frac{C_{j-sw0}}{\sqrt{1 + V_{DB}/\Phi_0}}$$

$$C_{D-sw} = P_D \frac{C_{j-sw0}}{\sqrt{1 + V_{SB}/\Phi_0}}$$

$P_{S(D)}$ – source (drain) perimeter

C_{j-sw0} – unit sidewall capacitance at 0V



- Bulk capacitances

$$C_{sb} = C_{sb}' + C_{s-sw} \qquad C_{db} = C_{db}' + C_{d-sw}$$

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MOS transistor model

- Triode region
  - Gain give as slope

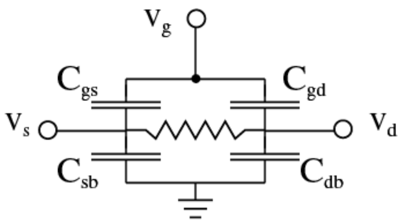
$$I_D = \mu_n C_{ox} \frac{W}{L}\left[(V_{GS} - V_{tn})V_{DS} - \frac{V_{DS}^2}{2}\right]$$

  - Output conductance

$$\frac{1}{r_{ds}} = g_{ds} = \frac{\partial I_D}{\partial V_{DS}} = \mu_n C_{ox} \frac{W}{L}(V_{GS} - V_{tn} - V_{DS})$$

    - V_{DS} is small and sometimes dropped

$$g_{ds} \approx \mu_n C_{ox} \frac{W}{L}(V_{GS} - V_{tn})$$



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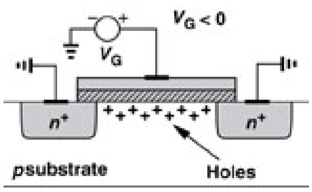
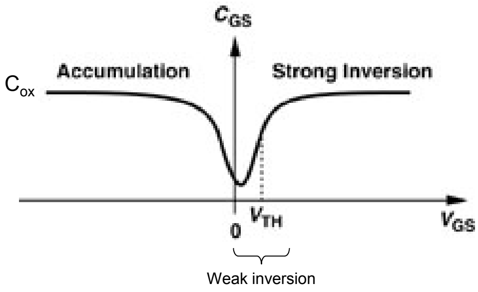
MOS Capacitor

- Nonlinear gate capacitance
  - Dips in weak inversion
  - Often used in accumulation





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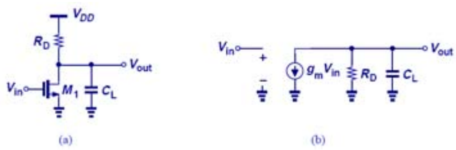
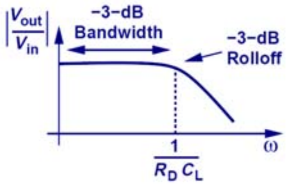
CS stage with load capacitor

- Finding transfer function



$$V_{out} = -g_m V_{in}\left(R_D \parallel \frac{1}{C_L s}\right)$$

  - Simply by adding capacitance impedance to output load
- magnitude



$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{g_m R_D}{\sqrt{R_D^2 C_L^2 \omega^2 + 1}}.$$

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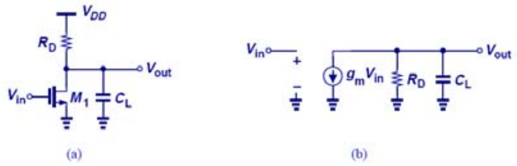
Bandwidth

- Example

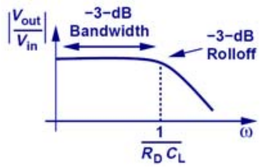


$$H(s) = \frac{V_{out}}{V_{in}}(s) = -g_m (R_D \parallel \frac{1}{C_L s})$$

$$= \frac{-g_m R_D}{R_D C_L s + 1}.$$

$$s = j\omega \rightarrow \left|\frac{V_{out}}{V_{in}}\right| = \frac{g_m R_D}{\sqrt{R_D^2 C_L^2 \omega^2 + 1}}.$$

$$\omega = 1/(R_D C_L) \quad \left|\frac{V_{out}}{V_{in}}\right| = \frac{g_m R_D}{\sqrt{2}}.$$



