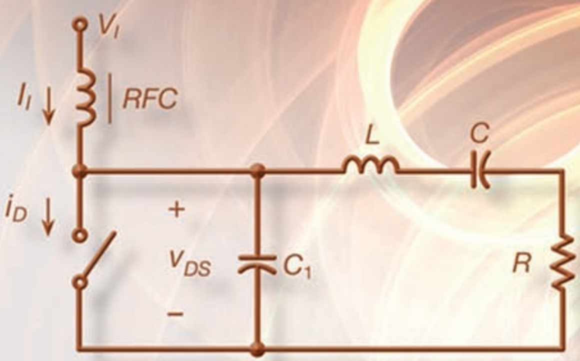
RF Power Amplifiers

MARIAN K. KAZIMIERCZUK



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To My Mother

Contents

Preface

This book is about RF power amplifiers used in wireless communications and many other RF applications. It is intended as a concept-oriented textbook at the senior and graduate levels for students majoring in electrical engineering, as well as a reference for practicing engineers in the area of RF power electronics. The purpose of the book is to provide foundations for RF power amplifiers, efficiency improvement, and linearization techniques. Class A, B, C, D, E, DE, and F RF power amplifiers are analyzed and design procedures are given. Impedance transformation is covered. Various linearization techniques are explored, such as predistortion, feedforward, and negative feedback techniques. Efficiency improvement methods are also studied, such as envelope elimination and restoration, envelope tracking, Doherty amplifier, and outphasing techniques. Integrated inductors are also studied.

The textbook assumes that the student is familiar with general circuit analysis techniques, semiconductor devices, linear systems, and electronic circuits. A communications course is also very helpful.

I wish to express my sincere thanks to Simone Taylor, Publisher, Engineering Technology, Jo Bucknall, Assistant Editor, and Erica Peters, Content Editor. It has been a real pleasure working with them. Last but not least, I wish to thank my family for the support.

I am pleased to express my gratitude to Nisha Das for the MATLAB figures. The author would welcome and greatly appreciate suggestions and corrections from readers, for improvements in the technical content as well as the presentation style.

Marian K. Kazimierczuk

About the Author

Marian K. Kazimierczuk is Professor of Electrical Engineering at Wright State University, Dayton, Ohio, USA. He is the author of five books, over 130 journal papers, over 150 conference papers, and seven patents. He is a Fellow of the IEEE and he received an Outstanding Teacher Award from the American Society for Engineering Education in 2008. His research interests are in the areas of RF power amplifiers, radio transmitters, power electronics, PWM dc-dc power converters, resonant dc-dc power converters, modeling and controls, semiconductor power devices, magnetic devices, and renewable energy sources.

List of Symbols

a	Coil mean radius
A_e	Effective area of antenna
A_v	voltage gain
B	Magnetic flux density
BW	Bandwidth
c_p	Output-power capability of amplifier
C	Resonant capacitance
C_c	Coupling capacitance
C_{ds}	Drain-source capacitance of MOSFET
$C_{ds(25V)}$	Drain-source capacitance of MOSFET at $V_{DS} = 25\,\mathrm{V}$
C_f	Filter capacitance
C_{fmin}	Minimum value of C_f
C_{gd}	Gate-drain capacitance of MOSFET
C_{gs}	Gate-source capacitance of MOSFET
C_{iss}	MOSFET input capacitance at $V_{DS} = 0$, $C_{iss} = C_{gs} + C_{gd}$
C_{oss}	MOSFET output capacitance at $V_{GD} = 0$, $C_{oss} = C_{gs} + C_{ds}$
C_{ox}	Oxide capacitance per unit area
C_{out}	Transistor output capacitance
C_{rss}	MOSFET transfer capacitance, $C_{rss} = C_{gd}$
C_B	Blocking capacitance
d	Coil inner diameter
D	Coil outer diameter
E	Electric field intensity
f	Operating frequency, switching frequency
f_c	Carrier frequency
f_{IF}	Intermediate frequency
f_{LO}	Local oscillator frequency
f_m	Modulating frequency
f_o	Resonant frequency
f_p	Frequency of pole of transfer function
f_r	Resonant frequency of L-C-R circuit
f_s	Switching frequency
g_m	Transconductance of transistor

H	Magnetic flux intensity
h	Trace thickness
i	Current
iC	Capacitor current
I_D	DC drain current
i_D	Large-signal drain current
i_d	Small-signal drain current
i_L	Inductor current
i_o	Ac output current
i_S	Switch current
I_m	Amplitude of current i
I_{rms}	Rms value of i
I_{DM}	Peak drain current
I_{SM}	Peak current of switch
J_n	Bessel's function of first kind and n-th order
K	MOSFET parameter
k_p	Power gain
l	Trace length, Winding length
L	Resonant inductance, channel length
L_f	Filter inductance
L_{fmin}	Minimum value of L_f
m	Modulation index
m_f	Index of frequency modulation
m_p	Index of phase modulation
N	Number of inductor turns
n	Transformer turns ratio
p	Perimeter enclosed by coil
P_G	Gate drive power
p_r	Power loss in resonant circuit
P_{tf}	Average power loss due to current fall time t_f
P_{AM}	Power of AM signal
P_C	Power of carrier
P_{LS}	Power of lower sideband
P_{Loss}	Power loss
P_D	Power dissipation
P_I	Dc supply (input) power
P_O	Ac output power
P_{US}	Power of upper sideband
$p_D(\omega t)$	Instantaneous drain power loss
Q_o	Unloaded quality factor at f_o
Q_{oL}	Quality factor of inductor
Q_L	Loaded quality factor at f_o
r	Total parasitic resistance
r_C	ESR of filter capacitor
r_{DS}	On-resistance of MOSFET
R	Overall resistance of amplifier
R_{DC}	DC input resistance of Amplifier
r_G	Gate resistance
R_L	Dc load resistance
R_{Lmin}	Minimum value of R_L

r_o	Ouput resistance of transistor
s	Trace-to-trace spacing
S_i	Current slope
S_v	Voltage slope
T	Operating temperature
THD	Total harmonic distortion
V_A	Channel-modulation voltage
v	Voltage
V_{DS}	DC drain-source voltage
V_{dsm}	Amplitude of small-signal drain-source voltage
v_{DS}	Large-signal drain-source voltage
v_{ds}	Small-signal drain-source voltage
V_{GS}	DC gate-source voltage
v_{GS}	Large-signal gare-source voltage
V_{gm}	Amplitude of small-signal gate-source voltage
v_{gs}	Small-simal gate-source voltage
v_m	Modulating voltage
v_c	Carrier voltage
v_m	Modulating voltage
v_o	Ac output voltage
V_c	Amplitude of carrier voltage
V_m	Amplitude of modulating voltage
V_m	Amplitude of voltage
V_n	n-th harmonic of the output voltage
V_{Cm}	Amplitude of the voltage across capacitance
V_{DS}	DC drain-source voltage
v_{DS}	Large-signal drain-source voltage
v_{ds}	Small-signal drain-source voltage
V_I	DC supply (input) voltage
V_{Lm}	Amplitude of the voltage across inductance
V_{DSM}	Peak voltage of switch
V_{rms}	Rms value of v
V_t	Threshold voltage of MoSFET
w	Trace width
W	Energy, channel width
X	Imaginary part of Z
Z	Input impedance of resonant circuit
Z_i	Input impedance
$\|Z\|$	Magnitude of Z
Z_o	Characteristic impedance of resonant circuit
α_n	Fourier coefficients of drain current
β	Gain of feedback network
δ	Dirac impulse function
Δf	Frequency deviation
ϵ_{ox}	Oxide permittivity
η	Efficiency of amplifier
η_{AV}	Average efficiency
η_D	Drain efficiency
η_r	Efficiency of resonant circuit
η_{PAE}	Power-aided efficiency

λ	Wavelength, channel length-modulation parameter
μ_0	Permeability of fee space
μ_n	Mobility of electrons
μ_r	Relative permeability
ρ	Resistivity
σ	Conductivity
ξ_n	Fourier coefficients of drain-source voltage
γ_n	Ratio Fourier coefficients of drain current
Φ	Phase, angle, magnetic flux
θ	Half of drain current conduction angle, mobility degration coefficient
ω	Operating angular frequency
ω_c	Carrier angular frequency
ω_m	Modulating angular frequency
ω_o	Resonant angular frequency

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Introduction

1.1 Block Diagram of RF Power Amplifiers

A power amplifier [1–15] is a key element to build a wireless communication system successfully. To minimize interference and spectral re-growth, transmitters must be linear. A block diagram of an RF power amplifier is shown in Figure 1.1. It consists of a transistor (MOSFET, MESFET, or BJT), output network, input network, and RF choke. In RF power amplifiers, a transistor can be operated

- as a dependent-current source;
- as a switch;
- in overdriven mode (partially as a dependent source and partially as a switch).

Figure 1.2(a) shows a model of an RF power amplifier in which the transistor is operated as a voltage- or current-dependent current source. When a MOSFET is operated as a dependent current source, the drain current waveform is determined by the gate-to-source voltage waveform and the transistor operating point. The drain voltage waveform is determined by the dependent current source and the load network impedance. When a MOSFET is operated as a switch, the switch voltage is nearly zero when the switch is ON and the drain current is determined by the external circuit due to the switching action of the transistor. When the switch is OFF, the switch current is zero and the switch voltage is determined by the external circuit response.

In order to operate the MOSFET as a dependent-current source, the transistor cannot enter the ohmic region. It must be operated in the active region, also called the pinch-off region or the saturation region. Therefore, the drain-to-source voltage v_{DS} must be kept higher than the minimum value V_{DSmin}, i.e., $v_{DS} > V_{DSmin} = V_{GS} - V_t$, where V_t is the transistor threshold voltage. When the transistor is operated as a dependent-current source, the magnitudes of the drain current i_D and the drain-to-source voltage v_{DS} are nearly proportional to the magnitude of the gate-to-source voltage v_{GS}. Therefore, this type

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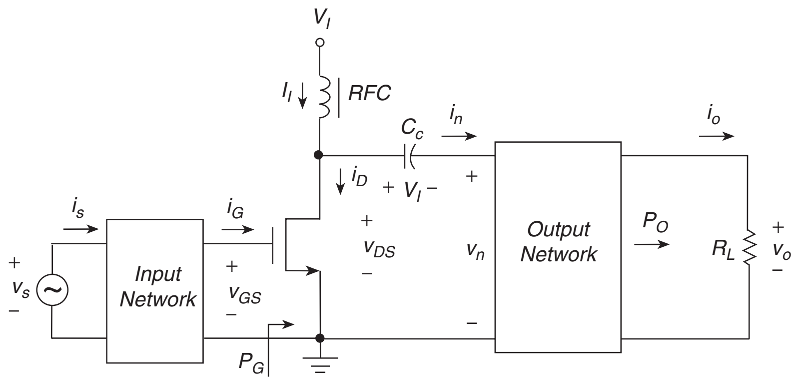

Figure 1.1 Block diagram of RF power amplifier.

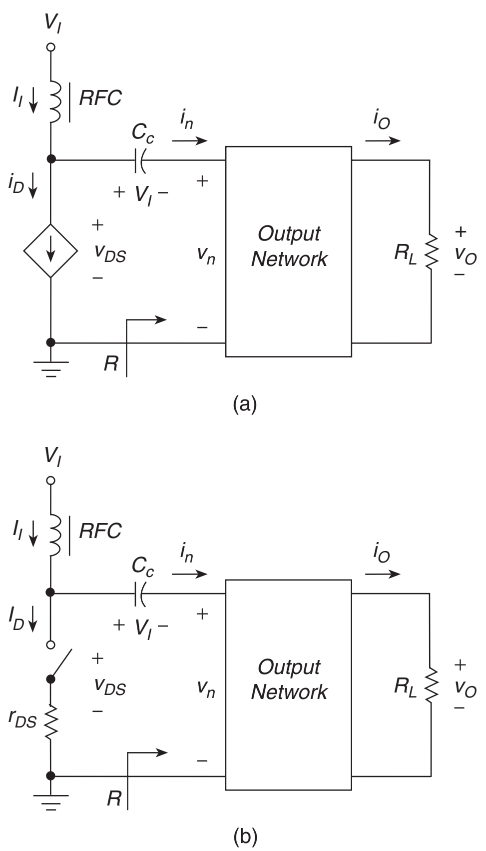

Figure 1.2 Modes of operation of transistor in RF power amplifiers. (a) Transistor as a dependent current source. (b) Transistor as a switch.

of operation is suitable for linear power amplifiers. Amplitude linearity is important for amplitude-modulated (AM) signals.

Figure 1.2(b) shows a model of an RF power amplifier in which the transistor is operated as a switch. To operate the MOSFET as a switch, the transistor cannot enter the active region, also called the pinch-off region. It must remain in the ohmic region when it is ON and in the cutoff region when it is OFF. To maintain the MOSFET in the ohmic region, it is required that $v_{DS} < V_{GS} - V_t$. If the gate-to-source voltage v_{GS} is increased at a given load impedance, the amplitude of the drain-to-source voltage v_{DS} will increase, causing the transistor to operate initially in the active region and then in the ohmic region. When the transistor is operated as a switch, the magnitudes of the drain current i_D and the drain-to-source voltage v_{DS} are independent of the magnitude of the gate-to-source voltage v_{GS}. In most applications, the transistor operated as a switch is driven by a rectangular gate-to-source voltage v_{GS}. A sinusoidal gate-to-source voltage V_{GS} is used to drive a transistor as a switch at very high frequencies, where it is difficult to generate rectangular voltages. The reason to use the transistors as switches is to achieve high amplifier efficiency. When the transistor conducts a high drain current i_D, the drain-to-source voltage v_{DS} is low v_{DS}, resulting in low power loss.

If the transistor is driven by sinusoidal voltage v_{GS} of high amplitude, the transistor is overdriven. In this case, it operates in the active region when the instantaneous values of v_{GS} are low and as a switch when the instantaneous values of v_{GS} are high.

The main functions of the output network are:

- impedance transformation;
- harmonic suppression;
- filtering of the spectrum of a signal with bandwidth BW to avoid interference with communication signals in adjacent channels.

1.2 Classes of Operation of RF Power Amplifiers

The classification of RF power amplifiers with a transistor operated as a dependent-current source is based on the conduction angle 2θ of the drain current. Waveforms of the drain current i_D of a transistor operated as a dependent-source in various classes of operation for sinusoidal gate-to-source voltage v_{GS} are shown in Figure 1.3. The operating points for various classes of operation are shown in Figure 1.4.

In Class A, the conduction angle 2θ is 360°. The gate-to-source voltage v_{GS} must be higher than the transistor threshold voltage V_t, i.e., $v_{GS} > V_t$. This is accomplished by choosing the dc component of the gate-to-source voltage V_{GS} sufficiently greater than the threshold voltage of the transistor V_t such that $V_{GS} - V_{gsm} > V_t$, where V_{gsm} is the amplitude of the ac component of v_{GS}. The dc drain current I_D must be greater than the amplitude of the ac component I_m of the drain current i_D. As a result, the transistor conducts during the entire cycle.

In Class B, the conduction angle 2θ is 180°. The dc component V_{GS} of the gate-to-source voltage v_{GS} is equal to V_t and the drain bias current I_D is zero. Therefore, the transistor conducts for only half of the cycle.

In Class AB, the conduction angle 2θ is between 180° and 360°. The dc component of the gate-to-source voltage V_{GS} is slightly above V_t and the transistor is biased at a small drain current I_D. As the name suggests, Class AB is the intermediate class between Class A and Class B.

In Class C, the conduction angle 2θ of the drain current is less than 180°. The operating point is located in the cutoff region because $V_{GS} < V_t$. The drain bias current I_D is zero. The transistor conducts for an interval less than half of the cycle.

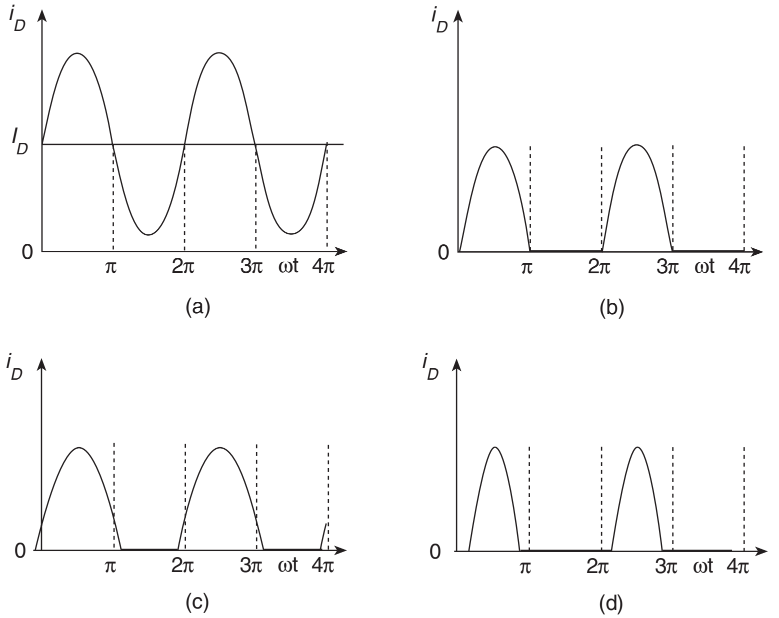

Figure 1.3 Waveforms of the drain current i_D in various classes of operation. (a) Class A. (b) Class B. (c) Class AB. (d) Class C.

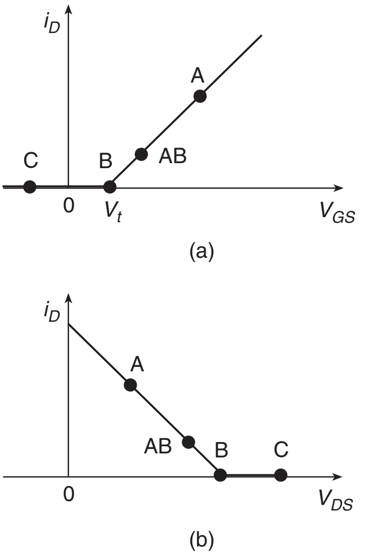

Figure 1.4 Operating points for Classes A, B, AB, and C.

Class A, AB, and B operations are used in audio and RF power amplifiers, whereas Class C is used only in RF power amplifiers.

The transistor is operated as a switch in Class D, E, and DE RF power amplifiers. In Class F, the transistor can be operated as either a dependent current source or a switch.

1.3 Parameters of RF Power Amplifiers

The dc supply power of an amplifier is

$$P_I = I_I V_I. \tag{1.1}$$

When the resonant frequency of the output network f_o is equal to the operating frequency f, the power delivered by the drain to the output network (the drain power) is given by

$$P_{DS} = \frac{1}{2} I_m V_m = \frac{1}{2} I_m^2 R = \frac{V_m^2}{2R} \tag{1.2}$$

where I_m is the amplitude of the fundamental component of the drain current i_D, V_m is the amplitude of the fundamental component of the drain-to-source voltage v_{DS}, and R is the input resistance of the output network at the fundamental frequency. If the resonant frequency f_o is not equal to the operating frequency f, the drain power of the fundamental component is given by

$$P_{DS} = \frac{1}{2} I_m V_m \cos\phi = \frac{1}{2} I_m^2 R \cos\phi = \frac{V_m^2 \cos\phi}{2R} \tag{1.3}$$

where ϕ is the phase shift between the fundamental components of the drain current and the drain-to-source voltage reduced by π.

The power level is often referenced to 1 mW and is expressed as

$$P = 10 \log \frac{[P(\mathrm{W})]}{0.001} \ (\mathrm{dBm}) = -30 + 10 \log[P(\mathrm{W})](\mathrm{dBW}). \tag{1.4}$$

A dBm value or dBW value represents an actual power, whereas a dB value represents a ratio of power, such as the power gain.

The instantaneous drain power dissipation is

$$p_D(\omega t) = i_D v_{DS}. \tag{1.5}$$

The drain power dissipation is

$$P_D = \frac{1}{2\pi} \int_0^{2\pi} p_D \, \mathrm{d}(\omega t) = \frac{1}{2\pi} \int_0^{2\pi} i_D v_{DS} \, \mathrm{d}(\omega t) = P_I - P_{DS}. \tag{1.6}$$

The drain efficiency is

$$\eta_D = \frac{P_{DS}}{P_I} = \frac{P_I - P_D}{P_I} = 1 - \frac{P_D}{P_I}. \tag{1.7}$$

The gate-drive power is

$$P_G = \frac{1}{2\pi} \int_0^{2\pi} i_G v_{GS} \, \mathrm{d}(\omega t). \tag{1.8}$$

For sinusoidal gate current and voltage,

$$P_G = \frac{1}{2} I_{gm} V_{gsm} \cos\phi_G = \frac{R_G I_{gm}^2}{2} \tag{1.9}$$

where I_{gm} is the amplitude of the gate current, V_{gsm} is the amplitude of the gate-to-source voltage, R_G is the gate resistance, and ϕ_G is the phase shift between the fundamental components of the gate current and the gate-to-source voltage.

The output power is

$$P_O = \frac{1}{2} I_{om} V_{om} = \frac{1}{2} I_{om}^2 R_L = \frac{V_{om}^2}{2R_L}. \tag{1.10}$$

The power loss in the resonant output network is

$$P_r = P_{DS} - P_O. \tag{1.11}$$

The efficiency of the resonant output network is

$$\eta_r = \frac{P_O}{P_{DS}}. \tag{1.12}$$

The overall power loss on the output side of the amplifier is

$$P_{Loss} = P_I - P_O = P_D + P_r. \tag{1.13}$$

The overall efficiency of the amplifier is

$$\eta = \frac{P_O}{P_I} = \frac{P_O}{P_{DS}} \frac{P_{DS}}{P_I} = \eta_D \eta_r. \tag{1.14}$$

The average efficiency is

$$\eta_{AV} = \frac{P_{O(AV)}}{P_{I(AV)}} \tag{1.15}$$

where $P_{O(AV)}$ is the average output power and $P_{I(AV)}$ is the average dc supply power over a specified period of time. The efficiency of power amplifiers in which transistors are operated as dependent-current sources increases with the amplitude of the output voltage V_m. It reaches the maximum value at the maximum amplitude of the output voltage, which corresponds to the maximum output power. In practice, power amplifiers are usually operated below the maximum output power. For example, the drain efficiency of the Class B power amplifier is $\eta_D = \pi/4 = 78.5\,\%$ at $V_m = V_I$, but it decreases to $\eta_D = \pi/8 = 39.27\,\%$ at $V_m = V_I/2$.

The power-added efficiency is the ratio of the difference between the output power and the gate-drive power to the supply power:

$$\eta_{PAE} = \frac{\text{Output power} - \text{Drive power}}{\text{DC supply power}} = \frac{P_O - P_G}{P_I} = \frac{P_O}{P_I}\left(1 - \frac{1}{k_p}\right) = \eta\left(1 - \frac{1}{k_p}\right). \tag{1.16}$$

If $k_p = P_O/P_G = 1$, $\eta_{PAE} = 0$. If $k_p \gg 1$, $\eta_{PAE} \approx \eta$.

The output power capability is given by

$$c_p = \frac{P_{O(max)}}{NI_{DM}V_{DSM}} = \frac{\eta P_I}{NI_{DM}V_{DSM}} = \frac{1}{2N}\left(\frac{I_I}{I_{DM}}\right)\left(\frac{V_I}{V_{DSM}}\right)$$
$$= \frac{1}{2N}\left(\frac{I_m}{I_I}\right)\left(\frac{I_I}{I_{DM}}\right)\left(\frac{V_m}{V_I}\right)\left(\frac{V_I}{V_{DSM}}\right) \tag{1.17}$$

where I_{DM} is the maximum value of the instantaneous drain current i_D, V_{DSM} is the maximum value of the instantaneous drain-to-source voltage v_{DS}, and N is the number of transistors in an amplifier, which are not connected in parallel or in series. For example, a push-pull amplifier has two transistors. The maximum output power of an amplifier with a transistor having the maximum ratings I_{DM} and V_{DSM} is

$$P_{O(max)} = c_p I_{DM} V_{DSM}. \tag{1.18}$$

As the output power capability c_p increases, the maximum output power $P_{O(max)}$ also increases. The output power capability is useful for comparing different types or families of amplifiers.

Typically, the output thermal noise of power amplifiers should be below -130 dBm. This requirement is to introduce negligible noise to the input of the low-noise amplifier (LNA) of the receiver.

Example 1.1

An RF power amplifier has $P_O = 10\,\mathrm{W}$, $P_I = 20\,\mathrm{W}$, $P_G = 1\,\mathrm{W}$. Find the efficiency, power-added efficiency, and power gain.

Solution. The efficiency of the power amplifier is

$$\eta = \frac{P_O}{P_I} = \frac{10}{20} = 50\,\%. \tag{1.19}$$

The power-added efficiency is

$$\eta = \frac{P_O - P_G}{P_I} = \frac{10-1}{20} = 45\,\%. \tag{1.20}$$

The power gain is

$$k_p = \frac{P_O}{P_G} = \frac{10}{1} = 10 = 10\,\log(10) = 10\,\mathrm{dB}. \tag{1.21}$$

1.4 Conditions for 100 % Efficiency of Power Amplifiers

The drain efficiency is given by

$$\eta_D = \frac{P_{DS}}{P_I} = 1 - \frac{P_D}{P_I}. \tag{1.22}$$

The condition for achieving a drain efficiency of 100 % is

$$P_D = \frac{1}{T}\int_0^T i_D v_{DS}\,\mathrm{d}t = 0. \tag{1.23}$$

For an NMOS, $i_D \geq 0$ and $v_{DS} \geq 0$ and for a PMOS, $i_D \leq 0$ and $v_{DS} \leq 0$. In this case, the condition for achieving a drain efficiency of 100 % becomes

$$i_D v_{DS} = 0. \tag{1.24}$$

Thus, the waveforms i_D and v_{DS} should be nonoverlapping for an efficiency of 100 %. Nonoverlapping waveforms i_D and v_{DS} are shown in Figure 1.5.

The drain efficiency of power amplifiers is less than 100 % for the following cases:

- The waveforms of $i_D > 0$ and $v_{DS} > 0$ are overlapping (e.g., like in a Class C amplifier).
- The waveforms of i_D and v_{DS} are adjacent, and the waveform v_{DS} has a jump at $t = t_o$ and the waveform i_D contains an impulse Dirac function, as shown in Figure 1.6(a).
- The waveforms of i_D and v_{DS} are adjacent, and the waveform i_D has a jump at $t = t_o$ and the waveform v_{DS} contains an impulse Dirac function, as shown in Figure 1.6(b).

For the case of Figure 1.6(a), an ideal switch is connected in parallel with a capacitor C. The switch is turned on at $t = t_o$, when the voltage v_{DS} across the switch is nonzero. At $t = t_o$, this voltage can be described by

$$v_{DS}(t_o) = \frac{1}{2}\left[\lim_{t\to t_O^-} v_{DS}(t) + \lim_{t\to t_o^-}(t)\right] = \frac{\Delta V}{2}. \tag{1.25}$$

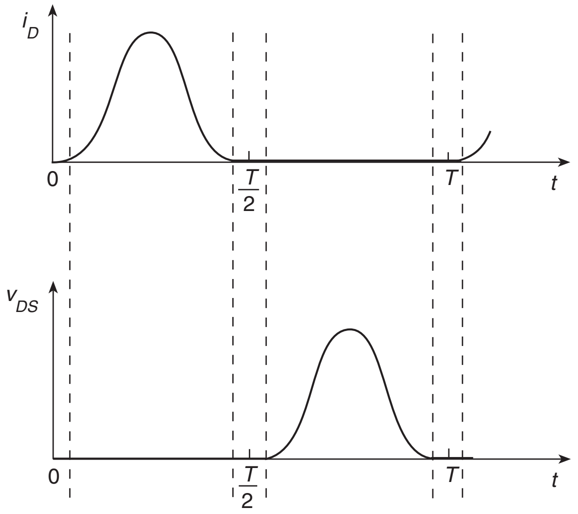

Figure 1.5 Nonoverlapping waveforms of drain current i_D and drain-to-source voltage v_{DS}.

