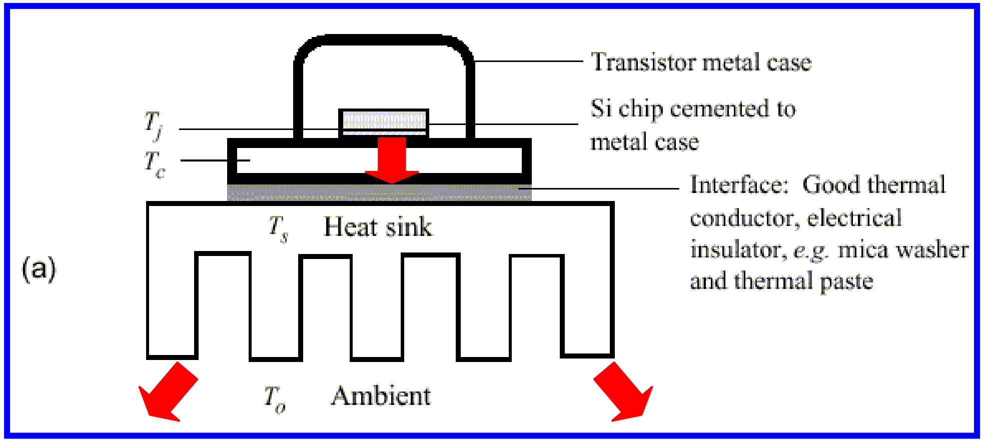
Heat transfer

Heat = amount of energy that is transferred from one system to another (or between system and surroundings) as a result of temperature difference

The origin of energy transfer is the ***random motion*** of molecules

Heat is transferred by
- thermal conduction
- convection
- radiation



Fourier's Law of Thermal Conduction

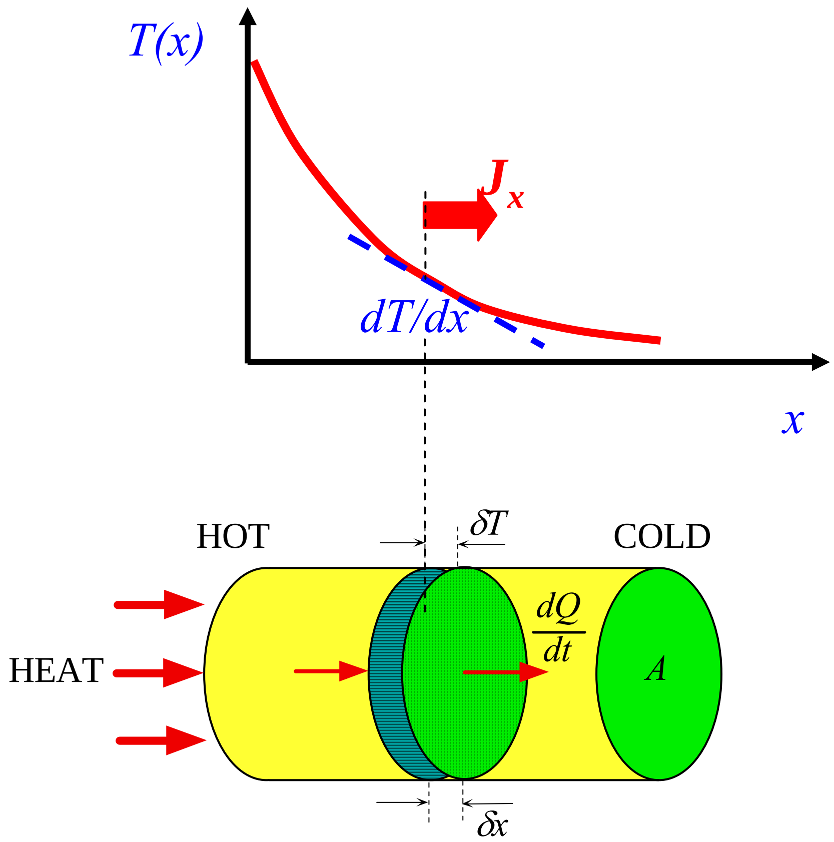


Fig. 2.19: Heat flow in a metal rod heated at one end. Consider the rate of heat flow, dQ/dt, across a thin section δx of the rod. The rate of heat flow is proportional to the temperature gradient $\delta T/\delta x$ and the cross sectional area A.

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$$J_x \equiv \frac{1}{A}\frac{dQ_x}{dt}$$

J_x = heat flux,

dQ_x/dt = the rate of heat flow

A = cross-sectional area

$$J_x = \frac{dQ}{dt} = -\kappa \frac{dT}{dx}$$

dT/dx = temperature gradient

κ = thermal conductivity

$[\kappa]$ = W m^{-1} K^{-1} or W m^{-1} ^{0}C^{-1}

Reminder: $\Gamma = -D\frac{dC}{dx}$ Fick's First Law

Thermal Conductivities of various materials

Table 1

Typical thermal conductivities of various classes of materials at 25 °C.

Pure metal	Nb	Sn	Fe	Zn	W	Al	Cu	Ag
κ (W m^{-1} K^{-1})	52	64	80	113	178	250	390	420
Metal alloys	Stainless Steel	55Cu-45Ni	Manganin (86Cu-12Mn-2Ni)	70Ni-30Cu	1080 Steel	Bronze (95Cu-5Sn)	Brass (63Cu-37Zn)	Dural (95Al-4Cu-1Mg)
κ (W m^{-1} K^{-1})	12 - 16	19.5	22	25	50	80	125	147
Ceramics and glasses	Glass-borosilicate	Silica-fused (SiO_2)	S_3N_4	Alumina (Al_2O_3)	Magnesia (MgO)	Saphire (Al_2O_3)	Beryllia (BeO)	Diamond
κ (W m^{-1} K^{-1})	0.75	1.5	20	30	37	37	260	1000
Polymers	Polypropylene	Polystyrene	PVC	Polycarbonate	Nylon 6,6	Teflon	Polyethylene low density	Polyethylene high density
κ (W m^{-1} K^{-1})	0.12	0.13	0.17	0.22	0.24	0.25	0.3	0.5

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κ (W m^{-1} K^{-1})	0.12	0.13	0.17	0.22	0.24	0.25	[illegible]	[illegible]

Strong metallic bonding

Strong covalent bonding

Weak Van-der-Waals bonding

Wiedemann – Franz - Lorenz Law

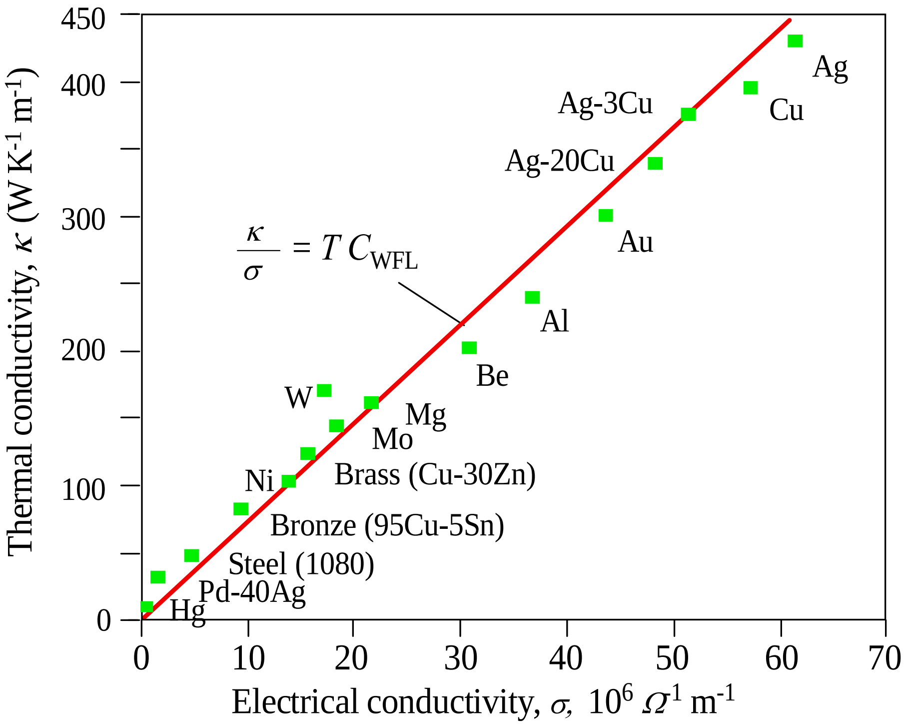

Fig. 2.20: Thermal conductivity, κ vs. electrical conductivity σ for various metals (elements and alloys) at 20 °C. The solid line represents the WFL law with $C_{WFL} \approx 2.44\times10^{8}$ W Ω K^{-2}.

$$\frac{\kappa}{\sigma T} = C_{\mathrm{WFL}} = 2.45\times10^{-8}\ \mathrm{W}\,\Omega\,\mathrm{K}^{-2}$$

κ = thermal conductivity

σ = electrical conductivity

T = temperature

C_{WFL} = Lorenz number

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Wiedemann – Franz - Lorenz Law

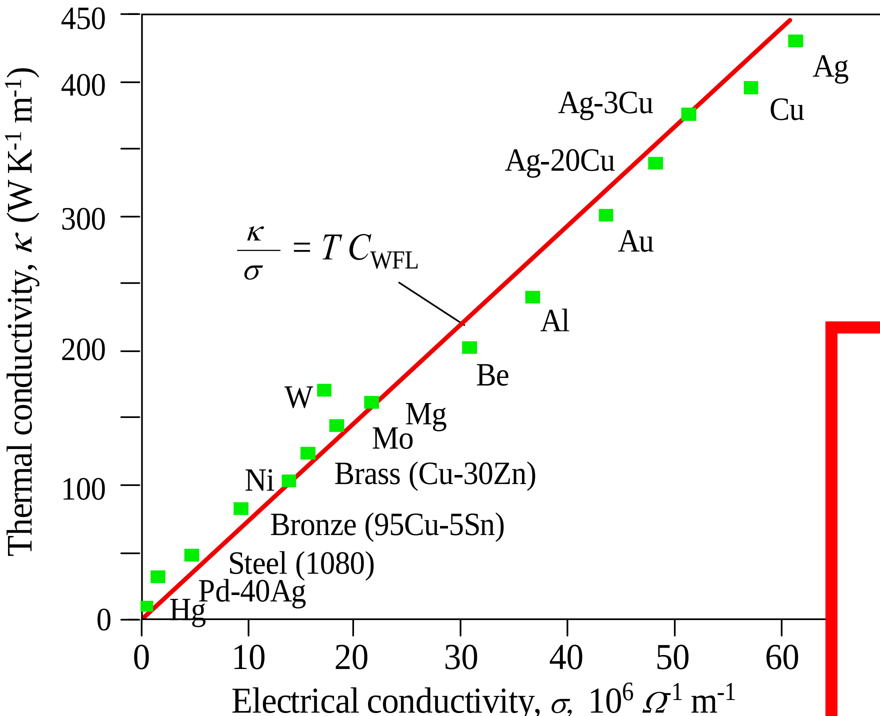

Fig. 2.20: Thermal conductivity, κ vs. electrical conductivit
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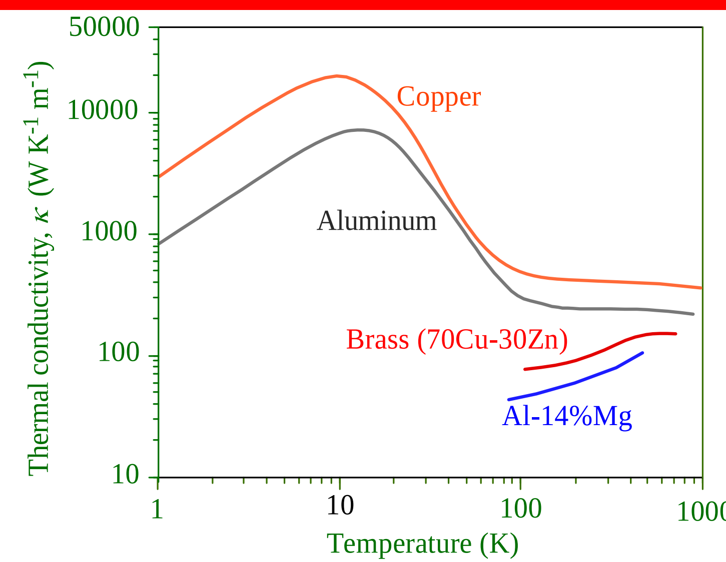
$$\frac{\kappa}{\sigma T} = C_{\mathrm{WFL}} = 2.45\times10^{-8}\ \mathrm{W\,\Omega\,K^{-2}}$$

κ = thermal conductivity



Thermal conductivity vs. temperature for two pure metals (Cu and Al) and two alloys (brass and Al-14%Mg). Data extracted from *Thermophysical Properties of Matter*, Vol. 1: Thermal Conductivity, Metallic Elements and Alloys, Y.S. Touloukian et. al (Plenum, New York, 1970).