Time constant or -3dB bandwidth

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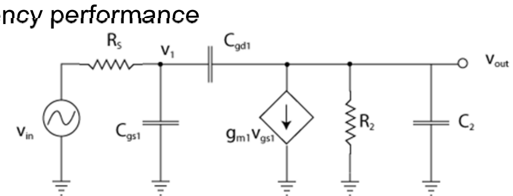
CS Frequency response

- Common source amp
  - Small signal, high frequency performance

$$C_2 = C_L + C_{db1} + C_{db2}$$

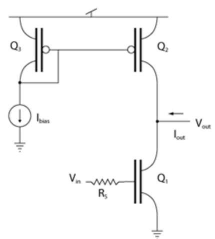
$$R_2 = r_{ds1} \| r_{ds2}$$



  - Nodal analysis
    - Sum currents

V_1: $$(v_1 - v_{in})G_{in} + v_1 sC_{gs1} + (v_1 - v_{out})sC_{gd1} = 0$$

$$v_1(G_S + sC_{gs1} + sC_{gd1}) - v_{in}G_S - v_{out}sC_{gd1} = 0$$



V_{out}: $$v_{out}G_2 + (v_{out} - v_1)sC_{gd1} - v_{out}sC_2 + g_{m1}v_1 = 0$$

$$v_{out}(G_2 + sC_{gd1} + sC_2) - v_1 sC_{gd1} + g_{m1}v_1 = 0$$

$$\frac{v_{out}}{v_{in}} = -\frac{g_{m1}R_2\left(1 - s\,{C_{gd1}}/{g_{m1}}\right)}{1 + sa + s^2 b}$$

$$a = R_S\left(C_{gs1} + C_{gd1}\left(1 + g_{m1}R_2\right)\right) + R_2\left(C_{gd1} + C_2\right)$$

$$b = R_S R_2\left(C_{gd1}C_{gs1} + C_{gs1}C_2 + C_{gd1}C_2\right)$$

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CS frequency response

- LF gain setting s=0 is giving: $A = -g_{m1}R_2$
- Assuming two poles $\omega_{p1} << \omega_{p2}$ we may write denominator:

$$D(s) = \left(1 + \frac{s}{\omega_{p1}}\right)\left(1 + \frac{s}{\omega_{p2}}\right) = 1 + \frac{s}{\omega_{p1}} + \frac{s^2}{\omega_{p1}\omega_{p2}}$$

- Giving ω_{p1} : $\omega_{p1} = \frac{1}{a} = \frac{1}{R_S\left(C_{gs1} + C_{gd1}\left(1 + g_{m1}R_2\right)\right) + R_2\left(C_{gd1} + C_2\right)}$
- And ω_{p2} : $\omega_{p2} = \frac{1}{\omega_{p1}b} \cong \frac{g_{m1}C_{gd1}}{C_{gs1}C_{gd1} + C_{gs1}C_2 + C_{gd1}C_2}$
- Zero at: $\omega_z = -\frac{g_{m1}}{C_{gd1}}$
  - May cause issues at high frequencies
  - Actually maintain 1.order transition

– With large load capacitance:

$$\omega_{p1} \cong \frac{1}{R_2C_2} \qquad \omega_{p2} \cong \frac{C_{gd1}}{C_{gs1} + C_{gd1}} \frac{g_{m1}}{C_2}$$

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Common source freq response

– At moderate frequencies $-s\, {C_{gd1}}/{g_{m1}}$ and s^2b ignored

$$\frac{v_{out}}{v_{in}} = \frac{-g_{m1}R_2}{1 + s\left(R_S\left(C_{gs1} + C_{gd1}\left(1 + g_{m1}R_2\right)\right) + R_2\left(C_{gd1} + C_2\right)\right)}$$

