

Lesson 8

Numerical Integration (Quadrature)

Most often, the integrand $f(x)$ is quite complicated and it may not be possible to carry out the integration analytically. In such cases, we resort to numerical integrations.

Numerical integration of the integral $\int f(x) dx$ is called **quadrature**.

MATLAB provides the following built-in functions for numerical integration:

quad:
It integrates the specified function over specified limits, based on adaptive Simpson's rule.

quadl:
It integrates a specified function over specified limits, based on adaptive Lobatto quadrature. This one is more accurate than quad but it also uses more function evaluations.

The general call syntax for both `quad` and `quadl` is as follows:

```
integral = quad('your_function', a, b)
```

To use `quadl`, you just replace `quad` with `quadl` in the syntax. `a` and `b` are the limits of the integral.

The steps involved in numerical integration using these built-in functions are:

Step 1

Write a function that returns the value of the integrand $f(x)$ given the value of x .

Step 2

Decide which function to use – `quad` or `quadl`. Find the integral.

Example 1

Let us compute the integral

$$\int_{1/2}^{3/2} e^{-x^2} dx.$$

Code:

```
>> F = 'exp(-x.^2)';
>> y = quad(F,1/2,3/2)

y =

    0.3949
```

Double Integration

To evaluate integrals of the form

$$\int_{y_{\min}}^{y_{\max}} \int_{x_{\min}}^{x_{\max}} f(x, y) dx dy$$

MATLAB provides a function `dblquad`. The calling syntax for `dblquad` is:

```
I = dblquad('function_xy', xmin, xmax, ymin, ymax)
```

Example 2

Let us compute the following integral

$$I = \int_{-1}^1 \int_0^2 (1 - 6x^2y) dx dy$$

Code:

```
>> F = '1-6*x.^2*y';
>> I = dblquad(F,0,2,-1,1)

I =

    4
```

Exercise 6 (32 marks)

1. Calculate the following integrals; by using the functions quad or quadl:

(a) $\int_0^2 xe^{x^2} dx$ [4 marks]

(b) $\int_{-\pi/4}^{\pi/4} x \tan x dx$ [4 marks]

(c) $\int_1^3 \frac{\ln x}{x} dx$ [4 marks]

(d) $\int_0^2 \frac{1}{x^3 - 2x - 5} dx$ [4 marks]

2. Calculate the following double integral for the functions:

(a) $\int_{-2}^2 \int_0^2 (x - 3y^2) dx dy$ [4 marks]

(b) $\int_{-3}^3 \int_{-3}^3 xe^{-(x-y^2)^2 + y^2} dx dy$ [4 marks]

(c) $\int_{-1}^1 \int_{-2}^2 \frac{-5}{1+x^2+y^2} dy dx$ [4 marks]

(d) $\int_0^2 \int_{-3}^2 (x+y) dy dx$ [4 marks]

Answer

Question 1(a)

```
>> F = 'x.*exp(x.^2)';  
>> quad(F,0,2)
```

ans =

26.7991

Question 1(b)

```
>> F = 'x.*tan(x)';  
>> quad(F,-pi/4,pi/4)
```

ans =

0.3716

Question 1(c)

```
>> F = 'log(x)./x';  
>> quad(F,1,3)
```

ans =

0.6035

Question 1(d)

```
>> F = '1./(x.^3 - 2*x - 5)';  
>> quad(F,0,2)
```

ans =

-0.4605

Question 2(a)

```
>> F = 'x-3*y.^2';  
>> dblquad(F,0,2,-2,2)
```

ans =

-24

Question 2(b)

```
>> F = 'x.*exp((x-y.^2).^2+y.^2)';  
>> dblquad(F,-3,3,-3,3)
```

Question 2(c)

```
>> F = '-5./(1 + x.^2 + y.^2)';  
>> dblquad(F,-1,1,-2,2)
```

ans =

-18.6143