Thermal Conductivities of various materials

Table 1

Typical thermal conductivities of various classes of materials at 25 °C.

Pure metal	Nb	Sn	Fe	Zn	W	Al	Cu	Ag
κ (W m^{-1} K^{-1})	52	64	80	113	178	250	390	420
Metal alloys	Stainless Steel	55Cu-45Ni	Manganin (86Cu-12Mn-2Ni)	70Ni-30Cu	1080 Steel	Bronze (95Cu-5Sn)	Brass (63Cu-37Zn)	Dural (95Al-4Cu-1Mg)
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Polymers	Polypropylene	Polystyrene	PVC	Polycarbonate	Nylon 6,6	Teflon	Pol[illegible]	Polyethylene
κ (W m^{-1} K^{-1})	0.12	0.13	0.17	0.22	0.24	0.25	[illegible]	[illegible]

Strong metallic bonding

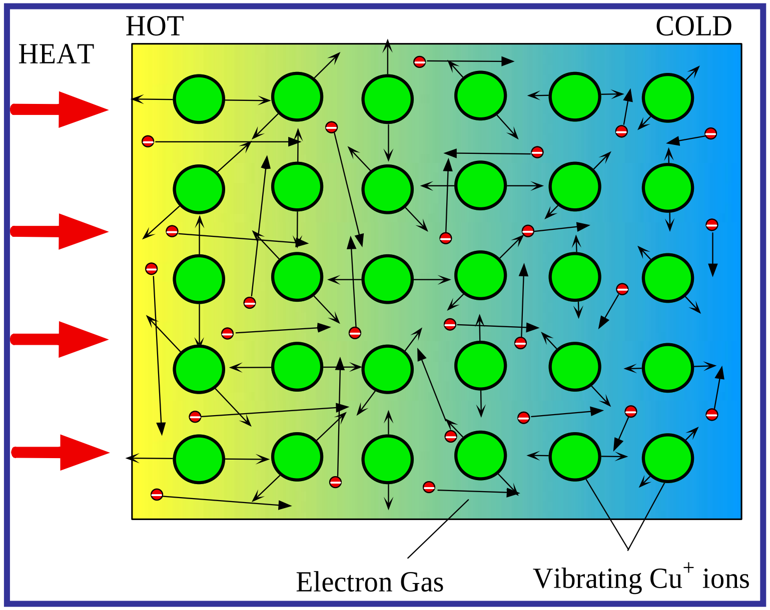
Strong covalent bonding

Weak Van-der-Waals bonding

Thermal conduction in metals and some insulators

Metals
Ag, Cu, Al ...

Insulators with _very_ strong covalent bonding
C (diamond), BeO (beryllia), ...



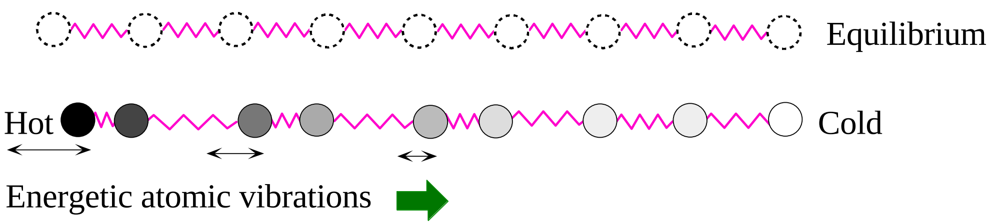


Fig. 2.22: Conduction of heat in insulators involves the generation and propogation of atomic vibrations through the bonds that couple the atoms. (An intuitive figure.)

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Heat is transferred by conduction electrons

Heat is transferred as atomic vibrations due to strong bonding between atoms

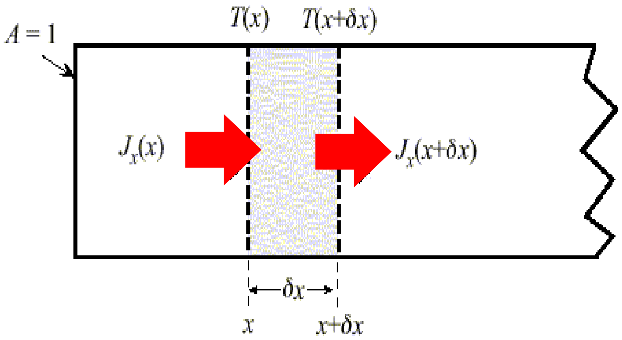
Parabolic heat equation

$$D_{th}\frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t}$$

where $D_{th} = \frac{\kappa}{c\rho}$ thermal diffusivity

ρ = density

c = specific heat capacity



Heat flow in – Heat flow out =

= Rate of heat accumulation in volume δx

Rate of heat accumulation in volume δx = $\delta x \times \rho \times c\frac{\partial T}{\partial t}$

Heat flow in – Heat flow out = $J_x(x)$- $J_x(x+\delta x) = \frac{\partial J_x}{\partial x}\delta x = -\kappa\frac{\partial^2 T}{\partial x^2}\delta x$

Reminder: $D\frac{\partial^2 C(x,t)}{\partial x^2} = \frac{\partial C(x,t)}{\partial t}$ Fick's Second Law

Fourier's Law

$$J_x = \frac{dQ}{dt} = -\kappa \frac{dT}{dx}$$

$$J_x \equiv \frac{1}{A}\frac{dQ_x}{dt}$$

$$Q' = A\kappa \frac{\Delta T}{L} = \frac{\Delta T}{(L/\kappa A)}$$

Q' = rate of heat flow or the heat current, A = cross-sectional area, κ = thermal conductivity (material-dependent constant of proportionality), ΔT = temperature difference between ends of component, L = length of component

Ohm's Law

$$I = \frac{\Delta V}{R} = \frac{\Delta V}{(L/\sigma A)}$$

I = electric current, ΔV = voltage difference across the conductor, R = resistance, L = length, σ = conductivity, A = cross-sectional area

Fourier's Law

$$Q' = A\kappa \frac{\Delta T}{L} = \frac{\Delta T}{(L/\kappa A)} = \theta$$

Q' = rate of heat flow or the heat current, A = cross-sectional area, κ = thermal conductivity (material-dependent constant of proportionality), ΔT = temperature difference between ends of component, L = length of component

Ohm's Law

$$I = \frac{\Delta V}{R} = \frac{\Delta V}{(L/\sigma A)} = R$$

I = electric current, ΔV = voltage difference across the conductor, R = resistance, L = length, σ = conductivity, A = cross-sectional area

Definition of Thermal Resistance

$$Q' = \frac{\Delta T}{\theta}$$

Q' = rate of heat flow, ΔT = temperature difference, θ = thermal resistance

Thermal Resistance

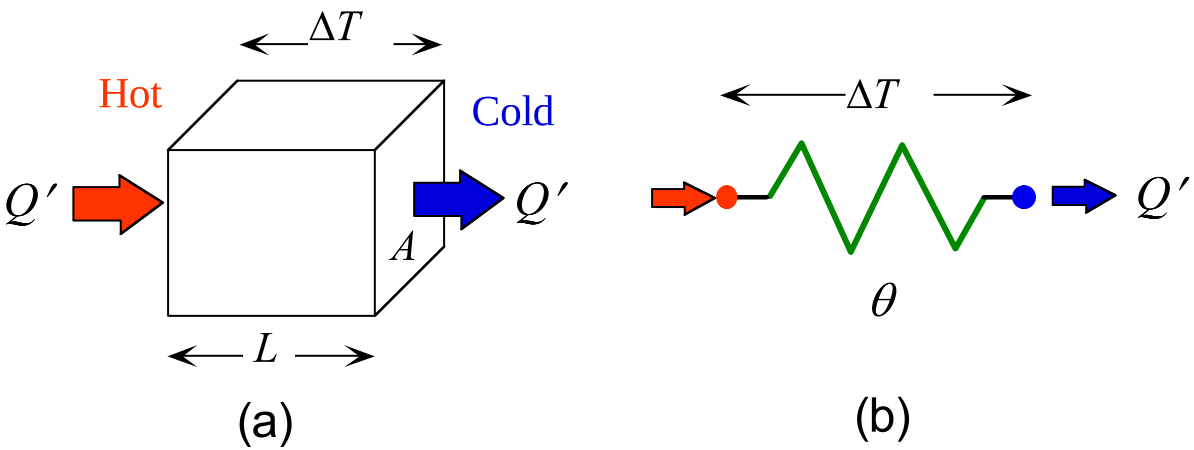
$$\theta = \frac{L}{A\kappa}$$

θ = thermal resistance, L = length, A = cross-sectional area, κ = thermal conductivity

Analogy between thermal and electrical phenomena

THERMAL PHENOMENA	ELECTRICAL PHENOMENA
Q = rate of heat flow	I = Current
ΔT = temperature difference	ΔV = bias (voltage)
Θ = *thermal resistance*	R = resistance

$$Q' = \frac{\Delta T}{\theta}$$

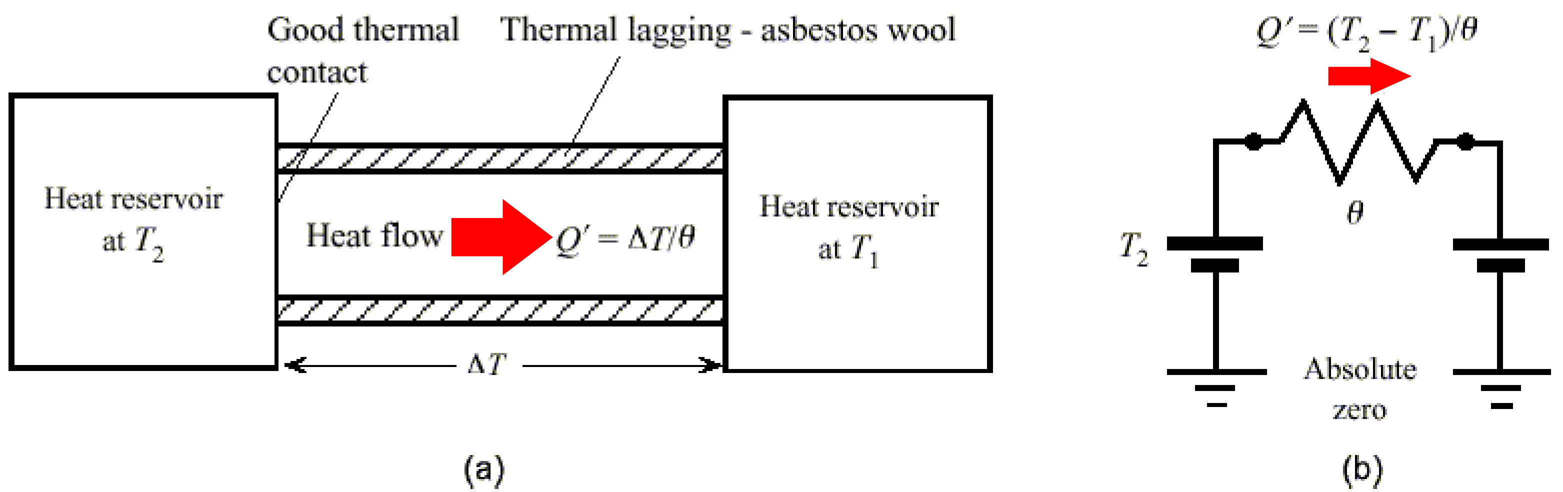


$$I = \frac{\Delta V}{R}$$

Fig. 2.23: Conduction of heat through a component in (a) can be modeled as a thermal resistance θ shown in (b) where $Q' = \Delta T/\theta$.