– Magnitude response at -3dB (ω_0)

$$\left|A(j\omega)\right| = \frac{1}{\sqrt{2}} \Rightarrow 1 + \left(\omega/\omega_0\right)^2 = 2 \Rightarrow \omega_0 = \frac{1}{a}$$

$$\left|A(j\omega)\right| = \frac{|K|}{\sqrt{1 + (\omega/\omega_0)^2}}$$

$$\omega_0 = \frac{1}{R_S\left(C_{gs1} + C_{gd1}\left(1 + g_{m1}R_2\right)\right) + R_2\left(C_{gd1} + C_2\right)}$$

– First term in denominator dominates unless $R_S << R_2$

$$\omega_0 = \frac{1}{R_S\left(C_{gs1} + C_{gd1}(1 + A)\right)} \quad \text{for } A = g_{m1}R_2$$

Miller capacitance → C_{gd1} is multiplied by 1 plus the LF gain

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CS frequency response

- Keypoints
  - Simple single transistor CS amplifier → complicated to analyze
  - Simplifications are important
  - CS stage
    - 2 poles
    - 1 zero
  - Gate-drain capacitance "amplified"
    - Miller-capacitance
    - Small, but significant

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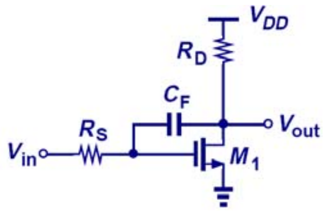
What about this?

- CS stage with feedback



  - Nodes connected by floating capacitor
  - Simple node for each pole does not work
  - Must transform before analysis
    - Caps to ground…..

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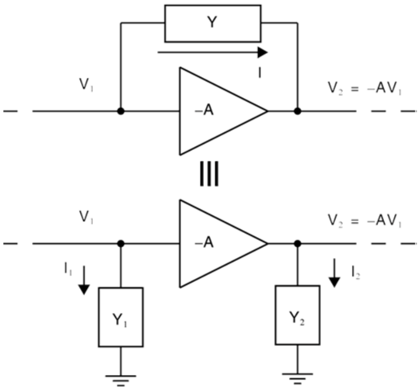
Miller Theorem

- Simplifying conversion
  - Coupling → decoupling
  - Let

$$Y_1(s) = Y(s)(1+A)$$

$$Y_2(s) = Y(s)\left(1+\frac{1}{A}\right)$$

  - Then same system behavior
  - Frequently used for capacitors
    - Coupling capacitors → decoupling capacitors



Approximation!

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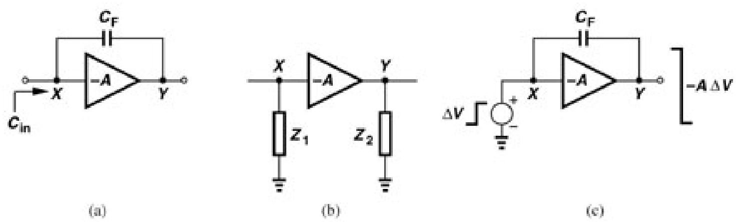
Example

- Ideal amp with feedback capacitor



$$Z = 1/sC_F \Rightarrow Z_1 = (1/sC_F)/(1+A)$$

$$\Rightarrow C_1 = C_F(1+A)$$

$$Z = 1/sC_F \Rightarrow Z_2 = (1/sC_F)/(1+1/A)$$

$$\Rightarrow C_2 = C_F(1+A^{-1}) \approx C_F$$

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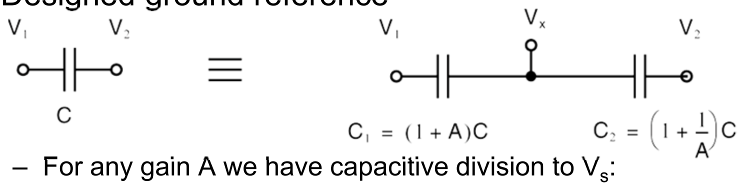
Capacitive ratios