At $t = t_o$, the drain current is given by

$$i_D(t_o) = C\,\Delta V\,\delta(t - t_o). \tag{1.26}$$

Hence, the instantaneous power dissipation is

$$p_D(t) = i_D v_{DS} = \begin{cases} \frac{1}{2}C\,\Delta V^2\delta(t - t_o) & \text{for} \quad t = t_o \\ 0 & \text{for} \quad t \neq t_o, \end{cases} \tag{1.27}$$

resulting in the time average power dissipation

$$P_D = \frac{1}{T}\int_0^T i_D v_{DS}\,\mathrm{d}t = \frac{1}{2}fC\,\Delta V^2 \tag{1.28}$$

and the drain efficiency

$$\eta_D = 1 - \frac{P_D}{P_I} = 1 - \frac{fC\,\Delta V^2}{2P_I}. \tag{1.29}$$

In a real circuit, the switch has a small series resistance, and the current through the switch is an exponential function of time of finite peak value.

Example 1.2

An RF power amplifier has a step change in the drain-to-source voltage at the MOSFET turn-on $V_{DS} = 5\,\text{V}$, the transistor capacitance is $C = 100\,\text{pF}$, the operating frequency is $f = 2.4\,\text{GHz}$, and the dc supply power is $P_I = 5\,\text{W}$. Assume that all parasitic resistances are zero. Find the efficiency of the power amplifier.

Solution. The switching power loss is

$$P_D = \frac{1}{2}fC\,\Delta V_{DS}^2 = \frac{1}{2} \times 2.4 \times 10^9 \times 100 \times 10^{-12} \times 5^2 = 3\,\text{W}. \tag{1.30}$$

Hence, the drain efficiency of the amplifier is

$$\eta_D = \frac{P_O}{P_I} = \frac{P_I - P_D}{P_I} = 1 - \frac{P_D}{P_I} = 1 - \frac{3}{5} = 40\,\%. \tag{1.31}$$

For the amplifier of Figure 1.6(b), an ideal switch is connected in series with an inductor L. The switch is turned on at $t = t_o$, when the current i_D through the switch is nonzero. At $t = t_o$, the switch current can be described by

$$i_D(t_o) = \frac{1}{2}\left[\lim_{t\to t_o^-} i_D(t) + \lim_{t\to t_o^+} i_D(t)\right] = \frac{\Delta I}{2}. \tag{1.32}$$

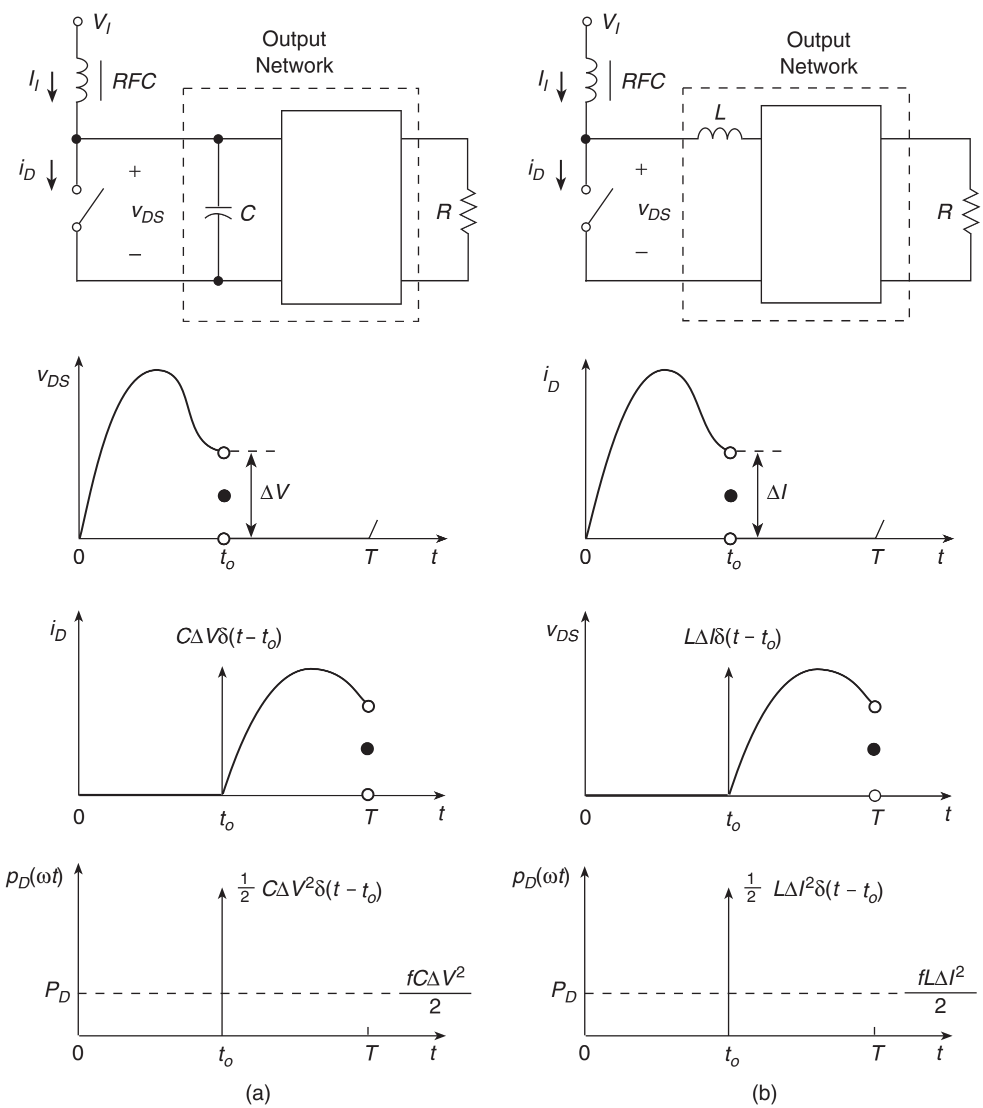

Figure 1.6 Waveforms of drain current i_D and drain-to-source voltage v_{DS} with delta Dirac functions. (a) Circuit with the switch in parallel with a capacitor. (b) Circuit with the switch in series with an inductor.

At $t = t_o$, the drain current is given by

$$v_{DS}(t_o) = L\Delta I \delta(t - t_o). \tag{1.33}$$

Hence, the instantaneous power dissipation is

$$p_D(t) = i_D v_{DS} = \begin{cases} \frac{1}{2} L\Delta I^2 \delta(t - t_o) & \text{for} \quad t = t_o \\ 0 & \text{for} \quad t \neq t_o, \end{cases} \tag{1.34}$$

resulting in the time average power dissipation

$$P_D = \frac{1}{T}\int_0^T i_D v_{DS}\, \mathrm{d}t = \frac{1}{2} fL\Delta I^2 \tag{1.35}$$

and the drain efficiency

$$\eta_D = 1 - \frac{P_D}{P_I} = 1 - \frac{fL\Delta I^2}{2P_I}. \tag{1.36}$$

In reality, the switch in the off-state has a large parallel resistance and voltage with a finite peak value developed across the switch.

1.5 Conditions for Nonzero Output Power at 100 % Efficiency of Power Amplifiers

The drain current and drain-to-source voltage waveforms have fundamental limitations for simultaneously achieving 100 % efficiency and $P_O > 0$ [9, 10]. The drain current i_D and the drain-to-source voltage v_{DS} can be represented by the Fourier series

$$i_D = I_I + \sum_{n=1}^{\infty} i_{dn} = I_I + \sum_{n=1}^{\infty} I_{dn} \sin(n\omega t + \psi_n) \tag{1.37}$$

and

$$v_{DS} = V_I + \sum_{n=1}^{\infty} v_{dsn} = V_I + \sum_{n=1}^{\infty} V_{dsn} \sin(n\omega t + \vartheta_n). \tag{1.38}$$

The derivatives of these waveforms with respect to time are

$$i'_D = \frac{\mathrm{d}i_D}{\mathrm{d}t} = \omega \sum_{n=1}^{\infty} nI_{dn} \cos(n\omega t + \psi_n) \tag{1.39}$$

and

$$v'_{DS} = \frac{\mathrm{d}v_{DS}}{\mathrm{d}t} = \omega \sum_{n=1}^{\infty} nV_{dsn} \cos(n\omega t + \vartheta_n). \tag{1.40}$$

Hence, the average value of the product of the derivatives is

$$\frac{1}{T}\int_0^T i'_D v'_{DS}\, \mathrm{d}t = -\frac{\omega^2}{2}\sum_{n=1}^{\infty} n^2 I_{dn} V_{dsn} \cos\phi_n = -\omega^2 \sum_{n=1}^{\infty} n^2 P_{dsn} \tag{1.41}$$

where $\phi_n = \vartheta_n - \psi_n - \pi$. Next,

$$\sum_{n=1}^{\infty} n^2 P_{dsn} = -\frac{1}{4\pi^2 f}\int_0^T i'_D v'_{DS}\, \mathrm{d}t. \tag{1.42}$$

If the efficiency of the output network is $\eta_n = 1$ and the power at harmonic frequencies is zero, i.e., $P_{ds2} = 0$, $P_{ds3} = 0, \ldots$, then

$$P_{ds1} = P_{O1} = -\frac{1}{4\pi^2 f}\int_0^T i'_D v'_{DS}\, \mathrm{d}t. \tag{1.43}$$

For multipliers, if $\eta_n = 1$ and the power at the fundamental frequency and at harmonic frequencies is zero except that of the n-th harmonic frequency, then the power at the n-th harmonic frequency is

$$P_{dsn} = P_{On} = -\frac{1}{4\pi^2 n^2}\int_0^T i'_D v'_{DS}\, \mathrm{d}t. \tag{1.44}$$

If the output network is passive and linear, then

$$-\frac{\pi}{2} \le \phi_n \le \frac{\pi}{2}. \tag{1.45}$$

In this case, the output power is nonzero

$$P_O > 0 \tag{1.46}$$

if

$$\frac{1}{T}\int_0^T i'_D v'_{DS}\, \mathrm{d}t < 0. \tag{1.47}$$

If the output network and the load are passive and linear and

$$\frac{1}{T}\int_0^T i'_D v'_{DS}\, \mathrm{d}t = 0 \tag{1.48}$$

then

$$P_O = 0 \tag{1.49}$$

for the following cases:

- The waveforms i_D and v_{DS} are nonoverlapping, as shown in Figure 1.5.
- The waveforms i_D and v_{DS} are adjacent and the derivatives at the joint time instants t_j are $i'_D(t_j) = 0$ and $v'_{DS}(t_j) = 0$, as shown in Figure 1.7(a).
- The waveforms i_D and v_{DS} are adjacent and the derivative $i'_D(t_j)$ at the joint time instant t_j has a jump and $v'_{DS}(t_j) = 0$, or vice versa, as shown in Figure 1.7(b).
- The waveforms i_D and v_{DS} are adjacent and their both derivatives $i_D(t_j)$ and $V_{DS}(t_j)$ have jumps at the joint time instant t_j, as shown in Figure 1.7(c).

1.6 Output Power of Class E ZVS Amplifier

The Class E zero-voltage switching (ZVS) RF power amplifier is shown in Figure 1.8. Waveforms for the Class E amplifier under zero-voltage switching and zero-derivative switching (ZDS) conditions are shown in Figure 1.9. Ideally, the efficiency of this amplifier is 100 %. Waveforms for the Class E amplifier are shown in Figure 1.9. The drain current i_D has a jump at $t = t_o$. Hence, the derivative of the drain current at $t = t_o$ is given by

$$i'_D(t_o) = \Delta I\, \delta(t - t_o) \tag{1.50}$$

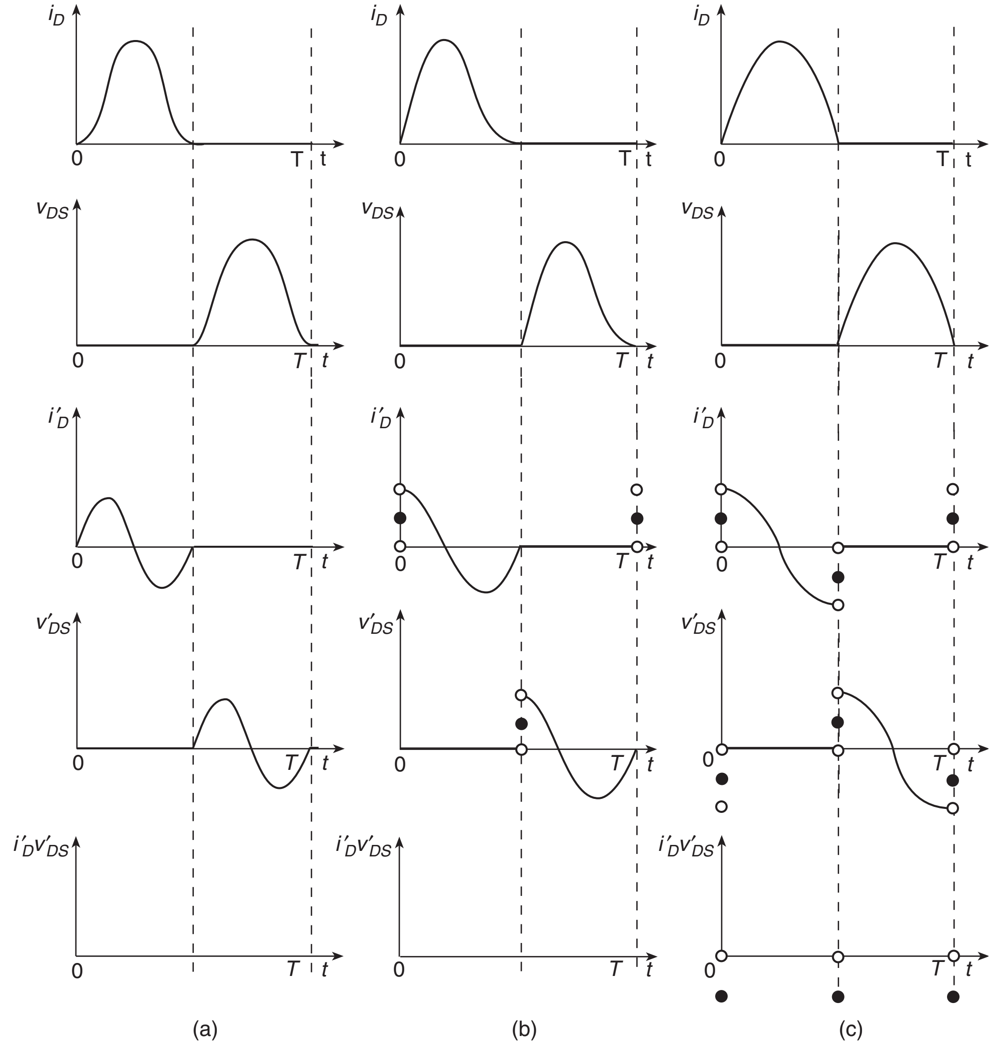

Figure 1.7 Waveforms of power amplifiers with $P_O = 0$.

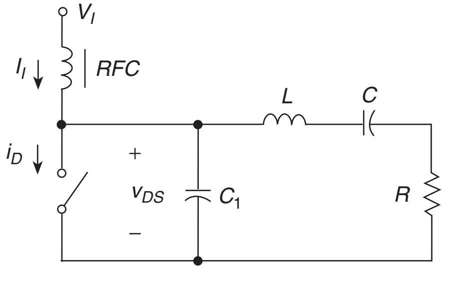

Figure 1.8 Class E ZVS power amplifier.

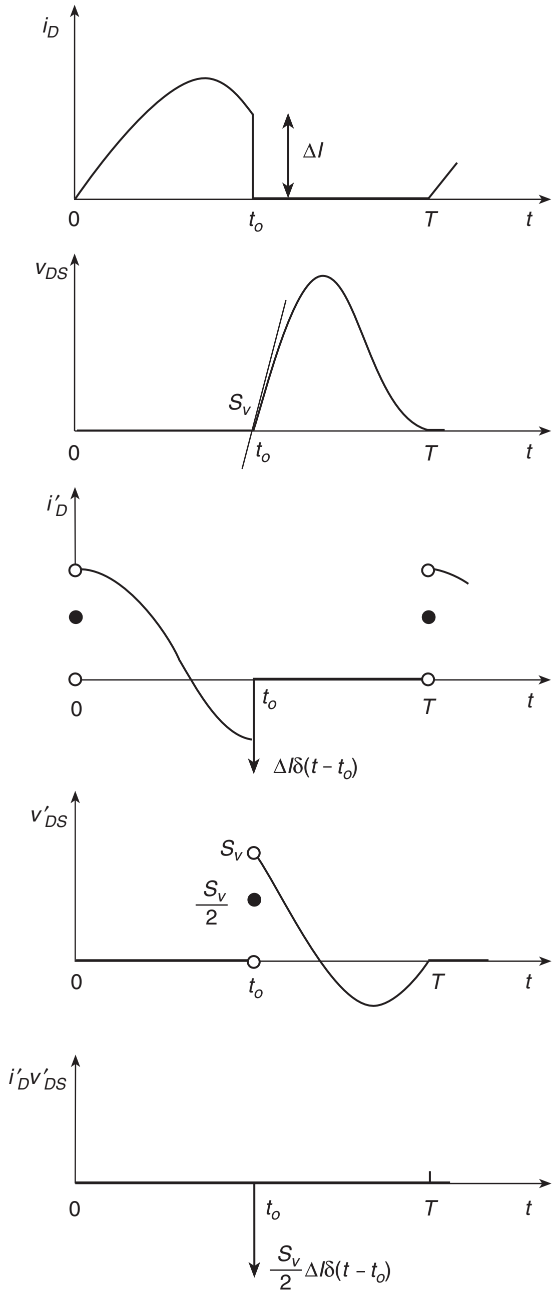

Figure 1.9 Waveforms of Class E ZVS power amplifier.

and the derivative of the drain-to-source voltage at $t = t_o$ is given by

$$v'_{DS}(t_o) = \frac{1}{2}\left[\lim_{t \to t_o^-} v'_{DS}(t) + \lim_{t \to t_o^+} v'_{DS}(t)\right] = \frac{S_v}{2}. \tag{1.51}$$

The output power of the Class E ZVS amplifier is

$$P_{ds} = P_O = -\frac{1}{4\pi^2 f}\int_0^T i'_D v'_{DS}\,\mathrm{d}t = -\frac{1}{4\pi^2 f}\int_0^T v'_{DS}\,\Delta I\,\delta(t-t_o)\,\mathrm{d}t$$
$$= -\frac{\Delta I S_v}{8\pi^2 f}\int_0^T \delta(t-t_o)\,\mathrm{d}t = -\frac{\Delta I S_v}{8\pi^2 f} \quad (1.52)$$

where $\Delta I < 0$. Since

$$\Delta I = -0.6988 I_{DM} \quad (1.53)$$

and

$$S_v = 11.08 f V_{DSM} \quad (1.54)$$

the output power is

$$P_O = -\frac{\Delta I S_v}{8\pi^2 f} = 0.0981 I_{DM} V_{DSM}. \quad (1.55)$$

Hence, the output power capability is

$$c_p = \frac{P_O}{I_{DM} V_{DSM}} = 0.0981. \quad (1.56)$$

Example 1.3

A Class E ZVS RF power amplifier has a step change in the drain current at the MOSFET turn-off $\Delta I_D = -1$ A, a slope of the drain-to-source voltage at the MOSFET turn-off $S_v = 11.08 \times 10^8$ V/s, and the operating frequency is $f = 1$ MHz. Find the output power of the amplifier.

Solution. The output power of the power amplifier is

$$P_O = -\frac{\Delta I S_v}{8\pi^2 f} = -\frac{-1 \times 11.08 \times 10^8}{8\pi^2 \times 10^6} = 14.03\,\mathrm{W}. \quad (1.57)$$

1.7 Class E ZCS Amplifier

The Class E zero-current switching (ZCS) RF power amplifier is depicted in Figure 1.10. Current and voltage waveforms under zero-current switching and zero-derivative switching (ZDS) conditions are shown in Figure 1.11. The efficiency of this amplifier with perfect

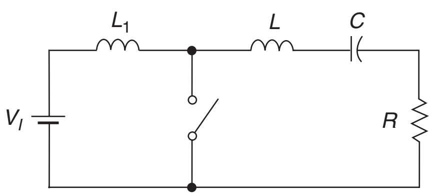

Figure 1.10 Class E ZCS power amplifier.

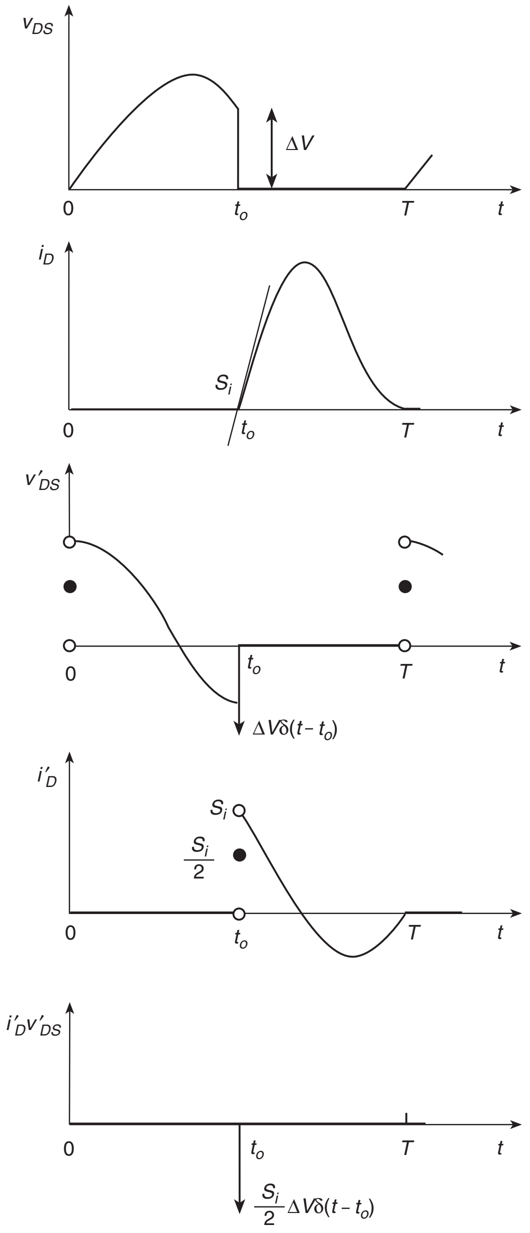

Figure 1.11 Waveforms of Class E ZCS power amplifier.

components and under ZCS condition is 100 %. The drain-to-source voltage v_{DS} has a jump at $t = t_o$. The derivative of the drain-to-source voltage at $t = t_o$ is given by

$$v'_{DS}(t_o) = \Delta V\,\delta(t - t_o) \tag{1.58}$$

and the derivative of the drain current at $t = t_o$ is given by

$$i'_D(t_o) = \frac{1}{2}\left[\lim_{t \to t_o^-} i'_D(t) + \lim_{t \to t_o^+} i'_D(t)\right] = \frac{S_i}{2}. \tag{1.59}$$

The output power of the Class E ZCS amplifier is

$$P_{ds} = P_O = -\frac{1}{4\pi^2 f}\int_0^T i'_D v'_{DS}\,\mathrm{d}t = -\frac{1}{4\pi^2 f}\int_0^T i'_D \Delta V\,\delta(t-t_o)\,\mathrm{d}t$$
$$= -\frac{\Delta V S_i}{8\pi^2 f}\int_0^T \delta(t-t_o)\,\mathrm{d}t = -\frac{\Delta V S_i}{8\pi^2 f}. \tag{1.60}$$

Since

$$\Delta V = -0.6988 V_{DSM} \tag{1.61}$$

and

$$S_i = 11.08 f I_{DM} \tag{1.62}$$

the output power is

$$P_O = -\frac{\Delta V S_i}{8\pi^2 f} = 0.0981 I_{DM} V_{DSM}. \tag{1.63}$$

Hence, the output power capability is

$$c_p = \frac{P_O}{I_{DM} V_{DSM}} = 0.0981. \tag{1.64}$$

Example 1.4

A Class E ZCS RF power amplifier has a step change in the drain-to-source voltage waveform at the MOSFET turn-on $\Delta V_{DS} = -100$ V, a slope of the drain current at the MOSFET turn-on $S_i = 11.08 \times 10^3$ V/s, and the operating frequency is $f = 1$ MHz. Find the output power of the amplifier.

Solution. The output power of the power amplifier is

$$P_O = -\frac{\Delta V_{DS} S_i}{8\pi^2 f} = -\frac{-100 \times 11.08 \times 10^7}{8\pi^2 \times 10^6} = 140.320\,\mathrm{W}. \tag{1.65}$$

1.8 Propagation of Electromagnetic Waves

An antenna is a device for radiating or receiving electromagnetic radio waves. A transmitting antenna converts an electrical signal into an electromagnetic wave. It is a transition structure between a guiding device (such as a transmission line) and free space. A receiving antenna converts an electromagnetic wave into an electrical signal. The electromagnetic wave travels at the speed of light in free space. The wavelength of an electromagnetic wave in free space is given by

$$\lambda = \frac{c}{f} \tag{1.66}$$

where $c = 3 \times 10^8$ m/s is the velocity of light in free space.

An isotropic antenna is a theoretical point antenna that radiates energy equally in all directions with its power spread uniformly on the surface of a sphere. This results in a spherical wavefront. The uniform radiated power density at a distance r from a transmitter with the output power P_T is given by

$$p(r) = \frac{P_T}{4\pi r^2}. \tag{1.67}$$

The power density is inversely proportional to the square of the distance r. The hypothetical isotropic antenna is not practical, but is commonly used as a reference to compare with other antennas. If the transmitting antenna has directivity in a particular direction and efficiency, the power density in that direction is increased by a factor called the antenna gain G_T. The power density received by a receiving directive antenna is

$$p_r(r) = G_T \frac{P_T}{4\pi r^2}. \tag{1.68}$$

The antenna efficiency is the ratio of the radiated power to the total power fed to the antenna.

A receiving antenna pointed in the direction of the radiated power gathers a portion of the power that is proportional to its cross-sectional area. The antenna effective area is given by

$$A_e = G_R \frac{\lambda^2}{4\pi} \tag{1.69}$$

where G_R is the gain of the receiving antenna and λ is the free-space wavelength. Thus, the power received by a receiving antenna is given by the Herald Friis formula for free-space transmission as

$$P_{REC} = A_e p(r) = G_T G_R P_T \frac{\lambda^2}{(4\pi r)^2} = G_T G_R P_T \left(\frac{c}{4\pi r f_c}\right)^2. \tag{1.70}$$

The received power is proportional to the gain of either antenna and inversely proportional to r^2. For example, the gain of the dish (parabolic) antenna is given by

$$G_T = G_R = 6\left(\frac{D}{\lambda}\right)^2 = 6\left(\frac{Df_c}{c}\right)^2 \tag{1.71}$$

where D is the mouth diameter of the primary reflector. For $D = 3\,\text{m}$ and $f = 10\,\text{GHz}$, $G_T = G_R = 60{,}000 = 47.8\,\text{dB}$.