Analogy between thermal and electrical phenomena

THERMAL PHENOMENA	ELECTRICAL PHENOMENA
Q = rate of heat flow	I = Current
ΔT = temperature difference	ΔV = bias (voltage)
Θ = *thermal resistance*	R = resistance
Heat reservoir	EMF (Electromotive Force)
Absolute zero	Ground

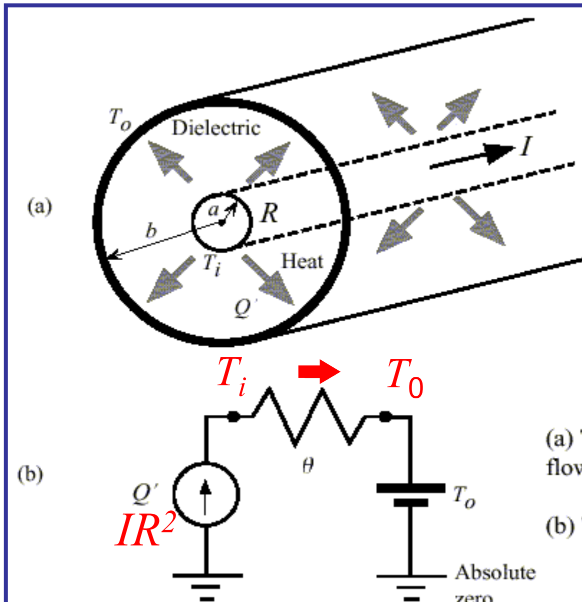


(a) A conductor between two heat reservoirs T_2 and T_1, $T_2 > T_1$. There is no heat loss from the surface. Steady state heat conduction through this component is given by $Q' = \Delta T/\theta$. (b) We can model the heat flow using two thermal EMFs and a thermal resistance between them.

Figure 4

Analogy between thermal and electrical phenomena

THERMAL PHENOMENA	ELECTRICAL PHENOMENA
Q = rate of heat flow	I = Current
ΔT = temperature difference	ΔV = bias (voltage)
Θ = *thermal resistance*	R = resistance
Heat reservoir	EMF (Electromotive Force)
Absolute zero	Ground
Heat generator	Current supply



$$Q' = (T_i - T_0)/\Theta$$

(a) The joule heat generated in the core conductor flows outwards radially through the dielectric material.

(b) The equivalent circuit

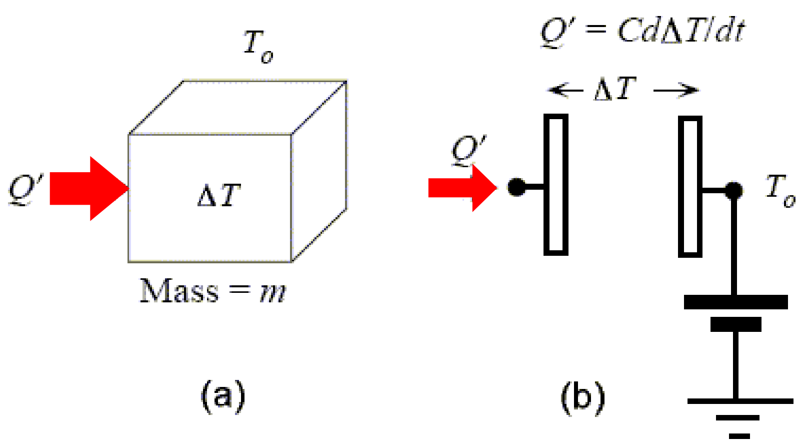
Figure 5

Analogy between thermal and electrical phenomena

THERMAL PHENOMENA	ELECTRICAL PHENOMENA
Q = rate of heat flow	I = Current
ΔT = temperature difference	ΔV = bias (voltage)
Θ = *thermal resistance*	R = resistance
Heat reservoir	EMF (Electromotive Force)
Absolute zero	Ground
Heat generator	Current supply
C = thermal capacitance	C = capacitance

$$\delta Q = C\,\delta T$$

$$Q' = C\frac{\delta T}{\delta t}$$



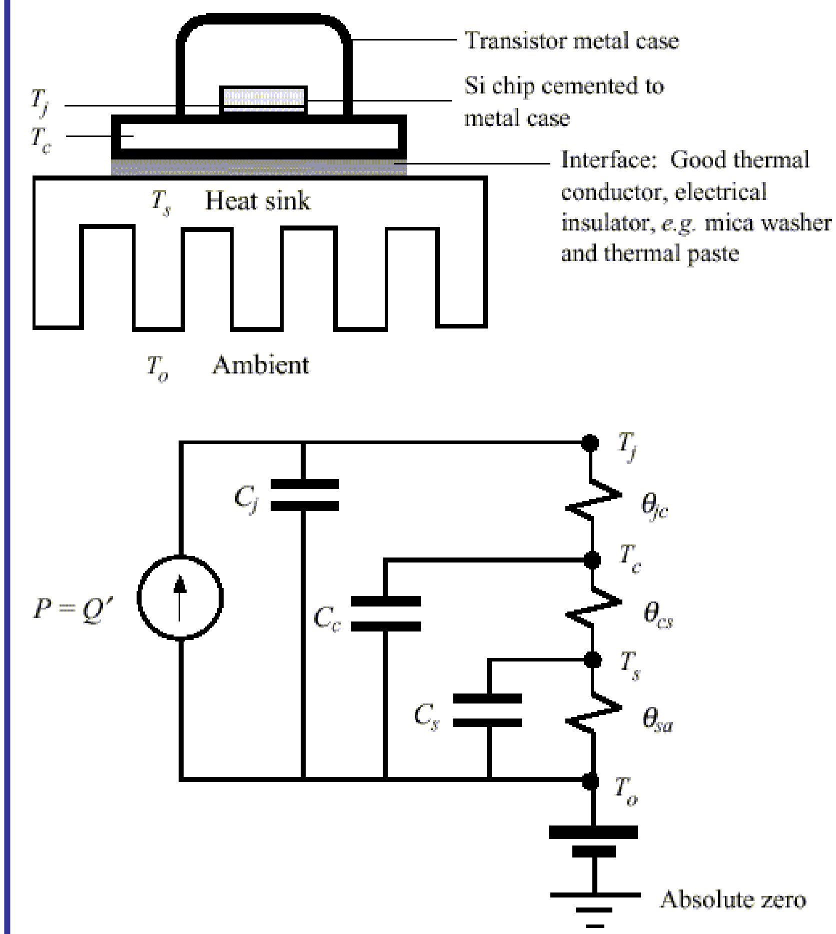
$$\delta Q = C\,\delta V$$

$$I = C\frac{\delta V}{\delta t}$$

(a) When heat flows into a body, its temperature increases. The rate of increase in the temperature difference ΔT between the body and its environment is determined by the heat current flowing into the body, that is, Q'. (b) Heating of a body uniformly can be represented as a thermal capacitance C into which a heat current flows and changes the temperature difference ΔT across C.

Analogy between thermal and electrical phenomena. Equivalent circuit of transistor

THERMAL PHENOMENA	ELECTRICAL PHENOMENA
Q = rate of heat flow	I = Current
ΔT = temperature difference	ΔV = bias (voltage)
Θ = *thermal resistance*	R = resistance
C = thermal capacitance	C = capacitance
Heat reservoir	EMF (Electromotive Force)
Absolute zero	Ground
Heat generator	Current supply

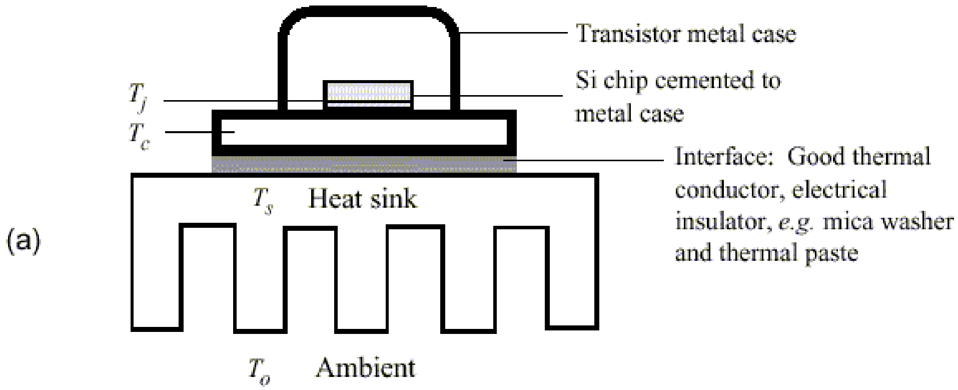


(a) A heat sinked transistor. The transistor has a metal case and is mounted on a heat sink using an insulating washer and thermal paste (grease). Note: The heat sink is normally placed with the fins facing up, upside down to that shown, to allow heated air to rise and set a convective flow. This sketch is for convenience only.)

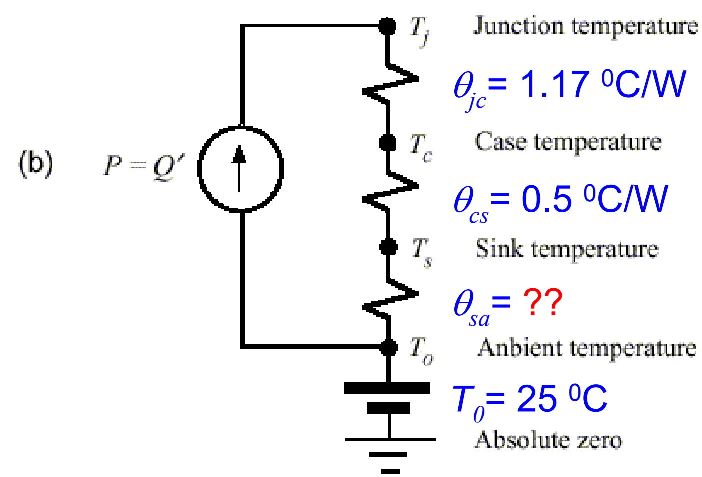
The equivalent circuit of a heat sinked transistor with its thermal resistances and capacitances.

Transistor specifications: estimation of required heat sink

BJT(2N3716) P_d = 15 W



T_j = 110 ^{0}C

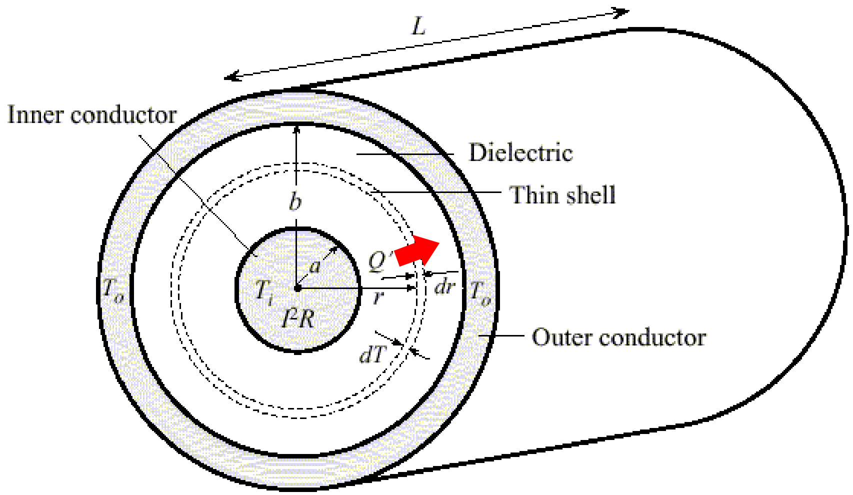


$$\theta_{ja} = \theta_{jc} + \theta_{cs} + \theta_{sa}$$

$$P_d = Q' = \frac{T_j - T_0}{\theta_{ja}} = \frac{T_j - T_0}{\theta_{jc} + \theta_{cs} + \theta_{sa}}$$

$$\theta_{ja} = \frac{T_j - T_0}{P_d} = \frac{110^0 C - 25^0 C}{15W} = 5.67^0 C/W$$

$$\theta_{ca} = \theta_{ja} - \theta_{jc} - \theta_{cs} = 5.67 - 1.17 - 0.5 = 4\ ^0\text{C/W}$$



Thermal resistance of a hollow cylindrical shell. Consider an infinitesimally thin cylindrical shell of radius r and thickness dr in the dielectric and concentrically around the inner conductor. The surface area is $2\pi rL$.

