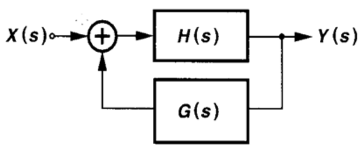
Feedback

- Most important techniques in analog circuits
  - Principle
  - Circuit analysis with feedback
  - Features and pitfalls
- History
  - R. Goddard patent of 1912
    - Theoretical without any practical implication
  - Armstrong Regenerative amplifier paper (1915)
    - exploring positive feedback
  - Harold Black negative feedback
    - August 2, 1927

Feedback principle

- Formal definition



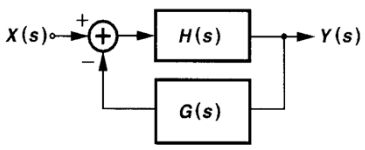
$$Y(s) = H(s)[X(s) + G(s)Y(s)]$$

$$\frac{Y(s)}{X(s)} = \frac{H(s)}{1 - G(s)H(s)}$$

-- Closed-loop transfer function

$H(s)$ -- Open-loop transfer function

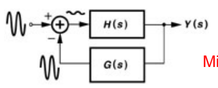
- Negative feedback



$$Y(s) = H(s)[X(s) - G(s)Y(s)]$$

$$\frac{Y(s)}{X(s)} = \frac{H(s)}{1 + G(s)H(s)}$$

1. Feed forward amplifier
2. Output sensing
3. Feedback network
4. Finding feedback error



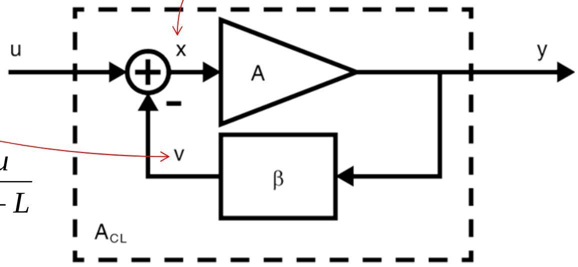
Minimizing error by correct feedback

Feedback in amplifiers

- Negative feedback
  - L - Loop Gain

$x = u - v$

$v = A\beta x = Lx \text{ with } L = A\beta$



$x = u - Lx \Rightarrow x = \dfrac{u}{1+L}$

- Closed loop gain

$$A_{CL} = \frac{y}{u} = \frac{A}{1+A\beta} = \frac{A}{1+L}$$

substituting $A = \dfrac{L}{\beta} \Rightarrow$ $A_{CL} = \dfrac{1}{\beta} \cdot \dfrac{L}{1+L}$

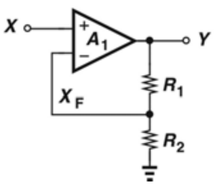
Assuming L>>1 $A_{CL} \approx \dfrac{1}{\beta}$

Gain desensitization

- Assuming poor components
  - Both transistor transconductance and resistors are imprecise
  - CS stage gain of $g_m R_D$ both $\pm 20\%$

$$A_{CL} = \frac{Y}{X} = \frac{A}{1+A\beta}$$

  - May improve gain
    - assuming $A\beta >> 1$ ⇨ $A_{CL} = \dfrac{Y}{X} \approx \dfrac{1}{\beta}$
    - Imprecision in A does not affect overall gain → reduced mismatch error
  - Example:



$A_1 R_2/(R_1+R_2) \gg 1$

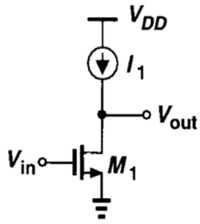
$\dfrac{Y}{X} \approx \dfrac{1}{K} \approx 1 + \dfrac{R_1}{R_2}$

Resistive dividers may be implemented quite accurate

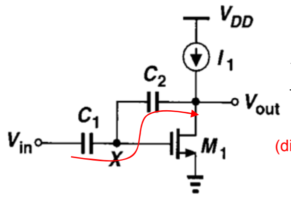
Example: Active CS stage

- Gain desensitization
  - CS example

$$A_V = g_{m1} r_{ds1}$$



  - Gain poorly defined
    - Temperature, mismatch
  - CS with feedback



$$\frac{V_{OUT}}{V_X} = -g_{m1} r_{ds1}$$

(disregard C_2 output load)

$$\frac{V_{OUT}}{V_{IN}} = -\frac{1}{\left(1 + \frac{1}{g_{m1} r_{ds1}}\right)\frac{C_2}{C_1} + \frac{1}{g_{m1} r_{ds1}}}$$

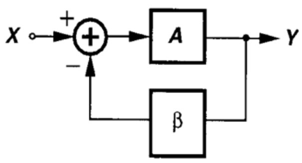
$$\approx -\frac{C_1}{C_2} \qquad (1/\ g_{m1} r_{ds1} \approx 0)$$

KCL: $(V_{OUT} - V_X) C_2 s = (V_X - V_{in}) C_1 s$

Gain determined by capacitive ratio
- high precision devices

Gain desensitization

- Negative feedback major advantage

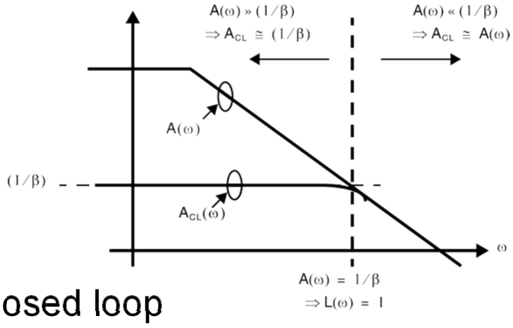


$$A_{CL} = \frac{Y}{X} = \frac{A}{1 + A\beta} = \frac{1}{\beta} \cdot \frac{A\beta}{1 + A\beta} \approx \frac{1}{\beta} \qquad A\beta >> 1$$

  - Feedback factor β and βA is loop gain
    - Major reduction of gain errors
    - Trading gain for precision

Feedback and bandwidth

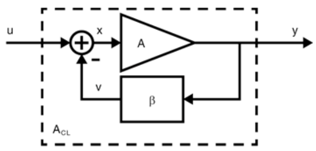
- Open loop
  - high gain and low frequency roll-off