- Designed ground reference



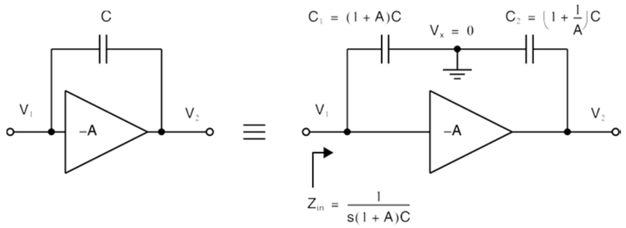
  - For any gain A we have capacitive division to V_s:

$$\frac{C_1C_2}{C_1+C_2}=\frac{C(1+A)C(1+\frac{1}{A})}{C(1+A)+C(1+\frac{1}{A})}=\frac{(A+2+\frac{1}{A})C^2}{(A+2+\frac{1}{A})C}=C$$

$$V_s=V_1+\frac{C_1C_2}{C_1+C_2}\left(V_2-V_1\right)$$

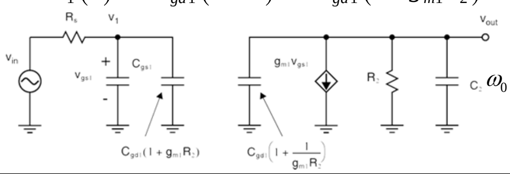
  - Inserting values give: $V_s=0$
    - With capacitive ratio match to gain → no signal on (virtual) middle node
    - May as well ground…..

Miller simplification in amps



- Keypoints
  - Miller effect → dominant pole
  - Quick and easy estimate with capacitor in inverting amplifiers
    - Assume LF gain: $A=g_{m1}R_2$
    - Admittance:

$$Y_1\left(s\right)=sC_{gd1}\left(1+A\right)=sC_{gd1}\left(1+g_{m1}R_2\right) \qquad Y_2\left(s\right)=sC_{gd1}\left(1+\frac{1}{g_{m1}R_2}\right)$$



$$\omega_0=\frac{1}{R_S\left(C_{gs1}+C_{gd1}\left(1+A\right)\right)} \quad \text{for } A=g_{m1}R_2$$

Miller effect

Zero-value time-constant analysis

- Powerful high frequency estimation
  1. Set all independent source to zero.
  2. For each capacitor, C_k, determine time-constant with all other caps open circuit. Replace C_k with a voltage source and determine resistance.
  3. Dominant pole (-3dB) found by summing poles:

$$\omega_{-3dB} = \frac{1}{\sum \tau_k} = \frac{1}{\sum R_k C_k}$$

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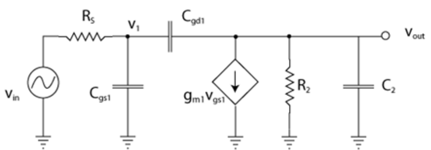
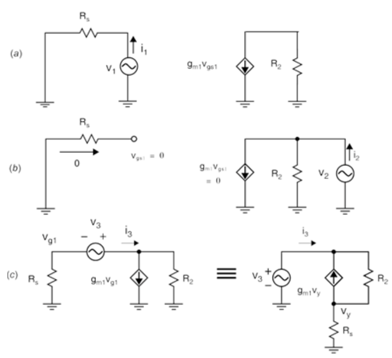
CS stage as example





- Three capacitors
  - Timeconstant for C_{gs1}: $\tau_1 = R_S C_{gs1}$
  - Timeconstant for C_2: $\tau_2 = R_2 C_2$
  - Timeconstant for C_{gd1}:
    - KCL at v_y:

$$\frac{(v_3 - v_y)}{R_2} - g_{m1} v_y - i_3 = 0 \text{ and } v_y = i_3 R_S \Rightarrow$$

$$R_3 = \frac{v_3}{i_3} = R_S (1 + g_{m1} R_2) + R_2$$

$$\tau_3 = \left[R_S (1 + g_{m1} R_2) + R_2 \right] C_{gd1}$$

$$\omega_{-3dB} = \frac{1}{\sum \tau_k} = \frac{1}{R_S \left(C_{gs1} + C_{gd1} (1 + g_{m1} R_2) \right) + R_2 \left(C_{gd1} + C_2 \right)}$$