Figure 6

$$Q' = -\kappa(2\pi rL)\frac{dT}{dr}$$

$$Q'\frac{dr}{r} = -\kappa(2\pi L)dT$$

$$Q'\int_a^b \frac{dr}{r} = -\kappa(2\pi L)\int_{T_i}^{T_0} dT$$

$$Q'\ln(\frac{b}{a}) = -\kappa 2\pi L(T_i - T_0)$$

$$Q' = \frac{\kappa 2\pi L(T_i - T_0)}{\ln(\frac{b}{a})} \Rightarrow \theta = \frac{T_i - T_0}{Q'} = \frac{\ln(\frac{b}{a})}{2\pi\kappa L}$$

a = 5 mm
b = 3 mm
ρ = 27 nΩ m – aluminum
κ = 0.3 W m^{-1} K^{-1} – polyethylene
I = 500 A
L = 1 m

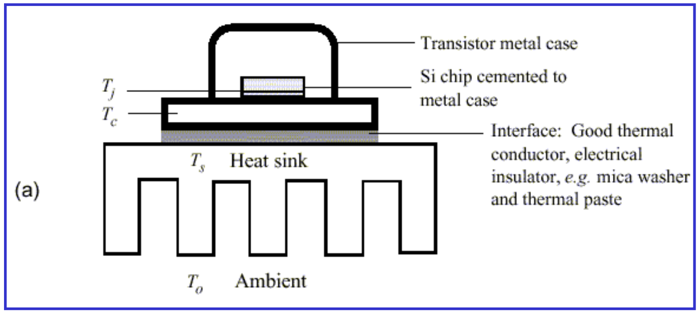
$$Q' = I^2\frac{\rho L}{\pi a^2} = 85.9W$$

$$\theta = \frac{\ln(\frac{b}{a})}{2\pi\kappa L} = 0.25^0 C/W$$

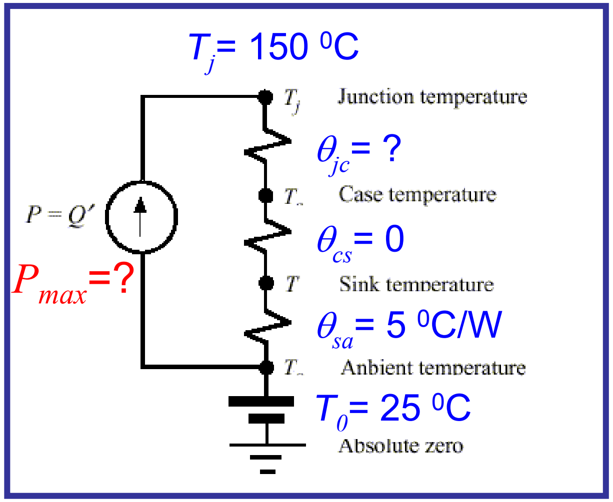
$$\Delta T = Q'\theta = 21.5^0 C$$

$$T_i = 41.5^0 C$$

Transistor specifications: derated power



(b)



Example 3.3: Derated power

A power transistor has a specification of power dissipation of 20 W at a case temperature of 25 °C. The maximum junction temperature is 150 °C. The transistor is mounted on a heat sink that has a thermal resistance of 5 °C/W. Neglect the thermal resistance of the washer and determine the maximum power that this transistor can dissipate.

Solution

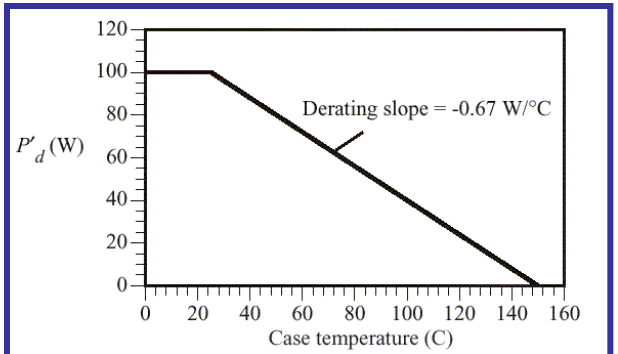
The thermal resistance from the junction-to-case is

$$\theta_{jc} = (T_j - T_c) / P_d = (150 - 25\ °C)/(20\ W) = 6.25\ °C/W.$$

With the transistor mounted on the heat sink, the maximum power P'_d flows from the junction to the ambient. Suppose that T_o = 25 °C. The case-to-ambient thermal resistance θ_{ca} is the heat sink resistance and is given as 5 °C /W. Thus, the derated power is,

$$P'_d = \frac{(T_j - T_a)}{\theta_{jc} + \theta_{ca}} = \frac{(150-25)}{6.25+5} = 11.1\ W.$$

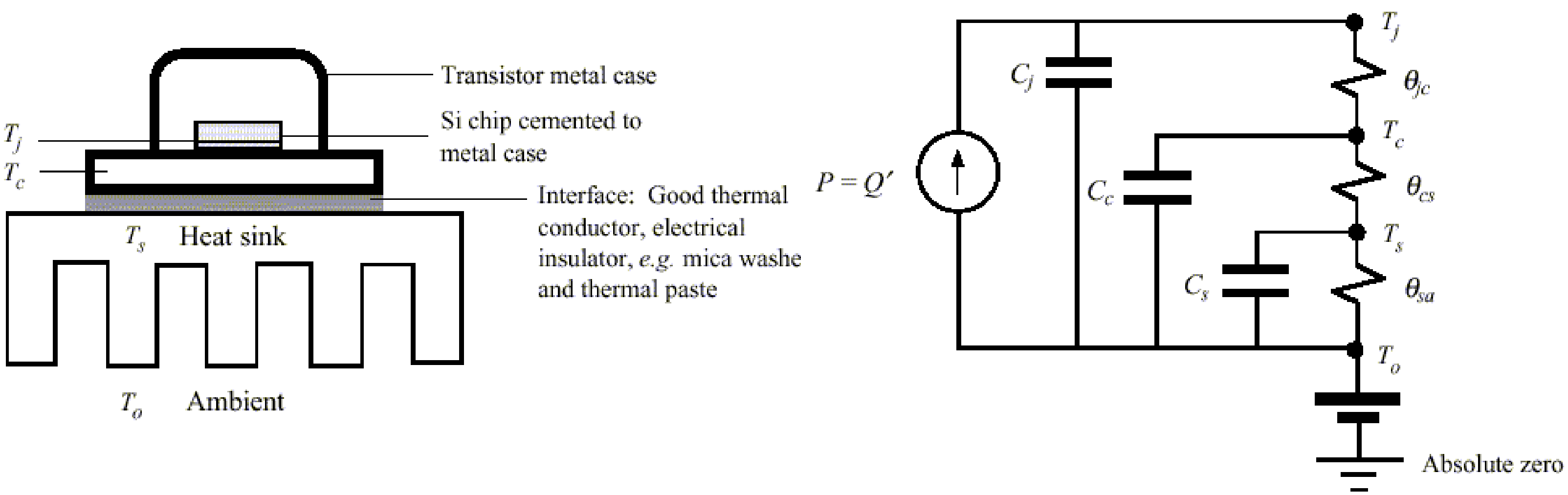
This heat sink choice limits the allowed dissipation to half the rated value. The case temperature, by the way, is 80.5 °C. (Why?)



A power dissipation derating curve as a function of transistor case temperature

Figure 10

Transistor specifications: non-steady-state regime



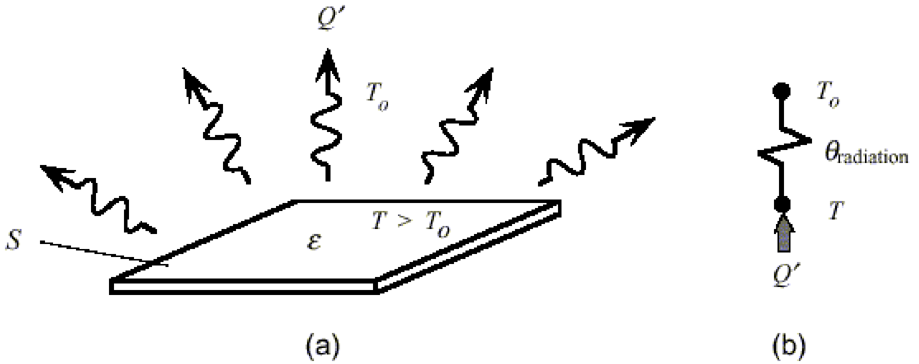
The equivalent circuit of a heat sinked transistor with its thermal resistances and capacitances.

Figure 9

Stefan's law

$$P_{radiated} = \varepsilon\sigma_s S(T^4 - T_0^4)$$

σ_s = 5.67×10^{-8} Wm^{-2}K^{-4} – Stefan's constant,
ε = emissivity of the surface,
S = surface area emitting the radiation,
T = temperature of the surface,
T_0 = ambient temperature



(a) Radiation loss from a surface area S that an emissivity of ε.(b) Equivalent circuit representation using an effective thermal resistance.

Figure 11

Effective thermal resistance

$$\theta_{\text{radiation}} = \frac{T - T_o}{Q'} = \frac{T - T_o}{\varepsilon\sigma_S S(T^4 - T_o^4)}$$

$$\theta_{\text{radiation}} \cong \frac{1}{\varepsilon\sigma_S S(T + T_o)(T + T_o^2)}$$

$T >> T_0$

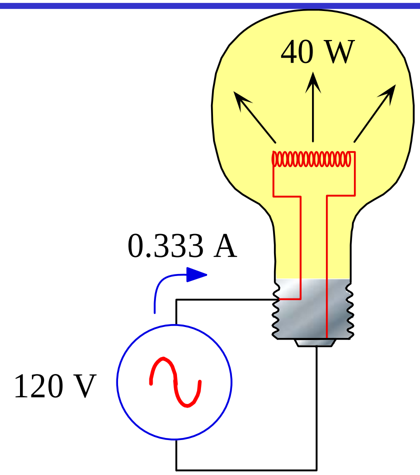
Emissivities of different materials

Table 2

Typical emissivity. Actual values may vary substantially.

Surface	ε	Surface	ε	Surface	ε
Al, foil	0.04	Black paint, nonglossy	0.8–0.96	Iron, cast polished	0.21
Al, oxidized at 316 °C	0.05	Brass polished	0.03–0.1	Iron, cast oxidized	0.60
Al, anodized	0.2–0.8	Brick, red rough	0.93	Iron, completely rusted	0.69
Al, oxidized, black, heat sink	0.9	Brick, building	0.45	Iron, wrought, oxidized	0.94
Asbestos	0.93–0.96	Concrete tile	0.63	Tungsten filament	0.35–0.39
Beryllium	0.16–0.30	Gold coating	0.05–0.1	White paint, nonglossy	0.8–0.9

What is the temperature of the filament?



Power radiated from a light bulb at 2408 °C is equal to the electrical power dissipated in the filament.

P = 40 W

V = 120 V

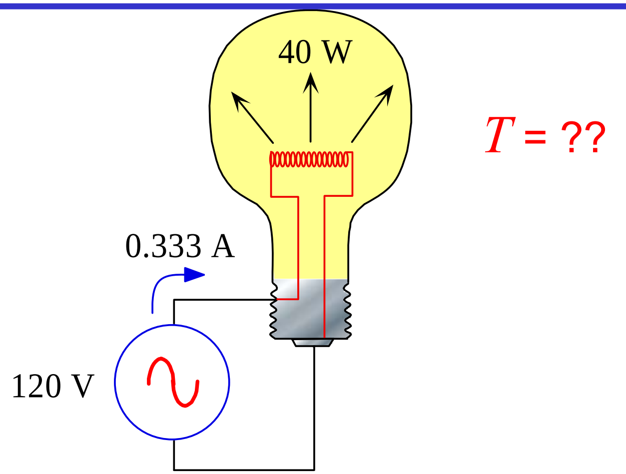
L = 38.1 cm

D = 33 μm

$\rho\,(273\text{K})$ = 5.51×10-8 Ωm

$\rho\,(T) \sim T^{1.2}$

What is the temperature of filament in electric bulb ?