- Closed loop
  - Reduced gain and increased bandwidth

$$\rightarrow A(s) = \frac{1}{\beta}$$

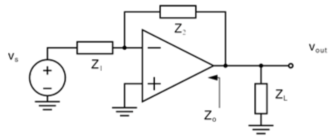
Amplifier linearity



- Feedback equation is linear
  - Limited by transistor linearity
- Feedback improve linearity
  - Input signal to amp reduce by feedback factor $x \cong \frac{u}{L}$
  - Feed amp stage within smaller signals
- Summary
  - Negative feedback has accurate gain, high bandwidth and high linearity
  - The loop gain L=Aβ must be large
  - The feedback loop is stable

Gain of feedback circuits

- Open loop gain A
  - Hard to estimate/simulate/measure
  - Affected by load and feedback
- Feedback factor β
  - Simply $\beta = -\dfrac{Z_1}{Z_1}$
- Loop gain L
  - Controlled signal
    - If stable
  - Unstable circuits amplifier → oscillator
  - Simulations and measurements are feasible
- Determining gain as



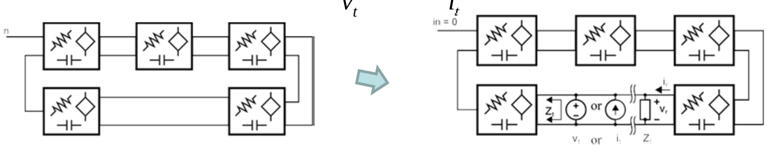
$$A \approx \frac{L}{\beta}$$

Writing: $A_{CL}(s) = \dfrac{1}{\beta} \cdot \dfrac{L(s)}{1 + L(s)}$

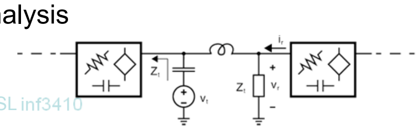
Determine L(s)

1. Set all independent sources to zero
   - Voltage source → short circuit
   - Current sources → open circuit
2. Break the loop
   - Determine impedance Z_t at break point
   - Terminate with Z_t at breakpoint
3. Insert some signal (voltage, v_t or current, i_t)
   - Determine signal over the added impedance Z_t (v_r or current, i_r)

- Loop gain given by: $L = \dfrac{v_r}{v_t}$ or $L = \dfrac{i_r}{i_t}$

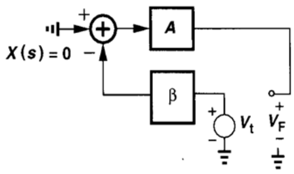


- Valid only for small signal analysis
  - Simulations:



Finding feedback

- Split the loop
- Follow the signal to break point



$$V_t\beta(-1)A = V_F \Rightarrow$$

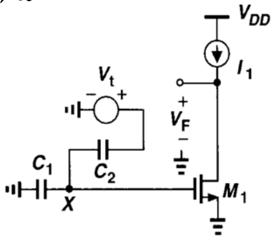
$$\frac{V_F}{V_t} = -\beta A$$

- CS example
  - Cs with ideal source $r_{ds2} \rightarrow \infty$

$$A_V = \frac{v_{out}}{v_{in}} = -g_{m1}\left(r_{ds1} \| r_{ds2}\right) \approx -g_{m1} r_{ds1}$$

$$V_t \frac{C_2}{C_1 + C_2}\left(-g_{m1} r_{ds1}\right) = V_F \Rightarrow$$

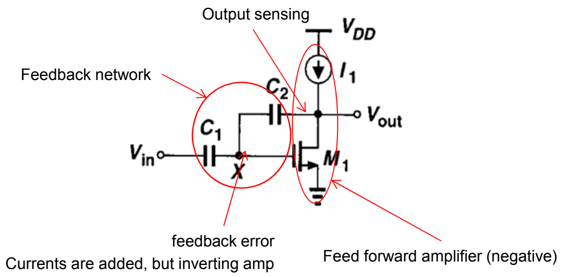
$$\frac{V_F}{V_t} = -\frac{C_2}{C_1 + C_2} g_{m1} r_{ds1}$$



Output loading is ignored!

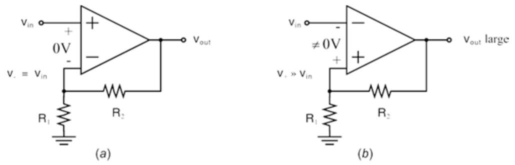
Identifying feedback elements

- Negative feedback circuits



Dynamics of feedback amps

- Stability
  - Sometimes hard to discover

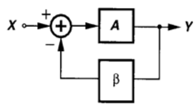


$$v_{in}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)-\frac{v_{out}}{R_2}=0 \Rightarrow \frac{v_{out}}{v_{in}}=1+\frac{R_2}{R_1}$$

  - a) and b) → same transfer function
  - a) is popular and well known op-amp architecture
  - b) just switched inputs, but is unstable
  - Due to phase shifts, instability may occur at higher frequencies
    - Feedback make stable circuits unstable….

Stability analysis techniques

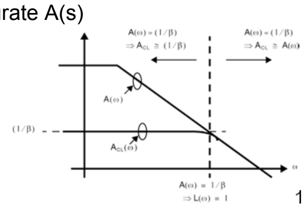
- Barkhausen criterion
  1. $|A\beta| = 1$
  2. $\angle A\beta = 2\pi n \quad n \in (1, 2, \ldots)$
  - Must be satisfied for oscillation



- Avoiding this condition
  - analyzing poles of closed-loop system
    - Must be in left half-plane

$$A_{CL}(s) = \frac{A(s)}{1+\beta A(s)}$$

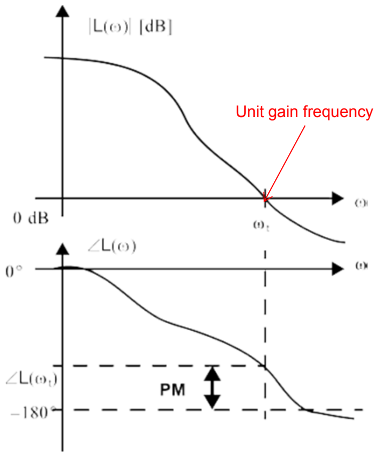
    - Quite some work to figure out due to inaccurate A(s)
- Amps have characteristic behavior
  - Poles giving roll-off at HF
  - Also give negative phase shift