Same as Already found

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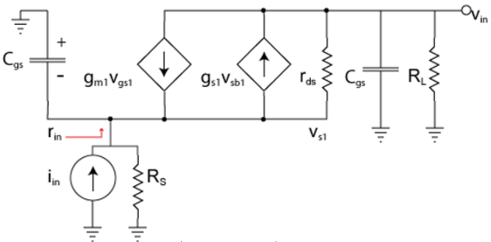
Common-gate AMP

- Superior to CS amp
  - No Miller cap
  - Low impedance input
    - If matched
  - Modeling with current in
    - Norton



- Find time-constants:
  - Timeconstant for C_{gs1}: $\tau_1 = (r_{in} \parallel R_S) C_{gs} \approx \left(\frac{1}{g_{m1}} \parallel R_S\right) C_{gs} \approx \frac{C_{gs}}{g_{m1}}$
  - Timeconstant for C_2:

    $\tau_2 = (r_{d1} \parallel R_L) C_2 = (r_{ds}(1 + g_m R_S) \parallel R_L) C_2$
    - R_L often dominating giving

$$\tau_2 \approx R_L C_2 \qquad \omega_{-3dB} = \frac{1}{\tau_1 + \tau_2} = \frac{1}{\frac{C_{gs}}{g_{m1}} + R_L C_2}$$

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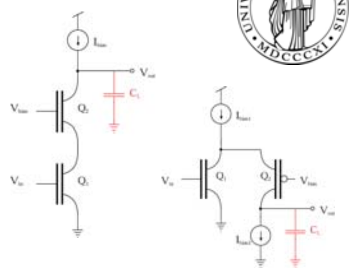
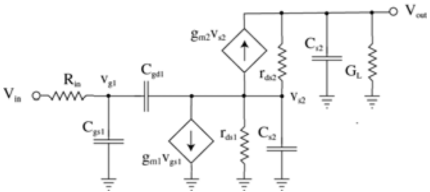
Cascode gain stage

- Large gain, single stage solution
  - PMOS in folded cascode→ lower mobility and lower frequency
  - Current source on output not highres
    - Handled well by cascode stage





  - Excact analysis complicated, use simulations
    - Approximation by zero-value time-constants
      - 4 capacitors → 4 time-constants
      - Parallel caps merged

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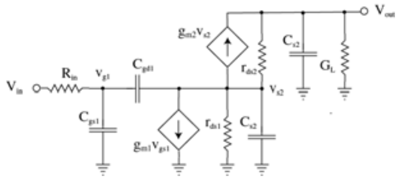
- Time-constant at output C_{out}:

$$\tau_{out} = (r_{d2} \parallel R_L) C_{out} = (g_m r_{ds1} r_{ds2} \parallel R_L) C_{out}$$



- Time-constant at output C_{gs1}:

$$\tau_{gs1} = R_S C_{gs1}$$

- Time-constant at output C_2:
  - Cascode input resistance determined earlier (g_{in2})

$$\tau_{s2} = r_{in2} C_2$$

- Time-constant at output C_{gd1}:
  - Like Miller cap in CS stage $\quad \tau_{gd1} = \left[R_S \left(1 + g_{m1} r_{in2}\right) + r_{in2} \right] C_{gd1}$
  - If $\quad g_{m1} R_S >> 1 \quad \tau_{gd1} \approx R_S \left(1 + g_{m1} r_{in2}\right) C_{gd1}$
- Total time constant:

$$\tau_{total} = (g_m r_{ds1} r_{ds2} \parallel R_L) C_{out} + R_S C_{gs1} + r_{in2} C_2 + R_S \left(1 + g_{m1} r_{in2}\right) C_{gd1}$$