Power radiated from a light bulb at 2408 °C is equal to the electrical power dissipated in the filament.

P = 40 W
V = 120 V
L = 38.1 cm
D = 33 μm

σ_S = 5.67×10^{-8} Wm^{-2}K^{-4}
ε = 0.35 - 0.39

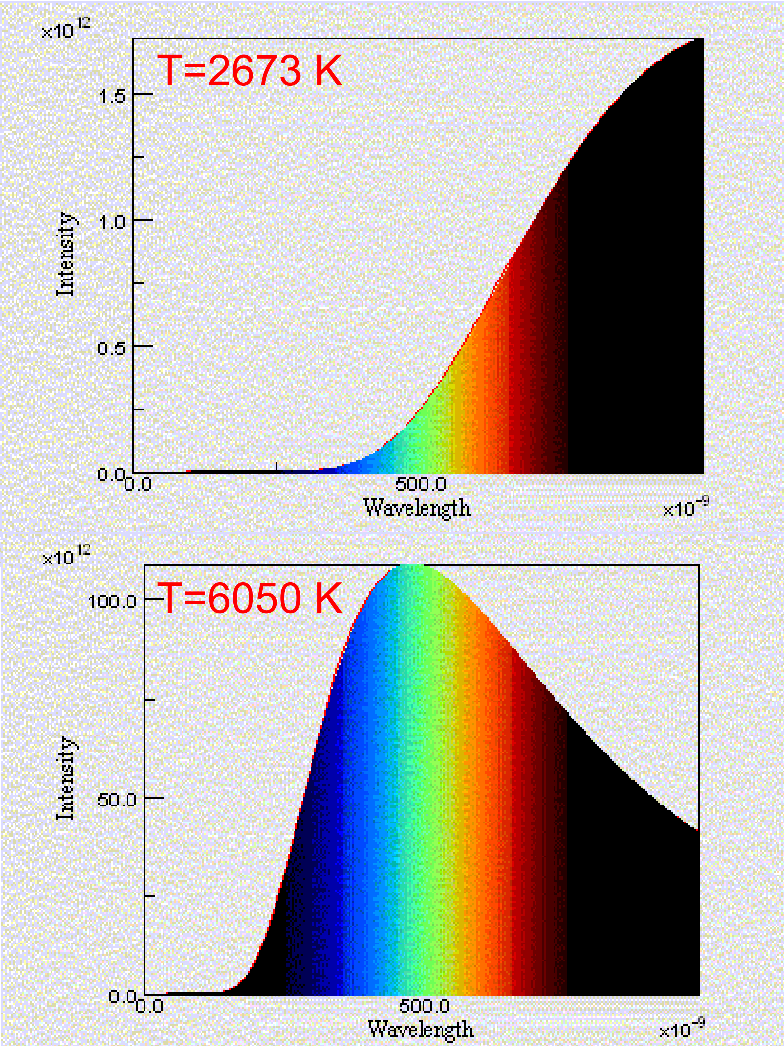
$$P = P_{radiated} = \varepsilon\sigma_s S(T^4 - T_0^4)$$

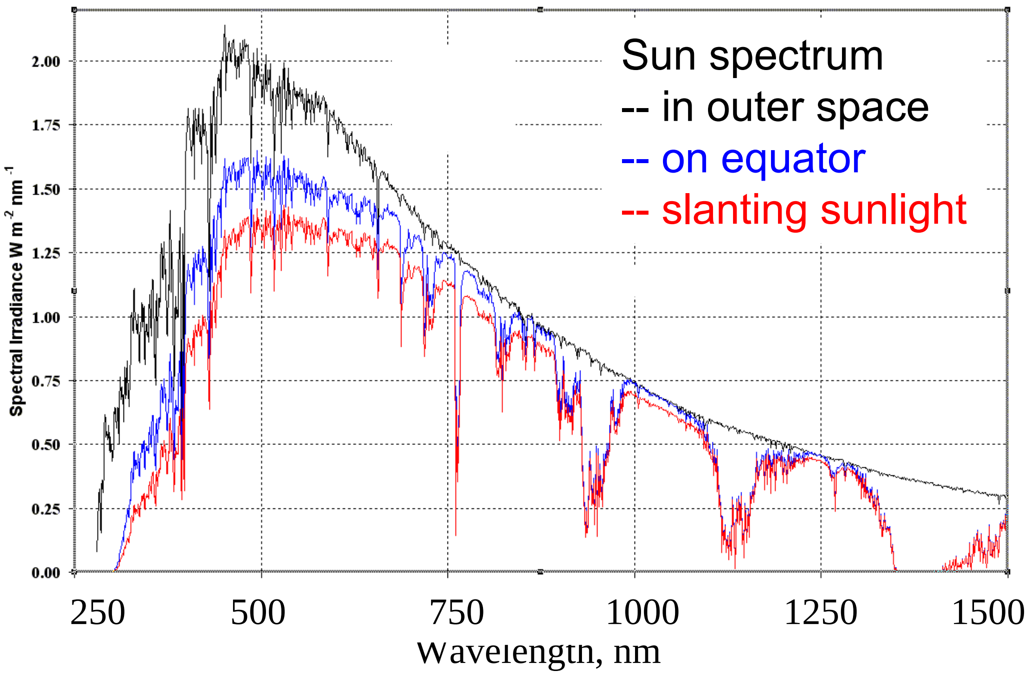
$$S = \pi DL = 3.14\times 33\times 10^{-6}\times 0.381 m^2 = 3.95\times 10^{-5} m^2$$

$$40W = (0.35)(5.67\times 10^{-8})(3.95\times 10^{-5})(T^4 - (293K)^4) \quad \Rightarrow \quad T = 2673K$$

T_W = 3680 K

Emission spectra of heated bodies





How long does it take to light an electric bulb ?

Consider a 40 W, 120 V incandescent General Electric light bulb. The tungsten filament is of length 0.381 m and diameter 33 μm. Its resistivity at room temperature is $5.7 \times 10^{-8}\ \Omega$ m. Tungsten has a density d = 19300 kg m^{-3}, specific heat capacity c = 130 J kg^{-1} K^{-1}; temperature coefficient of resistivity (TCR) α = 0.005 °C^{-1}. What is the time it takes for the filament to reach the operating temperature?

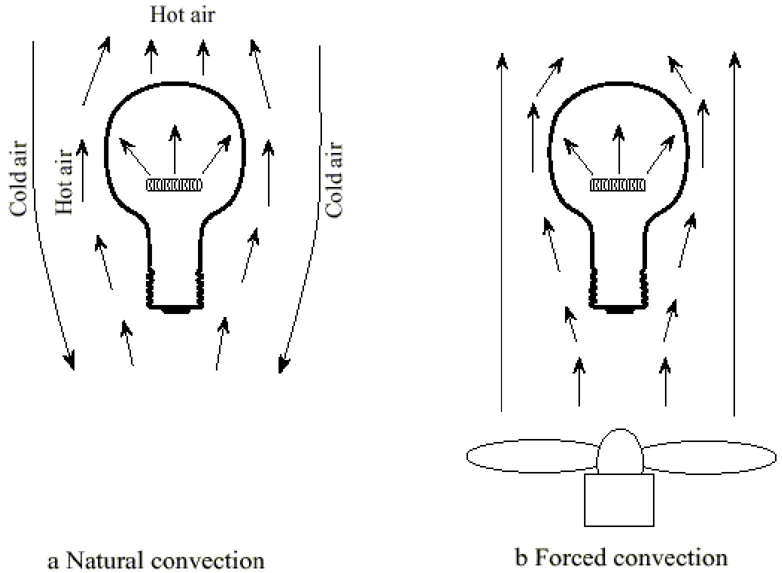
Electrical energy dissipated = Increase in heat content of filament
+ Energy radiated from filament surface

$$\left(\frac{V^2}{R}\right)dt = mcdT + \varepsilon\sigma_S S\left(T^4 - T_o^4\right)dt$$

$$t_f = cdL\pi r^2 \int_{T_o}^{T_f} \frac{dT}{\dfrac{\pi r^2 V^2}{L\rho_o[1+\alpha_o(T-T_o)]} - 2\pi L r\varepsilon(T^4 - T_o^4)}$$

t_f = 0.042 s = 42 ms

Convection



Convection is the transfer of heat from a hot surface by the motion of a fluid in contact with the surface. (a) natural convection. (b) forced convection.

Figure 13

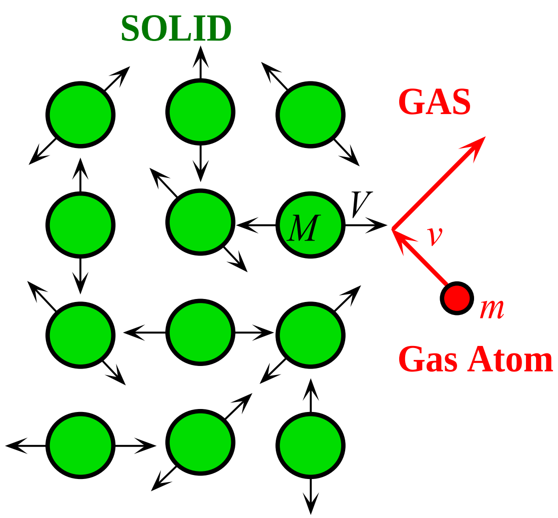
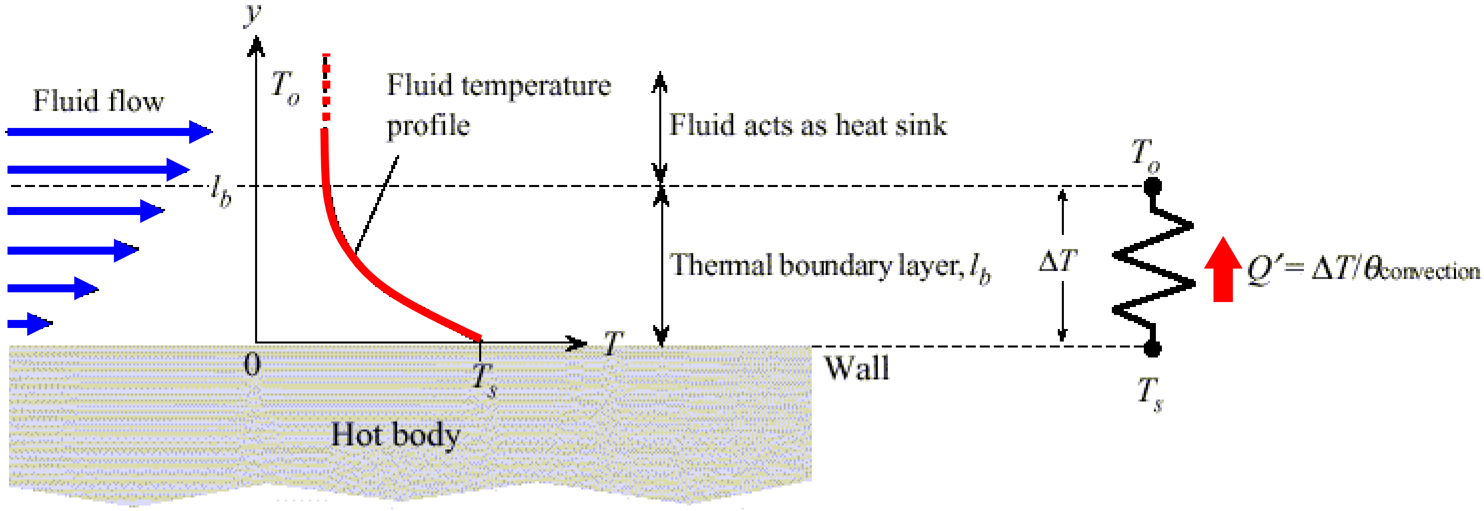


Fig. 1.23: Solid in equilibrium in air. During collisions between the gas and solid atoms, kinetic energy is exchanged.