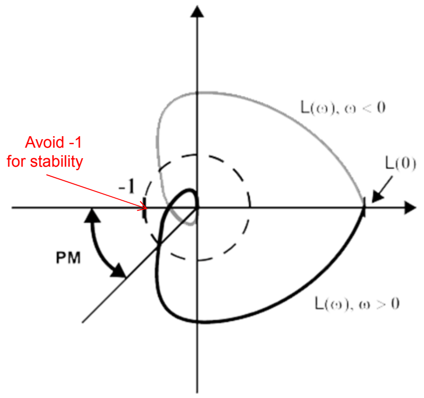


Typical feedback amp

- Bode plot

Polar plot





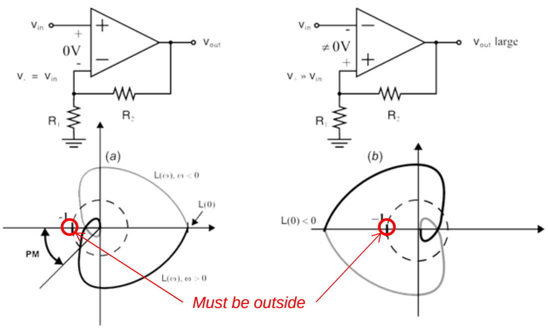
Inverting amps already -180!!!!

Stable if $\angle L(\omega_t) > -180$ at unit-gain frequency

-180 degrees due to inverting amp = -360

Phase margins

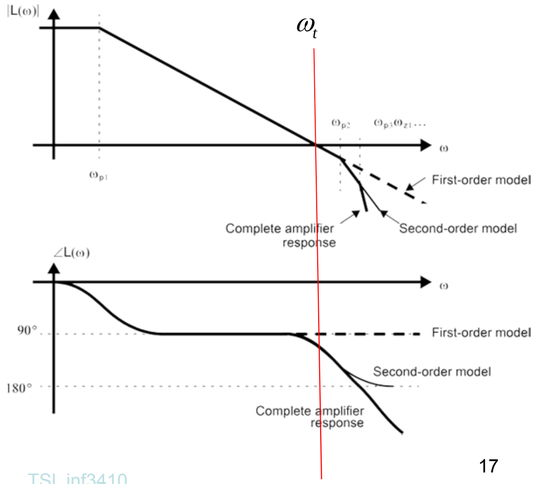
- Measure of stability $PM = \angle L(\omega_t) + 180$
  - Must account for variations and need margins
- Positive feedback:



  - Never positive feedback in amps!
    - Fine in oscillators

First and second order response

- Which poles to consider?
  - Those closest to signal band
  - ω_{p1} and maybe ω_{p2}
  - What happens beyond ω_t really do not matter



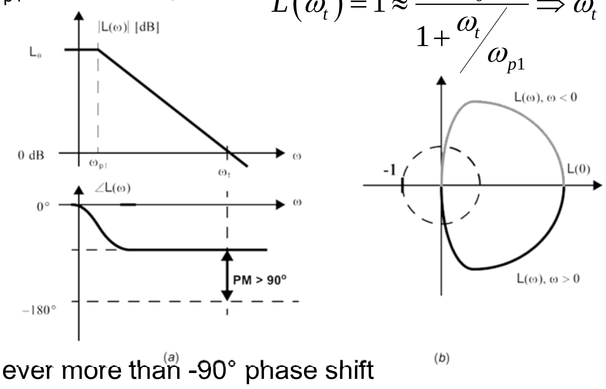
First order system

- Transfer function
  - L_0 – DC gain
  - ω_{p1} – dominant pole

$$A_{CL}(s) = L(s) = \frac{L_0}{1 + s/\omega_{p1}}$$

$$L(\omega_t) = 1 \approx \frac{L_0}{1 + \omega_t/\omega_{p1}} \Rightarrow \omega_t \cong L_0 \omega_{p1}$$

$$\omega_{-3dB} \cong \omega_t$$



  - Never more than -90° phase shift
  - Unconditionally stable

1. order system

- Midband frequencies - roll off $\omega >> \omega_{p1}$
  - Loop gain:

$$L(s) = \frac{\omega_t}{s}$$

  - Closed loop amp transfer

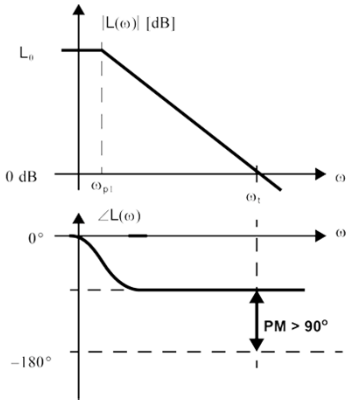
$$A_{CL}(s) \cong \frac{1}{\beta} \cdot \frac{1}{1 + s/\omega_t} \qquad A_0 = \frac{1}{\beta}$$

    - Giving ultimately

$$\omega_{-3dB} \cong \omega_t \qquad \tau \approx \frac{1}{\omega_{-3dB}} = \frac{1}{\omega_t}$$

  - Increased bandwidth

$$\omega_{-3dB} = \omega_{p1}(1 + \beta A_0)$$



Step input

- Settling time
    - Time to a given goal (percentage) of steady state

$$v_{out}(t) = V_{step}\left(1 - e^{-\frac{t}{\tau}}\right)$$

  - Assuming unlimited power we may find
    - 1% of input step → $e^{-\frac{t}{\tau}} = 0.01$ giving $t = 4.6\tau$
    - 0.1% → $e^{-\frac{t}{\tau}} = 0.001$ giving $t = 7\tau$
  - For slur-rate limited amps (always)
    - Finite charging current of parasitics
      - Distortion
      - Longer settling time

Second order systems

- First order may not give sufficient performance
  - Insufficient gain
  - Multiple transistors required, but will give more poles…
  - Often second order (two pole) systems analysis sufficient

$$L(s) = \frac{L_0}{\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{eq}}\right)} \qquad \omega_{eq} > \omega_{p1}$$