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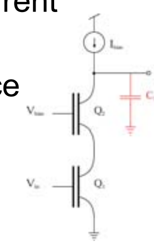
Cascode approximations

- Assuming high quality current mirror for bias current
  - In the order of: $\quad g_{m-p} r_{ds-p}^2$
- Assuming approximately same transconductance



$$A \cong -\frac{1}{2} g_m^2 r_{ds}^2$$

- Output resistance

$$R_{out} = r_{d2} \parallel R_L \approx \frac{1}{2} g_m r_{ds}^2$$

- Giving
  - With large load cap, the output pole is dominating

$$\tau_{out} = (r_{d2} \parallel R_L) C_{out} = \frac{1}{2} g_m r_{ds}^2 C_{out}$$

$$\omega_{-3dB} = \frac{1}{\tau_{out}} \approx \frac{2 g_{ds}^2}{g_m C_{out}}$$

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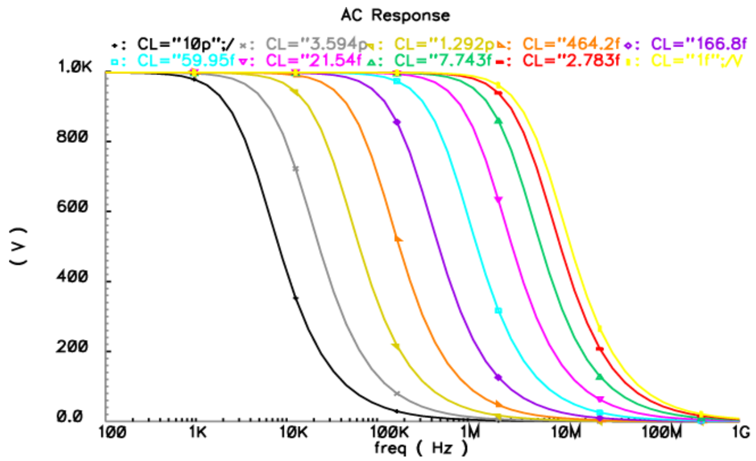
Cascode gain stage

- Cadence simulation
  - Load capacitor 1fF – 10pF



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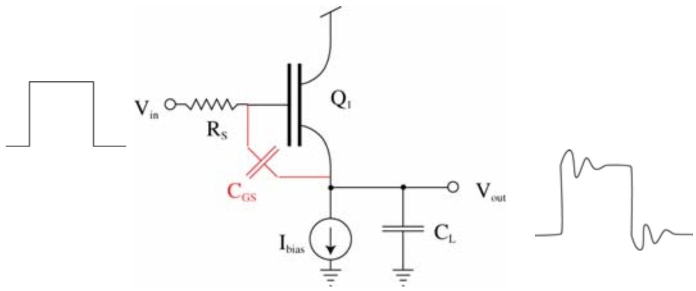
Source follower amp

- Intuitive understanding



•The gate-source capacitance is feeding back output changes

• High input resistance, R_S, increase output ringing

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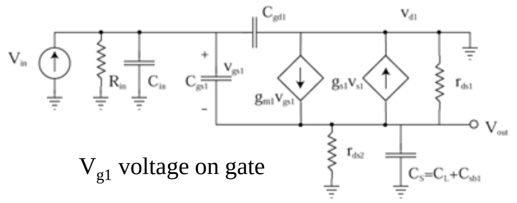
Source follower freq response

- Analysis for stability
  - Reduced overshoot

$$R_{S1} = r_{ds1} \| r_{ds2} \| \frac{1}{g_{s1}}$$

$$C_{in}^{'} = C_{in} + C_{gd1}$$



V_{g1} voltage on gate

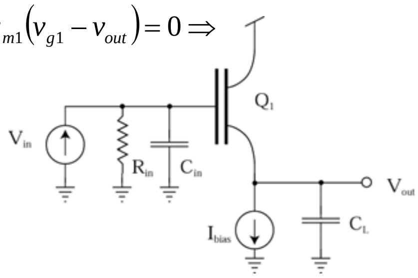
  - 1. Gain from input

V_{out}:

$$v_{out}G_{S1} + (v_{out} - v_{g1})sC_{gs1} + v_{out}sC_S - g_{m1}(v_{g1} - v_{out}) = 0$$

$$v_{out}(G_{S1} + sC_{gs1} + sC_S) - v_{g1}sC_{gs1} - g_{m1}(v_{g1} - v_{out}) = 0 \Rightarrow$$

$$\frac{v_{out}}{v_{g1}} = \frac{sC_{gs1} + g_{m1}}{s(C_{gs1} + C_S) + g_{m1} + G_{S1}}$$



