Summary inf3410

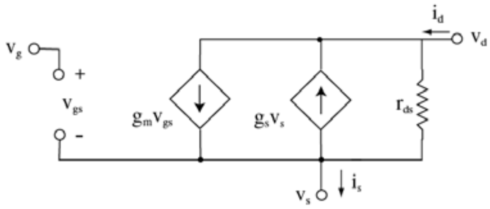
- CMOS amplifiers
  - Continuous time
  - Integrated modern technology
  - Single stage amps
  - Two-stage amps
  - Frequency response
  - Feedback topologies

MOS models

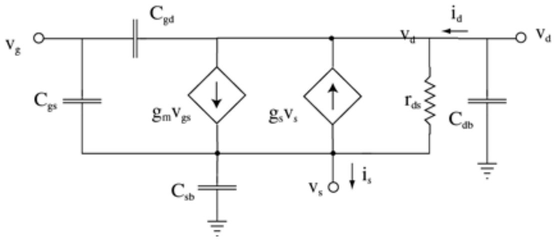
- Transistor operating regions
  - Drain-source biasing
    - Triode (linear) region $V_{DS} < V_{DS-sat} = V_{GS} - V_{tn} = V_{eff}$
    - Active (saturation) region $V_{DS} > V_{DS-sat} = V_{GS} - V_{tn} = V_{eff}$
  - Gate biasing in active region
    - Weak inversion/subthreshold $V_{GS} < V_{tn}$
    - Strong inversion $V_{GS} > V_{tn}$
    - Mobility degradation
      - At high current densities

MOS modeling

- Small signal models
  - Active region
  - Low frequencies

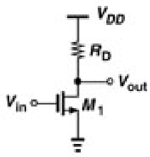


  - Medium frequencies



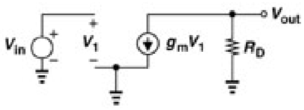
CS-stage with passive load

- Simple analysis



$$g_m = \sqrt{2\mu_n C_{ox} \tfrac{W}{L} I_D}$$

  - Normally in saturation region
  - Small-signal gain:

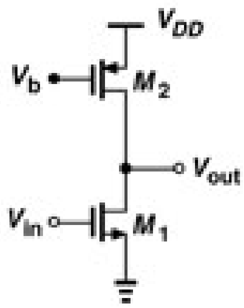


Inverting amplifier

$$A_v = -g_m R_D$$

CS-stage with current load

- Small signal gain
  - Dominated by output impedance



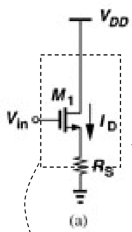
$$A_v = -g_m r_{o1} \parallel r_{o2} \qquad r_o = \frac{1}{\lambda I_D}$$

Assuming r_{o2} large,

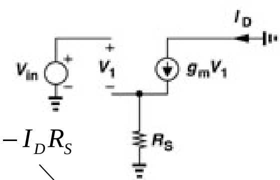
$$A_v = -g_m r_{o1} = -\sqrt{2\mu_n C_{ox} I_D \left(\frac{W}{L}\right)_1} \frac{1}{\lambda I_D}$$

CS-stage with source degeneration

- Improve linearity by weaker source



$V_{GS} = V_{in} - I_D R_S$



$$G_m = \left(1 - R_S \frac{\partial I_D}{\partial V_{in}}\right) \frac{\partial f}{\partial V_{GS}}$$

G_m g_m

$I_D = f(V_{GS})$

$$G_m = \frac{\partial I_D}{\partial V_{in}} = \frac{\partial f}{\partial V_{GS}} \frac{\partial V_{GS}}{\partial V_{in}}$$

$$\partial I_D / \partial V_{GS} = g_m \qquad \partial V_{GS} / \partial V_{in} = 1 - R_S \, \partial I_D / \partial V_{in}$$

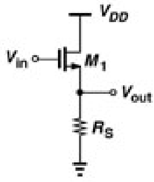
$$G_m = \frac{g_m}{1 + g_m R_S}$$

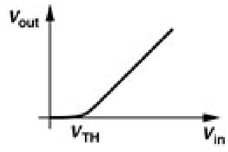
When $R_S >> 1/g_m \rightarrow G_m \approx 1/R_S$
giving lower gain and even more noise….