  - For frequencies $\omega >> \omega_{p1}$ we have $1+\frac{j\omega}{\omega_{p1}} \cong \frac{j\omega}{\omega_{p1}}$ giving:

$$L(s) \cong \frac{\omega_t}{s\left(1+\frac{s}{\omega_{eq}}\right)}$$

  - LF gain

$$L(s) \approx \beta A(s) \Rightarrow A_0 \approx L_0/\beta$$

$\omega_{ta} = A_0\omega_{p1}$ → unit-gain approximation of A(s) (not L(s)) assuming a dominant pole

Second order amp

- May write $L(s) = \beta A(s) = \frac{\beta\omega_{ta}}{s\left(1+ {s}/{\omega_{eq}}\right)}$ $\qquad \omega_{ta} = A_0\omega_{p1}$

- Unit gain frequency ω_t found:

$$L(j\omega_t) = \beta A(j\omega_t) = \frac{\beta\omega_{ta}}{s\left(1+ {j\omega_t}/{\omega_{eq}}\right)} \Rightarrow \beta\frac{\omega_{ta}}{\omega_{eq}} = \frac{\omega_t}{\omega_{eq}}\sqrt{1+\left(\frac{\omega_t}{\omega_{eq}}\right)^2}$$

  - For $\omega_t >> \omega_{eq}$ we may determine amp unit-gain freq ω_{ta}

$$\omega_{ta} = \frac{\omega_t}{\beta}\sqrt{1+\left(\frac{\omega_t}{\omega_{eq}}\right)^2} \cong \frac{\omega_t}{\beta}$$

The amp overall unit-gain frequency is
1/β fraction of the loop unit-gain frequency

  - Phase response

$$\angle L(\omega) = -90^\circ - \tan^{-1}\frac{\omega}{\omega_{eq}}$$

  - Phase margin at ω_t :

Loop unit-gain frequency independent of feedback factor β

$$PM = \angle L(\omega_t) - (-180^\circ) = 90^\circ - \tan^{-1}\frac{\omega_t}{\omega_{eq}} \Rightarrow \omega_t = \tan(90^\circ - PM)\,\omega_{eq}$$

Designing amp for sufficient phase margin (PM)
For proper ω_t/ω_{eq} is called COMPENSATION

Second order, closed loop

- Combining $L(s)=\dfrac{L_0}{\left(1+\dfrac{s}{\omega_{p1}}\right)\left(1+\dfrac{s}{\omega_{eq}}\right)}$ and $A_{CL}(s)=\dfrac{1}{\beta}\cdot\dfrac{L(s)}{1+L(s)}$
  - Closed loop response:

$$L(s)=\frac{A_{CL0}}{1+\dfrac{s\left(1/\omega_{p1}+1/\omega_{eq}\right)}{1+L_0}+\dfrac{s^2}{(1+L_0)\,\omega_{p1}\cdot\omega_{eq}}} \qquad A_{CL0}=\frac{1}{\beta}$$

  - Closed loop response is also second order
    - Rewriting to:

$$H(s)=\frac{K\omega_0^2}{\omega_0^2+s\dfrac{\omega_0}{Q}+s^2}=\frac{K}{1+\dfrac{s}{\omega_0 Q}+\dfrac{s^2}{\omega_0^2}}$$

    - We find:

$$\omega_0=\sqrt{(1+L_0)\,\omega_{p1}\cdot\omega_{eq}}\cong\sqrt{\omega_{p1}\omega_{eq}}$$

$$Q=\frac{\sqrt{(1+L_0)\,\omega_{p1}\omega_{eq}}}{1/\omega_{p1}+1/\omega_{eq}}\cong\sqrt{\frac{L_0\omega_{p1}}{\omega_{eq}}}=\sqrt{\frac{\beta\omega_{ta}}{\omega_{eq}}}$$

Approximation assuming:

$\omega_{p1}<<\omega_{eq}$

$L_0>>1$

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Q ↔ phase margin

1. Specify PM and determine: $\dfrac{\omega_t}{\omega_{eq}}=\tan\left(90^\circ-PM\right)$
2. Substitute into: $\beta\dfrac{\omega_{ta}}{\omega_{eq}}=\dfrac{\omega_t}{\omega_{eq}}\sqrt{1+\left(\dfrac{\omega_t}{\omega_{eq}}\right)^2}$
3. Determine Q: $Q=\sqrt{\dfrac{\beta\omega_{ta}}{\omega_{eq}}}$

PM (Phase margin)	ω_t/ω_{eq}	Q factor	% overshoot
55°	0.700	0.925	13.3%
60°	0.580	0.817	8.7%
65°	0.470	0.717	4.7%
70°	0.360	0.622	1.4%
75°	0.270	0.527	0.008%
80°	0.175	0.421	
85°	0.087	0.284	

Often better than: $Q=\sqrt{\dfrac{1}{2}}$

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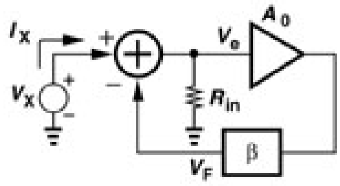
Impedance modifications

- Input impedance

$$V_e = V_X - V_F = V_X - \beta A_0 I_X R_{in} = I_X R_{in}$$

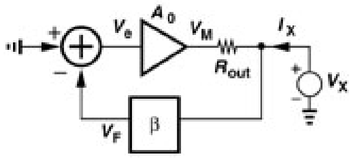
$$\frac{V_X}{I_X} = R_{in,CL} = R_{in}(1+\beta A_0)$$



- Output impedance

$$I_X = \frac{V_X - V_M}{R_{out}} = \frac{V_X - (0 - \beta V_X)A_0}{R_{out}}$$

$$\frac{V_X}{I_X} = R_{out,CL} = \frac{R_{out}}{1+\beta A_0}$$



Impedance modifications

- Determine input impedance
  - Broken feedback → $R_{in,open} = \dfrac{1}{g_{m1} + g_{mb1}}$