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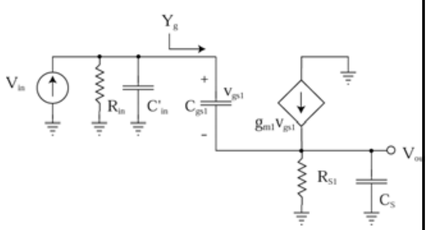
- 2. Admittance at input
  - No C_{gd1}

$$i_{g1} = (v_{g1} - v_{out})sC_{gs1}$$

    - Combining with previous

$$Y_{g1} = \frac{i_{g1}}{v_{g1}} = \frac{sC_{gs1}(sC_S + g_{m1})}{s(C_{gs1} + C_S) + g_{m1} + G_{S1}}$$



- 3. Transresistance

$$i_{in} = v_{g1}(sC_{in}^{'} + G_{in} + Y_g)$$

$$\frac{v_{g1}}{i_{in}} = \frac{s(C_{gs1} + C_S) + g_{m1} + G_{S1}}{a + sb + s^2c}$$

$$a = G_{in}(g_{m1} + G_{S1})$$

$$b = G_{in}(C_{gs1} + C_S) + C_{in}^{'}(g_{m1} + G_{S1}) + C_{gs1}G_{S1}$$

$$c = C_{gs1}G_{S1} + C_{in}^{'}(C_{gs1} + C_S)$$

- 4. Transfer function
  - 2. order

$$A(s) = \frac{v_{out}}{i_{in}} = \frac{sC_{gs1} + g_{m1}}{a + sb + s^2c}$$

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Source-follower analysis

- Equating coefficients

$$\omega_0 = \sqrt{\frac{G_{in}(g_{m1} + G_{S1})}{C_{gs1}G_{S1} + C_{in}^{'}(C_{gs1} + C_S)}}$$

$$Q = \frac{\sqrt{G_{in}(g_{m1} + G_{S1})[C_{gs1}G_{S1} + C_{in}^{'}(C_{gs1} + C_S)]}}{G_{in}(C_{gs1} + C_S) + C_{in}^{'}(g_{m1} + G_{S1}) + C_{gs1}G_{S1}}$$

  - Q<0.5: $C_{in}^{'}$ and/or C_S large and stable
  - Large Q: $G_{S1}, C_{in}^{'}$ and $G_{in} = \frac{1}{R_{in}}$ small

Source follower may exhibit overshoot and ringing.

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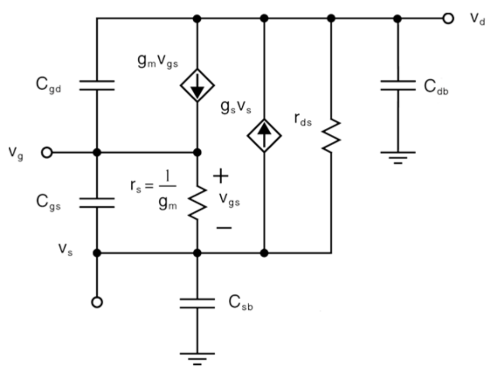
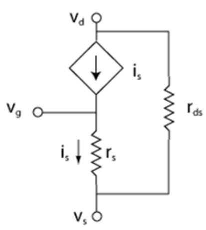
Diff pair frequency analysis