CD-stage voltage gain

Voltage follower

- Source follower



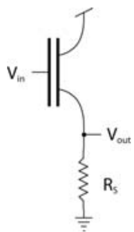


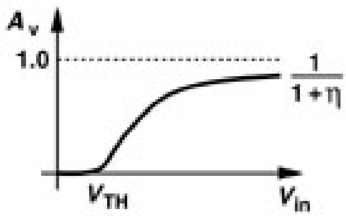
- Assuming saturation

$$\Rightarrow A_v = \frac{g_m R_S}{1 + (g_m + g_{mb}) R_S}$$

Follower behavior

- Voltage offset dependant on current





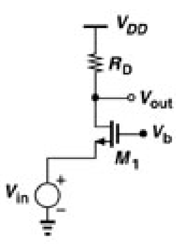
$$V_{out} = V_{in} - V_{GS} \Rightarrow$$

$$V_{out} = V_{in} - \left(V_{tn} + \sqrt{\frac{2}{\mu_n C_{ox} (W/L)}} \cdot \sqrt{I_D} \right)$$

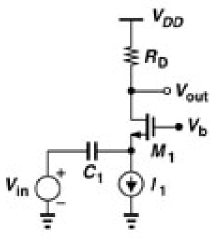
- 1. order approximation w/o body-effect and channel length modulation
- Current through resistor changes linear with voltage

Common-gate stage (CG)

- Main configuration



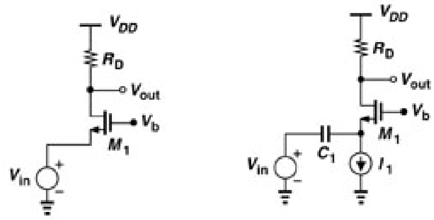
(a)

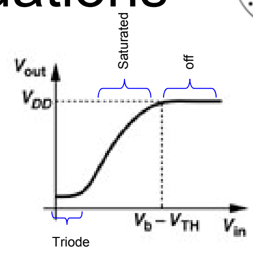


(b)

  - Input at source
  - Output at drain
  - Gate at some suitable bias voltage

CG- stage main equations



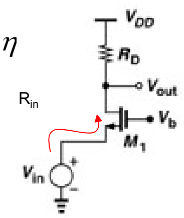


$$V_{out} = V_{DD} - \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_b - V_{in} - V_{TH})^2 R_D$$

$$A_V = g_m(1+\eta)R_D$$

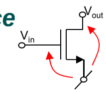
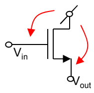
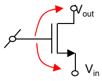
$$A_V = \frac{\partial V_{out}}{\partial V_{in}} = -\mu_n C_{ox}\frac{W}{L}(V_b - V_{in} - V_{TH})\left(-1 - \frac{\partial V_{TH}}{\partial V_{in}}\right)R_D$$

η



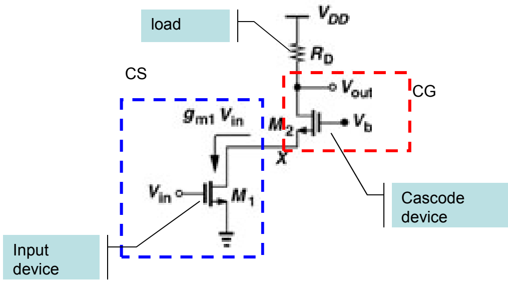
$$\frac{V_{in}}{I_{in}} = R_{in} = \frac{1}{g_m + g_{mb}} = \frac{1}{g_m(1+\eta)}$$

Single transistor amps

	Common Source 	*Common Drain* 	*Connom Gate* 
A_V	high $A_v = -g_m(r_o \parallel R_D)$	≈ 1 $A_v = \frac{g_m}{(g_m + g_{mb})}$	high $(g_m + g_{mb})(r_o \parallel R_D)$
A_I	"infinite"	"infinite"	=1
R_{in}	"infinite"	"infinite"	low $\frac{1}{g_m + g_{mb}}$
R_o	high $r_o \parallel R_D$	low $\frac{1}{g_m + g_{mb}}$	high $r_o \parallel R_D$