Common-gate as example

  - Closed loop $\quad V_{OUT} = (g_{m1} + g_{mb1})V_X R_D$ and $V_P = \dfrac{C_1}{C_1 + C_2} V_{out} \Rightarrow$
  - Determine I_x

$$= (g_{m1} + g_{mb1})V_X R_D \frac{C_1}{C_1 + C_2}$$

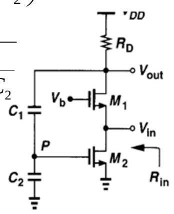
Gate voltage of M_2

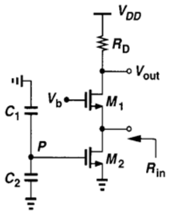
$$I_X = g_{m2}(g_{m1} + g_{mb1})V_X + g_{m2}(g_{m1} + g_{mb1}) \frac{C_1}{C_1 + C_2} V_X R_D$$

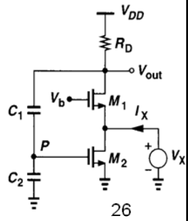
$$= (g_{m1} + g_{mb1})\left(1 + g_{m2} R_D \frac{C_1}{C_1 + C_2}\right) V_X$$

$$R_{in,closed} = \frac{1}{(g_{m1} + g_{mb1})} \frac{1}{1 + g_{m2} R_D \dfrac{C_1}{C_1 + C_2}}$$

- Input resistance reduced
- Four elements?







Output resistance also reduced

Bandwidth modifications

– Frequency response $A = \dfrac{A_0}{\left(1 + j\,\omega/\omega_p\right)}$

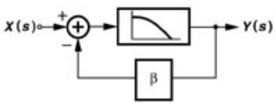
- Gain with feedback

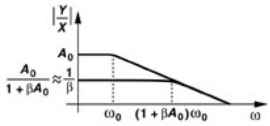
$$\frac{Y}{X} = \frac{A}{(1+A\beta)} = \frac{\dfrac{A_0\beta}{\left(1+s/\omega_p\right)}}{1+\dfrac{A_0\beta}{\left(1+s/\omega_0\right)}} = \frac{A_0}{1+s/\omega_0 + A_0\beta}$$

Increased bandwidth
Trading in reduced gain

- Numerator – closed loop gain at LF
- a pole at $(1+A_0\beta)\omega_0$
- The -3dB frequency increased by $1+A_0\beta$

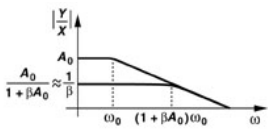
$$= \frac{\dfrac{A_0}{1+\beta A_0}}{1+\dfrac{s}{(1+A_0\beta)\omega_0}}$$

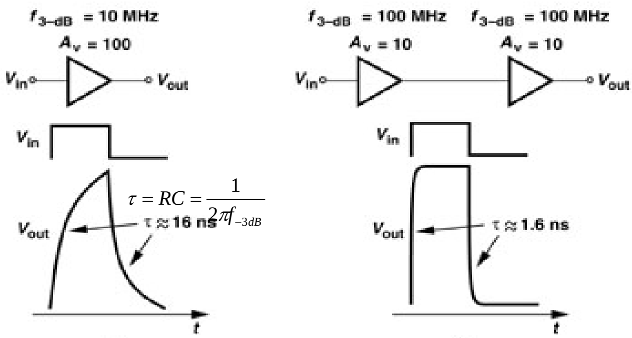




Gain•Bandwidth product

- Reduced gain → increased bandwidth
  – No magic
  – Moving corner frequency
  – Example:



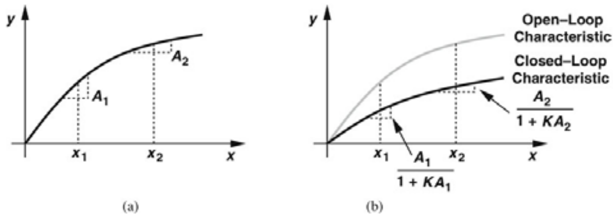


- gaining speed
- more area
- more power

Gain•bandwidth: characteristic measure of performance

Linearity improvements

- Gain variations → distortion



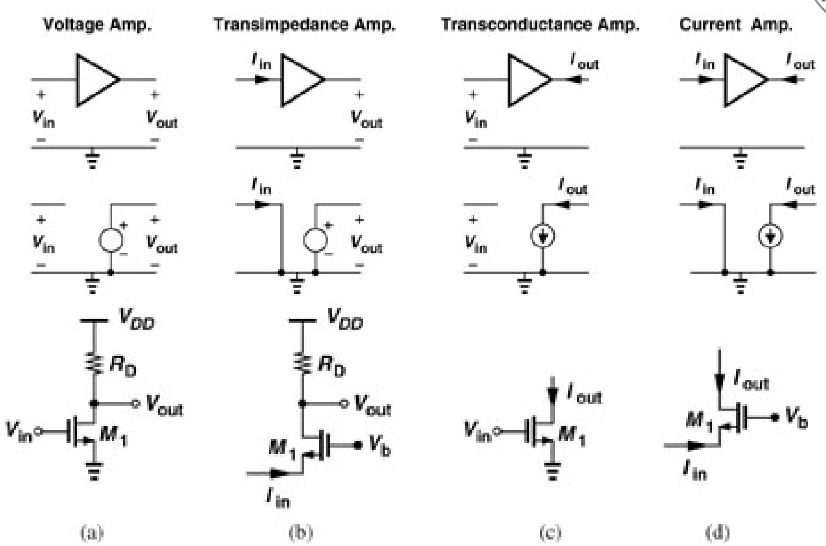
  – Less gain at x_2 than at x_1

- Improved with feedback

$$x_1 = \frac{A_1}{1+KA_1} \approx \frac{1}{K}\left(1-\frac{1}{KA_1}\right) \qquad x_2 = \frac{A_2}{1+KA_2} \approx \frac{1}{K}\left(1-\frac{1}{KA_2}\right)$$