- Using MOS T-model with caps





  - T-model used for simpler diff pair analysis

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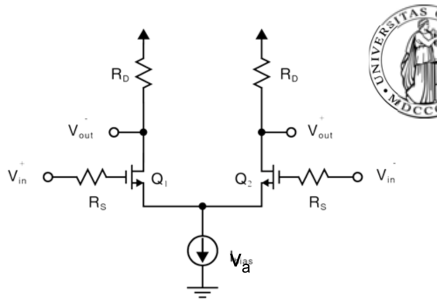
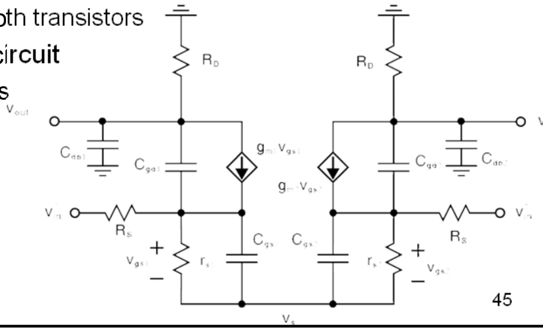
Diff-par HF model



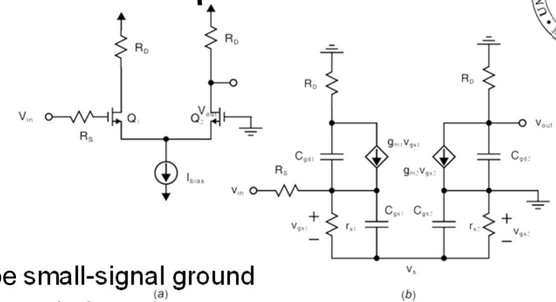
- Symmetric circuit
  - Transistors and resistors are the same as indicated
  - Input voltages balanced

$$V_{in}^{+} = -V_{in}^{-}$$

  - Common v_a voltage unchanged
    - Virtual ground
    - May ignore C_{sb} of both transistors
  - May analyze as half-circuit
    - → CS stage analysis



Single-ended diff amp



- v_s considered to be small-signal ground
  - Source-capacitors may be ignored
  - The diff-pair → 2 x common source stages
  - Dominant pole:

$$\omega_{-3dB} \approx \frac{1}{R_S\left(C_{gs1} + C_{gd1}\left(1 + g_{m1}R_2\right)\right)}$$

  - Giving bandwidth approximation assuming C_{db} to be small

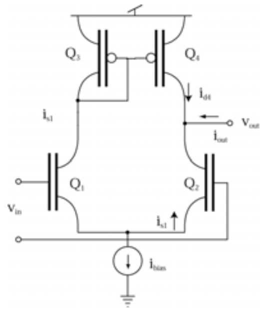
Differential amp with active load

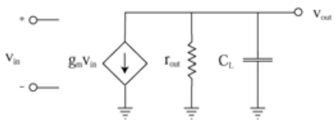
- Active current-mirror load
  - Used as input stage in opamps
  - From chapter 3.8:

$$A_V = \frac{v_{out}}{v_{in}} = \underline{g_m r_{out}}$$

  - With capacitive load
    - Substitute r_{out} with z_{out} with:

$$\omega_{-3dB} = \left(r_{out} \parallel \frac{1}{sC_L} \right) = \frac{1}{(r_{ds2} \parallel r_{ds4}) C_L}$$





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Chapter 4 Keypoints

- Important fundamentals for circuit analysis!
- Limited analysis with real coefficients
- Sinusoidal in → Sinusoidal out
  - Phase and magnitude may still change
- Transfer functions with real poles and zero order numerator have monotonically decreasing magnitude (20dB/decade) and phase up to -90 degrees
- Big caps dominates small caps
- CS-stage → 2 poles and 1 zero
- Caps over inverting amps blow up in effective value (Miller caps)
- Zero-value time-constants method estimate circuit bandwidth
- Common-gate used in cascode stage increase gain and reduce Miller caps
- Source follower may be unstable (oversshoot)
- Diff-pairs may be analyzed as half circuits → common source stage

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