Convection : Newton's law of cooling



Heat transfer by convection from a hot surface to a cold fluid.

Figure 14

Newton's law of cooling

$$Q' = hS(T_s - T_0)$$

Q' = heat flow,
h = coefficient of convective heat transfer,
S = surface area ,
T_s = temperature of the surface,
T_0 = ambient temperature

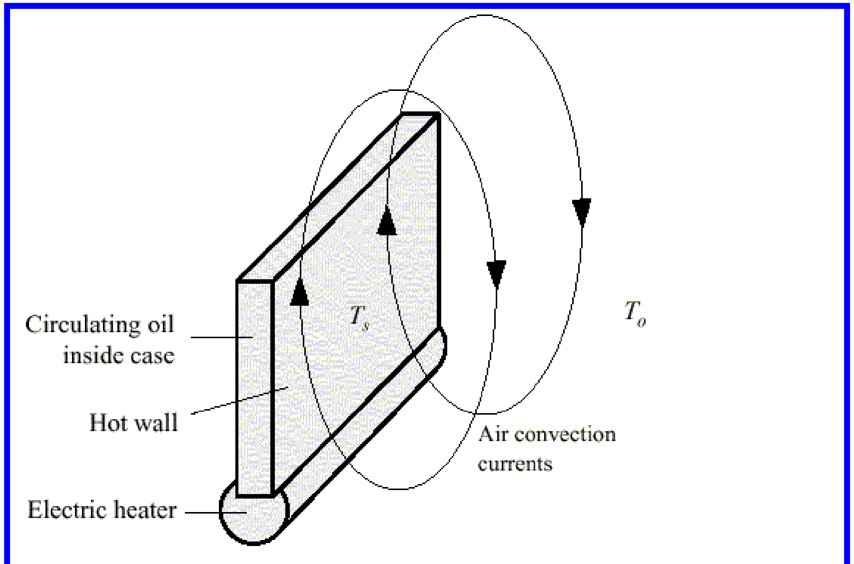
Table 3

Convection coefficient h for various modes of fluid flow. L_{ch} is a parameter called the *characteristics length* and depends on the geometry of the object.

Free convection	h (W m^{-2} °C^{-1})
Atmospheric air	5 - 25
Water	400 - 1000
Forced convection	h (W m^{-2} °C^{-1})
Air	15 - 500
Water	1000 - 15,000
Engine oil	1000 - 2000
Flow system in natural Cooling	h (W m^{-2} °C^{-1})
Vertical thin plate of height $d_{vertical}$.	$h = 1.51(\Delta T/d_{vertical})^{1/4}$
	$1.012(\Delta T/d_{vertical})^{0.35}$; $d_{vertical} < 0.1$
Horizontal thin plate, heated face up.	$h = 1.38(\Delta T/L_{ch})^{1/4}$; $L_{ch} = WL/[2(W+L)]$
	W = width; L = length of plate
Horizontal thin plate, heated face down.	$h = 0.69(\Delta T/L_{ch})^{1/4}$; $L_{ch} = WL/[2(W+L)]$
	W = Width; L = Length of plate
Flow system in forced cooling	h (W m^{-2} °C^{-1})
Forced air flow (laminar flow) along a	$h = 3.9[v_{flow}/d_{flow}]^{0.5}$
surface with length d_{flow} along the flow.	d_{flow} = Length along the flow direction (m)
	v_{flow} = Flow velocity (m/s)

Example 5.1: Convective heat transfer

Consider the electric convection heater shown in Figure 15 that is operating under steady state conditions. The electrical power P generated by the heater inside the cylindrical metal case is circulated by oil to the whole body. This heat is then carried from the surface of the heater to the ambient by air convection. Suppose that the dimensions of a particular 750 W heater is 1 m × 0.75 m. Using an average h of about 6 W m^{-2} °C^{-1} for this particular thermal system, estimate the temperature of the heater wall.



Heat transfer by convection from an electric heater to the ambient.

Figure 15

P = 750 W

S = 1m × 0.75m

$h \cong$ 6 W m^{-2} ^{0}C^{-1}

T = ??

Solution

$$P = Q' = \frac{\Delta T}{\Theta_{convection}} = hS \times \Delta T$$

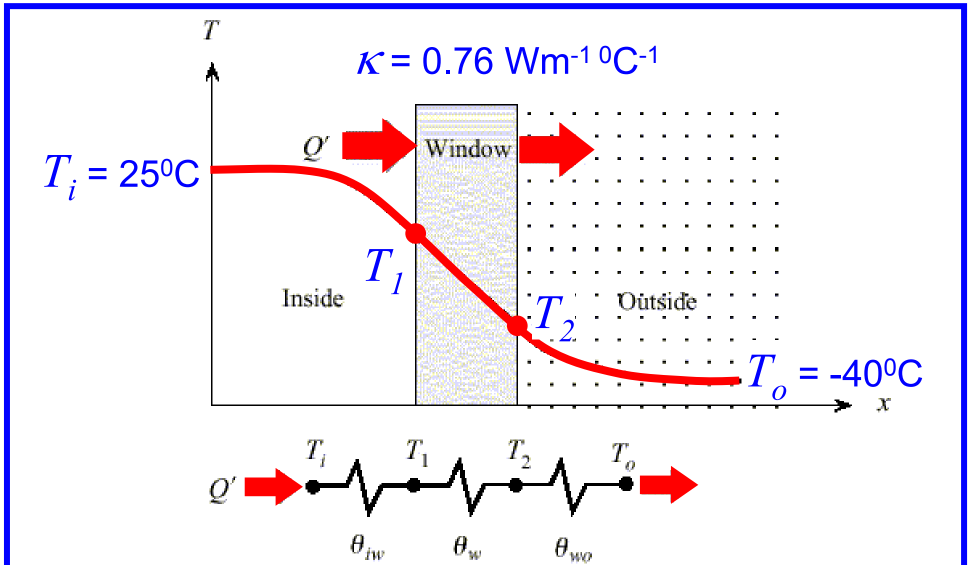
$$\Delta T = \frac{P}{hS} = \frac{750W}{(6Wm^{-2}\,{}^0C^{-1}) \times 2 \times (1m \times 0.75m)} = 83.3^0$$

=>

$$T = 83.3 + 25 = 108.3^0 C$$

Example 5.3: Convective and conductive heat transfer

Consider the escape of heat from a perfectly insulated room to the cold whether outside through a glass window as shown Figure 16. The temperature on the inside face of the window glass is not at room temperature; a fact easily verified by touching the glass surface with our finger (it always feels colder than the room temperature whenever the outside is cold). The heat flows by convection to the glass window, by conduction through the glass and by convection to the outside ambient as shown Figure 16. Consider a 1 m × 0.75 m window with a glass of thickness 10 mm. Window glass has $\kappa = 0.76$ W m^{-1} °C^{-1}. Suppose that the inside and outside ambient temperatures are $T_i = 25$ °C and $T_o = -40$ °C. Taking $h_i \approx 15$ and $h_o \approx 25$ W m^{-2} °C^{-1}, calculate the temperature of the inside and outside surfaces of the window and also the rate of heat escape.



Heat transfer transfer from a heated room to the cold outside through a glass window.

Figure 16

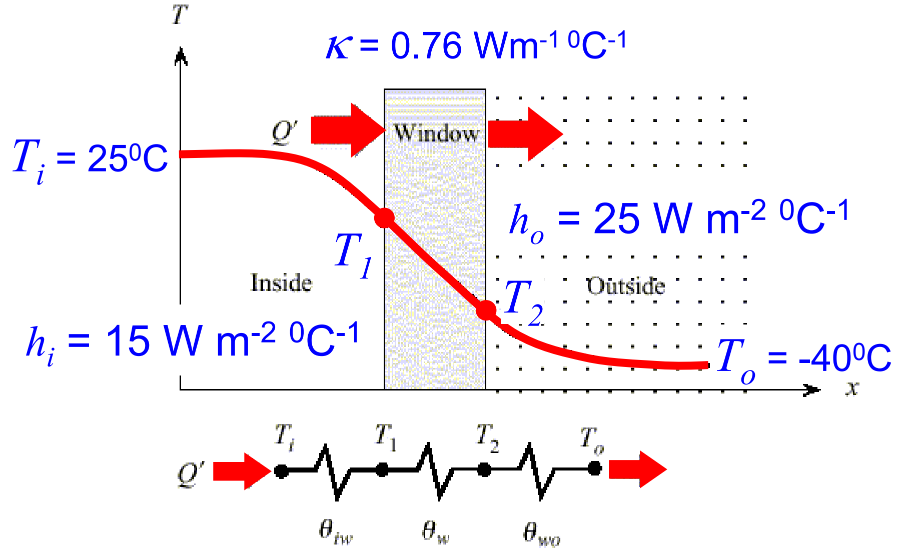
$S = 1\text{m} \times 0.75\text{m}$

$l = 10$ mm

$h_i = 15$ W m^{-2} ^{0}C^{-1}

$h_o = 25$ W m^{-2} ^{0}C^{-1}

$T_1 = ??$ $T_2 = ??$ $Q' = ??$



Heat transfer transfer from a heated room to the cold outside through a glass window.

Figure 16

S = 1m × 0.75m

l = 10 mm

h_i = 15 W m^{-2} ^{0}C^{-1}

h_o = 25 W m^{-2} ^{0}C^{-1}

T_1 = ?? T_2 = ?? Q' = ??

$$\theta_{iw} = \frac{1}{h_i S} = \frac{1}{(15)(1\times 0.75)} = 0.089\ ^\circ\text{C/W}$$

$$\theta_w = \frac{\ell}{\kappa A} = \frac{10\times 10^{-3}}{(0.76)(1\times 0.75)} = 0.018\ ^\circ\text{C/W}$$

$$\theta_{wo} = \frac{1}{h_o S} = \frac{1}{(25)(1\times 0.75)} = 0.053\ ^\circ\text{C/W}$$

$$\theta = \theta_{iw} + \theta_w + \theta_{wo} = 0.089 + 0.018 + 0.053 = 0.16\ ^\circ\text{C/W}\ .$$

⇨

$$Q' = \frac{(T_i - T_o)}{\theta} = \frac{25-(-40)}{0.16} = 406.25\ \text{W}.$$

$$T_2 - T_o = Q'\theta_{wo} = (406.25\ \text{W})(0.053\ ^\circ\text{C/W}) = 21.5\ ^\circ\text{C}$$

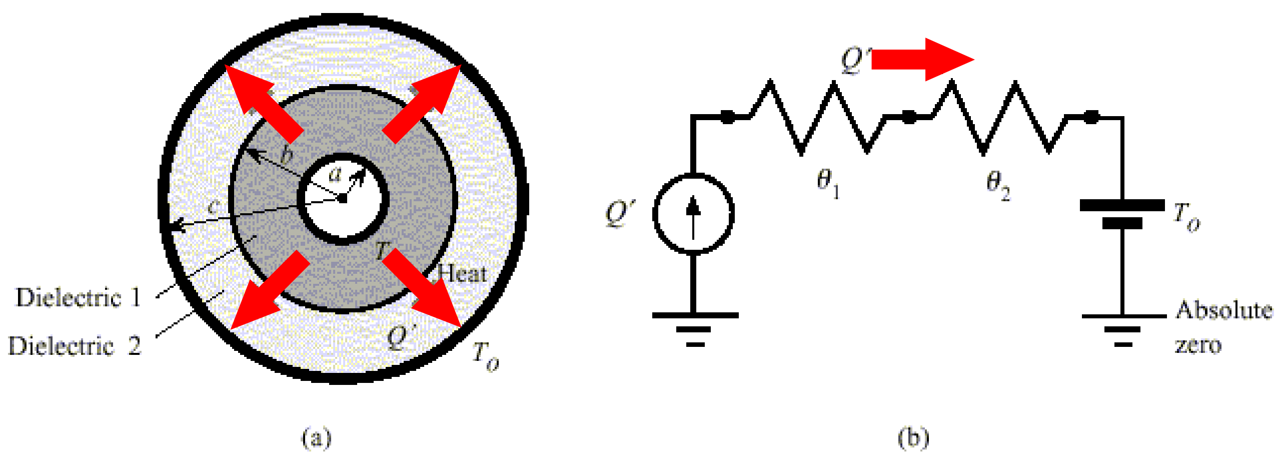
⇨ T_2 = -18.5 ^{0}C

$$T_1 - T_2 = Q'\theta_w = (406.25\ \text{W})(0.018\ ^\circ\text{C/W}) = 7.31\ ^\circ\text{C}$$

$$T_i - T_1 = Q'\theta_{iw} = (406.25\ \text{W})(0.089\ ^\circ\text{C/W}) = 36.16\ ^\circ\text{C}$$