  – When KA_1 and KA_2 is large, minor gain errors introduced

Amplifier types

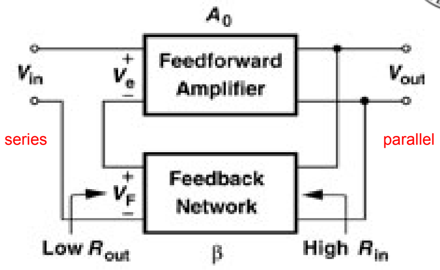


Simple circuits may have limited performance

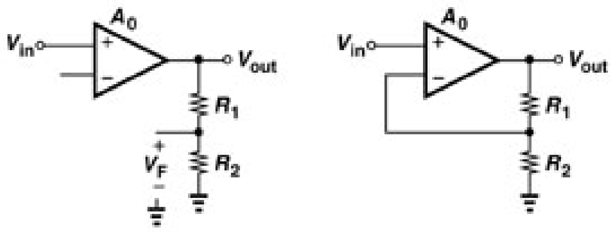
Feedback topologies

- Voltage-to-voltage

$$V_F = \beta V_{out},\ V_e = V_{in} - V_F,$$
$$V_{out} = A_0\left(V_{in} - \beta V_{out}\right) \Rightarrow$$
$$\frac{V_{out}}{V_{in}} = \frac{A_0}{1 + A_0\beta}$$



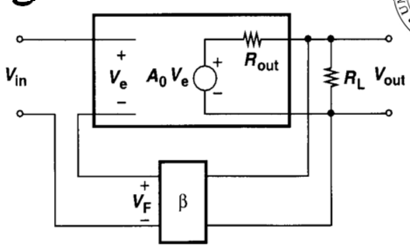
- Example:



Impedance changes

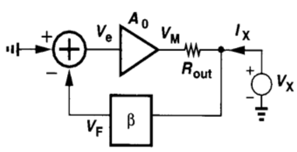
– Output load resistor

$$R_{out,CL} = \frac{R_L}{R_L + R_{out}}$$



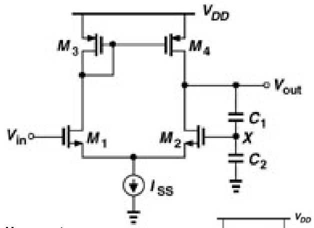
– Analysis

$$V_F = \beta V_X,\ V_e = -\beta V_X, V_M = -A_0\beta V_X \Rightarrow$$
$$I_X = \left[V_X - \left(-A_0\beta V_X\right)\right] / R_{out}$$
$$\frac{V_X}{I_X} = \frac{R_{out}}{1 + \beta A_0}$$

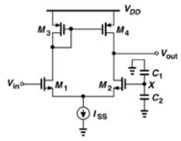


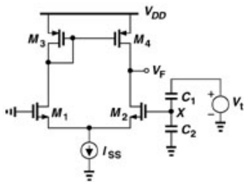
Example

- Capacitive feedback
  - Bias network not included
  - Close loop gain?
  - Output resistance?



- Breaking the loop
  - Open loop gain: $A_{V,OP} = g_{m1}(r_{ds4} \| r_{ds2})$
  - Closed loop gain





$$V_F = V_1 \frac{C_1}{C_1 + C_2} g_{m1}(r_{ds4} \| r_{ds2})$$

$$\beta A_0 = \frac{C_1}{C_1 + C_2} g_{m1}(r_{ds4} \| r_{ds2})$$

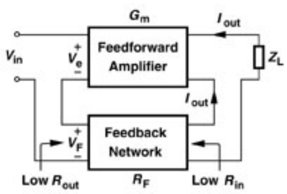
$$A_{CL} = \frac{g_{m1}(r_{ds4} \| r_{ds2})}{1 + \frac{C_1}{C_1 + C_2} g_{m1}(r_{ds4} \| r_{ds2})}$$

$$R_{out,closed} = \frac{(\cancel{r_{ds4} \| r_{ds2}})}{1 + \frac{C_1}{C_1 + C_2} g_{m1}(\cancel{r_{ds4} \| r_{ds2}})} \approx \left(1 + \frac{C_2}{C_1}\right)\frac{1}{g_{m1}}$$

Output impedance independent of $(r_{ds4} \| r_{ds2})$

Current-voltage feedback

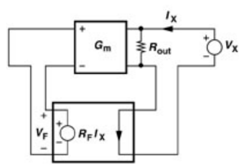
- Series-series feedback
- Resistor in series



$$V_F = R_F I_{out} \qquad V_e = V_{in} - R_F I_{out}$$

$$I_{out} = G_m(V_{in} - R_F I_{out}) \qquad \frac{I_{out}}{V_{in}} = \frac{G_m}{1 + G_m R_F}$$

- Output impedance

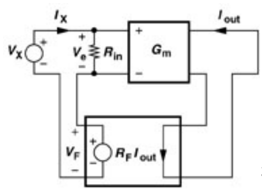


$$V_F = R_F I_X$$

$$G_m V_F = G_m R_F I_X = I_X - V_X / R_{out} \qquad \frac{V_X}{I_X} = R_{out}(1 + G_m R_F)$$

increase

- Input impedance



$$I_X R_{in} G_m = I_{out} \quad V_e = I_X R_{in} = V_X - R_F I_X R_{in} G_m$$

$$\frac{V_X}{I_X} = R_{in}(1 + G_m R_F)$$

increase

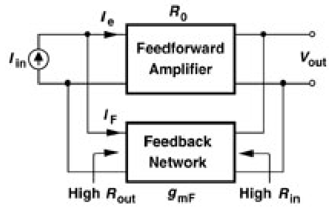
Voltage-current feedback

– Shunt-shunt feedback

$$I_F = g_{mF} V_{out}, \quad I_e = I_{in} - I_F$$

$$V_{out} = R_0 I_e = R_0 (I_{in} - g_{mF} V_{out})$$

$$\frac{V_{out}}{I_{in}} = \frac{R_0}{1 + g_{mF} R_0}$$

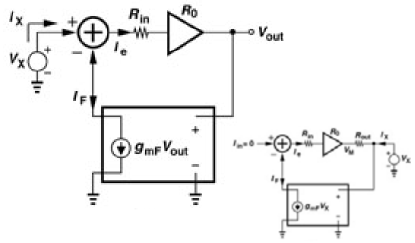


- Impedances

$$I_F = I_X - V_X / R_{in}, \quad (V_X / R_{in}) R_0 g_{mF} = I_F$$

$$\frac{V_X}{I_X} = \frac{R_{in}}{1 + g_{mF} R_0}$$

– Similar for output impedance



Decreasing both input and output impedance

Example

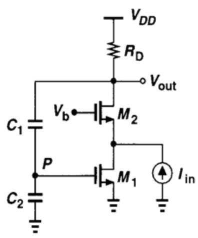
- Determine transimpedance

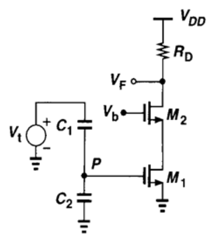
$$\frac{V_{out}}{I_{in}}$$

- Breaking the loop gives:
  - CG open-loop transimpedance: R_D
  - Setting $I_{in} = 0$ give loop gain:

$$-V_1 \frac{C_1}{C_1 + C_2} g_{m1} R_D = V_F$$

$$R_{tot} = \frac{R_D}{1 + g_{m1} R_D \frac{C_1}{C_1 + C_2}}$$

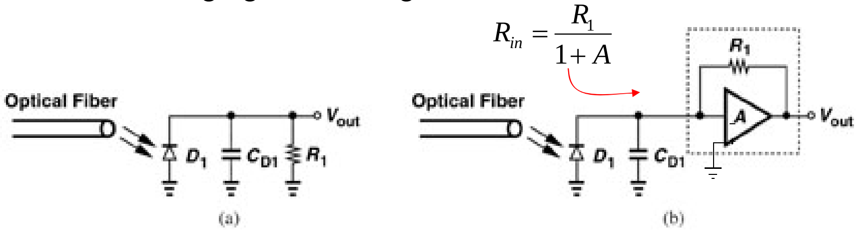




Transimpedance application

- Optical fiber sensor
  - Converting light to voltage

$$R_{in} = \frac{R_1}{1+A}$$



$$\omega_p = \frac{1}{R_1 C_{D1}}$$

$$V_{out} = -I_{D1} R_1$$

$$\omega_p = \frac{1}{\frac{R_1}{(1+A)} C_{D1}}$$

$$V_{out} = I_{D1} R_1$$

Improving signal bandwidth

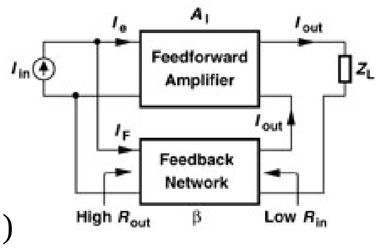
Current-current feedback

- Shunt-series feedback

$$\frac{I_{out}}{I_{in}} = \frac{A_I}{1+\beta A_I}$$

$$R_{in,CL} = \frac{R_{in}}{1+\beta A_I}$$

$$R_{out,CL} = R_{out}(1+\beta A_I)$$

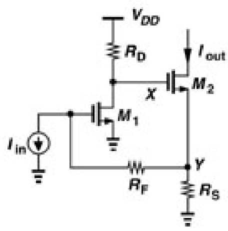


Current ratio β

- Example:

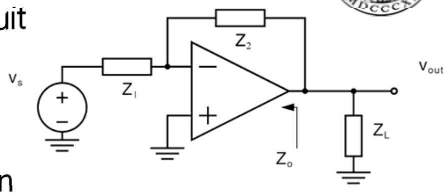
$$R_S << R_F$$

$$\beta = -\frac{R_S}{R_F}$$



Common feedback systems

- Not easy to find in actual circuit
  - Both A and β
  - Voltage or current?
  - Load impedance impact?



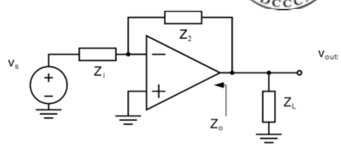
- Assuming high open-loop gain
  - Closed loop gain set by external components
  - Expecting: $A_{desired} = -\frac{Z_2}{Z_1} = \frac{1}{\beta} \Rightarrow \beta = -\frac{Z_1}{Z_2}$
  - β is inverse of the closed loop gain
  - L(s) may be determined by simple procedure (s. 10) and

$$A_{CL}(s) = \frac{1}{\beta}\frac{L(s)}{1+L(s)}$$

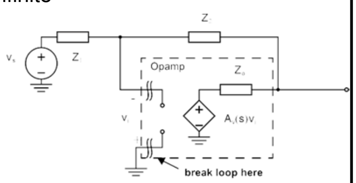
  - Loop-gain "normalized" to the desired gain with 1/β

Loop-gain example

- Determine l(s) and A_{CL}(s) of inv amp
  - Z_L – infinite
  - Set v_s=0
  - Break the loop at input
    - Input impedance of MOS is close enough to infinite
  - Testsignal, v_t, injected in new circuit



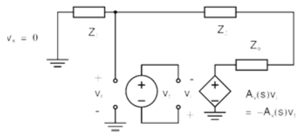
$$v_r = A_V(s)\frac{Z_1}{Z_1+Z_2+Z_o}v_t$$



$$L(s) = -\frac{v_r}{v_t} = A_V(s)\frac{Z_1}{Z_1+Z_2+Z_o}$$

  - Normalized closed-loop gain:

$$\frac{L(s)}{1+L(s)} = \frac{A_V(s)Z_1}{A_V(s)Z_1+Z_1+Z_2+Z_o}$$



    - giving

$$A_{CL}(s) = -\frac{Z_2}{Z_1}\frac{A_V(s)Z_1}{A_V(s)Z_1+Z_1+Z_2+Z_o}$$

Inv Amp example

- V_{in} grounded
- Breaking at gate of M2
  - Terminating Z_1
- Test source at break
- Determine L(s)

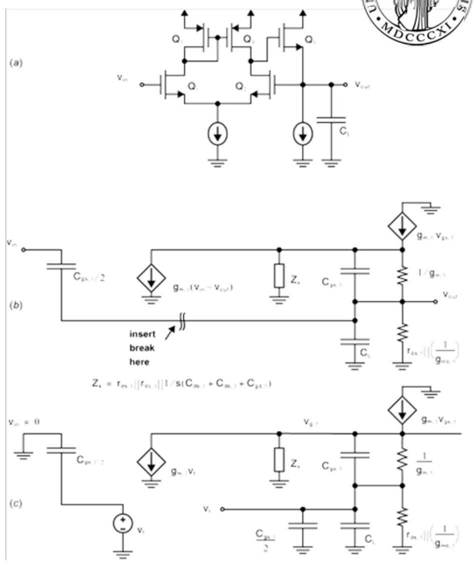
$$v_{g5} = -g_{m1} v_t (r_{ds1} \| r_{ds3})$$