The *space loss* is the loss due to spreading the RF energy as it propagates through free space and is defined as

$$S_L = \frac{P_T}{P_{REC}} = \left(\frac{4\pi r}{\lambda}\right)^2 = 10\log\left(\frac{P_T}{P_{REC}}\right) = 20\log\left(\frac{4\pi r}{\lambda}\right). \tag{1.72}$$

There are also other losses such as atmospheric loss, polarization mismatch loss, impedance mismatch loss, and pointing error denoted by L_{syst}. Hence, the link equation is

$$P_{REC} = \frac{L_{syst} G_T G_R P_T}{(4\pi)^2}\left(\frac{\lambda}{r}\right)^2. \tag{1.73}$$

The maximum distance between the transmitting and receiving antennas is

$$r_{max} = \frac{\lambda}{4\pi}\sqrt{L_{syst} G_T G_R \left(\frac{P_T}{P_{REC(min)}}\right)}. \tag{1.74}$$

Antennas are used to radiate the electromagnetic waves and transmit through the atmosphere. The radiation efficiency of antennas is high only if their dimensions are of the same order of magnitude as the wavelength of the carrier frequency f_c. The length of antennas is usually $\lambda/2$ (a half-dipole antenna) or $\lambda/4$ (quarter-wave antenna) and it should be higher than $\lambda/10$. For example, the height of a quarter-wave antenna $h_a = 750$ m at $f_c = 100$ kHz, $h_a = 75$ m at $f_c = 1$ MHz, $h_a = 7.5$ cm at $f_c = 1$ GHz, and $h_a = 7.5$ mm at $f_c = 10$ GHz.

There are three groups of electromagnetic waves based on their propagation properties:

- ground waves (below 2 MHz);
- sky waves (2–30 MHz);
- line-of-sight waves, also called space waves or horizontal waves (above 30 MHz).

Wave propagation is illustrated in Figure 1.12. The ground waves travel parallel to the Earth's surface and suffer little attenuation by smog, moisture, and other particles in the lower part of the atmosphere. Very high antennas are required for transmission of these low-frequency waves. The approximate transmission distance of ground waves is about 1600 km (1000 miles). Ground wave propagation is much better over water, especially salt water, than over a dry desert terrain. Ground wave propagation is the only way to communicate into the ocean with submarines. Extremely low frequency (ELF) waves (30–300 Hz) are used to minimize the attenuation of the waves by sea water. A typical frequency is 100 Hz.

The sky waves leave the curved surface of the Earth and are refracted by the ionosphere back to the surface of the Earth and therefore are capable of following the Earth's curvature. The altitude of refraction of the sky waves varies from 50 to 400 km. The transmission distance between two transmitters is 4000 km. The ionosphere is a region above the atmosphere, where free ions and electrons exist in sufficient quantity to affect the wave propagation. The ionization is caused by radiation from the Sun. It changes as the position of a point on the Earth with respect to the Sun changes daily, monthly, and yearly. After sunset, the lowest layer of the ionosphere disappears because of rapid recombination of its ions. The higher the frequency, the more difficult is the refracting (bending) process. Between the point where the ground wave is completely attenuated and the point where the first wave returns, no signal is received, resulting in the skip zone.

The line-of-sight waves follow straight lines. There are two types of line-of-sight waves: direct waves and ground reflected waves. The direct wave is by far the most widely used for propagation between antennas. Signals of frequencies above HF band cannot be propagated

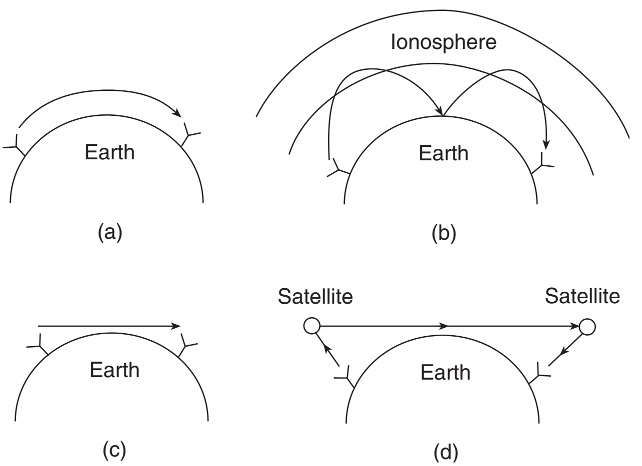

Figure 1.12 Electromagnetic wave propagation. (a) Ground wave propagation. (b) Sky wave propagation. (c) Horizontal wave propagation. (d) Wave propagation in satellite communications.

for long distances along the surface of the Earth. However, it is easy to propagate these signals through free space.

1.9 Frequency Spectrum

Table 1.1 gives the frequency spectrum. In the United States, the allocation of carrier frequencies, bandwidths, and power levels of transmitted electromagnetic waves is regulated by the Federal Communications Commission (FCC) for all nonmilitary applications. The communication must occur in a certain part of the frequency spectrum. The carrier frequency f_c determines the channel frequency.

Low-frequency (LF) electromagnetic waves are propagated by the ground waves. They are used for long-range navigation, telegraphy, and submarine communication. The medium-frequency (MF) band contains the commercial radio band from 535 to 1705 kHz. This band is used for radio transmission of amplitude-modulated (AM) signals to general audiences. The carrier frequencies are from 540 to 1700 kHz. For example, one carrier frequency is at $f_c = 550$ kHz, and the next carrier frequency $f_c = 560$ kHz. The modulation bandwidth is 5 kHz. The average power of local stations is from 0.1 to 1 kW. The average power of regional stations is from 0.5 to 5 kW. The average power of clear stations is from 0.25 to 50 kW. A radio receiver may receive power as low as of 10 pW, 1 μV/m, or 50 μV across a 300-Ω antenna. Thus, the ratio of the output power of the transmitter to the input power of the receiver is on the order of $P_T/P_{REC} = 10^{15}$.

The range from 1705 to 2850 kHz is used for short-distance point-to-point communications for services such as fire, police, ambulance, highway, forestry, and emergency services. The antennas in this band have reasonable height and radiation efficiency. Aeronautical frequency range starts in MF range and ends in HF range. It is from 2850 to 4063 kHz and is used for short-distance point-to-point communications and ground-air-ground communications. Aircraft flying scheduled routes are allocated specific channels. The high-frequency (HF) band contains the radio amateur band from 3.5 to 4 MHz in the United States. Other countries use this band for mobile and fixed services. High frequencies are also used for long-distance point-to-point transoceanic ground-air-ground communications. High frequencies are propagated by sky waves. The frequencies from 1.6 to 30 MHz are called short waves.

The very high frequency (VHF) band contains commercial FM radio and most TV channels. The commercial frequency-modulated (FM) radio transmission is from 88 to

Table 1.1 Frequency spectrum.

Frequency range	Band name	Wavelength range
30–300 Hz	Extremely Low Frequencies (ELF)	10 000–1000 km
300–3000 Hz	Voice Frequencies (VF)	1000–100 km
3–30 kHz	Very Low Frequencies (VLF)	100–10 km
30–300 kHz	Low Frequencies (LF)	10–1 km
0.3–3 MHz	Medium Frequencies (MF)	1000–100 m
3–30 MHz	High Frequencies (HF)	100–10 m
30–300 MHz	Very High Frequencies (VHF)	100–10 cm
0.3–3 GHz	Ultra High Frequencies (UHF)	100–10 cm
3–30 GHz	Super High Frequencies (SHF)	10–1 cm
30–300 GHz	Extra High Frequencies (EHF)	10–1 mm

Table 1.2 Broadcast frequency allocation.

Radio or TV	Frequency range	Channel spacing
AM Radio	535–1605 kHz	10 kHz
TV (channels 2–6)	54–72 MHz	6 MHz
TV	76–88 MHz	6 MHz
FM Radio	88–108 MHz	200 kHz
TV (channels 7–13)	174–216 MHz	6 MHz
TV (channels 14–83)	470–806 MHz	6 MHz

Table 1.3 UHF and SHF frequency bands.

Band Name	Frequency Range	Units
L	1–2	GHz
S	2–4	GHz
C	4–8	GHz
X	8–12.4	GHz
Ku	12.4–18	GHz
K	18–26.5	GHz
Ku	26.5–40	GHz

108 MHz. The modulation bandwidth is 15 kHz. The average power is from 0.25 to 100 kW. The transmission distance of VHF TV signals is 160 km (100 miles).

TV channels for analog transmission range from 54 to 88 MHz and from 174 to 216 MHz in the VHF band and from 470 to 890 MHz in the ultrahigh-frequency (UHF) band. The average power is 100 kW for the frequency range from 54 to 88 MHz and 316 kW for the frequency range from 174 to 216 MHz. Broadcast frequency allocations are given in Table 1.2. The UHF and SHF frequency bands are given in Table 1.3. The cellular phone frequency allocation is given in Table 1.4.

The superhigh frequency (SHF) band contains satellite communications channels. Satellites are placed in orbits. Typically, these orbits are 37 786 km in altitude above the equator. Each satellite illuminates about one-third of the Earth. Since the satellites maintain the same position relative to the Earth, they are placed in geostationary orbits. These geosynchronous satellites are called GEO satellites. Each satellite contains a communication system that can receive signals from the Earth or from another satellite and transmit the received signal back to the Earth or to another satellite. The system uses two carrier frequencies. The frequency for transmission from the Earth to the satellite (uplink) is 6 GHz and the transmission from the satellite to the Earth (downlink) is at 4 GHz. The bandwidth of each channel is 500 MHz. Directional antennas are used for radio transmission through free space. Satellite communications are used for TV and telephone transmission. The electronic circuits in the satellite are powered by solar energy using solar cells that deliver the supply power of about 1 kW. The combination of a transmitter and a receiver is called a transponder. A typical satellite has 12 to 24 transponders. Each transponder has a bandwidth of 36 MHz. The total time delay for GEO satellites is about 400 ms and the power of received signals is very low. Therefore, a low Earth-orbit (LEO) satellite system was deployed for a mobile phone system. The orbits of LEO satellites are 500–1500 km above the Earth. These satellites are not synchronized with the Earth's rotation. The total delay time for LEO satellite is about 250 ms.

Table 1.4 Cellular phone frequency allocation.

System	Frequency range	Channel spacing	Multiple access
AMPS	M-B 824–849 MHz	30 MHz	FDMA
	B-M 869–894 MHz	30 MHz	
GSM-900	M-B 880–915 MHz	0.2 MHz	TDMA
	B-M 915–990 MHz	0.2 MHz	
GSM-1800	M-B 1710–1785 MHz	0.2 MHz	TDMA
	B-M 1805–1880 MHz	0.2 MHz	
PCS-1900	M-B 1850–1910 kHz	30 MHz	TDMA
	B-M 1930–1990 kHz	30 MHz	
IS-54	M-B 824–849 MHz	30 MHz	TDMA
	B-M 869–894 MHz	30 MHz	
IS-136	M-B 1850–1910 MHz	30 MHz	TDMA
	B-M 1930–1990 MHz	30 MHz	
IS-96	M-B 824–849 MHz	30 MHz	CDMA
	B-M 869–894 MHz	30 MHz	
IS-96	M-B 1850–1910 MHz	30 MHz	CDMA
	B-M 1930–1990 MHz	30 MHz	

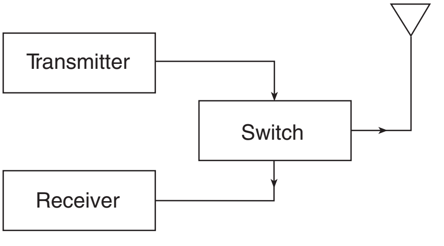

Figure 1.13 Block diagram of a transceiver.

1.10 Duplexing

In two-way communication, a transmitter and a receiver are used. The combination of a transmitter and a receiver is called a *transceiver*. A block diagram of the transceiver is shown in Figure 1.13. Duplexing techniques are used to allow for both users to transmit and receive signals. The most commonly used duplexing is called time-division duplexing (TDD). The same frequency channel is used for both transmitting and receiving signals, but the system transmits the signal for half of the time and receives for the other half.

1.11 Multiple-access Techniques

In multiple-access communications systems, information signals are sent simultaneously over the same channel. Cellular wireless mobile communications use the following multiple-access techniques to allow simultaneous communication among multiple transceivers:

- time-division multiple access (TDMA);

- frequency-division multiple access (FDMA);
- code-division multiple access (CDMA).

In the TDMA, the same frequency band is used by all the users, but at different time intervals. Each digitally coded signal is transmitted only during preselected time intervals, called the time slots T_{SL}. During every time frame T_F, each user has access to the channel for a time slot T_{SL}. The signals transmitted from different users do not interfere with each other in the time domain.

In the FDMA, a frequency band is divided into many channels. The carrier frequency f_c determines the channel frequency. Each baseband signal is transmitted with a different carrier frequency. One channel is assigned to each user for a connection period. After the connection is completed, the channel becomes available to other users. In FDMA, proper filtering must be carried out to provide channel selection. The signals transmitted from different users do not interfere with each other in the frequency domain.

In the CDMA, each user uses a different code (like a different language). CDMA allows the use of one carrier frequency. Each station uses a different binary sequence to modulate the carrier. The signals transmitted from different users overlap in both the frequency and time domains, but the messages are orthogonal.

There are several standards of cellular wireless communications:

- Advanced Mobile Phone Service (AMPS);
- Global System for Mobile Communications (GSM);
- CDMA wireless standard proposed by Qualcomm.

1.12 Nonlinear Distortion in Transmitters

Power amplifiers contain a transistor (MOSFET, MESFET, or BJT), which is a nonlinear device operated under large-signal conditions. The drain current i_D is a nonlinear function of the gate-to-source voltage v_{GS}. Therefore, power amplifiers produce components which are not present in the amplifier input signal. The relationship between the output voltage v_o and the input voltage v_s of a 'weakly nonlinear' or a 'nearly linear' power amplifier, such as the Class A amplifier, is nonlinear $v_o = f(v_s)$. This relationship can be expanded into Taylor's power series around the operating point

$$v_o = f(v_s) = V_{O(DC)} + a_1 v_s + a_2 v_s^2 + a_3 v_s^3 + a_4 v_s^4 + a_5 v_s^5 + \cdots \tag{1.75}$$

Thus, the output voltage consists of an infinite number of nonlinear terms. Taylor's power series takes into account only the amplitude relationships. Volterra's power series includes both the amplitude and phase relationships.

Nonlinearity of amplifiers produces two types of unwanted signals:

- harmonics of the carrier frequency;
- intermodulation products (IMPs).

Nonlinear distortion components may corrupt the desired signal. Harmonic distortion (HD) occurs when a single-frequency sinusoidal signal is applied to the power amplifier input. Intermodulation distortion (IMD) occurs when two or more frequencies are applied at the power amplifier input. To evaluate the linearity of power amplifiers, we can use (1) a single-tone test and (2) a two-tone test. In a single-tone test, a sinusoidal voltage source is used to drive a power amplifier. In a two-tone test, two sinusoidal sources connected in series are used as a driver of a power amplifier. The first test will produce harmonics and the second test will produce both harmonics and intermodulation products (IMPs).

1.13 Harmonics of Carrier Frequency

To investigate the process of generation of harmonics, let us assume that a power amplifier is driven by a single-tone excitation in the form of a sinusoidal voltage

$$v_s(t) = V_m \cos \omega t. \tag{1.76}$$

To gain an insight into generation of harmonics by a nonlinear transmitter, consider an example of a memoryless time-invariant power amplifier described by a third-order polynomial

$$v_o(t) = a_1 v_s(t) + a_2 v_s^2(t) + a_3 v_s^3(t). \tag{1.77}$$

The output voltage of the transmitter is given by

$$\begin{aligned} v_o(t) &= a_1 V_m \cos \omega t + a_2 V_m^2 \cos^2 \omega t + a_3 V_m^3 \cos^3 \omega t \\ &= a_1 V_m \cos \omega t + \frac{1}{2} a_2 V_m^2 (1 + \cos 2\omega t) + \frac{1}{4} a_3 V_m (3 \cos \omega t + \cos 3\omega t) \\ &= \frac{1}{2} a_2 V_m^2 + \left(a_1 V_m + \frac{3}{4} a_3 V_m^3 \right) \cos \omega t + \frac{1}{2} a_2 V_m^2 \cos 2\omega t + \frac{1}{4} a_3 V_m^3 \cos 3\omega t. \end{aligned} \tag{1.78}$$

Thus, the output voltage of the power amplifier contains the fundamental components of the carrier frequency $f_1 = f_c$ and harmonics $2f_1 = 2f_c$ and $3f_1 = 3f_c$, as shown in Figure 1.14. The amplitude of the n-th harmonic is proportional to V_m^n. The harmonics may interfere with other communication channels and must be filtered out to an acceptable level.

Harmonics are always integer multiples of the fundamental frequency. Therefore, the harmonic frequencies of the transmitter output signal with carrer frequency f_c are given by

$$f_n = nf_c. \tag{1.79}$$

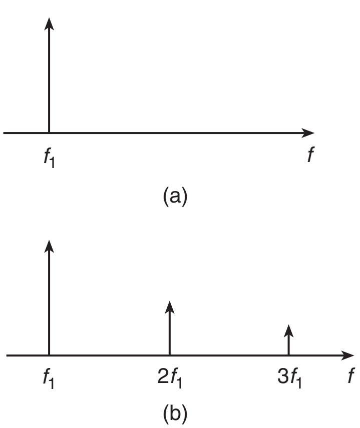

Figure 1.14 Spectrum of the input and output voltages of power amplifiers due to harmonics. (a) Spectrum of the input voltage. (b) Spectrum of the output voltage due to harmonics.

where $n = 2, 3, 4, \ldots$ is an integer. If a harmonic signal of a sufficiently large amplitude falls within the bandwidth of a nearby receiver, it may cause interference with the reception and cannot be filtered out in the receiver. The harmonics should be filtered out in the transmitter in its bandpass output network. For example, the output network of a transmitter may offer 37-dB second-harmonic suppression and 55-dB third-harmonic suppression.

Harmonic distortion is defined as the ratio of the amplitude of the n-th harmonic V_n to the amplitude of the fundamental V_1

$$HD_n = \frac{V_n}{V_1} = 20 \log\left(\frac{V_n}{V_1}\right) \text{ (dB)}. \tag{1.80}$$

The distortion by the second harmonic is given by

$$HD_2 = \frac{V_2}{V_1} = \frac{\frac{1}{2}a_2 V_m}{a_1 V_m + \frac{3}{4}a_3 V_m^3} = \frac{a_2 V_m}{2\left(a_1 + \frac{3}{4}V_m^2\right)}. \tag{1.81}$$

For $a_1 \gg 3a_3 V_m^2/4$,

$$HD_2 \approx \frac{a_2 V_m}{2a_1}. \tag{1.82}$$

The second harmonic distortion HD_2 is proportional to the input voltage amplitude V_m.

The distortion by the third harmonic is given by

$$HD_3 = \frac{V_3}{V_1} = \frac{\frac{1}{4}a_3 V_m^3}{a_1 V_m + \frac{3}{4}a_3 V_m^3} = \frac{a_3 V_m^2}{4a_1 + 3a_3 V_m^2}. \tag{1.83}$$

For $a_1 \gg 3a_3 V_m^2/4$,

$$HD_3 \approx \frac{a_3 V_m^2}{4a_1}. \tag{1.84}$$

The third-harmonic distortion HD_3 is proportional to V_m^2. Usually, the amplitudes of harmonics should be -50 to -70 dB below the amplitude of the carrier.

The ratio of the power of an n-th harmonic P_n to the power of the carrier P_c is

$$HD_n = \frac{P_n}{P_c} = 10\frac{P_n}{P_c} \text{ (dBc)}. \tag{1.85}$$

The term 'dBc' refers to ratio of the power of a spectral distortion component to the power of the carrer.

The harmonic content in a waveform is described by the total harmonic distortion (THD) defined as

$$\text{THD} = \sqrt{\frac{P_2 + P_3 + P_4 + \cdots}{P_1}} = \sqrt{\frac{\frac{V_2^2}{2R} + \frac{V_3^2}{2R} + \frac{V_4^2}{2R} + \cdots}{\frac{V_1^2}{2R}}} = \sqrt{\frac{V_2^2 + V_3^2 + V_4^2 + \cdots}{V_1^2}}$$
$$= \sqrt{HD_2^2 + HD_3^2 + HD_4^2 + \cdots}. \tag{1.86}$$

Higher harmonics ($n \geq 2$) are distortion terms.

1.14 Intermodulation

Intermodulation occurs when two or more signals of different frequencies are applied to the input of a nonlinear circuit, such as a nonlinear RF transmitter. This results in mixing the components of different frequencies. Therefore, the output signal contains components with additional frequencies, called *intromodulation products*. The frequencies of the intermodulation products are either the sums or the differences of the frequencies of input signals and their harmonics. For a two-frequency input excitation at frequencies f_1 and f_2, the frequencies of the output signal components are given by

$$f_{IM} = nf_1 \pm mf_2 \tag{1.87}$$

where $n = 0, 1, 2, 3, \ldots$ and $m = 0, 1, 2, 3, \ldots$ are integers. The order of an intermodulation product for a two-tone signal is the sum of the absolute values of coefficients n and n given by

$$k_{IMP} = n + m. \tag{1.88}$$

If the intermodulation products of sufficiently large amplitudes fall within the bandwidth of a receiver, they will degrade the reception quality. For example, $2f_1 + f_2$, $2f_1 - f_2$, $2f_2 + f_1$, and $2f_2 - f_1$ are the third-order intermodulation products. The third-order intermodulation products usually have components in the system bandwidth. In contrast, the second-order harmonics $2f_1$ and $2f_2$, and the second-order intermodulation products $f_1 + f_2$ and $f_1 - f_2$ are generally out of the system passband and are therefore not a serious problem.

A two-tone excitation test is used to evaluate intermodulation distortion of power amplifiers. In this test, the input voltage of a power amplifier is given by

$$v_s(t) = V_{m1} \cos \omega_1 t + V_{m2} \cos \omega_2 t. \tag{1.89}$$

If the power amplifier is a memoryless time-invariant circuit described by a third-order polynomial, the output voltage is given by

$$\begin{aligned}
v_o(t) &= a_1 v_s(t) + a_2 v_s^2(t) + a_3 v_s^3(t) \\
&= a_1 V_{m1} \cos \omega_1 t + a_1 V_{m2} \cos \omega_2 t + a_2 (V_{m2} \cos \omega_1 t + V_{m2} \cos \omega_2 t)^2 \\
&\quad + a_3 (V_{m1} \cos \omega_1 t + V_{m2} \cos \omega_2 t)^3 \\
&= a_1 V_{m1} \cos \omega_1 t + a_1 V_{m2} \cos \omega_2 t + a_2 V_{m1}^2 \cos^2 \omega_1 t + 2a_2 V_{m1} V_{m2} \cos \omega_1 t \cos \omega_2 t \\
&\quad + a_2 V_{m2}^2 \cos \omega_2 t \\
&\quad + a_3 V_{m1}^3 \cos^3 \omega_1 t + 3a_3 V_{m1}^2 V_{m2} \cos^2 \omega_1 t \cos \omega_2 t + 3a_3 V_{m1} V_{m2}^2 \cos \omega_1 t \cos^2 \omega_2 t \\
&\quad + a_3 V_{m2}^3 \cos^3 \omega_2 t.
\end{aligned} \tag{1.90}$$

Thus,

$$\begin{aligned}
v_o = &\left(a_1 V_{m1} + \frac{3}{2} a_3 V_{m1} V_{m2}^2 + \frac{3}{4} a_3 V_{m1}^3 \right) \cos \omega_1 t \\
&+ \left(a_1 V_{m2} + \frac{3}{2} a_3 V_{m2} V_{m1}^2 + \frac{3}{4} V_{m2}^3 \right) \cos \omega_2 t \\
&+ a_2 V_{m1} V_{m2} \cos(\omega_2 - \omega_1) t + a_2 V_{m1} V_{m2} \cos(\omega_1 + \omega_2) t
\end{aligned}$$

$$+\frac{3}{4}a_3V_{m1}^2V_{m2}\cos(2\omega_1-\omega_2)t+\frac{3}{4}a_3V_{m1}^2V_{m2}\cos(2\omega_1+\omega_2)t$$
$$+\frac{3}{4}a_3V_{m1}V_{m2}^2\cos(2\omega_2-\omega_1)t+\frac{3}{4}a_3V_{m1}V_{m2}^2\cos(2\omega_2+\omega_1)t+\cdots. \quad (1.91)$$

The output voltage contains:

- fundamental components f_1 and f_2;
- harmonics of the fundamental components $2f_1, 2f_2\ 3f_1, 3f_2, \ldots$;
- intermodulation products $f_2 - f_1$, $f_1 + f_2$, $2f_1 - f_2$, $2f_2 - f_1$, $2f_1 + f_2$, $2f_2 + f_1$, $3f_1 - 2f_2$, $3f_2 - 2f_1, \ldots$.

The spectrum of the two-tone input voltage of equal amplitudes and several components of the spectrum of the output voltage are depicted in Figure 1.15. If the difference between f_2 and f_1 is small, the intermodulation products appear in the close vicinity of f_1 and f_2. The third-order intromodulation products, which are at $2f_1 - f_2$ and $2f_2 - f_1$, are of the most interest because they are the closest to the fundamental components. The frequency difference of the IM product at $2f_1 - f_2$ from the fundamental component at f_1 is

$$\Delta f = f_1 - (2f_1 - f_2) = f_2 - f_1. \quad (1.92)$$

The frequency difference of the IM product at $2f_2 - f_1$ from the fundamental component at f_2 is

$$\Delta f = (2f_2 - f_1) - f_2 = f_2 - f_1. \quad (1.93)$$

For example, if $f_1 = 800\,\text{kHz}$ and $f_2 = 900\,\text{kHz}$, then the IMPs of particular interest are $2f_1 - f_2 = 2 \times 800 - 900 = 700\,\text{MHz}$ and $2f_2 - f_1 = 2 \times 900 - 800 = 1000\,\text{MHz}$. To filter out the unwanted components, filters with a very narrow bandwidth are required.

Assuming that $V_{m1} = V_{m2} = V_m$, the amplitudes of the third-order intermodulation product are

$$V_{2f_2-f_1} = V_{2f_1-f_2} = \frac{3}{4}a_3V_m^2. \quad (1.94)$$

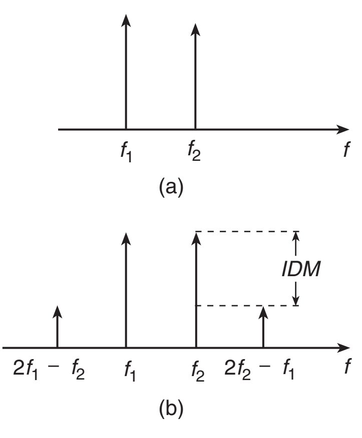

Figure 1.15 Spectrum of the input and output voltages in power amplifiers due to intermodulation. (a) Spectrum of the input voltage. (b) Several components of the spectrum of the output voltage due to intermodulation.

Assuming that $V_{m1} = V_{m2} = V_m$ and $a_3 \ll a_1$, The third-order intermodulation distortion by the IMP component at $2f_1 \pm f_2$ or the IMP component at $2f_2 \pm f_1$ is given by

$$IM_3 = \frac{V_{2f_2-f_1}}{V_{f_2}} = \frac{\frac{3}{4}a_3 V_m^3}{\left(a_1 + \frac{9}{4}a_3\right) V_m} \approx \frac{\frac{3}{4}a_3 V_m^3}{a_1 V_m} = \frac{3}{4}\left(\frac{a_3}{a_1}\right) V_m^2. \tag{1.95}$$

As V_m is increased, the amplitudes of the fundamental components V_{f_1} and V_{f_2} are directly proportional to V_m, whereas the amplitudes of the third-order IM products $V_{2f_1-f_2}$ and $V_{2f_2-f_1}$ are proportional to V_m. Therefore, the amplitudes of the IM products increase three times faster than the amplitudes of the fundamentals and have an intersection point. If the amplitudes are drawn on a log-log scale, they are linear functions of V_m.