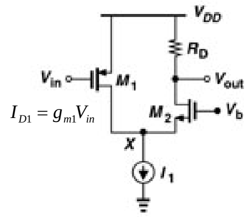
Cascode stage

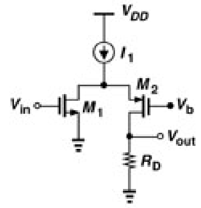
- Combining CS- and CG-stages
  - Cascaded CS and CG = cascode
  - CS → current
  - CG buffer current to load

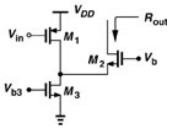


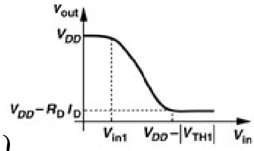
Folded cascode

- Combining nMOS CS-stage with pMOS CG-stage
  - Or vice versa
  - Same first-order behavior
  - Must feed both devices with current source

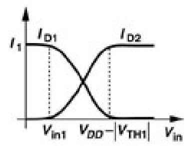








$$A_V = g_{m1}.\{[(g_{m2} + g_{mb2})r_{o2}]r_{o1} \parallel R_D\}$$



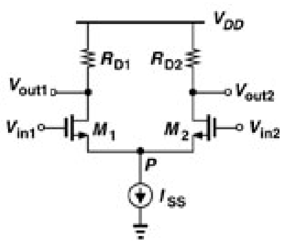
  - Increased bias current for same performance
  - No DC offset, important for low supply voltage

Small signal gain

- Apply small input signal
  - Assuming saturation
  - In equilibrium
    - Split tail current

$$G_{m-\max} = \sqrt{\mu_n C_{ox}\frac{W}{L} I_{SS}}$$ *Must be valid for small signals*

$$I_{SS} = 2I_{M1,2}$$



$$G_{m-\max} = g_m = \sqrt{2\mu_n C_{ox}\frac{W}{L} I_{M1,2}}$$

Common-source topology

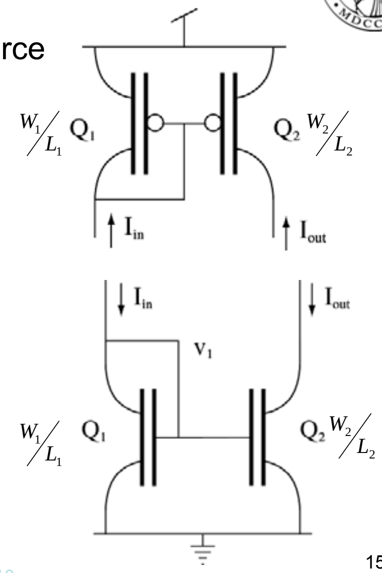
Current mirrors

- Current controlled current source
  - Equal transistor sizes with same gate voltage
    - Give same current

$$I_{out} = I_{in}$$

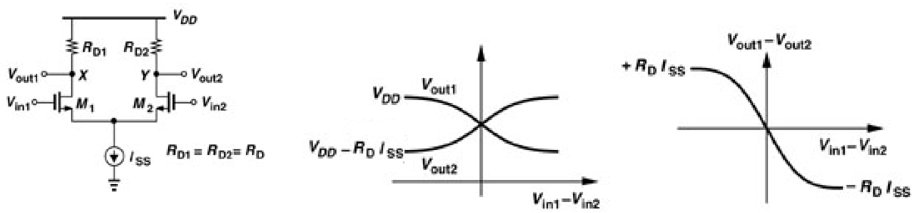
  - Sink or source
    - Assuming active region
    - Ignoring channel shortening
- Fixed gain current amplifier
  - Different transistor sizes