- Returned voltage by voltage division

$$v_r = v_{g5} \frac{g_{m1}(r_{ds1} \| r_{ds3}) g_{m5}}{g_{m5} + 1/r_{ds5} + g_{mb5}}$$

- combining to find

$$L(s) = -\frac{v_r}{v_t} = \frac{g_{m1}(r_{ds1} \| r_{ds3}) g_{m5}}{g_{m5} + 1/r_{ds5} + g_{mb5}}$$

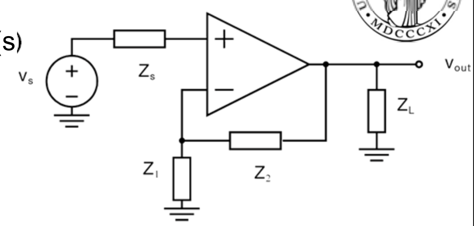


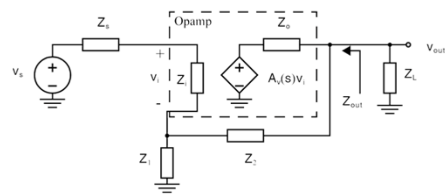
Non-inv amplifier

- Assuming high open-loop gain A(s)
  - Frequency-dependent
- Ideal Op-amp gain

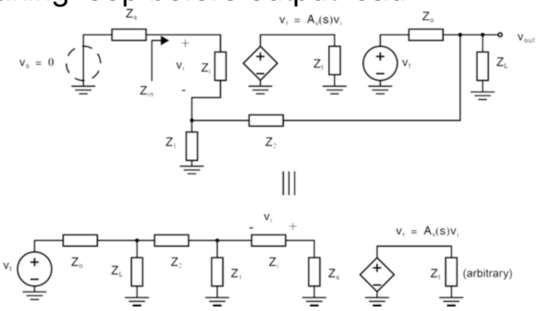
$$A_{desired} = \frac{Z_1 + Z_2}{Z_1} = \frac{1}{\beta} \Rightarrow \beta = \frac{Z_1}{Z_1 + Z_2}$$

- Determine loop gain by breaking loop ----





- Breaking loop before output load



  - Finding

$$v_r = A_V(s) v_i = -A_V(s)\frac{Z_i}{Z_o}\frac{Z_L \parallel Z_o}{Z_L \parallel Z_o + Z_2 + Z_1 \parallel (Z_i + Z_s)}\frac{Z_1}{Z_i + Z_s + Z_1} v_t$$

  - Giving

$$L(s) = A_V(s)\frac{Z_i}{Z_o}\frac{Z_L \parallel Z_o}{Z_L \parallel Z_o + Z_2 + Z_1 \parallel (Z_i + Z_s)}\frac{Z_1}{Z_i + Z_s + Z_1}$$

  - Assuming

$Z_i >> Z_s, Z_1$ High input-impedance

$Z_o << Z_L, Z_2$ Low output-impedance

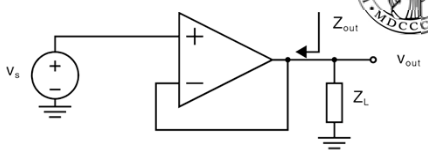
$$L(s) \approx A_V(s)\frac{Z_1}{Z_1 + Z_2} = A_V(s)\beta$$

Voltage follower

- Non-inv amp

$Z_1 = \infty, Z_2 = 0$

$Z_L << Z_i$



$$L(s) = A_V(s)\frac{Z_i}{Z_o}\frac{Z_L \parallel Z_o}{Z_L \parallel Z_o + Z_2 + Z_1 \parallel (Z_i + Z_s)}\frac{Z_1}{Z_i + Z_s + Z_1}$$

$$L(s) = A_V(s)\frac{Z_L}{Z_L + Z_o}\underbrace{\frac{Z_i}{Z_L \parallel Z_o + Z_i + Z_s}}_{\approx 1}\underbrace{\frac{Z_1}{Z_i + Z_s + Z_1}}_{\approx 1} \cong A_V(s)\frac{Z_L}{Z_L + Z_o}$$

- Unit-gain→1/β=1

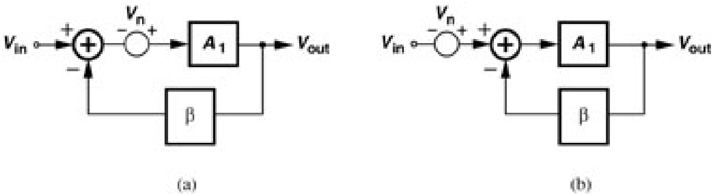
$$A_{CL} = \frac{L(s)}{1 + L(s)} = \frac{A_V(s) Z_L}{A_V(s) Z_L + Z_L + Z_o}$$

- Assuming ω_{eq} to be large→ $A_V \cong \omega_{ta}/s$

$$L(s) = \frac{A_{V0}}{\left(1 + \frac{s}{\omega_{p1}}\right)\left(1 + \frac{s}{\omega_{eq}}\right)}\frac{Z_L}{Z_L + Z_o} \qquad A_{CL} = \frac{1}{1 + \left(\frac{Z_L}{Z_L + Z_o}\right)\frac{s}{\omega_{ta}}}$$

Feedback and noise

- No noise improvements
  - Assuming no feedback noise



$$(V_{in} - \beta V_{out} + V_n)A_1 = V_{out} \Rightarrow V_{out} = (V_{in} + V_n)\frac{A_1}{1+\beta A_1}$$

  - Overall input referred noise is unchanged

Chap 5 keypoints

- L(s)>>1, large loop gain the amp gain is inverse proportional to the loop gain is insensitive to the open-loop gain A.
- Bandwidth estimate for |L(ω)|=1 improving A(ω) bandwidth
- Reducing signal swing by a factor of L
- Stability of higher order inverting amps may be checked with phase margins (PM)
- Not only stable but minimal overshoot as well
- 1. order systems unconditionally stable
- The closed-loop bandwidth is ω_t, independent of the -3dB amp pole ω_{p1}
- Designing for proper phase margin is called compensation
- Analyzing amps wit L(s), 1/β and desired gain
- L8(s) estimated by loop breaking
- Reduced output impedance with feedback (approx by the loop gain)