1.15 Dynamic Range of Power Amplifiers

Figure 1.16 shows the desired output power $P_O(f_2)$ and the undesired third-order intermodulation product output power $P_O(2f_2 - f_1)$ as functions of the input power P_i on a log-log scale. This characteristic exhibits a linear region and a nonlinear region. As the input power P_i increases, the output power reaches saturation, causing *power gain compression*. The point at which the power gain of the nonlinear amplifier deviates from the fictitious ideal linear amplifier by 1 dB is called the *1-dB compression point*. It is used as a measure of the power handling capability of the power amplifier. The output power at the 1-dB compression point is given by

$$P_{O(1\text{dB})}\ (\text{dBm}) = A_{1\text{dB}} + P_{i(1\text{dB})}\ (\text{dBm}) = A_{o(1\text{dB})} - 1\ \text{dB} + P_{i(1\text{dB})}\ (\text{dBm}) \tag{1.96}$$

where A_o is the power gain of an ideal linear amplifier and $A_{1\text{dB}}$ is the power gain at the 1-dB compression point.

The *dynamic range* of a power amplifier is the region where the amplifier has a linear power gain. It is defined as the difference between the output power $P_{O(1\text{dB})}$ and the minimum detectable power P_{Omin}

$$d_R = P_{O(1\text{dB})} - P_{Omin} \tag{1.97}$$

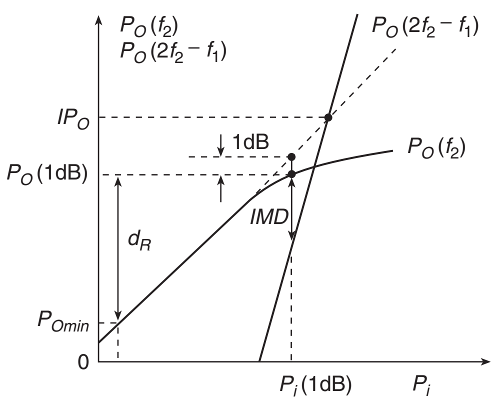

Figure 1.16 Output power $P_O(f_2)$ and $P_O(2f_2 - f_1)$ as functions of input power P_i of power amplifier.

where the minimum detectable power P_{Omin} is defined as an output power level x dB above the input noise power level P_{On}, usually $x = 3$ dB.

In a linear region of the amplifier characteristic, the desired output power $P_O(f_2)$ is proportional to the input power P_i, e.g., $P_O(f_2) = aP_i$. Assume that the third-order intermodulation product output power $P_O(2f_2 - f_1)$ increases proportionally to the third power, e.g., $P_O(2f_2 - f_1) = (a/8)^3 P_i$. Projecting the linear region of $P_O(f_2)$ and $P_O(2f_2 - f_1)$ results in an intersection point called the *intercept point* (IP), as shown in Figure 1.16, where the output power at the IP point is denoted by IP_O.

The intermodulation product is the difference between the desired output power $P_O(f_2)$ and the undesired output power of the intermodulation component $P_O(2f_2 - f_1)$ of a power amplifier

$$IMD \text{ (dB)} = P_O(f_2) \text{ (dBm)} - P_O(2f_2 - f_1) \text{ (dBm)}. \tag{1.98}$$

1.16 Analog Modulation

The function of a communications system is to transfer information from one point to another through a communication link. Block diagrams of a typical communication system depicted in Figure 1.17. It consists of a transmitter whose block diagram is shown in Figure 1.17(a) and a receiver whose block diagram is shown shown in in Figure 1.17(b). In a transmitting system, a radio-frequency signal is generated, amplified, modulated, and applied to the antenna. A local oscillator generates a signal with a frequency f_{LO}. The signals with an intermediate frequency f_{IF} and the local-oscillation frequency f_{LO} are applied to a mixer. The frequency of the output signal of the mixer and a bandpass filter is increased (up-conversion) from the intermediate frequency to a carrier frequency

$$f_c = f_{LO} + f_{IF}. \tag{1.99}$$

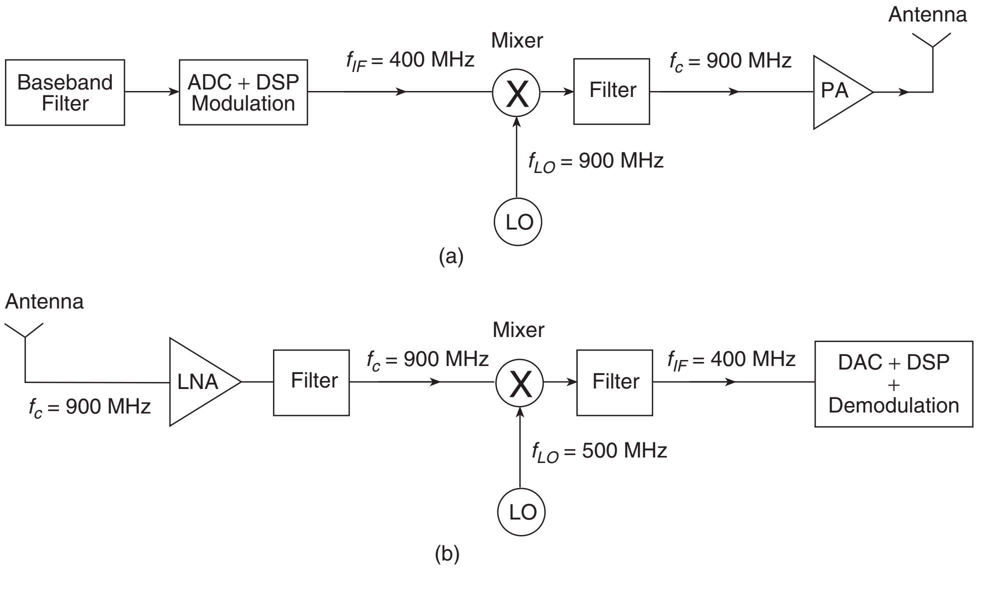

Figure 1.17 (a) Block diagram of a transmitter. (b) Block diagram of a receiver.

The RF current flows through the antenna and produces electromagnetic waves. Antennas produce or collect electromagnetic energy. The transmitted signal is received by the antenna, amplified by a low-noise amplifier (LNA), and applied to a mixer. The frequency of the output signal of the mixer and a bandpass filter is reduced (down-conversion) from the carrier frequency to an intermediate frequency

$$f_{IF} = f_c - f_{LO}. \tag{1.100}$$

The most important parameters of a transmitter are as follows:

- spectral efficiency;
- power efficiency;
- signal quality in the presence of noise and interference.

A 'baseband' signal (modulating signal or information signal) has a nonzero spectrum in the vicinity of $f = 0$ and negligible elsewhere. For example, a voice signal generated by a microphone or a video signal generated by a TV camera are baseband signals. The modulating signal may consist of many, e.g., 24 multiplexed telephone channels. In RF systems with analog modulation, the carrier is modulated by an analog baseband signal. The frequency bandwidth occupied by the baseband signal is called the baseband. The modulated signal consists of components with much higher frequencies than the highest baseband frequency. The modulated signal is an RF signal. It consists of components whose frequencies are very close to the frequency of the carrier.

RF modulated signals can be divided into two groups:

- variable-envelope signals;
- constant-envelope signals.

Modulation is the process of placing an information band around a high-frequency carrier for transmission. Modulation conveys information by changing some aspects of a carrier signal in response to a modulating signal. In general, a modulated output voltage is given by

$$v_o(t) = A(t)\cos[\omega_c t + \theta(t)] \tag{1.101}$$

where $A(t)$ is the amplitude of the voltage, f_c is the carrier frequency, and $\theta(t)$ is the phase of the carrier. If the amplitude of v_o is varied, it is called amplitude modulation (AM). If the frequency of v_o is varied, it is called frequency modulation (FM). If the phase of v_o is varied, it is called phase modulation (PM).

1.16.1 Amplitude Modulation

In amplitude modulation, the carrier envelope is varied by the modulating signal $v_m(t)$. A single-frequency modulating voltage is given by

$$v_m(t) = V_m \cos \omega_m t. \tag{1.102}$$

The carrier voltage is

$$v_c = V_c \cos \omega_c t. \tag{1.103}$$

The amplitude-modulated (AM) voltage is

$$v_o(t) = V(t)\cos\omega_c t = [V_c + v_m(t)]\cos\omega_c t = (V_c + V_m\cos\omega_m t)\cos\omega_c t$$
$$= V_c\left(1 + \frac{V_m}{V_c}\cos\omega_m t\right)\cos\omega_c t = V_c\,(1 + m\cos\,\omega_m t)\cos\omega_c t \quad (1.104)$$

where the *modulation index* is

$$m = \frac{V_m}{V_c} \leq 1. \quad (1.105)$$

Applying the trigonometric identity,

$$\cos\omega_c t\cos\,\omega_m t = \tfrac{1}{2}[\cos(\omega_c - \omega_m)t + \cos(\omega_c + \omega_m)t] \quad (1.106)$$

we can express v_o as

$$v_o = V_c\cos\omega_c t + \frac{mV_c}{2}\cos(\omega_c - \omega_m)t + \frac{mV_c}{2}\cos(\omega_c + \omega_m)t]. \quad (1.107)$$

The AM voltage modulated by a pure sine wave consists of carrier at f_c, component at $f_c - f_m$, and component at $f_c + f_m$. The AM waveform consists of (1) the carrier component at f_c, (2) the lower side component at $f_c - f_m$, and (3) the upper side component at $f_c + f_m$. If a band of frequencies is used as a modulating signal, we obtain the lower side band and the upper side band. A phasor diagram for amplitude modulation by a single-modulating frequency is shown in Figure 1.18.

The bandwidth of an AM signal is given by

$$BW_{AM} = (f_c + f_m) - (f_c - f_m) = 2f_m. \quad (1.108)$$

The frequency range of voice from 100 to 3000 Hz contains about 95 % of energy. The carrier frequency f_c is much higher than $f_{m(max)}$, e.g., $f_c/f_{m(max)} = 200$.

The power of the carrier is

$$P_C = \frac{V_c^2}{2R}. \quad (1.109)$$

The power of the lower sideband P_{LS} is equal to the power of the upper sideband P_{US}

$$P_{LS} = P_{US} = \frac{m^2V_c^2}{8R} = \frac{m^2}{4}P_C. \quad (1.110)$$

The total transmitted power of the AM signal is given by

$$P_{AM} = P_C + P_{LS} + P_{US} = \left(1 + \frac{m^2}{4} + \frac{m^2}{4}\right)P_C = \left(1 + \frac{m^2}{2}\right)P_C. \quad (1.111)$$

For $m = 1$, the total power is

$$P_{AMmax} = P_C + P_{LS} + P_{US} = \left(1 + \frac{1}{4} + \frac{1}{4}\right)P_C = \frac{3}{2}P_C. \quad (1.112)$$

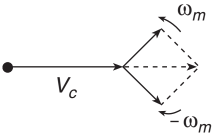

Figure 1.18 Phasor diagram for amplitude modulation by a single modulating frequency.

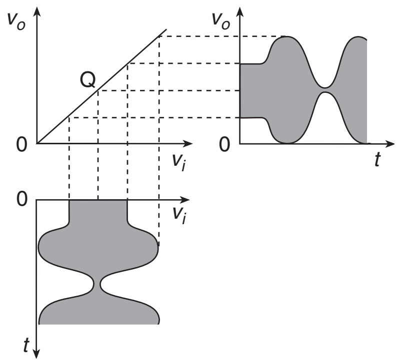

Figure 1.19 Amplification of AM signal by linear power amplifier.

A high-power AM voltage can be generated by the following methods:

- AM signal amplification using linear power amplifiers;
- drain amplitude modulation;
- gate-to-source bias operating point modulation.

Variable-envelope signals, such as AM signals, usually require linear power amplifiers. Figure 1.19 shows the amplification process of an AM signal by a linear power amplifier. In this case, the carrier and the sidebands are amplified by a linear amplifier. The drain AM signal can be generated by connecting a modulating voltage source in series with the drain dc voltage supply V_I. When the MOSFET is operated as a current source like in Class C amplifiers, it should be driven into the ohmic region. If the transistor is operated as a switch, the amplitude of the output voltage is proportional to the supply voltage and a high-fidelity AM signal is achieved. However, the modulating signal v_m must be amplified to a high power level before it is applied to modulate the supply voltage.

AM is used in commercial radio broadcasting and to transmit the video information in analog TV. Variable-envelope signals are also used in modern wireless communication systems.

Quadrature amplitude modulation (QAM) is a modulation method, which is based on modulating the amplitude of two carrier sinusoidal waves. The two waves are out of phase with each other by 90° and therefore are called quadrature carriers. The QAM signal is expressed as

$$v_{QAM} = I(t)\cos\omega_c t + Q(t)\sin\,\omega_c t \tag{1.113}$$

where $I(t)$ and $Q(t)$ are modulating signals. The received signal can be demodulated by multiplying the modulated signal v_{QAM} by a cosine wave of carrier frequency f_c

$$\begin{aligned} v_{DEM} &= v_{QAM}\cos\omega_c t = I(t)\cos\omega_c t\cos\,\omega_c t + Q(t)\sin\omega_c t\cos\,\omega_c t \\ &= \frac{1}{2}I(t) + \frac{1}{2}[I(t)\cos(2\omega_c t) + Q(t)\sin(2\omega_c t)]. \end{aligned} \tag{1.114}$$

A low-pass filter removes the $2f_c$ components, leaving only the $I(t)$ term, which is unefected by $Q(t)$. On the other hand, if the modulated signal v_{QAM} is multiplied by a sine wave, and transmitted through a low-pass filter, one obtains $Q(t)$.

1.16.2 Phase Modulation

The output voltage with angle modulation is described by

$$v_o = V_c \cos[\omega_c t + \theta(t)] = V_c \cos\phi(t) \tag{1.115}$$

where the instantaneous phase is

$$\phi(t) = \omega_c t + \theta(t) \tag{1.116}$$

and the instantaneous angular frequency is

$$\omega(t) = \frac{d\phi(t)}{dt} = \omega_c + \frac{d\theta(t)}{dt}. \tag{1.117}$$

Depending on the relationship between $\theta(t)$ and $v_m(t)$, the angle modulation can be categorized as:

- phase modulation (PM);
- frequency modulation (FM).

In PM systems, the phase of the carrier is changed by the modulating signal $v_m(t)$. In FM systems, the frequency of the carrier is changed by the modulating signal $v_m(t)$. The angle modulated systems are inherently immune to amplitude fluctuations due to noise. In addition to the high degree of noise immunity, they require less RF power because the transmitter output power contains only the power of the carrier. Therefore, PM and FM systems are suitable for high-fidelity music broadcasting and for mobile radio wireless communications. However, the bandwidth of an angle-modulated signal is much wider than that of an amplitude-modulated signal.

The modulating voltage, also called the message signal, is expressed as

$$v_m(t) = V_m \sin\omega_m t. \tag{1.118}$$

The phase of PM signal is given by

$$\theta(t) = k_p v_m(t) = k_p V_m \sin\omega_m t = m_p \sin\omega_m t. \tag{1.119}$$

Hence, the phase-modulated output voltage is

$$\begin{aligned} v_o &= V_c \cos[\omega_c t + k_p v_m(t)] = V_c \cos(\omega_c t + k_p V_m \sin\omega_m t) = V_c \cos(\omega_c t + m_p \sin\omega_m t) \\ &= V_c \cos(\omega_c t + \Delta\phi \sin\omega_m t) = V_c \cos\phi(t) \end{aligned} \tag{1.120}$$

where the *index of phase modulation* is the maximum phase shift caused by the modulating voltage V_m.

$$m_p = k_p V_m = \Delta\phi. \tag{1.121}$$

Hence, the maximum phase modulation is

$$\Delta\phi_{max} = k_p V_{m(max)}. \tag{1.122}$$

The instantaneous frequency is

$$\omega(t) = \frac{d\phi(t)}{dt} = \omega_c + \frac{d\theta(t)}{dt} = \omega_c + k_p V_m \omega_m \cos\omega_m t. \tag{1.123}$$

Thus, the phase change results in a frequency change. The frequency deviation is

$$\Delta f = f_{max} - f_c = f_c + k_p V_m f_m - f_c = k_p V_m f_m = m_p f_m. \tag{1.124}$$

The frequency deviation Δf is directly proportional to the modulating frequency f_m. Figure 1.20 shows plots of the frequency modulation Δf as functions of the modulating frequency f_m and the amplitude of the modulating voltage V_m.

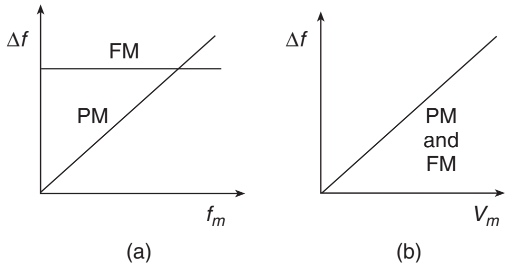

Figure 1.20 (a) Frequency deviation Δf as a function of the modulating frequency f_m. (b) Frequency deviation Δf as a function of the modulating voltage V_m.

1.16.3 Frequency Modulation

The modulating voltage is

$$v_m(t) = V_m \cos \omega_m t. \tag{1.125}$$

The derivative of the phase is

$$\frac{\mathrm{d}\theta(t)}{\mathrm{d}t} = 2\pi k_f v_m(t). \tag{1.126}$$

Thus, the phase is

$$\theta(t) = 2\pi \int k_f v_m(t)\,\mathrm{d}t = k_f V_m \int \cos \omega_m t\,\mathrm{d}t = \frac{k_f V_m}{f_m} \sin \omega_m t. \tag{1.127}$$

The frequency-modulated output voltage is given by

$$\begin{aligned} v_o &= V_c \cos[\omega_c t + \theta(t)] = V_c \cos\left(\omega_c t + \frac{k_f V_m}{f_m} \sin \omega_m t\right) \\ &= V_c \cos\left(\omega_c t + \frac{\Delta f}{f_m} \sin \omega_m t\right) = V_c \cos(\omega_c t + m_f \sin \omega_m t). \end{aligned} \tag{1.128}$$

The instantaneous frequency of an FM signal is given by

$$f(t) = f_c + k_f V_m \sin \omega_m t = f_c + \Delta f \sin \omega_m t \tag{1.129}$$

where the frequency deviation is

$$\Delta f = k_f V_m. \tag{1.130}$$

The frequency deviation of the FM signal is the maximum change of frequency caused by the modulating voltage V_m. The frequency deviation is

$$\Delta f = f_{max} - f_c = f_c + k_f V_m - f_c = k_f V_m. \tag{1.131}$$

The maximum frequency deviation is

$$\Delta f_{max} = k_f V_{M(max)}. \tag{1.132}$$

The frequency deviation of the FM signal is independent of the modulating frequency f_m. The *index of FM modulation* is defined as the ratio of the maximum frequency deviation Δf to the modulating frequency f_m

$$m_f = \frac{\Delta f}{f_m} = \frac{k_f V_m}{f_m}. \tag{1.133}$$

The angle-modulated signal contains components at $f_c \pm nf_m$, where $n = 0, 1, 2, 3, \ldots$. Therefore, the bandwidth of an angle-modulated signal is infinite. However, the amplitudes of components with large values of n is very small. Therefore, the effective bandwidth of the FM signal that contains 98 % of the signal power is given by Carson's rule

$$BW_{FM} = 2(m_f + 1)f_m = 2\left(\frac{\Delta f}{f_m} + 1\right)f_m = 2(\Delta f + f_m). \tag{1.134}$$

If the bandwidth of the modulating signal is BW_m, the frequency-modulation index is

$$m_f = \frac{\Delta f_{max}}{BW_m} \tag{1.135}$$

and the bandwidth of the frequency-modulated signal is

$$BW_{FM} = 2(m_f + 1)BW_m. \tag{1.136}$$

Commercial FM radio stations with an allowed maximum frequency deviation Δf of 75 kHz and a maximum modulation frequency f_m of 15 kHz require a bandwidth

$$BW_{FM} = 2 \times (75 + 15) = 180\,\text{kHz}. \tag{1.137}$$

For $f_c = 100$ MHz and $\Delta f = 75$ kHz,

$$\frac{\Delta f}{f_c} = \frac{75}{100 \times 10^6} = 0.075\,\%. \tag{1.138}$$

Standard broadcast FM uses a 200 kHz bandwidth for each station.

The only difference between PM and FM is that for PM the phase of the carrier varies with the modulating signal and for FM the carrier phase depends on the ratio of the modulating signal amplitude V_m to the modulating frequency f_m. Thus, FM is not sensitive to the modulating frequency f_m, but PM is. If the modulating signal is integrated and then used to modulate the phase of the carrier, the FM signal is obtained. This method is used in the Armstrong indirect FM system. The amount of deviation is proportional to the modulating frequency amplitude V_m. If the modulating signal is integrated and then used for phase modulation of the carrier, an FM signal is obtained. This method is used in Armstrong's indirect FM systems.

The relationship between PM and FM is illustrated in Figure 1.21. If the modulating signal amplitude applied to the phase modulator is inversely proportional to the modulating frequency f_m, the phase modulator produces an FM signal. The modulating voltage is given by

$$v_m(t) = \frac{V_m}{\omega_m} \sin \omega_m t. \tag{1.139}$$

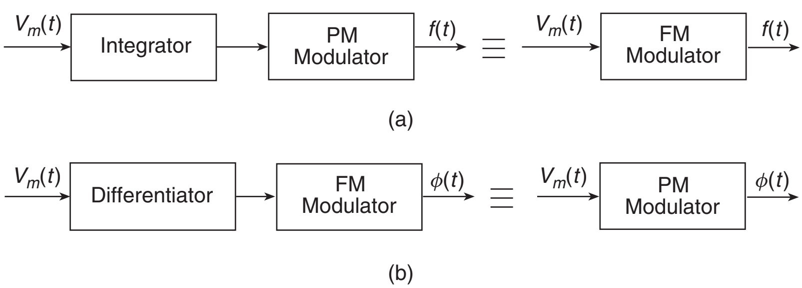

Figure 1.21 Relationship between FM and PM.

The modulated voltage is

$$v_o = V_v \cos\left(\omega_c t + \frac{k_p V_m}{\omega_m} \sin \omega_m t\right) = V_c \cos \phi(t). \tag{1.140}$$

Hence, the instantaneous frequency is

$$\omega(t) = \frac{d\phi(t)}{dt} = \omega_c + k_p V_m \cos \omega_m t \tag{1.141}$$

where

$$\Delta f = \frac{k_p V_m}{2\pi}. \tag{1.142}$$

This approach is applied in the Armstrong frequency modulator and is used for commercial FM transmission.

The output power of the transmitter with FM and PM modulation is constant and independent of the modulation index because the envelope is constant and equal to the amplitude of the carrier V_c. Thus, the transmitter output power is

$$P_O = \frac{V_c^2}{2R}. \tag{1.143}$$

FM is used for commercial radio and for audio in analog TV.

Both FM and PM signals modulated with a pure sine wave and having modulation index $m = m_f = m_p$ can be represented as

$$\begin{aligned} v_o(t) &= V_c \cos(\omega_c t + m \sin \omega_m t) \\ &= V_c\{J_0(m) \cos \omega_c t + J_1(m)[\cos(\omega_c + \omega_m)t - \cos(\omega_c - \omega_m)t] \\ &\quad + J_2(m)[\cos(\omega_c + 2\omega_m)t - \cos(\omega_c - 2\omega_m)t] \\ &\quad + J_3(m)[\cos(\omega_c + 3\omega_m)t - \cos(\omega_c - 3\omega_m)t] + \cdots\} \\ &= \sum_{n=0}^{\infty} A_c J_n(m) \cos[2\pi(f_c \pm n f_m)t] \end{aligned} \tag{1.144}$$

where $n = 0, 1, 2, \ldots$ and $J_n(m)$ are Bessel functions of the first kind of order n and can be described by

$$\begin{aligned} J_n(m) &= \left(\frac{m}{2}\right) n \left[\frac{1}{n!} - \frac{(m/2)^2}{1!(n+1)!} + \frac{(m/2)^4}{2!(n+2)!} - \frac{(m/2)^6}{3!(n+3)!} + \cdots\right] \\ &= \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{m}{2}\right)^{n+2k}}{k!(n+k)!}. \end{aligned} \tag{1.145}$$

For $m \ll 1$,

$$J_n(m) \approx \frac{1}{n!}\left(\frac{m}{2}\right)^n. \tag{1.146}$$

The angle-modulated voltage consists of a carrier and an infinite number of components (sidebands) placed at multiples of the modulating frequency f_m below and above the carrier frequency f_c, i.e., at $f_c \pm n f_m$. The amplitude of the sidebands decreases with n, which allows FM transmission within a finite bandwidth. Figure 1.22 shows the amplitudes of spectrum components for angle modulation as functions of modulation index m.

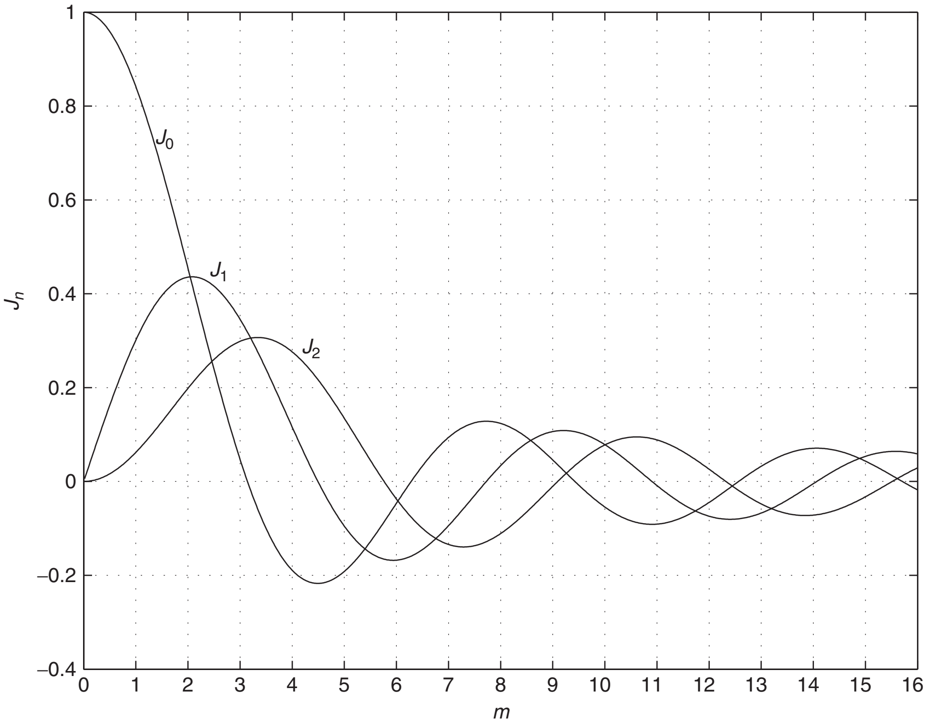

Figure 1.22 Amplitudes of spectrum components J_n for angle modulation as a function of modulation index m.