$$I_{out} = \frac{W_2 L_1}{W_1 L_2} I_{in}$$



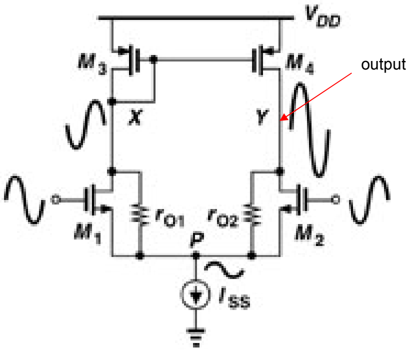
Differential amp

- Output peak values well-defined
  - Independent of CM level
- Maximum gain at $V_{in1}=V_{in2}$
  - Non-linear for large signals



Diff pair small signal

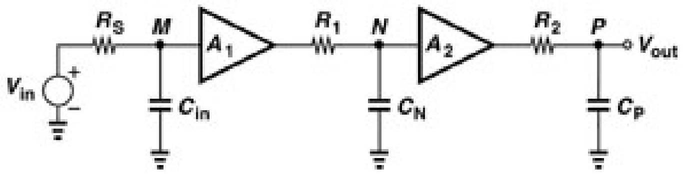
- Signal swing differ
  - X node much less signal sw
    - Nonlinear load (diode)
  - Affecting symmetry
  - Must be analyzed
    - compensated



$$A_{v-Y} \approx G_m R_{out}$$

Frequency response

- Assuming one pole for each node



  - Total gain by multiplying each stage gain

$$\frac{V_{out}}{V_{in}}(s) = \frac{A_1}{1+sR_sC_{in}} \cdot \frac{A_2}{1+sR_1C_N} \cdot \frac{1}{1+sR_2C_P}$$

Simple and easy 1. order approximation

Poles and zeros

$$H(s) = K\frac{(s+z_1)(s+z_2)...(s+z_m)}{(s+\omega_1)(s+\omega_2)...(s+\omega_n)} = a\frac{\left(1+\frac{s}{z_1}\right)\left(1+\frac{s}{z_2}\right)...\left(1+\frac{s}{z_m}\right)}{\left(1+\frac{s}{\omega_1}\right)\left(1+\frac{s}{\omega_2}\right)...\left(1+\frac{s}{\omega_n}\right)}$$

- Poles and zeros are real or complex conjugates
- The actual roots are the *negatives* of the poles and zeros
- Referred to as a positive frequency

- Magnitude and phase

  Euler's formula

  - Sinusoidal: $x_{in}(t) = A_{in}\cos(\omega_{in}t) = A_{in}\frac{e^{j\omega_{in}t} + e^{-j\omega_{in}t}}{2}$
  - Frequency domain two solutions $s = j\omega_m$ and $s = -j\omega_m$
  - May find for particular $s = j\omega_m$:

$$x_{out}(t) = \frac{A_{in}}{2}\left|H(j\omega_{in})\right|\left(e^{j(\omega_{in}t+\phi)} + e^{-j(\omega_{in}t+\phi)}\right) = A_{in}\left|H(j\omega_{in})\right|\cos(\omega_{in}t+\phi)$$

  - Where magnitude is $\left|H(j\omega_{in})\right|$ and phase $\phi = \angle H(j\omega_{in})$

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2. order transfer

$$H(s) = \frac{K\omega_0^2}{\omega_0^2 + s\frac{\omega_0}{Q} + s^2}$$

- Analyze some cases
  - May equate denominators

$$D(s) = \omega_0^2 + s\frac{\omega_0}{Q} + s^2 = (s+\omega_{p1})(s+\omega_{p2}) = \omega_{p1}\omega_{p2} + s(\omega_{p1}+\omega_{p2}) + s^2$$

  - Equating coefficients: $\omega_0^2 = \omega_{p1}\omega_{p2}$ $\quad \frac{\omega_0}{Q} = (\omega_{p1}+\omega_{p2})$
  - Solving yields

$$\omega_{p1}, \omega_{p2} = \frac{\omega_0}{2Q}\left(1 \pm \sqrt{1-4Q^2}\right)$$