⇨ T_1 = -11.2 ^{0}C

Consider a coaxial cable that has a copper core conductor with a resistivity ρ = 18 nΩ m. The core radius is 5 mm. The dielectric insulation consists of two different concentric layers of polymer insulation as indicated in Figure 17 (a). First polymer layer next to the inner core has a thickness of 1.5 mm and the second polymer layer, between the first layer and the outside conductor, has a thickness of 2 mm. The equivalent thermal circuit under state operation is shown in Figure 17 (b). The thermal conductivities of the first and the second layers are of 0.3 W m^{-1} K^{-1} and 0.25 W m^{-1} K^{-1} respectively. The outside (ambient) temperature T_o is 20 °C. Ambient convection is sufficiently strong (h very large, *i.e.* $h \to \infty$) to maintain T_o at 20 °C. The coaxial cable is carrying a current of 700 A. What is the temperature of the inner conductor? Sketch the temperature vs. distance profile from the inner conductor to the outside conductor. (Note: this is a sketch not a plot.) How would you modify your calculation to include a finite convection from the surface of the cable given a convective heat transfer coefficient h of 25 W m^{-2} K^{-1}?



(a) A coaxial cable with two layers of dielectric insulation. (b) Equivalent thermal circuit.

Figure 17

ρ = 18 nΩ m

I = 700 A

a = 5 mm

$b-a$ = 1.5 mm

$c-b$ = 2 mm

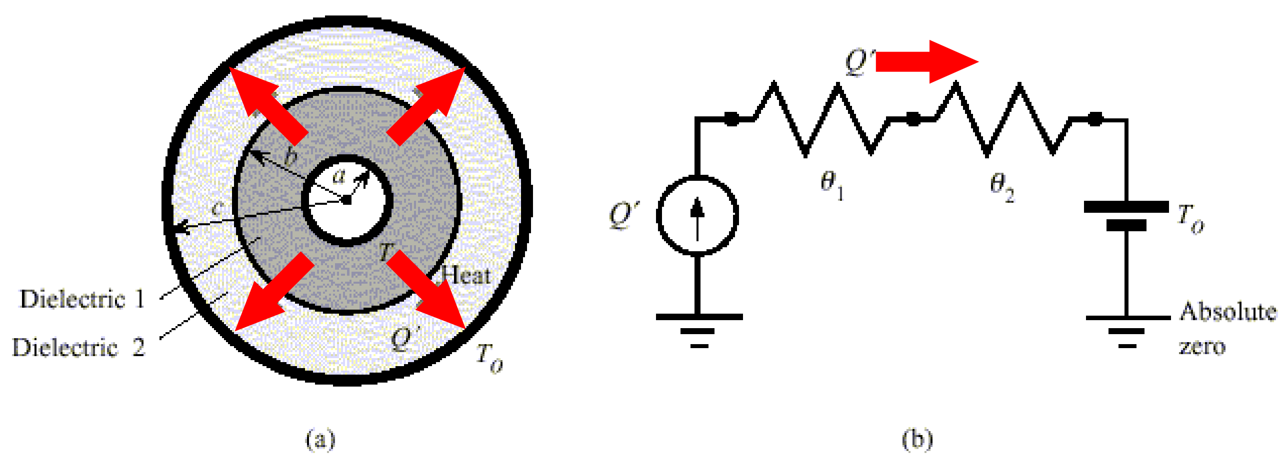
κ_1 = 0.3 W m^{-1} ^{0}C^{-1}

κ_2 = 0.25 W m^{-1} ^{0}C^{-1}

T_0 = 20^{0}C

h = 25 W m^{-2} K^{-1}

T_∞ = ?? T_h = ??



(a) A coaxial cable with two layers of dielectric insulation. (b) Equivalent thermal circuit.

Figure 17

ρ = 18 nΩ m
I = 700 A
a = 5 mm
b-a = 1.5 mm
c-b = 2 mm
κ_1 = 0.3 W m^{-1} ^{0}C^{-1}
κ_2 = 0.25 W m^{-1} ^{0}C^{-1}
T_0 = 20^{0}C
h = 25 W m^{-2} K^{-1}
T_∞ = ?? T_h = ??

ASSUMPTIONS

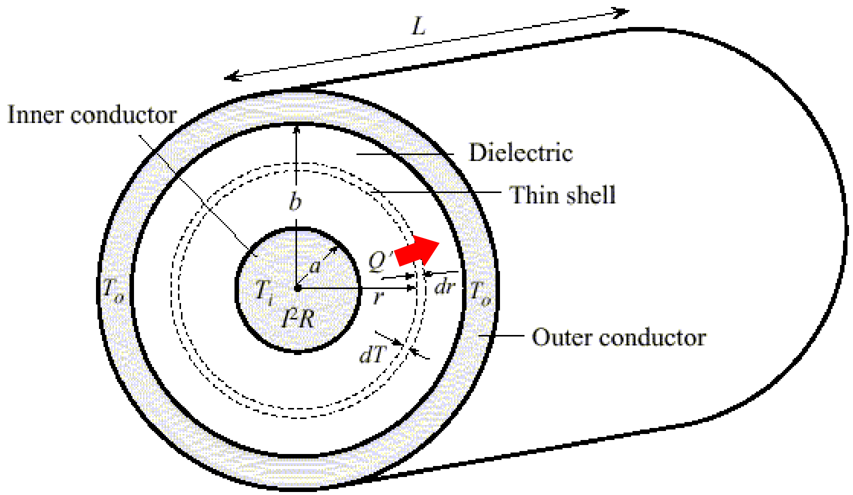
a. The operation has reached steady state so that the temperature anywhere inside the cable does not change with time.

b. The cable is sufficiently long so that there is no heat transfer along the cable, that is, the temperature does no vary along the cable. Thus, we only need to consider a portion of length L of a very long cable. For simplicity we can set L = 1 m.

c. The thermal conductivity of the core conductor is *very much greater* than the thermal conductivity of the region outside the core (the dielectric) so that for all practical purposes, the inner conductor is at a uniform temperature, T_i. (This is justified by the fact that κ_{Cu} = 400 W m^{-1} °C^{-1} for Cu and $\kappa_{dielectric} \approx$ 0.3 W m^{-1} °C^{-1} for the dielectric insulation, which means $\kappa_{Cu}/\kappa_{dielectric} \approx$ 1,300; three orders of magnitude!).

d. Assume that the thermal conductivities do not change with temperature. This assumption is not entirely true as κ for polymers increases with T.

e. We neglect the change in the electrical resistivity of the inner conductor with temperature



Thermal resistance of a hollow cylindrical shell. Consider an infinitesimally thin cylindrical shell of radius r and thickness dr in the dielectric and concentrically around the inner conductor. The surface area is $2\pi rL$.

Figure 6

$$Q' = -\kappa(2\pi rL)\frac{dT}{dr}$$

$$Q'\frac{dr}{r} = -\kappa(2\pi L)dT$$

$$Q'\int_a^b \frac{dr}{r} = -\kappa(2\pi L)\int_{T_i}^{T_0} dT$$

$$Q'\ln\left(\frac{b}{a}\right) = -\kappa 2\pi L(T_i - T_0)$$

$$Q' = \frac{\kappa 2\pi L(T_i - T_0)}{\ln\left(\frac{b}{a}\right)} \Rightarrow \quad \Theta = \frac{T_i - T_0}{Q'} = \frac{\ln\left(\frac{b}{a}\right)}{2\pi\kappa L}$$

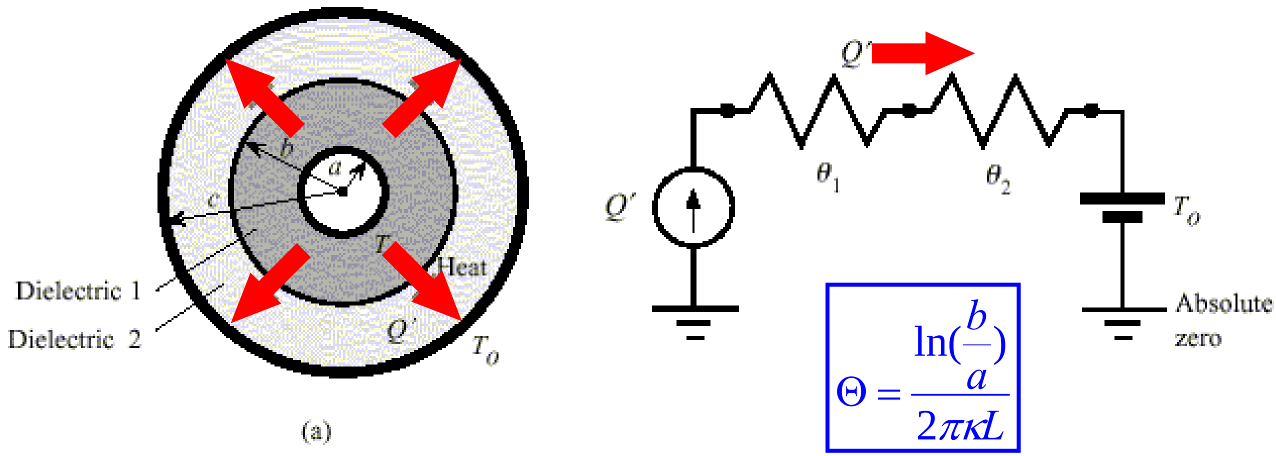
a = 5 mm
b = 3 mm
ρ = 27 nΩ m – aluminum
κ = 0.3 W m^{-1} K^{-1} – polyethylene
I = 500 A
L = 1 m

$$Q' = I^2\frac{\rho L}{\pi a^2} = 85.9W$$

$$\Theta = \frac{\ln\left(\frac{b}{a}\right)}{2\pi\kappa L} = 0.25^0 C/W$$

$$\Delta T = Q'\Theta = 21.5^0 C$$

$$T_i = 41.5^0 C$$



(a) A coaxial cable with two layers of dielectric insulation. (b) Equivalent thermal circuit.

Figure 17

ρ = 18 nΩ m
I = 700 A
a = 5 mm
b-a = 1.5 mm
c-b = 2 mm
κ_1 = 0.3 W m^{-1} ^{0}C^{-1}
κ_2 = 0.25 W m^{-1} ^{0}C^{-1}
T_0 = 20^{0}C
h = ∞
T_∞ = ?? T_h = ??