1.17 Digital Modulation

Digital modulation is special case of analog modulation. In RF systems with digital modulation, the carrier is modulated by a digital baseband signal, which consists of discrete values. Digital modulation offers many advantages over analog modulation and is widely used in wireless communications and digital TV. The main advantage is the quality of the signal, which is measured by the bit error rate (BER). BER is defined as the ratio of the average number of erroneous bits at the output of the demodulator to the total number of bits received in a unit of time in the presence of noise and other interference. It is expressed as a probability of error. Usually, BER $>10^{-3}$. A waveform of a digital binary baseband signal is given by

$$v_D = \sum_{n=1}^{n=m} b_n v(t - nT_c) \tag{1.147}$$

where b_n is the bit value, equal to 0 or 1. Counterparts of analog modulation are amplitude shift keying (ASK), frequency-shift keying (FSK), and phase-shift keying (PSK).

1.17.1 Amplitude-shift Keying

The digital amplitude modulation is called the *amplitude-shift keying* (ASK) or ON-OFF keying (OOK). It is the oldest type of modulation used in *radio-telegraph transmitters*. The

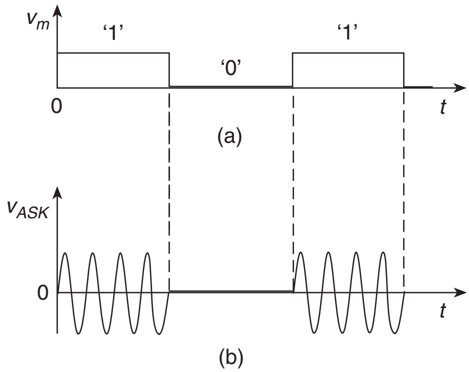

Figure 1.23 Waveform of amplitude-shift keying (ASK).

binary amplitude-modulated signal is given by

$$v_{BFSK} = \begin{cases} V_c \cos \omega_c t & \text{for} \quad v_m = 1 \\ 0 & \text{for} \quad v_m = -1. \end{cases} \tag{1.148}$$

Waveforms illustrating the amplitude-shift keying (ASK) are shown in Figure 1.23. The ASK is sensitive to amplitude noise and therefore is rarely used in RF applications.

1.17.2 Phase-shift Keying

The digital PM modulation is called *phase-shift keying* (PSK). One of the most fundamental types of PSK is the binary phase-shift keying (BPSK), where the phase shift is $\Delta\phi = 180^\circ$. The BPSK has two phase states. Waveforms for BPSK are shown in Figure 1.24. When the waveform is in phase with the reference waveform, it represents a logic '1'. When the

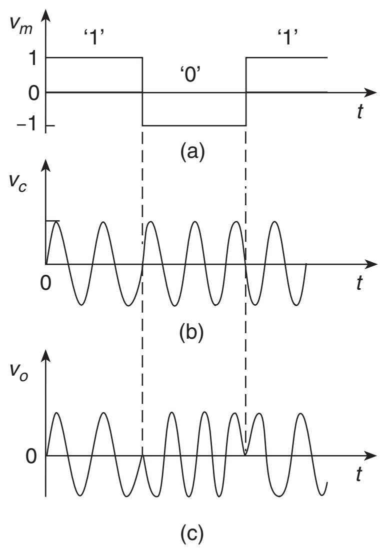

Figure 1.24 Waveforms for binary phase-shift keying (BPSK).

waveform is out of phase with respect to the reference waveform, it represents a logic '0'. The modulating binary signal is

$$v_m = \begin{cases} 1 \\ -1. \end{cases} \tag{1.149}$$

The binary phase-modulated signal is given by

$$v_{BPSK} = v_m \times v_c = (\pm 1) \times v_c = \begin{cases} v_c = V_c \cos \omega_c t & \text{for} \quad v_m = 1 \\ -v_c = -V_c \cos \omega_c t & \text{for} \quad v_m = -1. \end{cases} \tag{1.150}$$

Another type of PSK is the quanternary phase-shift keying (QPSK), where the phase shift is 90°. The QPSK uses four phase states. The systems 8PSK and 16PSK are also widely used. The PSK is used in transmission of digital signals such as digital TV.

In general, the output voltage with digital PM modulation is given by

$$v_o = V_c \sin \left[\omega_c t + \frac{2\pi(i-1)}{M} \right] \tag{1.151}$$

where $i = 1, 2, \ldots, M$, $M = 2^N$ is the number of phase states, and N is the number of data bits needed to determine the phase state. For $M = 2$ and $N = 1$, a binary phase-shift keying (BPSK) is obtained. For $M = 4$ and $N = 2$, we have a quadrature phase-shift keying (QPSK) with the following combinations of digital signal: (00), (01), (11), and (10). For $M = 8$ and $N = 3$, the 8PSK is obtained.

1.17.3 Frequency-shift Keying

The digital FM modulation is called *frequency-shift keying* (FSK). Waveforms for FSK are shown in Figure 1.25. FSK uses two carrier frequencies, f_1 and f_2. The lower frequency f_2

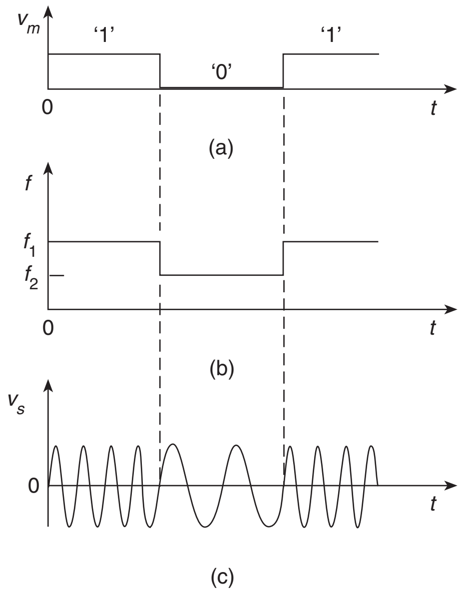

Figure 1.25 Waveforms for frequency-shift keying (FSK).

may represent a logic '0' and the higher frequency f_1 may represent a logic '1'. The binary frequency-modulated signal is given by

$$v_{BFSK} = \begin{cases} V_c \cos \omega_{c1} t & \text{for} \quad v_m = 1 \\ V_c \cos \omega_{c2} t & \text{for} \quad v_m = -1. \end{cases} \tag{1.152}$$

Gaussian frequency-shift keying (GFSK) is used in the Bluetooth. A binary 'one' is represented by a positive frequency deviation and a binary 'zero' is represented by a negative frequency deviation. The GFSK employs a constant envelope RF voltage. Thus, high-efficiency switching-mode RF power amplifiers can be used as transmitters.

1.18 Radars

The term 'radar' is derived from *ra*dio *d*etection *a*nd *r*anging. Radars are used in military applications and in a variety of commercial applications, such as navigation, air traffic control, meteorology as weather radars, automobile collision avoidance, law enforcement (in speed guns), astronomy, radio-frequency identification (RFID), and automobile automatic toll collection. Military applications of radars include surveillance and tracking of land, air, sea, and space objects from land, air, ship, and space platforms. In monostatic radar systems, the transmitter and the receiver are at the same location. The process of locating an object requires three coordinates: distance, horizontal direction, and angle of elevation. The radar antenna transmits a pulse of electromagnetic energy toward a target. The antenna has two basic purposes: it radiates RF energy and provides beam focusing and energy focusing. A wide beam pattern is required for search and a narrow beam pattern is neded for tracking. A typical waveform of the radar signal is shown in Figure 1.26. The duty ratio of the waveform is very low, e.g., 0.1 %. Therefore, the ratio of the peak power to the average power is very large. For example, the peak RF power is $P_{pk} = 200\,\text{kW}$ and the average power is $P_{AV} = 200\,\text{W}$. Some portion of this energy is reflected (or scattered) by the target. The echo signal (some of the reflected energy) is received by the radar antenna. The direction of the antennas main beam determines the location of the target. The distance to the target is determined by the time between transmitting and receiving the electromagnetic pulse. The speed of the target with respect to the radar antenna is determined by the frequency shift in the electromagnetic signal, i.e., the Doppler effect. A highly directive antenna is necessary. The radar equation is

$$\frac{P_{rec}}{P_{rad}} = \frac{\sigma_s \lambda^2}{(4\pi)^3 R^4} D^2 \tag{1.153}$$

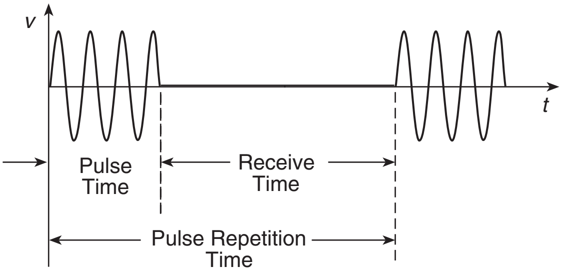

Figure 1.26 Waveform of radar signal.

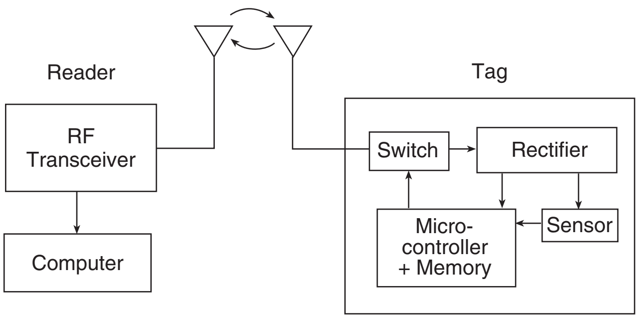

Figure 1.27 Block diagram of an RFID sensor system with a passive tag.

where P_{rad} is the power transmitted by the radar antenna, P_{rec} is the power received by the radar antenna, σ_s is the radar cross section, and D is the directive gain of the antenna.

1.19 Radio-frequency Identification

A radio-frequency identification (RFID) system consists of a *tag* placed on the item to be identified and a *reader* used to read the tag. The most common carrier frequency of RFID systems is 13.56 MHz, but it can be in the range of 135 kHz to 5.875 GHz. The tags are passive or active. An passive tag contains an RF rectifier, which rectifies a received RF signal from the reader. The rectified voltage is used to supply the tag circuitry. They exhibit long lifetime and durability. An active tag is supplied by an on-board battery. The reading range is of active tags is much larger that of passive tags and are more expensive. A block diagram of an RFID sensor system with a passive tag is shown in Figure 1.27. The reader transmits a signal to to the tag. The memory in the tag contains identification information unique to the particular tag. The microcontroller exports this information to the switch on the antenna. A modulated signal is transmitted to the reader and the reader decodes the identification information.

The tag may be placed in a plastic bag and attached to store merchandise to prevent theft or checking the store inventory. Tags may be mounted on windshields inside cars and used for automatic toll collection or checking the parking permit without stopping the car. Passive tags may be used for checking the entire inventory in libraries in seconds or locating a misplaced book. Tags along with pressure sensors may be placed in tires to alert the driver if the tire looses pressure. Small tags may be inserted beneath the skin for animal tracking. IRFD systems are capable of operating in adverse environments.

1.20 Summary

- In RF power amplifiers, the transistor can be operated as a dependent-current source or as a switch.
- When the transistor is operated as a dependent current-source, the drain current depends on the gate-to-source voltage. Therefore, variable-envelope signals can be amplified by the amplifiers with current-source mode of operation.

- When the transistor is operated as a switch, the drain current is independent of the gate-to-source voltage.
- When the transistor is operated as a switch, the drain-to-source voltage in the on-state is low, yielding low conduction loss and high efficiency.
- When the transistor is operated as a switch, switching losses limit the efficiency and the maximum operating frequency of RF power amplifiers.
- Switching power loss can be reduced by using the zero-voltage switching (ZVS) technique or zero-current switching (ZCS) technique.
- In Class A, B, and C power amplifiers, the transistor is operated as a dependent current source.
- In Class D, E, and DE power amplifiers, the transistor is operated as a switch.
- When the drain current and the drain-to-source voltage are completely displaced, the output power of an amplifier is zero.
- RF modulated signals can be classified as variable-envelope signals and constant-envelope signals.
- The most important parameters of wireless communications systems are spectral efficiency, power efficiency, and signal quality.
- dBm is a method of rating power with respect to 1 mW of power.
- The carrier is a radio wave of constant amplitude, frequency, and phase.
- Power and bandwidth are two scarce resources in RF systems.
- The major analog modulation schemes are AM, FM, and PM.
- AM is the process of superimposing a low-frequency modulating signal on a high-frequency carrier so that the instantaneous changes in the amplitude of the modulating signal produce corresponding changes in the amplitude of the high-frequency carrier.
- FM is the process of superimposing the modulating signal on a high-frequency carrier so that the carrier frequency departs from the the carrier frequency by an amount proportional to the amplitude of the modulating signal.
- Frequency deviation is the amount of carrier frequency increase or decrease around its center reference value.
- The major digital modulation schemes are ASK, FSK, and PSK.
- BPSK is a form of PSK in which the binary ‘1’ is represented as no phase shift and the binary ‘0’ is represented by phase inversion of the carrier signal.
- A transceiver is a combination of a transmitter and a receiver.
- Duplexing techniques allow for users to transmit and receive signals simultaneously.
- The major multiple-access techniques are: time-division multiple access (TDMA) technique, frequency-division multiple access (FDMA) technique, and code-division multiple access (CDMA) technique.

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1.22 Review Questions

1.1 What are the modes of operation of transistors in RF power amplifiers?

1.2 What are the classes of operation of RF power amplifiers?

1.3 What are the necessary conditions to achieve 100 % efficiency of a power amplifier?

1.4 What are the necessary conditions to achieve nonzero output power in RF power amplifiers?

1.5 Explain the principle of ZVS operation of power amplifiers.

1.6 Explain the principle of ZCS operation of power amplifiers.

1.7 What is a ground wave?

1.8 What is a sky wave?

1.9 What is a line-of-sight wave?

1.10 What is an RF transmitter?

1.11 What is a transceiver?

1.12 What is duplexing?

1.13 List multiple-access techniques.

1.14 When are harmonics generated?

1.15 What is the effect of harmonics on communications channels?

1.16 Define THD.

1.17 What are intermodulation products?

1.18 When intermodulation products are generated?

1.19 What is the effect of intermodulation products on communications channels?

1.20 What is intermodulation distortion?

1.21 Define the 1-dB compression point.

1.22 What is the interception point?

1.23 What is the dynamic range of power amplifiers?

1.24 What determines the bandwidth of emission for AM transmission?

1.25 Define the modulation index for FM signals.

1.26 Give an expression for the total power of an AM signal modulated with a single-modulating frequency.

1.27 What is QAM?

1.28 What is phase modulation?

1.29 What is frequency modulation?

1.30 What is the difference between frequency and phase modulation?

1.31 What is FSK?

1.32 What is PSK?

1.33 Explain the principle of operation of a radar.

1.34 Explain the principle of operation of an RFID system.

1.23 Problems

1.1 The rms value of the voltage at the carrier frequency is 100 V. The rms value of the voltage at the second harmonic of the carrier frequency is 1 V. The load resistance is 50 Ω. Neglecting all other harmonics, find THD.

1.2 The carrier frequency of an RF transmitter is 2.4 GHz. What is the height of a quarter-wave antenna?

1.3 An AM transmitter has an output power of 10 kW at the carrier frequency. The modulation index at $f_m = 1$ kHz is $m = 0.5$. Find the total output power of the transmitter.

1.4 The dc voltage of an RF transmitter with a Class C power amplifier is 24 V. The minimum voltage of the transistor is $V_{DSmin} = 2$ V. Find the maximum value of the index modulation.

1.5 An intermodulation product occurs at (a) $3f_1 - 2f_2$ and (b) $3f_1 + 2f_2$. What is the order of intermodulation?

1.6 An FM signal of a transmitter is applied to a 50 Ω antenna. The amplitude of the signal is $V_m = 1000$ V. What is the output power of the transmitter?

1.7 An FM signal is modulated with the modulating frequency $f_m = 10$ kHz and has a maximum frequency deviation $\Delta f = 20$ kHz. Find the modulation index.

1.8 An FM signal has a carrier frequency $f_c = 100.1$ MHz, modulation index $m_f = 2$, and modulating frequency $f_m = 15$ kHz. Find the bandwidth of the FM signal.

1.9 How much bandwidth is necessary to transmit a Chopin's sonata through an FM radio broadcasting system with high fidelity?

1.10 The peak power of a radar transmitter is 100 kW. The duty ratio is 0.1 %. What is the average power of the radar transmitter?

2

Class A RF Power Amplifier

2.1 Introduction

The Class A power amplifier [1–13] is a linear amplifier. A linear amplifier is supposed to produce an amplified replica of the input voltage or current waveform. It provides an accurate reproduction of both the envelope and the phase of the input signal. The input signal may contain voice information or image information. The transistor in the Class A amplifier is operated as a dependent-current source. The conduction angle of the drain or collector current is 360°. The efficiency of the Class A amplifier is very low. In this chapter, the basic characteristics of the Class A RF power amplifier are analyzed. The amplifier circuits, biasing, current and voltage waveforms, power losses, efficiency, and impedance matching are studied.

2.2 Circuit of Class A RF Power Amplifier

The Class A amplifier is shown in Figure 2.1. It consists of a transistor, a parallel-resonant circuit L-C, an RF choke L_f, and a coupling capacitor C_c. The operating point of the transistor is located in the active region (the pinch-off region or the saturation region). The dc component of the gate-to-source voltage V_{GS} is higher than the transistor threshold voltage V_t. The transistor is operated as a voltage-controlled dependent-current source. The ac component of the gate-to-source voltage v_{gs} can have any shape. The drain current i_D is in phase with the gate-to-source voltage v_{GS}. The ac component of drain current i_d has the same shape as the ac component of the gate-to-source voltage v_{gs} as long as the transistor is operated in the pinch-off region, i.e., for $v_{DS} > v_{GS} - V_t$. Otherwise, the drain current waveform flattens at the crest. We will assume a sinusoidal gate-to-source voltage in the subsequent analysis. At the resonant frequency f_0, the drain current i_D and the drain-to-source voltage v_{DS} are shifted in phase by 180°. The large-signal operation of the Class A power amplifier is similar to that of the small-signal operation. The main difference is the level of current and voltage amplitudes. The operating point is chosen in such a way

RF Power Amplifiers Marian K. Kazimierczuk

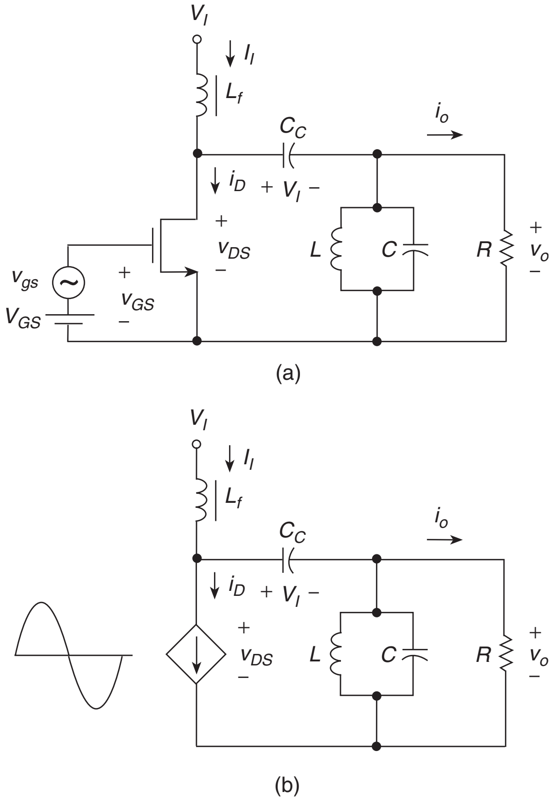

Figure 2.1 Class A RF power amplifier.

that the conduction angle of the drain current 2θ is 360°. The v_{GS}-v_o characteristic of the Class A power amplifier is nearly linear, yielding low harmonic distortion (HD) and low intermodulation distortion (IMD). The level of harmonics in the output voltage is very low. Therefore, the Class A power amplifier is a linear amplifier. It is suitable for amplification of AM signals. In narrowband Class A amplifiers, a parallel-resonant circuit is used as a bandpass filter to suppress harmonics and select a narrowband spectrum of a signal. In wideband Class A amplifiers, the filters are not needed.

The RF choke can be replaced by a quarter-wavelength transmission line, as shown in Figure 2.2. The input impedance of the transformer is given by

$$Z_i = \frac{Z_o^2}{Z_L}. \tag{2.1}$$

The load impedance of the transmission line in the form of a filter capacitor of the power supply at the operating frequency is very low, nearly a short circuit. Therefore, the input impedance of the transmission line at the drain for the fundamental component of the drain current is very high, nearly an open circuit.

The current ripple of the choke inductor L_f is low, at least 10 times lower than the dc component I_I, which is equal to the dc component of the drain current I_D. Hence, $X_{Lf} \gg R$, resulting in the design equation

$$X_{Lf} = \omega L \geq 10R. \tag{2.2}$$

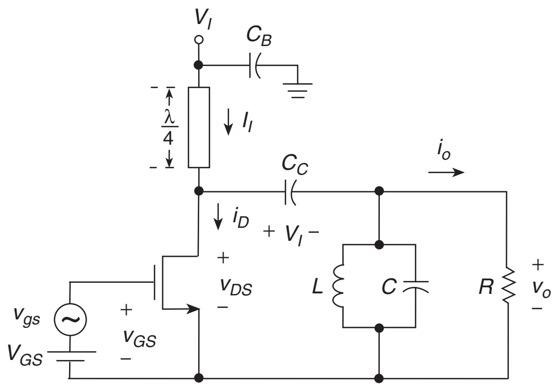

Figure 2.2 Class A RF power amplifier with a quarter-wavelength transformer used in place of an RF choke.

The coupling capacitor is high enough so that its ac component is nearly zero. Since the dc component of the voltage across the inductor L in steady state is zero, the voltage across the coupling capacitor is

$$V_{Cc} = V_I. \tag{2.3}$$

The ripple voltage across the coupling capacitor is low if $X_{Cc} \ll R$. The design equation is

$$X_{Cc} = \frac{1}{\omega C_c} = \frac{R}{10}. \tag{2.4}$$

2.3 Power MOSFET Characteristics

For low drain currents, the drain current of an n-channel enhancement-type MOSFET with a long channel and operating in the pinch-off region (or the saturation region) with a low-field and constant carrier mobility in the channel is given by a *square law*

$$i_D = \frac{1}{2}\mu_{n0} C_{ox}\left(\frac{W}{L}\right)(v_{GS} - V_t)^2$$
$$= K(v_{GS} - V_t)^2 \quad \text{for} \quad v_{GS} \geq V_t \quad \text{and} \quad v_{DS} \geq v_{GS} - V_t \tag{2.5}$$

where

$$K = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right), \tag{2.6}$$

μ_{n0} is the low-field surface electron mobility in the channel, $C_{ox} = \epsilon_{ox}/t_{ox}$ is the oxide capacitance per unit area of the gate capacitor, t_{ox} is the oxide thickness, $\epsilon_{ox} = 0.345$ pF/cm is the silicon oxide permittivity, L is the channel length, W is the channel width, and V_t is the threshold voltage. Typically, $t_{ox} = 0.1\ \mu\text{m}$, $\mu_{n0} C_{ox} = 20\ \mu\text{A/V}^2$, $C_{ox} = \frac{1}{3}$ mF/m^2, and $\mu_{n0} = 600\ \text{cm}^2/\text{Vs}$ for electrons in silicon at room temperature. The electron mobility in the channel is substantially lower than that in bulk silicon and can be as low as $200\ \text{cm}^2/\text{Vs}$. The low-frequency large-signal model of the long-channel MOSTETs for the pinch-off region at low drain currents is shown in Figure 2.3.

The drain current i_D is inversely proportional to the channel length L. Once the channel pinches off at $v_{DSsat} = v_{GS} - V_t$, the channel charge nearly vanishes at the drain end and

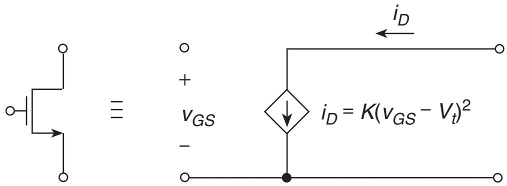

Figure 2.3 Low-frequency large-signal model of power MOSFETs for the pinch-off region at low drain currents (i.e., at constant carrier mobility).

the lateral electric field becomes very high. An increase in v_{DS} beyond v_{DSsat} causes the high-field region at the drain end to widen by a small distance ΔL. The voltage drop $v_{DS} - v_{DSsat}$ is across ΔL. As the drain-to-source voltage v_{DS} increases in the pinch-off region, the channel pinch-off point is moved slightly away from the drain toward the source. Thus, the effective channel length L_e is shortened to

$$L_e = L - \Delta L. \tag{2.7}$$

This phenomenon is known as *channel-length modulation* and is more significant for short channels. The drain current for MOSFETs with a short channel is given by

$$i_D = \frac{1}{2}\mu_{n0}C_{ox}\left(\frac{W}{L-\Delta L}\right)(v_{GS} - V_t)^2 = \frac{1}{2}\mu_{n0}C_{ox}\left[\frac{W}{L\left(1-\frac{\Delta L}{L}\right)}\right](v_{GS} - V_t)^2$$
$$\approx \frac{1}{2}\mu_{n0}C_{ox}\left(\frac{W}{L}\right)(v_{GS} - V_t)^2\left(1+\frac{\Delta L}{L}\right) = K(v_{GS} - V_t)^2(1+\lambda v_{DS}). \tag{2.8}$$

where the fractional change in the channel length is proportional to the drain-to-source voltage v_{DS} and is given by

$$\frac{\Delta L}{L} = \lambda v_{DS} = \frac{v_{DS}}{V_A}, \tag{2.9}$$

$\lambda = 1/V_A$ is the channel-length modulation parameter, and V_A is a voltage similar to the Early voltage. Another short-channel effect is the reduction of the threshold voltage V_t as the channel length decreases.

The transconductance for low drain currents at a given operating point determined by the dc components of gate-to-source voltage V_{GS}, drain current I_D, and drain-to-source voltage V_{DS} is

$$g_m = \left.\frac{di_D}{dv_{GS}}\right|_{v_{GS}=V_{GS}} = 2K(V_{GS} - V_t) = 2\sqrt{KI_D} = \sqrt{2\mu_{n0}C_{ox}\left(\frac{W}{L}\right)I_D}. \tag{2.10}$$

Hence, the ac small-signal component of the drain current of an ideal MOSFET with horizontal i_D-v_{DS} characteristics for low dc drain currents I_D is

$$i_d' = g_m v_{gs} = 2K(V_{GS} - V_t)v_{gs} = \mu_{n0}C_{ox}\left(\frac{W}{L}\right)(V_{GS} - V_t)v_{gs} = 2\sqrt{KI_D}v_{gs}. \tag{2.11}$$

The horizontal i_D-v_{DS} characteristics of real MOSFETs have a slope in the pinch-off region. As the drain-to-source voltage v_{DS} increases, the drain current i_D slightly increases. The small-signal output conductance of a MOSFET at a given operating point is

$$g_o = \frac{1}{r_o} = \left.\frac{di_D}{d_{DS}}\right|_{v_{GS}=V_{GS}} = \frac{1}{2}\mu_{n0}C_{ox}\left(\frac{W}{L}\right)(V_{GS} - V_t)^2\lambda \approx \lambda I_D = \frac{I_D}{V_A} \tag{2.12}$$

resulting in the small-signal output resistance of the MOSFET

$$r_o = \frac{\Delta v_{DS}}{\Delta i_D} = \frac{1}{\lambda I_D} = \frac{V_A}{I_D} = \frac{V_A}{\frac{1}{2}\mu_{n0} C_{ox} \left(\frac{W}{L}\right) (V_{GS} - V_t)^2}. \tag{2.13}$$

Hence, the ac small-signal component of the drain current in the pinch-off region is

$$i_d = g_m v_{gs} + g_o v_{ds} = g_m v_{gs} + \frac{v_{ds}}{r_o}. \tag{2.14}$$

A low-frequency small-signal model of power MOSFETs in the pinch-off region (with neglected transistor capacitances) representing (2.14) is shown in Figure 2.4.