  - Assuming roots $\omega_{p1} << \omega_{p2}$ giving $Q<<1$ and $\sqrt{1-4Q^2} \cong 1-2Q^2$

$$\omega_{p1} \approx \omega_0 Q \qquad \omega_{p2} \approx \frac{\omega_0}{Q}$$

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Slur rate

- Frequency → time
  - 1. order LP filter
  - Applied step response

$$H(s) = \frac{A_0}{1 + \frac{s}{\omega_0}}$$

$$X_{out}(s) = A_{in} \frac{1}{s} \frac{A_0}{1 + \frac{s}{\omega_0}}$$

  - Residue method

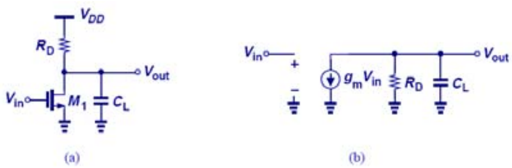
$$X_{out}(s) = A_{in} A_0 \left[\frac{1}{s} - \frac{1}{s + \omega_0} \right]$$

  - Inverse Laplace give time domain

$$x_{out}(t) = u(t) A_{in} A_0 \left(1 - e^{-\frac{t}{\tau}} \right) \qquad \tau = \frac{1}{\omega_0}$$

Bandwidth

- Example: determine time-constant

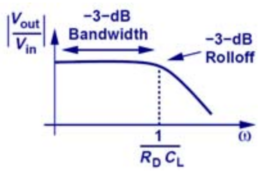


$$H(s) = \frac{V_{out}}{V_{in}}(s) = -g_m (R_D \| \frac{1}{C_L s})$$

$$= \frac{-g_m R_D}{R_D C_L s + 1}.$$

$$s = j\omega \rightarrow \quad \left| \frac{V_{out}}{V_{in}} \right| = \frac{g_m R_D}{\sqrt{R_D^2 C_L^2 \omega^2 + 1}}$$

$$\omega = 1/(R_D C_L) \quad \left| \frac{V_{out}}{V_{in}} \right| = \frac{g_m R_D}{\sqrt{2}}$$



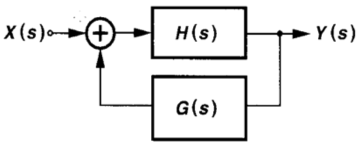
Time constant or -3dB bandwidth

Frequency response

- Basic amp circuits
  - CS stage
  - CD stage
  - CG stage
  - Diff pair
  - Cascode

Feedback principle

- Formal definition



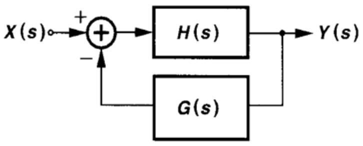
$$Y(s) = H(s)[X(s) + G(s)Y(s)]$$

$$\frac{Y(s)}{X(s)} = \frac{H(s)}{1 - G(s)H(s)}$$

-- Closed-loop transfer function

$H(s)$ -- Open-loop transfer function

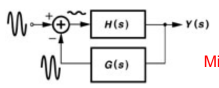
- Negative feedback



$$Y(s) = H(s)[X(s) - G(s)Y(s)]$$

$$\frac{Y(s)}{X(s)} = \frac{H(s)}{1 + G(s)H(s)}$$

1. Feed forward amplifier
2. Output sensing
3. Feedback network
4. Finding feedback error

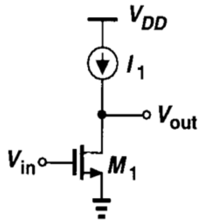


Minimizing error by correct feedback

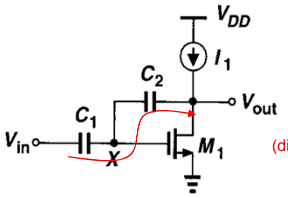
Negative feedback properties

- Gain desensitization
  - CS example

$$A_V = g_{m1} r_{o1}$$



  - Gain poorly defined
    - Temperature, mismatch
  - CS with feedback



$$\frac{V_{OUT}}{V_X} = -g_{m1} r_{o1}$$

(disregard C_2 output load)

$$\frac{V_{OUT}}{V_{IN}} = -\frac{1}{\left(1 + \frac{1}{g_{m1} r_{o1}}\right)\frac{C_2}{C_1} + \frac{1}{g_{m1} r_{o1}}}$$