Thermal Resistance

$$\theta_1 = \frac{\ln\left(\frac{b}{a}\right)}{2\pi\kappa_1 L} = \frac{\ln\left(\frac{(5+1.5)\times10^{-3}}{5\times10^{-3}}\right)}{2\pi(0.3)(1)} = 0.139\ ^\circ\text{C/W}$$

$$\theta_2 = \frac{\ln\left(\frac{c}{b}\right)}{2\pi\kappa_2 L} = \frac{\ln\left(\frac{(5+1.5+2)\times10^{-3}}{(5+1.5)\times10^{-3}}\right)}{2\pi(0.25)(1)} = 0.171\ ^\circ\text{C/W}$$

$$\theta = \theta_1 + \theta_2 = (0.139\ ^\circ\text{C/W}) + (0.171\ ^\circ\text{C/W}) = 0.310\ ^\circ\text{C/W}$$

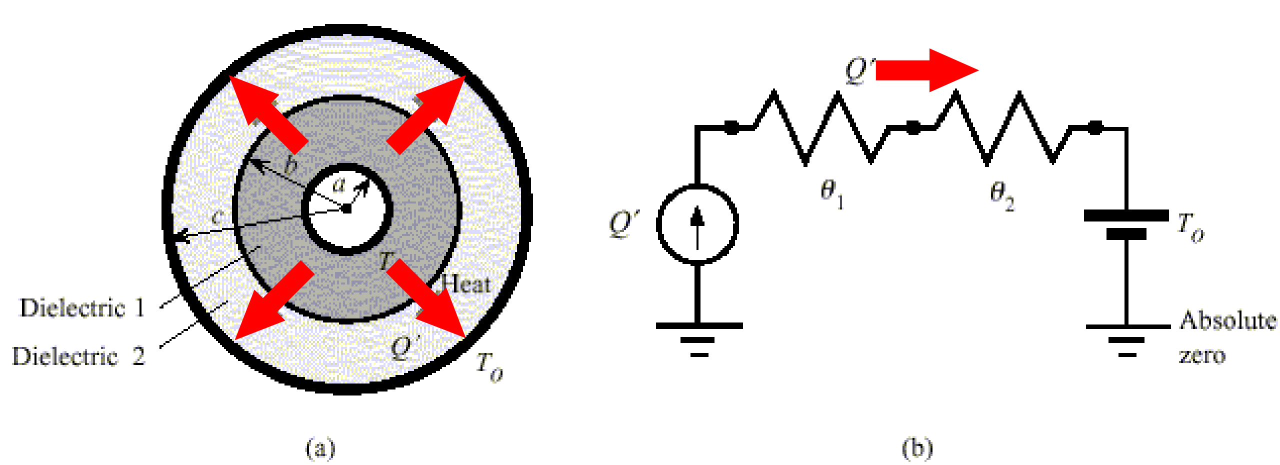
$$Q' = I^2\frac{\rho L}{\pi a^2} = (700^2)\frac{(18\times10^{-9})(1)}{\pi(5\times10^{-3})^2} = 112.5\text{ W}$$

$$\Delta T = Q'\theta = (112.5\text{ W})(0.310\ ^\circ\text{C/W}) = 34.9\ ^\circ\text{C}.$$

$$T_i = T_o + \Delta T = 20 + 34.9 = \mathbf{54.9\ ^\circ C.}$$

$$\frac{Q'_{54.9}}{Q'_{20}} = \frac{I^2 R_{54.9}}{I^2 R_{20}} = \frac{54.9+273\text{ K}}{20+273\text{ K}} = 1.116$$

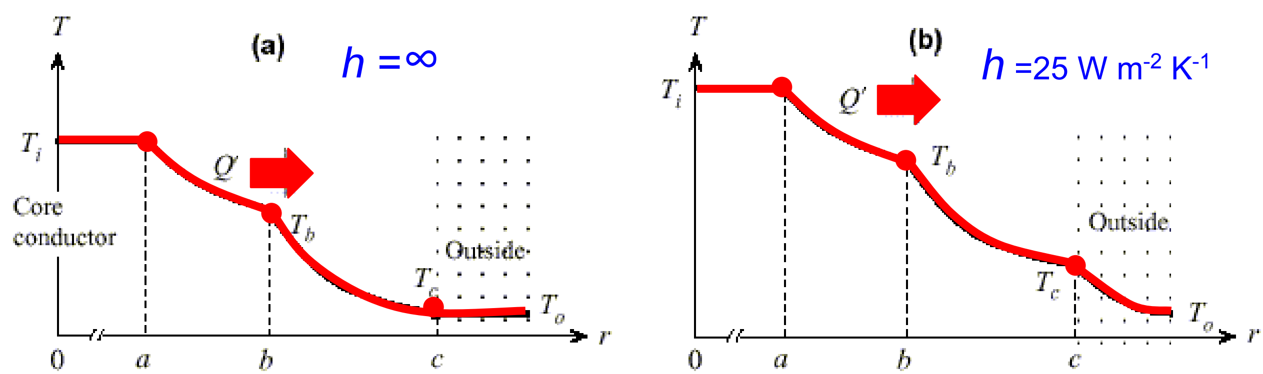
$$\Delta T' = \Delta T\frac{Q'_{54.9}}{Q'_{20}} = (34.9)(1.116) = \mathbf{38.9\ ^\circ C}$$ ⇨ T_i = 58.9 ^{0}C



(a) A coaxial cable with two layers of dielectric insulation. (b) Equivalent thermal circuit.

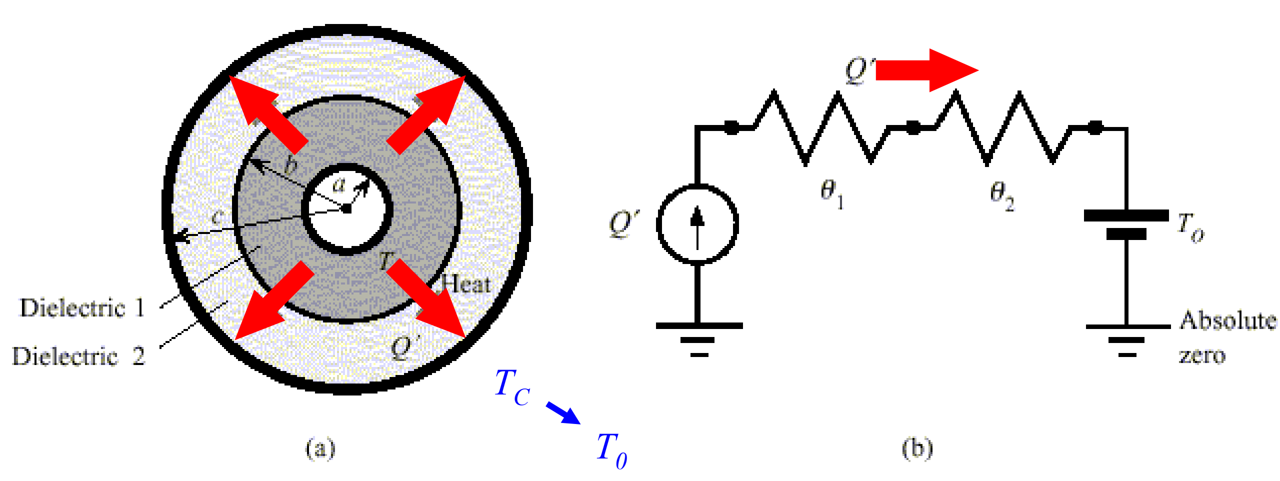
Figure 17

ρ = 18 nΩ m
I = 700 A
a = 5 mm
b-a = 1.5 mm
c-b = 2 mm
κ_1 = 0.3 W m^{-1} ^{0}C^{-1}
κ_2 = 0.25 W m^{-1} ^{0}C^{-1}
T_0 = 20^{0}C
h = 25 W m^{-2} K^{-1}
T_∞ = 58.9^{0}C T_h = ??



Heat transfer and temperature profile in a coaxial cable from the inner conductor to the outer conductor through two concentrentic layers of different insulation. The surface convection maintains the outer conductor temperature at T_o. (a) $h = \infty$. (b) h is finite and there is thermal boundary layer around the outer conductor.

Figure 18



(a) A coaxial cable with two layers of dielectric insulation. (b) Equivalent thermal circuit.

Figure 17

ρ = 18 nΩ m
I = 700 A
a = 5 mm
$b-a$ = 1.5 mm
$c-b$ = 2 mm
κ_1 = 0.3 W m^{-1} ^{0}C^{-1}
κ_2 = 0.25 W m^{-1} ^{0}C^{-1}
T_0 = 20^{0}C
h = 25 W m^{-2} K^{-1}
T_∞ = 58.9^{0}C T_h = ??

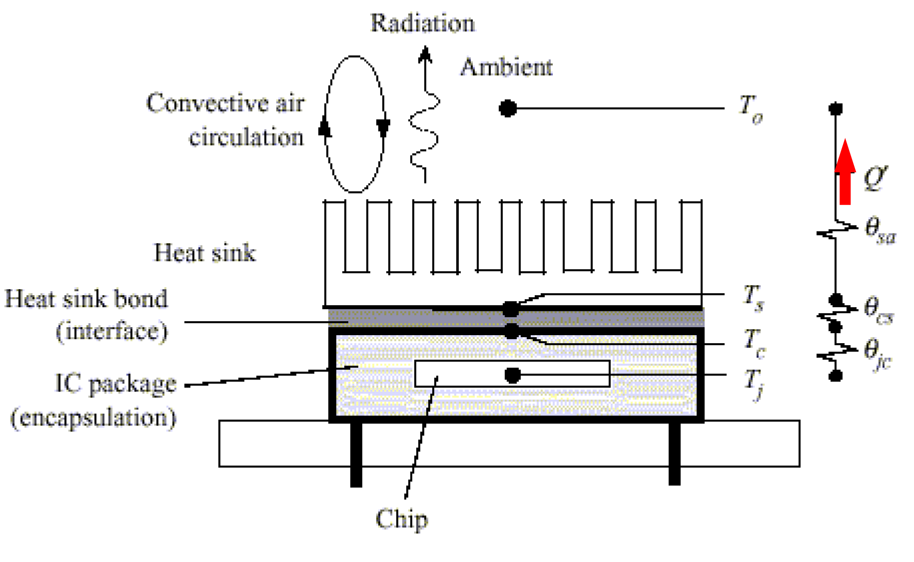
$$\theta_{th} = \frac{1}{hS} = \frac{1}{h(2\pi cL)} = \frac{1}{(25)(2\pi)[(5+3.5)\times10^{-3}](1)} = 0.75\ ^\circ\text{C/W}$$

$$Q' = I^2\frac{\rho L}{\pi a^2} = (700^2)\frac{(18\times10^{-9})(1)}{\pi(5\times10^{-3})^2} = 112.5\ \text{W}$$

$$\theta_1 = \frac{\ln\left(\frac{b}{a}\right)}{2\pi\kappa_1 L} = \frac{\ln\left(\frac{(5+1.5)\times10^{-3}}{5\times10^{-3}}\right)}{2\pi(0.3)(1)} = 0.139\ ^\circ\text{C/W}$$

$$\theta_2 = \frac{\ln\left(\frac{c}{b}\right)}{2\pi\kappa_2 L} = \frac{\ln\left(\frac{(5+1.5+2)\times10^{-3}}{(5+1.5)\times10^{-3}}\right)}{2\pi(0.25)(1)} = 0.171\ ^\circ\text{C/W}$$

$$T_c - T_o = Q'\theta_{th} = (112.5\ \text{W})\ (0.75\ ^\circ\text{C/W}) = 64.4\ ^\circ\text{C.}$$!!!



A heat sinked IC.

Figure 19

$$\theta_{\text{sink}} = \frac{T_s - T_o}{Q'} \qquad Q' = \varepsilon\sigma_S S(T_s^{\,4} - T_o^{\,4}) + hS(T_s - T_o)$$

$$\frac{1}{\theta_{\text{sink}}} = \varepsilon\sigma_S S\frac{T_s^4 - T_o^4}{T_s - T_o} + hS = \frac{1}{\theta_{\text{radiation}}} + \frac{1}{\theta_{\text{convection}}}$$

T_0 = 25 ^{0}C
T_j = 100 ^{0}C
S = 100 cm^2 = 0.01 m^2
ε = 0.75
h = 10 W m^{-2} K^{-1}
Θ_{jc} = 15 ^{0}C/W
Θ_{cs} = 1 ^{0}C/W

P_d = ??

$\Theta_{radiation}$ = 15.4 ^{0}C/W > $\Theta_{convection}$ = 10 ^{0}C/W ⇨ { convective transfer is more important; $\Theta_{sink} \cong$ 6.0 ^{0}C/W }

$$P_d = (T_j - T_o)/(\theta_{jc} + \theta_{cs} + \theta_{sa}) = (100\ °\text{C} - 25\ °\text{C})/\ (15\ °\text{C/W} + 1\ °\text{C/W} + 6.0\ °\text{C/W}) = 5\text{W}$$