A high-frequency model of power MOSFETs for the pinch-off region is depicted in Figure 2.5, where $C_{gd} = C_{rss}$ is the gate-to-drain capacitance, $C_{gs} = C_{iss} - C_{rss}$ is the gate-to-source capacitance, and $C_{ds} = C_{oss} - C_{rss}$ is the drain-to-source capacitance. Capacitances C_{rss}, C_{iss}, and C_{oss} are given in manufacturer's data sheets of power MOSFETs.

The average drift velocity of current carriers in a semiconductor caused by the electric field intensity E is given by

$$v = \frac{\mu_0 E}{1 + \frac{\mu_0 E}{v_{sat}}} \tag{2.15}$$

where μ_0 is the low-field carrier mobility and v_{sat} is the saturation carrier drift velocity. Typically, $v_{sat} = 8 \times 10^6$ cm/s for electrons in silicon (Si) and $v_{sat} = 2.7 \times 10^7$ cm/s for electrons in silicon carbide (SiC).

The average mobility of current carriers μ decreases when electric field intensity E, the doping concentration, and temperature T increase. The mobility as a function of the electric field intensity E is given by

$$\mu = \frac{v}{E} = \frac{\mu_0}{1 + \frac{\mu_0 E}{v_{sat}}} \tag{2.16}$$

The range of the electric field intensity E from point of view of the average mobility μ can be divided into three ranges: low-field range, intermediate-field range, and high-field range [13].

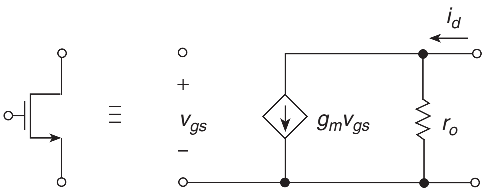

Figure 2.4 Low-frequency small-signal model of power MOSFETs for the pinch-off region.

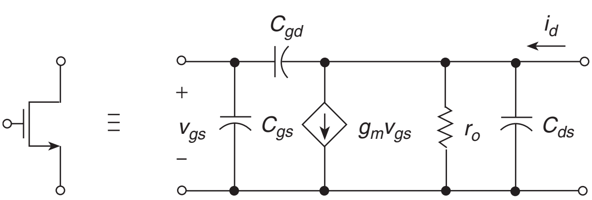

Figure 2.5 High-frequency small-signal model of power MOSFETs for the pinch-off region.

In the low-field range, the average carrier mobility is approximately constant

$$\mu \approx \mu_0 \quad \text{for} \quad E < 3 \times 10^3 \text{ V/cm}. \tag{2.17}$$

Therefore, the low-field average carrier drift velocity is directly proportional to the electric field intensity E

$$v \approx \mu_{n0} E \quad \text{for} \quad E < 3 \times 10^3 \text{ V/cm}. \tag{2.18}$$

For intermediate-field range, the carrier mobility μ decreases when the electric field intensity E increases and is given by (2.16). Consequently, the intermediate-field average drift carrier velocity increases at a lower rate than that of the low-field average drift velocity. This range of the electric field intensity is

$$3 \times 10^3 \text{ V/cm} \le E \le 6 \times 10^4 \text{ V/cm}. \tag{2.19}$$

As the electric field intensity enters the intermediate range, the collisions of electrons with the semiconductor lattice are so frequent and the time between the collisions is so short that the current carriers do not accelerate as much as at low electric field intensity, slowing down the movement of the current carriers as a group.

In the high-field range, the average carrier drift velocity reaches a constant value called the saturation drift velocity and is independent of the electric field intensity E

$$v \approx v_{sat} \quad \text{for} \quad E > 6 \times 10^4 \text{ V/cm}. \tag{2.20}$$

In this region, the carrier mobility μ is inversely proportional to the electric field intensity E. The collisions rate is so dramatically increased and the time between the collisions is so much reduced that the velocity of the movement of current carriers is independent of the electric field intensity.

At the boundary between the ohmic region and the saturation region,

$$v_{DS} = v_{GS} - V_t. \tag{2.21}$$

The electric field intensity in the channel is

$$E = \frac{v_{DS}}{L} = \frac{v_{GS} - V_t}{L} \tag{2.22}$$

and the average carrier drift velocity in an n-channel is

$$v = \mu_n E = \mu \frac{v_{DS}}{L} = \mu \frac{v_{GS} - V_t}{l}. \tag{2.23}$$

The drain current at the intermediate electric field intensity for the saturation region is

$$\begin{aligned}
i_D &= \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)(v_{GS} - V_t)^2 = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)v_{DS}^2 = \frac{1}{2}C_{ox} W v_{DS} \mu_n \left(\frac{v_{DS}}{L}\right) \\
&= \frac{1}{2}C_{ox} W v_{DS} \mu_n E = \frac{1}{2}C_{ox} W v_{DS} v = \frac{1}{2}C_{ox} W v_{DS} \frac{\mu_{n0} E}{1 + \dfrac{\mu_{n0} E}{v_{sat}}} \\
&= \frac{1}{2}\mu_{n0} C_{ox}\left(\frac{W}{L}\right) v_{DS}^2 \frac{1}{1 + \dfrac{\mu_{n0} v_{DS}}{L v_{sat}}} = \frac{1}{2}\frac{\mu_{n0}}{1 + \dfrac{\mu_{n0}(v_{GS} - V_t)}{L v_{sat}}} C_{ox}\left(\frac{W}{L}\right)(v_{GS} - V_t)^2 \\
&= \frac{1}{2}\frac{\mu_{n0}}{1 + \theta(v_{GS} - V_t)} C_{ox}\left(\frac{W}{L}\right)(v_{GS} - V_t)^2 \\
&\qquad \text{for} \quad v_{GS} \ge V_t \quad \text{and} \quad v_{DS} \ge v_{GS} - V_t
\end{aligned} \tag{2.24}$$

where the *mobility degradation coefficient* is

$$\theta = \frac{\mu_{n0}}{L v_{sat}}. \tag{2.25}$$

The shorter the channel, the higher is the mobility degradation coefficient. The drain current i_D follows the average carrier drift velocity v. The decrease in the current carrier mobility μ_n in the channel is called the 'short-channel effect.' The carrier mobility in the channel for the n-channel MOSFET can be expressed as

$$\mu_n = \frac{\mu_{n0}}{1 + \theta(V_{GS} - V_t)}. \tag{2.26}$$

Taking into account the channel-length modulation, the drain current for the intermediate electric field intensity in the pinch-off region is

$$i_D = \frac{1}{2}\frac{\mu_{n0}}{1 + \theta(v_{GS} - V_t)} C_{ox}\left(\frac{W}{L}\right)(v_{GS} - V_t)^2(1 + \lambda v_{DS})$$
$$\text{for} \quad v_{GS} \geq V_t \quad \text{and} \quad v_{DS} \geq v_{GS} - V_t. \tag{2.27}$$

The i_D-v_{GS} characteristics are displayed in Figure 2.6.

For high drain currents, $v = v_{sat}$ and therefore the i_D-v_{GS} characteristic of an n-channel enhancement-type power MOSFET for the pinch-off region (or the saturation region) is nearly linear and is given by a *linear law* [13]

$$i_D = \frac{1}{2} C_{ox} W v_{sat}(v_{GS} - V_t) = K_{sat}(v_{GS} - V_t)(1 + \lambda v_{DS})$$
$$\text{for} \quad v_{GS} \geq V_t \quad \text{and} \quad v_{DS} \geq v_{GS} - V_t \tag{2.28}$$

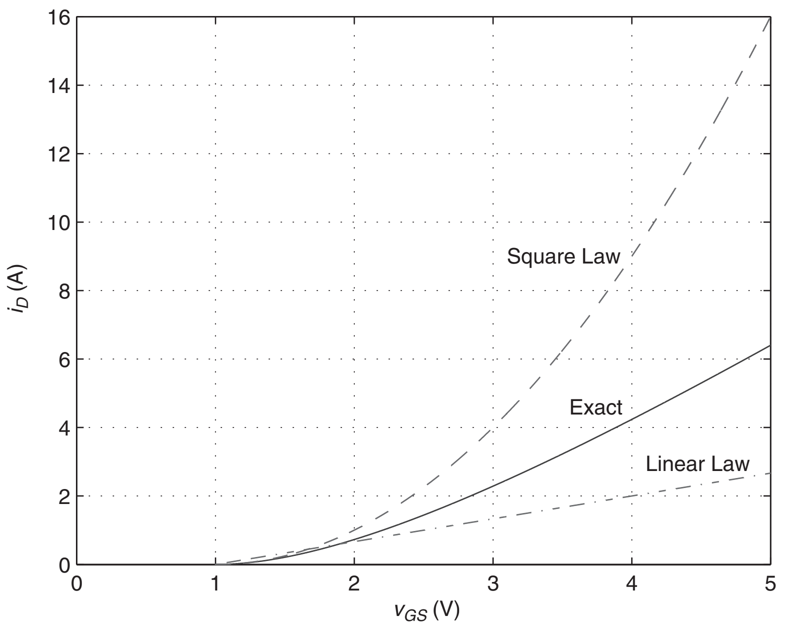

Figure 2.6 MOSFET i_D-v_{GS} characteristics described by a square low, linear law, and exact equation for $V_t = 1\,\text{V}$, $\mu_{n0} = 600\,\text{cm}^2/\text{s}$, $C_{ox} = \frac{1}{3}\,\text{mF/m}^2$, $W/L = 10^5$, $v_{sat} = 8 \times 10^6\,\text{cm/s}$, and $\lambda = 0$. Exact: $L = 2\,\mu\text{m}$ and $\theta = 0.375$. Linear: $L = 0.5\,\mu\text{m}$ and $W = 0.5 \times 10^5\,\mu\text{m}$.

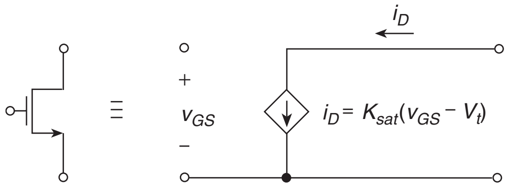

Figure 2.7 Low-frequency large-signal model of power MOSFETs for constant carrier drift velocity $v \approx v_{sat}$.

where

$$K_{sat} = \frac{1}{2} C_{ox} W v_{sat}. \tag{2.29}$$

Thus, the large-signal model of the MOSFET for high drain currents in the constant dift velocity range consists of the voltage-dependent current source driven by the voltage $v_{GS} - V_t$ for $v_{GS} > V_t$ and the coefficient of this source is K_{sat}. The low-frequency large-signal model of power MOSFETs (with neglected capacitances) for constant carrier drift velocity $v \approx v_{sat}$ is shown in Figure 2.7.

The transconductance of the MOSFET for high drain currents is

$$g_m = \left. \frac{\mathrm{d}i_D}{\mathrm{d}v_{GS}} \right|_{v_{GS}=V_{GS}} = K_{sat} = \frac{1}{2} C_{ox} W v_{sat}. \tag{2.30}$$

For high drain currents, g_m is constant and independent of the operating point if $v_{GS} > V_t$. The ac model of the MOSFET consists of a voltage-dependent current source i_d driven by the gate-to-source voltage v_{gs}. The ac component of the drain current is

$$i_d' = g_m v_{gs} = K_{sat} v_{gs} = \frac{1}{2} C_{ox} W v_{sat} v_{gs}. \tag{2.31}$$

The small-signal output conductance of a MOSFET at a given operating point is

$$g_o = \frac{1}{r_o} = \left. \frac{\mathrm{d}i_D}{\mathrm{d}_{DS}} \right|_{v_{GS}=V_{GS}} = K_{sat}(V_{GS} - V_t)^2 \lambda = \frac{1}{2} C_{ox} W v_{sat} (V_{GS} - V_t)^2 \lambda \approx \lambda I_D = \frac{I_D}{V_A} \tag{2.32}$$

resulting in the small-signal output resistance of the MOSFET

$$r_o = \frac{\Delta v_{DS}}{\Delta i_D} = \frac{1}{\lambda I_D} = \frac{V_A}{I_D} = \frac{V_A}{\frac{1}{2} C_{ox} W v_{sat} (V_{GS} - V_t)}. \tag{2.33}$$

The drain current is given by (2.14) and the low-frequency small-signal model for $v \approx v_{sat}$ is displayed in Figure 2.4, where g_m and r_o are given by (2.30) and (2.32), respectively.

Power MOSFETs operate with either characteristics intermediate between the constant carrier mobility and the constant drift velocity regions (like in Class A power amplifiers) or in all the three regions (like in Class B and C power amplifiers). A SPICE model of a MOSFET can be used for high-frequency large-signal computer simulation.

2.4 Waveforms of Class A RF Amplifier

Figure 2.8 shows current and voltage waveforms in the Class A power amplifier. The gate-to-source voltage is given by

$$v_{GS} = V_{GS} + v_{gs} = V_{GS} + V_{gsm} \sin \omega t \tag{2.34}$$

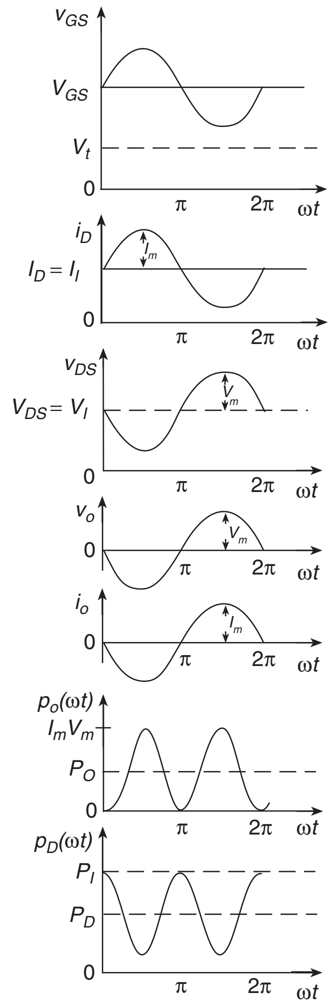

Figure 2.8 Waveforms in Class A RF power amplifier.

where V_{GS} is the dc gate-to-source bias voltage, V_{gsm} is the amplitude of the ac component of the gate-to-source voltage v_{gs}, and $\omega = 2\pi f$ is the operating angular frequency. The dc component of the voltage V_{GS} is higher than the MOSFET threshold voltage V_t such that the conduction angle of the drain current is $2\theta = 360^\circ$. To keep the transistor in the active region at all times, the following condition must be satisfied:

$$V_{GS} - V_{gsm} > V_t. \tag{2.35}$$

The drain current in the Class A power amplifier is

$$i_D = K_{sat}(V_{GS} + V_{gsm}\cos\omega t - V_t) = K_{sat}(V_{GS} - V_t) + K_{sat}V_{gsm}\cos\omega t \tag{2.36}$$

where K_{sat} is the MOSFET parameter and V_t is the MOSFET threshold voltage. The drain current is expressed as

$$i_D = I_D + i_d = I_D + I_m\cos\omega t = I_I + I_m\cos\omega t \tag{2.37}$$

where the dc component of the drain current I_D is equal to the dc supply current I_I

$$I_D = I_I. \tag{2.38}$$

The ac component of the drain current is

$$i_d = I_m \cos \omega t \tag{2.39}$$

where I_m is the amplitude of the ac component of the drain current. To avoid distortion of the sinusoidal ac component of the drain current, the maximum amplitude of the ac component of the drain current is given by

$$I_{m(max)} = I_I = I_D. \tag{2.40}$$

An ideal parallel-resonant circuit L-C presents an infinite reactance at the resonant frequency $f_0 = 1/(2\pi\sqrt{LC})$. Therefore, the output current at the resonant frequency f_0 of the parallel-resonant circuit is

$$i_o = i_{C_c} = I_I - i_D = I_I - I_I - i_d = -i_d = -I_m \cos \omega t. \tag{2.41}$$

The fundamental component of the drain-to-source voltage is

$$v_{ds} = -Ri_o = -RI_m \cos \omega t = -V_m \cos \omega t \tag{2.42}$$

where $V_m = RI_m$. The drain-to-source voltage at the resonant frequency f_0 of the parallel-resonant circuit is

$$v_{DS} = V_{DS} + v_{ds} = V_{DS} - V_m \cos \omega t = V_I - V_m \cos \omega t \tag{2.43}$$

where the dc component of the drain-to-source bias voltage V_{DS} is equal to the dc supply voltage V_I

$$V_{DS} = V_I. \tag{2.44}$$

Since operation of an ideal transistor as a dependent-current source can be maintained only when v_{DS} is positive, it is necessary to limit the amplitude V_m to a value lower than V_I

$$V_{m(max)} = RI_{m(max)} = V_I = V_{DS}. \tag{2.45}$$

The output voltage at the resonant frequency f_0 is

$$v_o = v_{DS} - V_{Cc} = V_I + v_{ds} - V_I = v_{ds} = -V_m \cos \omega t. \tag{2.46}$$

in which the amplitude of the ac component of the drain-to-source voltage and the output voltage is

$$V_m = RI_m. \tag{2.47}$$

The instantaneous drain power at the operating frequency $f = f_0 = 1/(2\pi\sqrt{LC})$ is given by

$$p_o(\omega t) = i_o v_o = I_m V_m \cos^2 \omega t = \frac{I_m V_m}{2}(1 + \cos 2\omega t). \tag{2.48}$$

The instantaneous power dissipation in the transistor at $f = f_0$ is

$$\begin{aligned} p_D(\omega t) &= i_D v_{DS} = (I_I + I_m \cos \omega t)(V_I - V_m \cos \omega t) \\ &= I_I \left(1 + \frac{I_m}{I_I} \cos \omega t\right) V_I \left(1 - \frac{V_m}{V_I} \cos \omega t\right) \end{aligned}$$

$$= P_I \left(1 + \frac{I_m}{I_I}\cos\omega t\right)\left(1 - \frac{V_m}{V_I}\cos\omega t\right). \tag{2.49}$$

For $I_m = I_I$ and $V_m = V_I$, the instantaneous power dissipation in the transistor at $f = f_0$ becomes

$$p_D(\omega t) = I_I(1 + \cos\omega t)V_I(1 - \cos\omega t) = P_I(1 - \cos^2\omega t) = \frac{P_I}{2}(1 - \cos 2\omega t). \tag{2.50}$$

The normalized instantaneous drain power loss $p_D(\omega t)/P_I$ in the Class A power amplifier at $I_m = I_I, f = f_0$, and selected values of V_m/V_I are shown in Figure 2.9.

When the operating frequency f is different than the resonant frequency f_0, the drain-to-source voltage is

$$v_{DS} = V_I - V_m \cos(\omega t + \phi) \tag{2.51}$$

and the instantaneous power dissipation in the transistor is

$$\begin{aligned} p_D(\omega t) &= i_D v_{DS} = (I_I + I_m\cos\omega t)[V_I - V_m\cos(\omega t + \phi)] \\ &= I_I\left(1 + \frac{I_m}{I_I}\cos\omega t\right)V_I\left[1 - \frac{V_m}{V_I}\cos(\omega t + \phi)\right] \\ &= P_I\left(1 + \frac{I_m}{I_I}\cos\omega t\right)\left[1 - \frac{V_m}{V_I}\cos(\omega t + \phi)\right]. \end{aligned} \tag{2.52}$$

The normalized instantaneous drain power loss $p_D(\omega t)/P_I$ in the Class A power amplifier at $I_m = I_I$, $\phi = 10^\circ$, and selected values of V_m/V_I are shown in Figure 2.10.

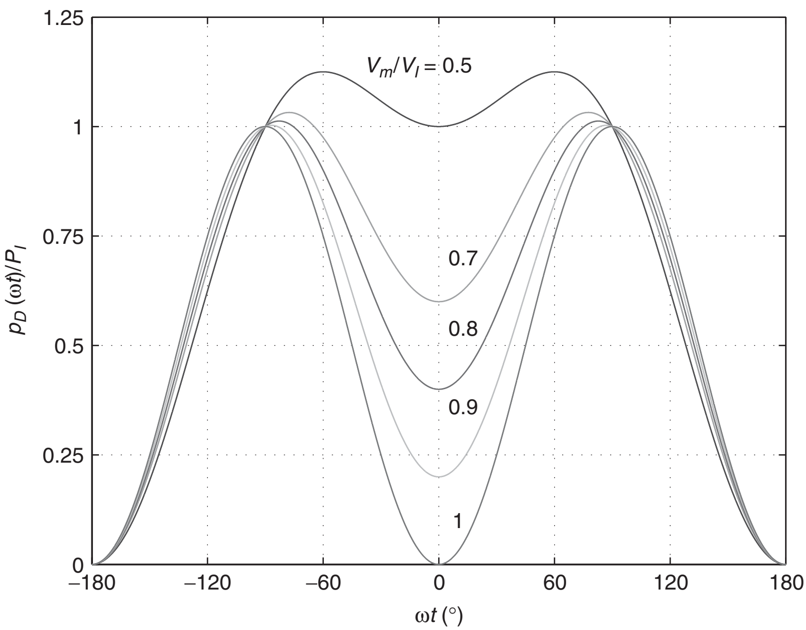

Figure 2.9 Normalized instantaneous drain power loss $p_D(\omega t)/P_I$ in the Class A amplifier for $I_m = I_I, f = f_0$, and selected values of V_m/V_I.

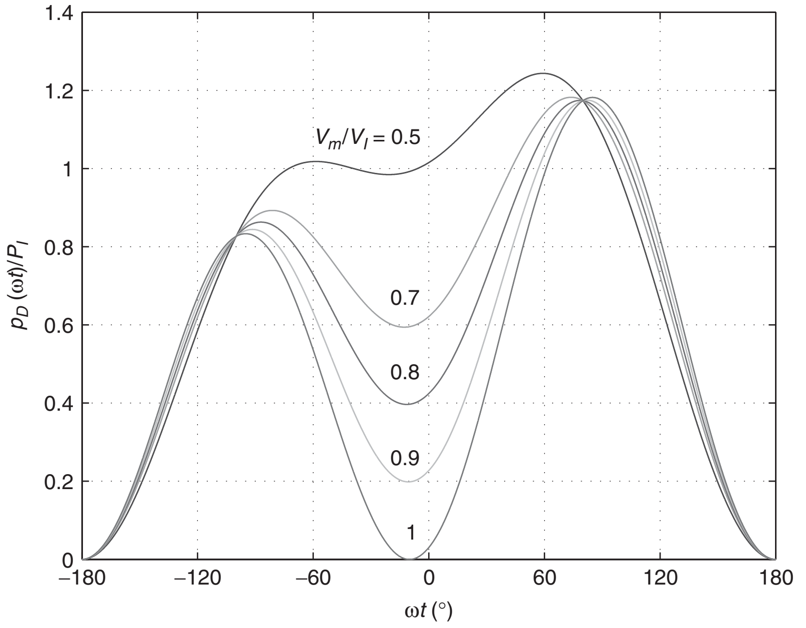

Figure 2.10 Normalized instantaneous drain power loss $p_D(\omega t)/P_I$ in the Class A amplifier for $I_m = I_I$, $\phi = 10^\circ$, and selected values of V_m/V_I.

2.5 Parameters of Class A RF Power Amplifier

The dc supply current is

$$I_I = I_{m(max)} = \frac{V_{m(max)}}{R} = \frac{V_{DS}}{R} = \frac{V_I}{R}. \tag{2.53}$$

The dc resistance seen by the power supply V_I is

$$R_{DC} = \frac{V_I}{I_I} = R. \tag{2.54}$$

The dc supply power (or the dc input power) is given by

$$P_I = I_D V_{DS} = I_{m(max)} V_I = I_I V_I = \frac{V_I^2}{R}. \tag{2.55}$$

The dc supply power is constant and independent of the amplitude of the output voltage V_m. The power dissipated in the drain at $f = f_0$ is given by

$$P_{DS} = \frac{I_m V_m}{2} = \frac{R I_m^2}{2} = \frac{V_m^2}{2R} = \frac{1}{2}\left(\frac{V_I^2}{R}\right)\left(\frac{V_m}{V_I}\right)^2. \tag{2.56}$$

For $V_{m(max)} = V_I$, the drain power reaches its maximum value

$$P_{DSmax} = \frac{V_I^2}{2R} = \frac{P_I}{2}. \tag{2.57}$$

The output power is proportional to the square of the output voltage amplitude V_m. The power dissipated in the transistor (excluding the gate-drive power) is

$$P_D = P_I - P_{DS} = I_I V_I - \frac{V_m^2}{2R} = \frac{V_I^2}{R} - \frac{V_m^2}{2R} = \frac{V_I^2}{R}\left[1 - \frac{1}{2}\left(\frac{V_m}{V_I}\right)^2\right]. \tag{2.58}$$

The maximum power dissipation in the transistor occurs at $P_{DS} = 0$ (i.e., for $V_m = 0$)

$$P_{Dmax} = P_I = I_I V_I = \frac{V_I^2}{R}. \tag{2.59}$$

A heat sink for the transistor should be designed for this power dissipation. The minimum power dissipation in the transistor occurs at P_{DSmax}

$$P_{Dmin} = P_I - P_{DSmax} = \frac{V_I^2}{R} - \frac{V_I^2}{2R} = \frac{V_I^2}{2R} = \frac{P_I}{2}. \tag{2.60}$$

Figure 2.11 shows plots of P_I, P_O, and P_D as a function of the amplitude of the output voltage V_m.

The drain efficiency of the Class A power amplifier at $f = f_0$ is given by

$$\eta_D = \frac{P_{DS}}{P_I} = \frac{1}{2}\left(\frac{I_m}{I_I}\right)\left(\frac{V_m}{V_I}\right) = \frac{\frac{V_m^2}{2R}}{\frac{V_I^2}{R}} = \frac{V_m^2}{2RV_I I_{m(max)}} = \frac{V_m^2}{2RV_I\frac{V_{m(max)}}{R}} = \frac{1}{2}\left(\frac{V_m}{V_I}\right)^2. \tag{2.61}$$

The maximum theoretical efficiency of the Class A RF amplifier with choke L_f is

$$\eta_{D(max)} = \frac{V_{m(max)}}{2V_I} = 0.5. \tag{2.62}$$

The drain efficiency η_D as a function of V_m is shown in Figure 2.12.