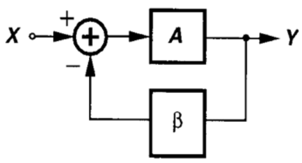
$$\approx -\frac{C_1}{C_2} \qquad (1/g_{m1} r_{o1} \approx 0)$$

$$(V_{OUT} - V_X) C_2 s = (V_X - V_{in}) C_1 s$$

Gain determined by capacitive ratio
- high precision devices

Gain desensitization

- Negative feedback major advantage

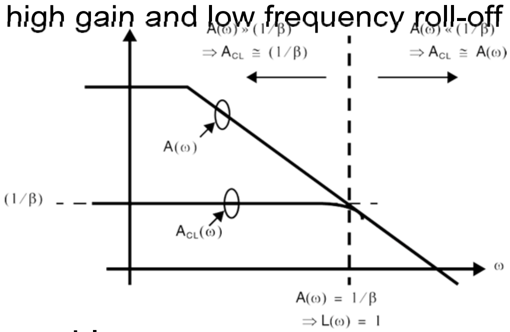


$$A_{CL} = \frac{Y}{X} = \frac{A}{1 + A\beta} = \frac{1}{\beta} \cdot \frac{A\beta}{1 + A\beta} \approx \frac{1}{\beta} \cdot \left(1 - \frac{1}{A\beta}\right) \qquad A\beta >> 1$$

  - Feedback factor β and βA is loop gain
    - Major reduction of gain errors
    - Trading gain for precision

Feedback and bandwidth

- Open loop
  - high gain and low frequency roll-off



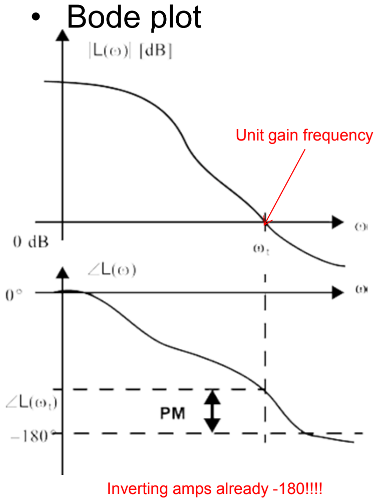
- Closed loop
  - Reduced gain and increased bandwidth