In reality, a MOSFET behaves like a dependent current source above a minimum drain-to-source voltage

$$V_{DSmin} = v_{GS} - V_t = V_{GS} + V_{gsm} - V_t \tag{2.63}$$

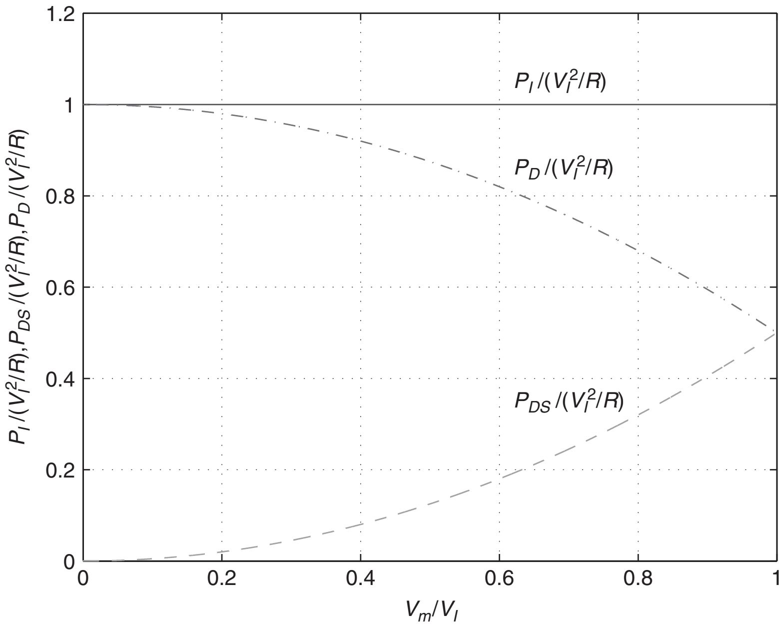

Figure 2.11 DC supply power P_I, output power P_O, and dissipated power P_D for Class A RF power amplifier with choke inductor as functions of the output voltage amplitude V_m.

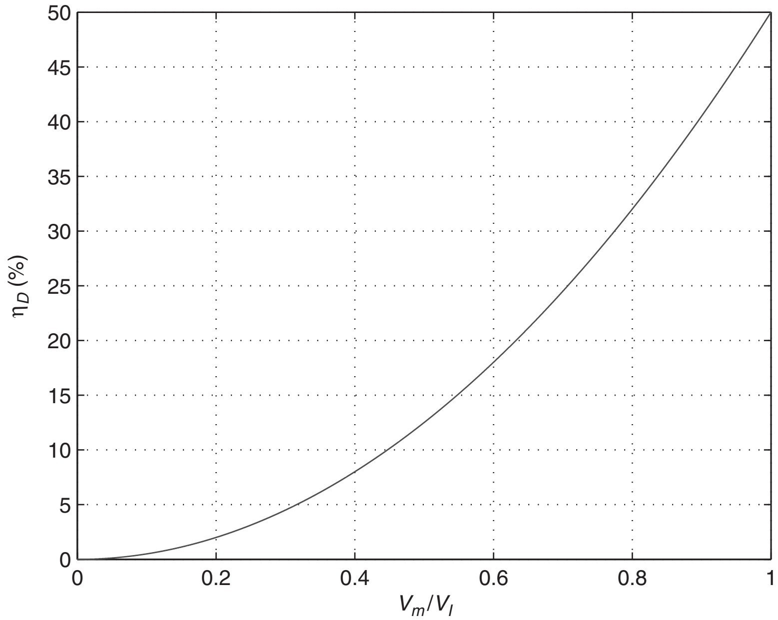

Figure 2.12 Drain efficiency η_D of Class A RF power amplifier (with choke inductor) as a function of the output voltage amplitude V_m.

and therefore the maximum amplitude of the ac component of v_{DS} is given by

$$V_{m(max)} = V_I - V_{DSmin} = V_I - V_{GS} - V_{gsm} + V_t \tag{2.64}$$

resulting in the maximum drain efficiency

$$\eta_{Dmax} = \frac{V_{m(max)}}{2V_I} = \frac{V_I - V_{DSmin}}{2V_I} = \frac{1}{2}\left(1 - \frac{V_{DSmin}}{V_I}\right) < 0.5. \tag{2.65}$$

The efficiency of the Class A RF power amplifier is low. Therefore, this amplifier is used in low-power applications or as an intermediate stage in a cascaded amplifier. The efficiency of Class A audio power amplifiers is below 25% because the RF choke is replaced by a resistor at low frequencies, which dissipates power.

Assuming that $I_m = I_I$ and $V_m = V_I$, the maximum drain current is

$$I_{DM} = I_I + I_m = 2I_I \tag{2.66}$$

and the maximum drain-to-source voltage is

$$V_{DSM} = V_I + V_m = 2V_I. \tag{2.67}$$

The output power capability is

$$c_p = \frac{P_{O(max)}}{I_{DM}V_{DSM}} = \frac{\eta_D P_I}{I_{DM}V_{DSM}} = \eta_D\left(\frac{I_I}{I_{DM}}\right)\left(\frac{V_I}{V_{DSM}}\right) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8} = 0.125 \tag{2.68}$$

or

$$c_p = \frac{P_{O(max)}}{V_{DSM}I_{DM}} = \frac{\frac{V_I^2}{2R}}{(2V_I)(2I_I)} = \frac{\frac{V_I^2}{2R}}{(2V_I)\left(\frac{2V_I}{R}\right)} = \frac{1}{8} = 0.125 \tag{2.69}$$

If the operating frequency is not equal to the resonant frequency, the drain power is

$$P_{DS} = \frac{1}{2} I_m V_m \cos\phi \tag{2.70}$$

where ϕ is the phase shift between the drain current and the drain-to-source voltage. As a result, some equations should be modified.

The gate-drive power is

$$P_G = \frac{1}{2} I_{gm} V_{gsm} \cos\phi_G = \frac{1}{2} I_{gm}^2 R_G \cos\phi_G = \frac{V_{gsm}^2}{2R_G} \cos\phi_G \tag{2.71}$$

where R_G is the gate resistance and ϕ_G is the phase shift between the gate voltage and current. The gate impedance consists of a gate resistance R_G and a gate input capacitance $C_g = C_{gs} + (1 - A_m)C_{gs}$, where C_{gs} is the gate-to-source capacitance, C_{gd} is the gate-to-drain capacitance, and A_m is the gate-to-drain voltage gain. The gate input capacitance is increased by the Miller effect. The power gain of the amplifier is

$$k_p = \frac{P_O}{P_G} = \left(\frac{V_m}{V_{gsm}}\right)^2 \frac{1}{\cos\phi_G} \frac{R_G}{R}. \tag{2.72}$$

2.6 Parallel-resonant Circuit

The resonant frequency of the resonant circuit is

$$\omega_0 = \frac{1}{\sqrt{LC}}. \tag{2.73}$$

The characteristic impedance of the parallel-resonant circuit is

$$Z_o = \sqrt{\frac{L}{C}} = \omega_0 L = \frac{1}{\omega_0 C}. \tag{2.74}$$

The loaded-quality factor is

$$Q_L = 2\pi \frac{\text{Maximum energy stored at } f_0}{\text{Total energy lost per cycle at } f_0} = 2\pi \frac{[w_L(\omega_0 t) + w_C(\omega_0 t)]_{max}}{P_O T} \tag{2.75}$$

where the instantaneous energy stored in the capacitor C is

$$w_C(\omega_0 t) = \frac{1}{2} C v_o^2 = \frac{1}{2} C V_m^2 \sin^2 \omega_0 t \tag{2.76}$$

and the instantaneous energy stored in the inductor L is

$$w_L(\omega_0 t) = \frac{1}{2} L i_L^2 = \frac{1}{2} L I_m^2 \cos^2 \omega_0 t = \frac{1}{2} L \left(\frac{V_m}{\omega_0 L}\right)^2 \cos^2 \omega_0 t = \frac{1}{2} C V_m^2 \cos^2 \omega_0 t. \tag{2.77}$$

Hence,

$$Q_L = 2\pi \frac{\frac{1}{2} C V_m^2 (\sin^2 \omega_0 t + \cos^2 \omega_0 t)}{V_m^2 / 2R f_0} = \omega_0 C R = \frac{R}{Z_o} = \frac{R}{\omega_0 L}. \tag{2.78}$$

The amplitude of the current through the inductor L and the capacitor C is

$$I_{Lm} = I_{Cm} = \frac{V_m}{\omega_0 L} = \frac{R I_m}{\omega_0 L} = Q_L I_m. \tag{2.79}$$

For the basic parallel-resonant circuit, the output voltage v_o is equal to the fundamental component of the drain-to-source voltage v_{ds1}

$$v_o = v_{ds1}. \tag{2.80}$$

The phasor of the output voltage V_o in terms of the phasor of the fundamental component of the drain current I_d is given by

$$V_o = -ZI_d = -\frac{I_d}{Y} \tag{2.81}$$

where the admittance of the parallel-resonant circuit is

$$Y = \frac{I_d}{V_o} = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right) = \frac{1}{R}\left[1 + jQ_L\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)\right] = |Y|e^{j\phi_Y} \tag{2.82}$$

in which

$$|Y| = \frac{1}{R}\sqrt{1 + Q_L^2\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)^2} \tag{2.83}$$

and

$$\phi_Y = \arctan\left[Q_L\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)\right]. \tag{2.84}$$

The impedance of the parallel-resonant circuit is

$$Z = \frac{V_o}{I_d} = \frac{1}{Y} = \frac{1}{\frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)} = \frac{R}{1 + jQ_L\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)} = |Z|e^{j\phi_Z} \tag{2.85}$$

where

$$|Z| = \frac{R}{\sqrt{1 + Q_L^2\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)^2}} \tag{2.86}$$

and

$$\phi_Z = -\arctan\left[Q_L\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)\right]. \tag{2.87}$$

Figures 2.13 and 2.14 show the normalized input impedance $|Z|/R$ and phase ϕ_Z of the parallel-resonant circuit.

The parallel-resonant circuit acts like a bandpass filter. The voltage gain of the amplifier decreases by 3 dB when

$$\frac{|Z|}{R} = \frac{1}{\sqrt{2}} \tag{2.88}$$

yielding

$$Q_L\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) = -1 \tag{2.89}$$

and

$$Q_L\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) = 1. \tag{2.90}$$

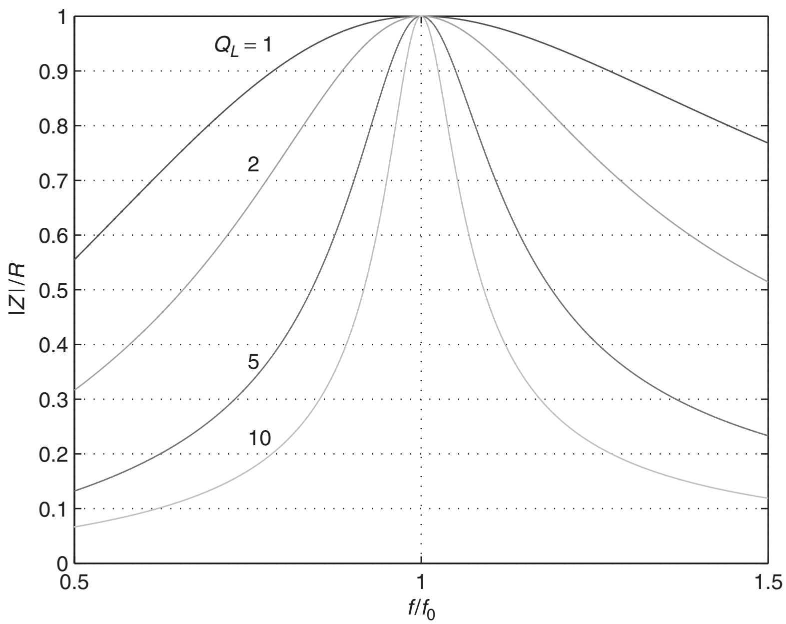

Figure 2.13 Normalized magnitude of the input impedance of the parallel-resonant circuit $|Z|/R$.

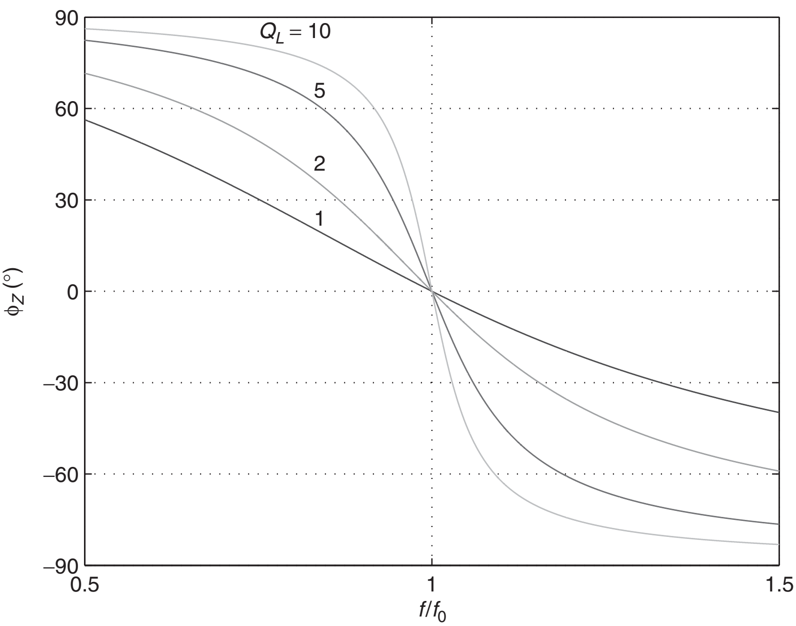

Figure 2.14 Phase of the input impedance of the parallel-resonant circuit ϕ_Z.

Hence, the lower 3-dB frequency is

$$f_L = f_0 \left[\sqrt{1 + \left(\frac{1}{2Q_L} \right)^2} - \frac{1}{2Q_L} \right] \tag{2.91}$$

and the upper 3-dB frequency is

$$f_H = f_0 \left[\sqrt{1 + \left(\frac{1}{2Q_L} \right)^2} + \frac{1}{2Q_L} \right]. \tag{2.92}$$

The 3-dB bandwidth of the resonant circuit is

$$BW = f_H - f_L = \frac{f_0}{Q_L}. \tag{2.93}$$

2.7 Power Losses and Efficiency of Parallel Resonant Circuit

Figure 2.15 shows the equivalent circuit of the Class A RF power amplifier with parasitic resistances. The power loss in the equivalent series resistance (ESR) of the inductor r_L is

$$P_{rL} = \frac{r_L I_{Lm}^2}{2} = \frac{r_L Q_L^2 I_m^2}{2} = \frac{r_L Q_L^2}{R} P_O. \tag{2.94}$$

Similarly, the power loss in the ESR of the capacitor r_C is

$$P_{rC} = \frac{r_C I_{Cm}^2}{2} = \frac{r_C Q_L^2 I_m^2}{2} = \frac{r_C Q_L^2}{R} P_O. \tag{2.95}$$

The power loss in the ESR r_{Cc} of the coupling capacitor C_c is

$$P_{rCc} = \frac{r_{Cc} I_m^2}{2} = \frac{r_{Cc}}{R} P_O. \tag{2.96}$$

Neglecting the ac component of the current through the choke L_f and assuming that $I_I \approx I_m$, the power loss in the dc resistance of the choke is

$$P_{rLf} = r_{Lf} I_I^2 \approx \frac{r_{Lf}}{R} P_O. \tag{2.97}$$

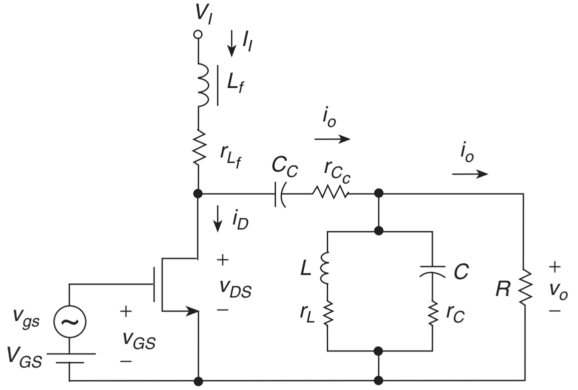

Figure 2.15 Equivalent circuit of the Class A RF power amplifier with parasitic resistances.

The overall power loss in the resonant circuit is

$$P_r = P_{rL} + P_{rC} + P_{rCc} + P_{Lf} = \frac{(r_L + r_C)Q_L^2 I_m^2}{2} + \frac{r_{Cc} I_m^2}{2} + r_{Lf} I_I^2$$
$$= \left[\frac{Q_L^2(r_L + r_C) + r_{Cc} + r_{Lf}}{R}\right] P_O \tag{2.98}$$

resulting in the efficiency of the resonant circuit

$$\eta_r = \frac{P_O}{P_O + P_r} = \frac{1}{1 + \frac{P_r}{P_O}} = \frac{1}{1 + \frac{Q_L^2(r_L + r_C) + r_{Cc} + r_{Lf}}{R}}. \tag{2.99}$$

The overall power loss of the transistor and the resonant circuit is

$$P_{LS} = P_D + P_r = P_D + P_{rL} + P_{rC} + P_{rCc} + P_{rLf}. \tag{2.100}$$

The overall efficiency of the amplifier is

$$\eta = \frac{P_O}{P_I} = \frac{P_O}{P_{DS}}\frac{P_{DS}}{P_I} = \eta_D \eta_r = \frac{P_O}{P_O + P_{LS}} = \frac{1}{1 + \frac{P_{LS}}{P_O}}. \tag{2.101}$$

The gate-drive power is

$$P_G = \frac{I_{gm} V_{gsm}}{2}. \tag{2.102}$$

where I_{gm} is the amplitude of the gate current. The power-added efficiency is

$$\eta_{PAE} = \frac{P_O - P_G}{P_I} = \frac{P_O - \frac{P_O}{k_p}}{P_I} = \frac{P_O}{P_I}\left(1 - \frac{1}{k_p}\right) = \eta\left(1 - \frac{1}{k_p}\right). \tag{2.103}$$

where the power gain is

$$k_p = \frac{P_O}{P_G}. \tag{2.104}$$

When $k_p = 1$, $\eta_{PAE} = 0$.

Example 2.1

Design a basic Class A RF power amplifier to meet the following specifications: $P_O = 0.25\,\text{W}$, $f = 1\,\text{GHz}$, $BW = 100\,\text{MHz}$, $V_I = 3.3\,\text{V}$, $r_L = 0.05\,\Omega$, $r_C = 0.01\,\Omega$, $r_{Cc} = 0.07\,\Omega$, and $r_{Lf} = 0.02\,\Omega$.

Solution. The loaded quality factor is

$$Q_L = \frac{f_0}{BW} = \frac{10^9}{10^8} = 10. \tag{2.105}$$

Assuming $V_{DSmin} = 0.5\,\text{V}$, the drain efficiency is

$$\eta_D = \frac{1}{2} - \frac{V_{DSmin}}{2V_I} = \frac{1}{2} - \frac{0.5}{2 \times 3.3} = 0.4242 = 42.42\%. \tag{2.106}$$

Assuming the efficiency of the resonant circuit $\eta_r = 0.6$, The total efficiency is

$$\eta = \eta_D \eta_r = 0.4242 \times 0.6 = 0.2545. \tag{2.107}$$

Hence, the dc supply power is

$$P_I = \frac{P_O}{\eta} = \frac{0.25}{0.2545} = 0.9823\,\mathrm{W}. \tag{2.108}$$

The drain ac power is

$$P_{DS} = \eta_D P_I = 0.4242 \times 0.9823 = 0.4167\,\mathrm{W}. \tag{2.109}$$

The power loss in the transistor is

$$P_D = P_I - P_{DS} = 0.9823 - 0.4167 = 0.5656\,\mathrm{W}. \tag{2.110}$$

The amplitude of the output voltage is

$$V_m = V_I - V_{DSmin} = 3.3 - 0.5 = 2.8\,\mathrm{V}. \tag{2.111}$$

The maximum drain-to-source voltage is

$$V_{DSmax} = V_I + V_m = 3.3 + 2.8 = 6.1\,\mathrm{V}. \tag{2.112}$$

The load resistance is

$$R = \frac{V_m^2}{2P_O} = \frac{2.8^2}{2 \times 0.25} = 15.68\,\Omega. \tag{2.113}$$

The resonant inductance and capacitance are

$$L = \frac{R}{\omega_0 Q_L} = \frac{15.68}{2\pi \times 10^9 \times 10} = 0.25\,\mathrm{nH} \tag{2.114}$$

and

$$C = \frac{Q_L}{\omega_0 R} = \frac{10}{2\pi \times 10^9 \times 15.68} = 101.5\,\mathrm{pF}. \tag{2.115}$$

The reactance of the RF choke is

$$X_{Lf} = 10R = 10 \times 15.68 = 156.8\,\Omega \tag{2.116}$$

resulting in the choke inductance

$$L_f = \frac{X_{Lf}}{\omega} = \frac{156.8}{2\pi \times 10^9} = 24.96\,\mathrm{nH}. \tag{2.117}$$

The reactance of the coupling capacitor is

$$X_{Cc} = \frac{R}{10} = \frac{15.68}{10} = 1.568\,\Omega \tag{2.118}$$

yielding

$$C_c = \frac{1}{\omega X_{Cc}} = \frac{1}{2\pi \times 10^9 \times 1.568} = 101.5\,\mathrm{pF}. \tag{2.119}$$

The amplitude of the load current and the ac component of the drain current is

$$I_m = \frac{V_m}{R} = \frac{2.8}{15.68} = 0.1786\,\mathrm{A}. \tag{2.120}$$

The dc component of the drain current is

$$I_D = I_m = 0.1786\,\mathrm{A}. \tag{2.121}$$

Let $I_D = I_I = 0.1786$ A. The maximum drain current is

$$I_{DM} = I_D + I_m = 0.1786 + 0.1786 = 0.3572\,\mathrm{A}. \tag{2.122}$$

The power loss in the inductor ESR is

$$P_{rL} = \frac{r_L I_{Lm}^2}{2} = \frac{r_L Q_L^2 I_m^2}{2} = \frac{0.05 \times 10^2 \times 0.1786^2}{2} = 0.07975\,\text{W} \tag{2.123}$$

and the power loss in the capacitor ESR is

$$P_{rC} = \frac{r_C I_{Cm}^2}{2} = \frac{r_C Q_L^2 I_m^2}{2} = \frac{0.01 \times 10^2 \times 0.1786^2}{2} = 0.0159\,\text{W}. \tag{2.124}$$

The power loss in the ESR r_{Cc} of the coupling capacitor C_c is

$$P_{rCc} = \frac{r_{Cc} I_m^2}{2} = \frac{0.07 \times 0.1786^2}{2} = 0.001\,\text{W}. \tag{2.125}$$

The power loss in the ESR of the choke is

$$P_{rLf} = r_{Lf} I_I^2 = 0.02 \times 0.1786^2 = 0.0006379\,\text{W}. \tag{2.126}$$

The power loss in the resonant circuit is

$$P_r = P_{rL} + P_{rC} + P_{rCc} + P_{rLf} = 0.07975 + 0.0159 + 0.001 + 0.0006379 = 0.09728\,\text{W}. \tag{2.127}$$

The efficiency of the resonant circuit is

$$\eta_r = \frac{P_O}{P_O + P_r} = \frac{0.25}{0.25 + 0.09728} = 71.98\%. \tag{2.128}$$

The efficiency of the amplifier is

$$\eta = \eta_D \eta_r = 0.4242 \times 0.7198 = 30.53\%. \tag{2.129}$$

The power loss in the transistor and the resonant circuit is

$$P_{LS} = P_D + P_r = 0.5656 + 0.09728 = 0.66288\,\text{W}. \tag{2.130}$$

The maximum power is dissipated in the transistor at $P_O = 0$. Therefore, the heat sink should be designed for $P_{LSmax} = P_I \approx 1\,\text{W}$.

Let us use a power MOSFET Q_2 with the following parameters: $K_n = \mu_n C_{ox} = 0.142$ mA/V^2, $V_t = 0.356\,\text{V}$, and $L = 0.35\,\mu\text{m}$. Assuming $V_{GS} = 1\,\text{V}$, the aspect ratio of the power MOSFET is given by

$$\frac{W}{L} = \frac{2I_D}{K_n(V_{GS} - V_t)^2} = \frac{2 \times 0.1786}{0.142 \times 10^{-3}(1 - 0.356)^2} = 6065 \tag{2.131}$$

yielding the MOSFET channel width

$$W = 6065L = 6065 \times 0.35 \times 10^{-6} = 2122.75\,\mu\text{m} = 2.12275\,\text{mm}. \tag{2.132}$$

A current mirror can be used to bias the power transistor, as shown in Figure 2.16. The reference current is

$$I_{ref} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W_1}{L_1}\right)(V_{GS} - V_t)^2 \tag{2.133}$$

and the drain current of the power transistor Q_2 is

$$I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)(V_{GS} - V_t)^2 = \frac{1}{2} \times 0.142 \times 6065 \times (1 - 0.356)^2 = 178.59\,\text{mA}. \tag{2.134}$$

Assuming the same length of both transistors ($L = L_1$) and the dc current gain $A_I = I_D/I_{ref} = 100$, we obtain the dc current gain of the current mirror

$$A_I = \frac{I_D}{I_{ref}} = \frac{W}{W_1} = 100. \tag{2.135}$$

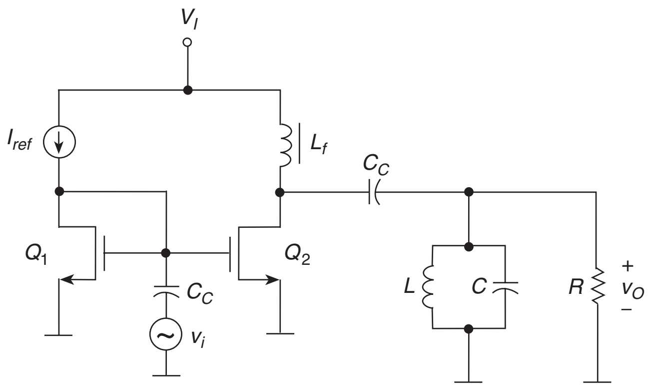

Figure 2.16 Class A RF power amplifier biased with a current mirror.

Let us select

$$I_{ref} = \frac{I_D}{100} = \frac{178.59}{100} = 1.786\,\text{mA}. \tag{2.136}$$

Hence, the width of the current mirror MOSFET Q_1 is

$$W_1 = \frac{W}{100} = \frac{2122.75}{100} = 21.2275\,\mu\text{m}. \tag{2.137}$$

The power consumption by the current mirror is

$$P_{Q1} = I_{ref} V_I = 1.786 \times 10^{-3} \times 3.3 = 5.8938\,\text{mW}. \tag{2.138}$$

2.8 Impedance Matching Circuits

Figure 2.17 shows series and parallel two-terminal networks. These two networks are equivalent at a given frequency. The reactance factor of these networks is given by [5]

$$q = \frac{x}{r} = \frac{R}{X}. \tag{2.139}$$

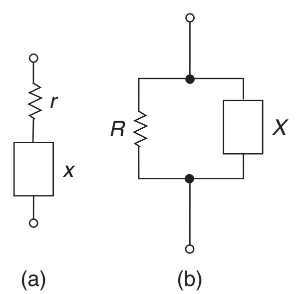

Figure 2.17 Series and parallel two-terminal equivalent networks. (a) Series two-terminal network. (b) Parallel two-terminal network.