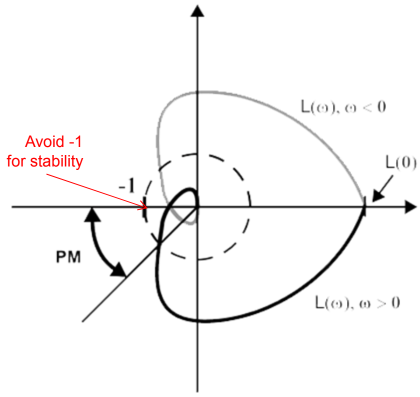
$$\rightarrow A(s) = \frac{1}{\beta}$$

Typical feedback amp

- Bode plot

Polar plot



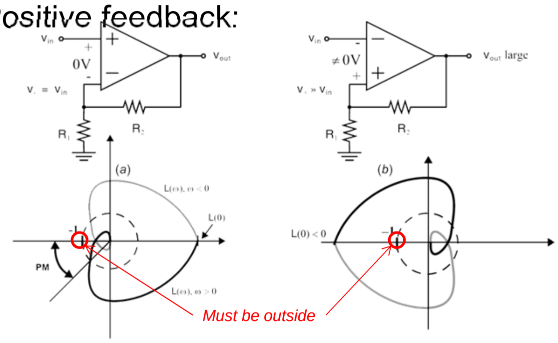


Stable if $\angle L(\omega_t) > -180$ at unit-gain frequency

-180 degrees due to inverting amp = -360

Phase margins

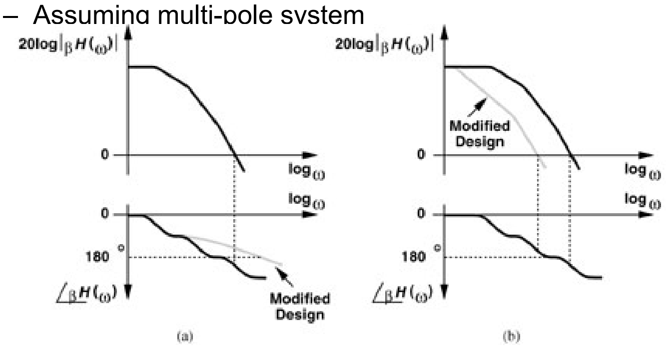
- Measure of stability $PM = \angle L(\omega_t) + 180$
  - Must account for variations and need margins
- Positive feedback:



  - Never positive feedback in amps!
    - Fine in oscillators

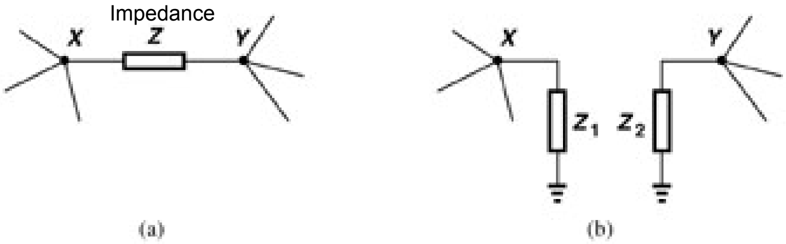
Frequency compensation

- Improving phase margins
  - Assuming multi-pole system



1. Reduce phase shift for sufficient PM
   - Hard since we must reduce number of poles → lower gain
2. Reduce gain for sufficient PM
   - Maintain gain and signal swing at LF, but reduce bandwidth

Miller Theorem



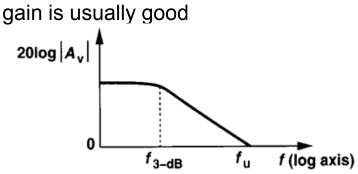
- If circuit (a) is converted to (b) then:

$$Z_1 = \frac{Z}{1 - A_v} \qquad Z_2 = \frac{Z}{\left(1 - \frac{1}{A_v}\right)} \qquad A_v = \frac{V_Y}{V_X}$$

Approximation!

Op Amp performance parameters

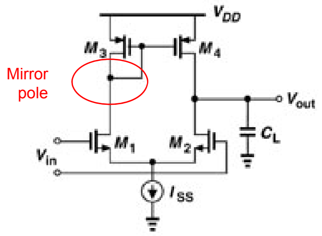
- Gain
  - May vary depending on application, but high gain is usually good
- Small-signal bandwidth
  - Gain drops with increased frequency
  - Unity-gain bandwidth; recurring measure
- Large-signal bandwidth
  - May introduce instability
  - Hard to analyze
- Output swing
  - "Rail-to-rail" operation
  - Differential topologies popular
- Linearity
- Noise and offset
- Supply rejection
  - Important in mixed-mode systems
  - Digital switching noise is showing up all over!

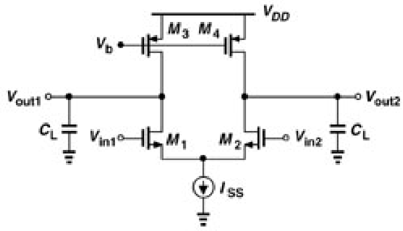


Trading off performance parameters
According to application and technology

Single-stage diff amplifiers

- Any diff amp may do…





$$A_v = g_{m1,2}(r_{o2} \| r_{o4})$$

Single ended output

$$A_v = g_{m1,2}(r_{o2} \| r_{o4})$$

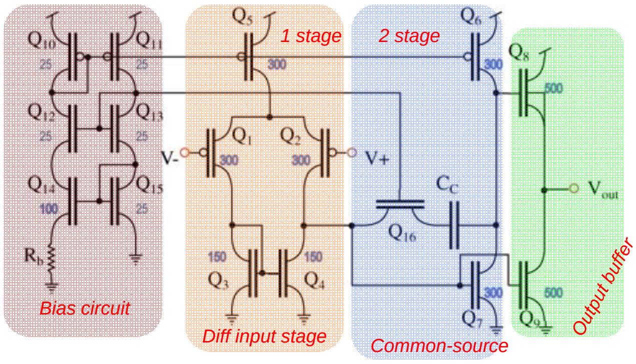
Differential output

- Output load capacitance determine bandwidth
- Gain around 20-50
- How do we improve?

Advanced mirrors and opamps

- Two-stage OPAMP
  - Compensation
- Advanced Current Mirrors
  - Wide-Swing Current Mirrors
  - Enhanced Output-Impedance Current Mirrors
- Advanced OTAs
  - Folded-Cascode OTA
  - Current-Mirror OTA
  - Fully Differential OTAs
    - Folded-Cascode
    - Current-Mirror
    - Common-Mode Feedback Circuits

CMOS OPAMP topology

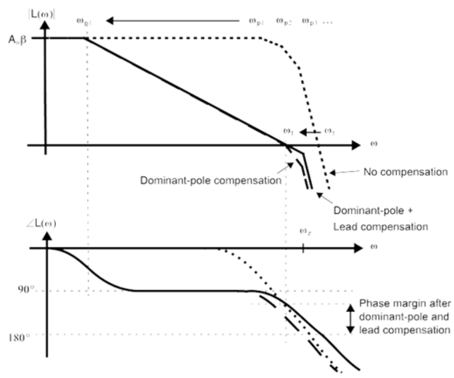


- PMOS diff input stage
- Numbers realistic transistor widths
  - Length 1-2 times minimum
- Output buffer dropp for capacitive loads

Opamp compensation

$$\omega_t \approx \beta\omega_{ta}$$

- Dominant-pole compensation
  - Forcing a feedback system to have 1. order response up to loop unit-gain frequency ω_t
  - Stable system with increased PM
- Lead compensation
  - Adding zero, ω_z, just above ω_t
  - May improve PM with 20°

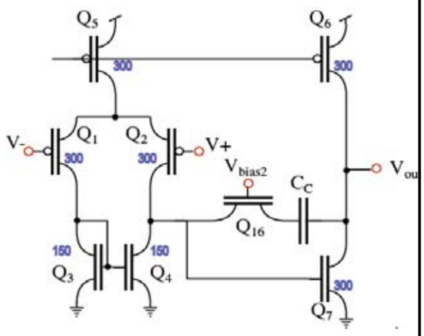


Two stage opamp compensation

- Dominant pole compensation
  - Setting the ω_{p1} and ω_t since
    $$\omega_t = A_0\omega_0$$
  - Q_{16} operate in triode region
    - Resistive element

$$R_C = r_{DS16} = \frac{1}{\mu_n C_{ox} \frac{W}{L} V_{eff16}}$$

    - Resistive element ensures left half-plane zero
      - Damping element
    - *Lead compensation*

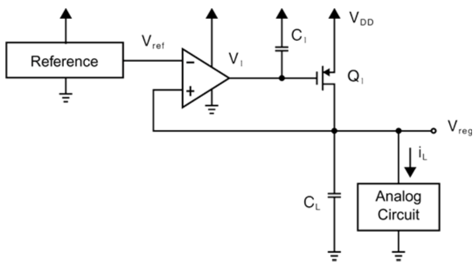


Topology comparison

	Gain	Output Swing	Speed	Power dissipation	Noise
Telescopic	Medium	Medium	Highest	Low	Low
Folded Cascode	Medium	Medium	High	Medium	Medium
Two-stage	High	Highest	Low	Medium	Low
Gain Boosted	High	Medium	Medium	High	Medium

Biasing and refs

- Distribute as current
- Combine mirrors and resistor for ref voltage
- Temperature compensate
  - Combine positive and negative temperature dependencies
- Low dropout reference



That’s it!

- Good luck!