The impedance of the two networks is

$$Z = \frac{RjX}{R+jX} = \frac{RjX(R-jX)}{(R+jX)(R-jX)} = \frac{RX^2+jR^2X}{R^2+X^2} = \frac{R}{1+\left(\frac{R}{X}\right)^2} + j\frac{X}{1+\left(\frac{X}{R}\right)^2}$$

$$= \frac{R}{1+q^2} + j\frac{X}{1+\frac{1}{q^2}} = r + jx. \tag{2.140}$$

Hence, the equivalent series components at a given frequency are given by

$$r = \frac{R}{1+q^2} \tag{2.141}$$

and

$$x = \frac{X}{1+\frac{1}{q^2}}. \tag{2.142}$$

Consider the matching circuit $\pi 1$ shown in Figure 2.18. The reactance factor of the section to the right of line A is

$$q_A = \frac{R_A}{X_A} = \frac{x_A}{r_A}. \tag{2.143}$$

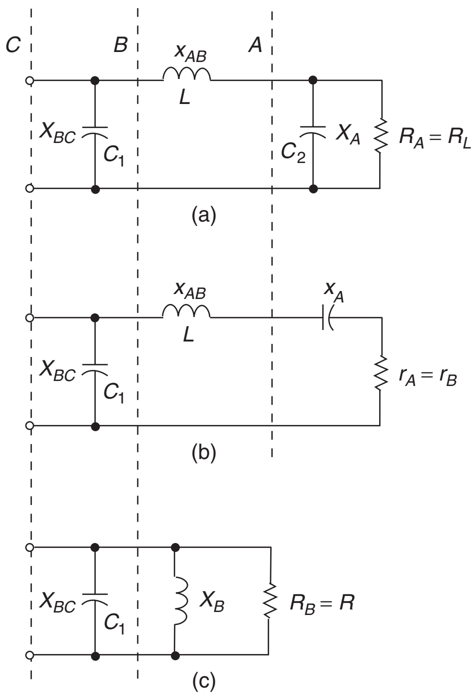

Figure 2.18 Impedance matching resonant circuit $\pi 1$. (a) Impedance matching circuit. (b) Equivalent circuit after the conversion of the parallel circuit R_A-X_A into the series circuit r_A-x_A. (c) Equivalent circuit after the conversion of the series circuit r_A-$(x_A + x_B)$ into the parallel circuit R_B-X_B.

The equivalent series resistance and reactance of the circuit to the right of line A are

$$r_A = r_B = \frac{R_A}{1+q_A^2} = \frac{R_L}{1+q_A^2} \tag{2.144}$$

and

$$x_A = \frac{X_A}{1+\frac{1}{q_A^2}}. \tag{2.145}$$

The loaded quality factor is

$$Q_L = \frac{x_{AB}}{r_A}. \tag{2.146}$$

The series reactance to the right of the B line is

$$x_B = x_{AB} - x_A = (Q_L - q_A)r_A. \tag{2.147}$$

The reactance factor of the circuit to the right of line B is

$$q_B = \frac{x_B}{r_B} = \frac{R}{X_B} = \frac{R}{X_{CD}} \tag{2.148}$$

resulting in

$$R_B = R = r_B(1+q_B^2) \tag{2.149}$$

and

$$X_B = x_B\left(1+\frac{1}{q_B^2}\right). \tag{2.150}$$

Example 2.2

Design an impedance matching circuit to meet the specifications: $R_L = 50\,\Omega$, $R = 15.68\,\Omega$, and $f = 1\,\text{GHz}$.

Solution. Assume $q_B = 3$. The reactance X_{CD} is

$$X_{BC} = \frac{1}{\omega_0 C_1} = \frac{R}{q_B} = \frac{15.68}{3} = 5.227\,\Omega \tag{2.151}$$

producing

$$C_1 = \frac{1}{\omega_0 X_{BC}} = \frac{1}{2\pi \times 10^9 \times 5.227} = 30.45\,\text{pF}. \tag{2.152}$$

Now

$$r_B = r_A = \frac{R}{1+q_B^2} = \frac{15.68}{3^2+1} = 1.568\,\Omega \tag{2.153}$$

which gives

$$q_A = \sqrt{\frac{R_L}{r_A} - 1} = \sqrt{\frac{50}{1.568} - 1} = 5.558. \tag{2.154}$$

The reactance of C_2 is

$$X_A = \frac{1}{\omega_0 C_2} = \frac{R_L}{q_A} = \frac{50}{5.558} = 8.996\,\Omega \tag{2.155}$$

resulting in

$$C_2 = \frac{1}{\omega X_A} = \frac{1}{2\pi \times 10^9 \times 8.996} = 17.69\,\text{pF}. \quad (2.156)$$

The loaded quality factor is

$$Q_L = \frac{x_{AB}}{r_A} = \frac{x_B}{r_B} + \frac{x_A}{r_A} = q_B + q_A = 3 + 5.558 = 8.558. \quad (2.157)$$

Thus

$$x_{AB} = \omega_0 L = Q_L r_A = 8.558 \times 1.568 = 13.419\,\Omega. \quad (2.158)$$

and

$$L = \frac{x_{AB}}{\omega_0} = \frac{13.419}{2\pi \times 10^9} = 2.136\,\text{nH}. \quad (2.159)$$

2.9 Class A RF Linear Amplifier

2.9.1 Amplifier of Variable-envelope Signals

Modern wireless communication systems use power amplifiers to amplify variable-envelope signals. Linear amplifiers are required to amplify variable-envelope signals, such as AM signals. The Class A amplifier is a linear amplifier. Figure 2.19 shows waveforms of gate-to-source voltage v_{GS} and drain-to-source voltage v_{DS} in an ideal linear power amplifier.

The modulating voltage at the amplifier input is given by

$$v_{im}(t) = V_{im} \cos \omega_m t. \quad (2.160)$$

The carrier voltage at the amplifier input is

$$v_{ic} = V_{ic} \cos \omega_c t. \quad (2.161)$$

The gate-to-source AM voltage at the input of a Class A RF amplifier is

$$\begin{aligned} v_{GS} &= V_{GSQ} + V(t) \cos \omega_c t = V_{GSQ} + [V_{ic} + v_m(t)] \cos \omega_c t \\ &= V_{GSQ} + [V_c + V_{im} \cos \omega_m t] \cos \omega_c t = V_{GSQ} + V_{ic}\,(1 + m \cos \omega_m t) \cos \omega_c t \end{aligned} \quad (2.162)$$

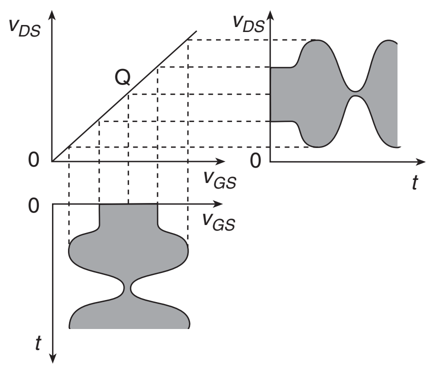

Figure 2.19 Amplification of AM signal in Class A linear amplifier.

where

$$m = \frac{V_{im}}{V_{ic}}. \tag{2.163}$$

To avoid distortion of the AM signal, the transistor must be ON at all times. The maximum amplitude of the ac component of the gate-to-source voltage occurs at $m = 1$:

$$V_{gsm(max)} = V_{ic}(1 + m_{max}) = V_{ci}(1 + 1) = 2V_{ic}. \tag{2.164}$$

The dc gate-to-source voltage at the operating point Q must satisfy the following condition:

$$V_{GSQ} > 2V_{ic}. \tag{2.165}$$

Assuming a linear $v_{DS} = A_v v_{GS}$ relationship, we obtain the drain-to-source AM voltage

$$v_{DS} = V_{DSQ} + A_v V_{ic}\,(1 + m\cos\omega_m t)\cos\omega_c t = V_{DSQ} + V_c\,(1 + m\cos\omega_m t)\cos\omega_c t. \tag{2.166}$$

where A_v is the voltage gain of the amplifier and the amplitude of the carrer at the amplifier output is

$$V_c = A_v V_{ic}. \tag{2.167}$$

To avoid distortion, the dc drain-to-source voltage at the operating point must satisfy the condition:

$$V_{DSQ} > 2V_c + V_{GSQ} - V_t. \tag{2.168}$$

The output AM voltage of the amplifier is

$$v_o = V_c\,(1 + m\cos\omega_m t)\cos\omega_c t. \tag{2.169}$$

2.9.2 Amplifiers of Constant-envelope Signals

The Class A amplifier can be used to amplify constant-envelope signals, such as angle-modulated signals (FM and PM signals). The gate-to-source FM voltage is

$$v_{gs} = V_{DSQ} + V_{ic}\cos(\omega_c t + m_f \sin\omega_m t) \tag{2.170}$$

where

$$V_{DSQ} > V_{ic}. \tag{2.171}$$

Hence, the drain-to-source FM voltage is

$$v_{DS} = V_{DSQ} + A_v V_c \cos(\omega_c t + m_f \sin\omega_m t) = V_{DSQ} + V_c \cos(\omega_c t + m_f \sin\omega_m t) \tag{2.172}$$

where

$$m_f = \frac{\Delta f}{f_m} \tag{2.173}$$

$$V_c = A_v V_{ic} \tag{2.174}$$

and

$$V_{DSQ} > V_{ic} + V_{GSQ} - V_t. \tag{2.175}$$

2.10 Summary

- The conduction angle of the drain current is $2\theta = 360^\circ$ in the Class A power amplifier.
- The operating point of the transistor in the Class A amplifier is above the transistor threshold voltage V_t such that the gate-to-source voltage waveform is above V_t for the entire voltage swing ($V_{gsm} < V_{GS} - V_t$). Therefore, the transistor never enters the cutoff region.
- The transistor is biased at a current greater than the peak value of the ac component of the drain current ($I_m < I_D$).
- There is no sharp dividing boundary between Class A power amplifiers and small-signal amplifiers.
- The dc supply power P_I is constant in Class A RF power amplifier and is independent of the amplitude of the output voltage V_m.
- Power loss in the transistor of the Class A RF power amplifier is high.
- The Class A power amplifier dissipates the maximum power at zero output power.
- The maximum drain efficiency of Class A RF power amplifiers with the RF choke is 50% at $I_m = I_I$ and $V_m = V_I$.
- The harmonic distortion in Class A power amplifiers is low. Therefore, these circuits are linear amplifiers, suitable for the amplification of AM signals.
- The Class A power amplifier suffers from low efficiency, but provides a higher linearity than other power amplifiers.
- Negative feedback can be used to suppress the nonlinearity.
- The amplitude of the current in the parallel-resonant circuit is high.
- Power losses in the parallel-resonant circuit are high because the amplitude of the current flowing through the resonant inductor and the resonant capacitor is Q_L times higher than that of the output current I_m.
- The maximum drain efficiency of the Class A audio amplifier, in which the RF choke is replaced by a resistor, is only 25% at $I_m = I_I$ and $V_m = V_I$.
- The linearity of the Class A RF power amplifier is very good.
- The amplitudes of harmonics in the load current in the Class A amplifier are low.
- Class A power amplifiers are used as low-power drivers of high-power amplifiers and as linear power amplifiers.

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2.12 Review Questions

2.1 What is the value of the conduction angle of the drain current in the Class A power amplifier?

2.2 What is the location of the transistor operating point for the Class A RF power amplifier?

2.3 Does the dc supply power depend on the output voltage amplitude V_m?

2.4 Is the power loss in the transistor of the Class A amplifier low?

2.5 Are the power losses in the parallel-resonant circuit high?

2.6 Is the efficiency of the Class A RF power amplifier high?

2.7 Is the linearity of the Class A RF power amplifier good?

2.8 Are the harmonics in the load current of the Class A RF power amplifier high?

2.9 What are the upper and lower limits of the output voltage in the Class A amplifier for linear operation?

2.13 Problems

2.1 Determine the drain efficiency of a Class A amplifier with an RF choke to meet the following specifications:

(a) $V_I = 20\,\text{V}$ and $V_m = 10\,\text{V}$.

(b) $V_I = 20\,\text{V}$ and $V_m = 18\,\text{V}$.

2.2 Determine the maximum power loss in the transistor of the Class A amplifier with an RF choke, which has $V_I = 10\,\text{V}$ and $I_I = 1\,\text{A}$.

2.3 Design a Class A RF power amplifier to meet the following specifications: $P_O = 0.25\,\text{W}$, $V_I = 1.5\,\text{V}$, $V_{DS(min)} = 0.2\,\text{V}$, and $f = 2.4\,\text{GHz}$.

2.4 Design an impedance matching circuit for the RF Class A amplifier with $R_L = 50\,\Omega$, $R = 25\,\Omega$, and $f = 2.4\,\text{GHz}$.

3

Class AB, B, and C RF Power Amplifiers

3.1 Introduction

The Class B RF power amplifier [1–7] consists of a transistor and a parallel-resonant circuit. The transistor is operated as a dependent current source. The conduction angle of the drain or collector current in the Class B power amplifier is 180°. The parallel resonant circuit acts like a bandpass filter and selects only the fundamental component. The efficiency of the Class B power amplifier is higher than that of the Class A power amplifier. The circuit of the Class C power amplifier is the same as that of the Class B amplifier. However, the operating point is such that the conduction angle of the drain current is less than 180°. Class B and C power amplifiers are usually used for RF amplification in radio and TV transmitters as well as in mobile phones. In this chapter, we will present Class AB, B, and C RF power amplifiers with their principle of operation, analysis, and design examples.

3.2 Class B RF Power Amplifier

3.2.1 Circuit of Class B RF Power Amplifier

The circuit of a Class B RF power amplifier is shown in Figure 3.1. It consists of a transistor (MOSFET, MESFET, or BJT), parallel-resonant circuit, and RF choke. The operating point of the transistor is located exactly at the boundary between the cutoff region and the active region (the saturation region or the pinch-off region). The dc component of the gate-to-source voltage V_{GS} is equal to the transistor threshold voltage V_t. Therefore, the conduction angle of the drain current 2θ is 180°. The transistor is operated as a voltage-controlled dependent-current source. Voltage and current waveforms in the Class B power amplifier

RF Power Amplifiers Marian K. Kazimierczuk

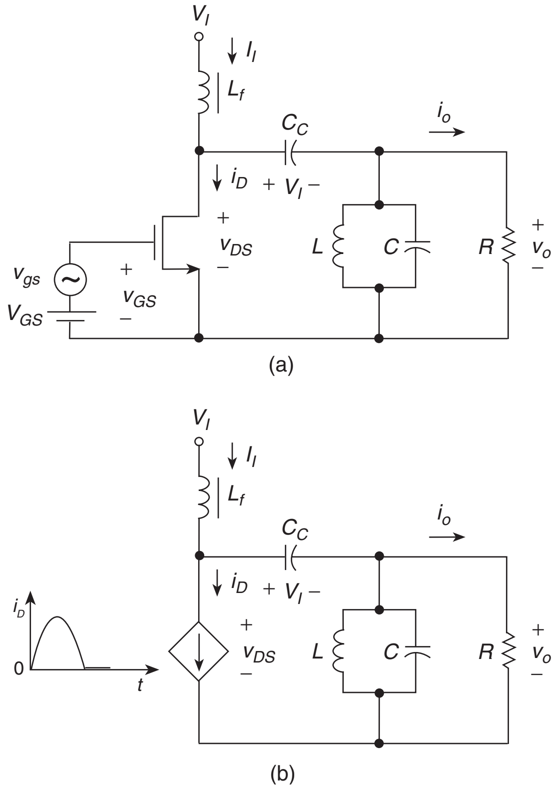

Figure 3.1 Class AB, B, or C RF power amplifiers.

are shown in Figure 3.2. The ac component of the gate-to-source voltage v_{gs} is a sine wave. The drain current is a half sine wave and contains the dc component, fundamental component, and even harmonics. The parallel-resonant circuit acts as a bandpass filter, which attenuates all the harmonics. The 'purity' of the output sine wave is a function of the selectivity of the bandpass filter. The higher the loaded quality factor Q_L, the lower is the harmonic content of the sine wave output current and voltage. The parallel-resonant circuit may be more complex to serve as an impedance matching network.

3.2.2 Waveforms of Class B Amplifier

The gate-to-source voltage is given by

$$v_{GS} = V_t + V_{gsm} \cos \omega t. \tag{3.1}$$

For large-signal operation, the drain current is nearly proportional to the gate-to-source voltage v_{GS}, when v_{GS} is above V_t:

$$i_D = K(v_{GS} - V_t) = KV_{gsm} \cos \omega t \quad \text{for} \quad v_{GS} > V_t \tag{3.2}$$

and

$$i_D = 0 \quad \text{for} \quad v_{GS} < V_t. \tag{3.3}$$

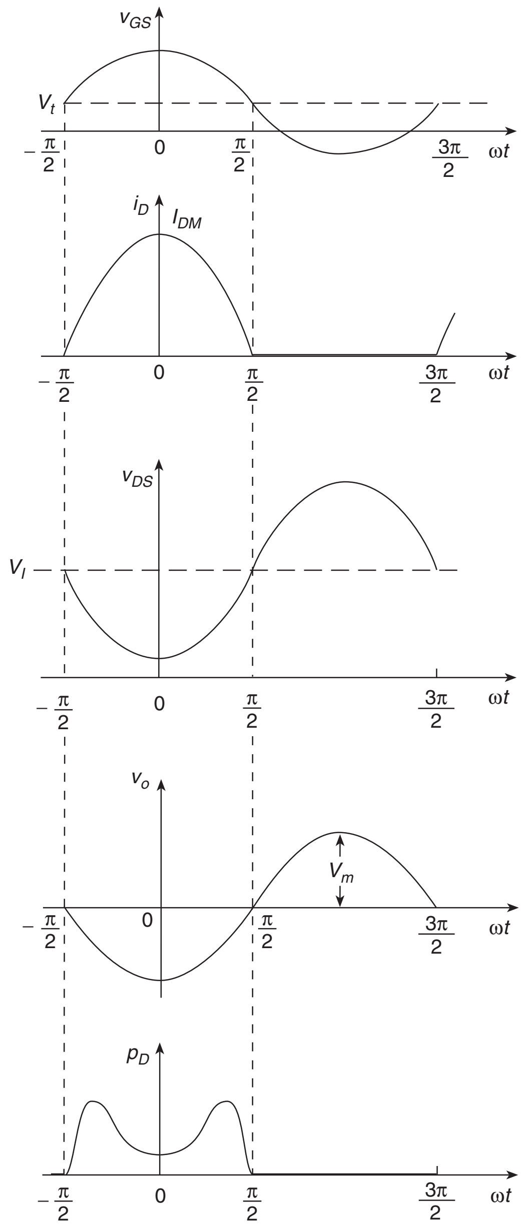

Figure 3.2 Waveforms in Class B RF power amplifier.

The drain current in the Class B amplifier is a half sine wave and is given by

$$i_D = \begin{cases} I_{DM} \cos \omega t & \text{for} \quad -\dfrac{\pi}{2} < \omega t \le \dfrac{\pi}{2} \\ 0 & \text{for} \quad \dfrac{\pi}{2} < \omega t \le \dfrac{3\pi}{2} \end{cases} \tag{3.4}$$

where I_{DM} is the peak value of the drain current. The drain-to-source voltage is expressed as

$$v_{DS} = V_I - V_m \cos \omega t. \tag{3.5}$$

The instantaneous power dissipation in the transistor is

$$p_D(\omega t) = i_D v_{DS} = I_{DM} \cos \omega t (V_I - V_m \cos \omega t) \quad \text{for} \quad -\frac{\pi}{2} < \omega t \leq \frac{\pi}{2} \tag{3.6}$$

and

$$p_D(\omega t) = 0 \quad \text{for} \quad \frac{\pi}{2} < \omega t \leq \frac{3\pi}{2}. \tag{3.7}$$

The fundamental component of the drain current is

$$I_m = \frac{1}{\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} I_{DM} \cos^2 \omega t \, \mathrm{d}(\omega t) = \frac{I_{DM}}{2} = \frac{\pi}{2} I_I. \tag{3.8}$$

The drain current waveform is

$$i_D = I_{DM} \cos \omega t = \pi I_I \cos \omega t \quad \text{for} \quad -\frac{\pi}{2} < \omega t \leq \frac{\pi}{2} \tag{3.9}$$

and

$$i_D = 0 \quad \text{for} \quad \frac{\pi}{2} < \omega t \leq \frac{3\pi}{2}. \tag{3.10}$$

The drain-to-source voltage waveform is

$$v_{DS} = V_I - V_m \cos \omega t = V_I \left(1 - \frac{V_m}{V_I}\right) \cos \omega t. \tag{3.11}$$

The instantaneous power dissipation in the transistor at $f = f_0$ is

$$\begin{aligned} p_D(\omega t) &= i_D v_{DS} = I_I V_I \pi \cos \omega t \left(1 - \frac{V_m}{V_I} \cos \omega t\right) \\ &= P_I \pi \cos \omega t \left(1 - \frac{V_m}{V_I} \cos \omega t\right) \quad \text{for} \quad -\frac{\pi}{2} < \omega t \leq \frac{\pi}{2} \end{aligned} \tag{3.12}$$

and

$$p_D(\omega t) = 0 \quad \text{for} \quad \frac{\pi}{2} < \omega t \leq \frac{3\pi}{2}. \tag{3.13}$$

Hence, the normalized instantaneous power dissipation in the transistor is

$$\frac{p_D(\omega t)}{P_I} = \pi \cos \omega t \left(1 - \frac{V_m}{V_I} \cos \omega t\right) \quad \text{for} \quad -\frac{\pi}{2} < \omega t \leq \frac{\pi}{2} \tag{3.14}$$

and

$$\frac{p_D(\omega t)}{P_I} = 0 \quad \text{for} \quad \frac{\pi}{2} < \omega t \leq \frac{3\pi}{2}. \tag{3.15}$$

The normalized instantaneous power loss in the transistor at various values of V_m/V_I and at $f = f_0$ is shown in Figure 3.3 for the Class B power amplifier. As the ratio V_m/V_I increases, the peak values of $p_D(\omega t)/P_I$ decrease, yielding higher drain efficiency.

3.2.3 Power Relationships in Class B Amplifier

The dc supply current is

$$I_I = \frac{1}{2\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} i_D \, \mathrm{d}(\omega t) = \frac{1}{2\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} I_{DM} \cos \omega t \, \mathrm{d}(\omega t) = \frac{I_{DM}}{\pi} = \frac{2}{\pi} I_m = \frac{2}{\pi} \frac{V_m}{R}. \tag{3.16}$$

Hence, the dc resistance seen by the dc power supply V_I is

$$R_{DC} = \frac{V_I}{I_I} = \frac{\pi}{2} \frac{V_I}{V_m} R. \tag{3.17}$$

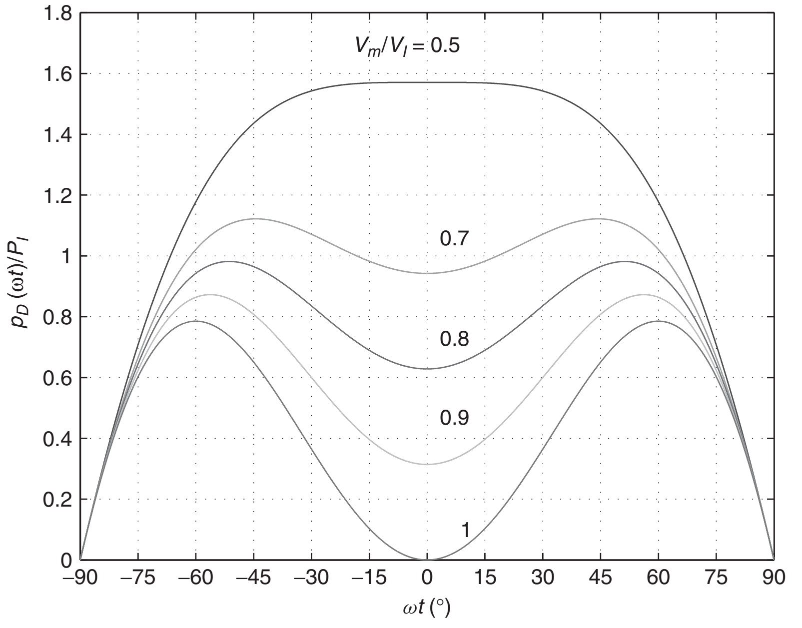

Figure 3.3 Normalized instantaneous power dissipation $p_D(\omega t)/P_I$ at various values of V_m/V_I and at $f = f_0$ for the Class B RF power amplifier.

For $V_m = V_I$,

$$I_{Imax} = \frac{2}{\pi}\frac{V_I}{R} \tag{3.18}$$

resulting in the dc resistance seen by the power supply V_I

$$R_{DCmin} = \frac{V_I}{I_{Imax}} = \frac{\pi}{2}R. \tag{3.19}$$

The maximum value of R_{DC} occurs at $V_m = 0$ and its value is $R_{DCmax} = \infty$. The amplitude of the output voltage is

$$V_m = RI_m. \tag{3.20}$$

The dc supply power is

$$P_I = I_I V_I = \frac{I_{DM}}{\pi}V_I = \frac{2}{\pi}V_I I_m = \frac{2}{\pi}\frac{V_I V_m}{R} = \frac{2}{\pi}\left(\frac{V_I^2}{R}\right)\left(\frac{V_m}{V_I}\right). \tag{3.21}$$

The output power is

$$P_O = \frac{I_m V_m}{2} = \frac{V_m^2}{2R} = \frac{1}{2}\left(\frac{V_I^2}{R}\right)\left(\frac{V_m}{V_I}\right)^2. \tag{3.22}$$

The power dissipation at the drain of the MOSFET is

$$P_D = \frac{1}{2\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} i_D v_{DS}\, \mathrm{d}(\omega t) = P_I - P_O = \frac{2}{\pi}\frac{V_I V_m}{R} - \frac{V_m^2}{2R}$$

$$= \frac{2}{\pi}\left(\frac{V_I^2}{R}\right)\left(\frac{V_m}{V_I}\right) - \frac{1}{2}\left(\frac{V_I^2}{R}\right)\left(\frac{V_m}{V_I}\right)^2. \tag{3.23}$$

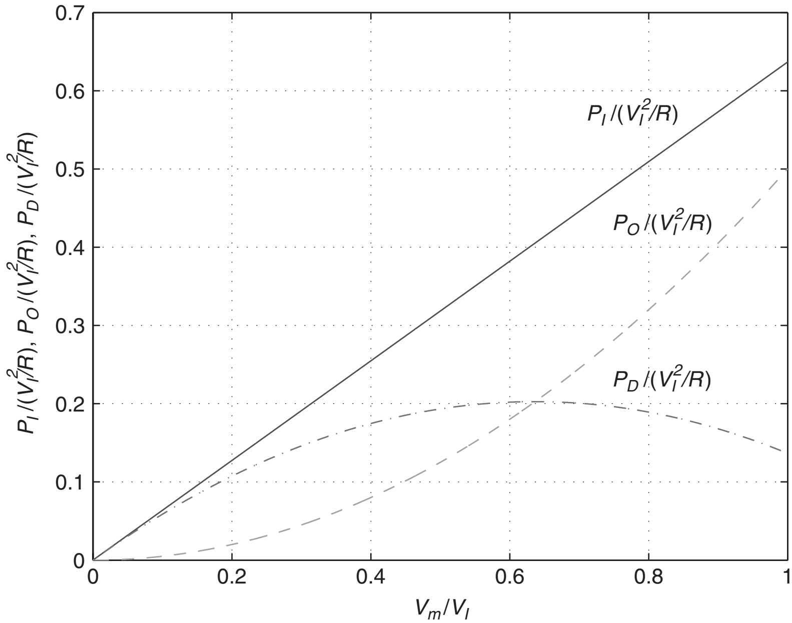

Figure 3.4 Normalized dc supply power $P_I/(V_I^2/R)$, normalized drain power $P_O/(V_I^2/R)$, and normalized drain power dissipation $P_D/(V_I^2/R)$ as a function of V_m/V_I for the Class B RF power amplifier.

The maximum power dissipation can be determined by taking the derivative of P_D with respect to V_m and setting it to zero. Thus,

$$\frac{dP_D}{dV_m} = \frac{2}{\pi}\frac{V_I}{R} - \frac{V_m}{R} = 0. \tag{3.24}$$

The critical value of V_m at which the maximum power dissipation occurs is

$$V_{m(cr)} = \frac{2V_I}{\pi}. \tag{3.25}$$

Hence, the maximum power dissipated in the drain of the transistor is

$$P_{Dmax} = \frac{4}{\pi^2}\frac{V_I^2}{R} - \frac{2}{\pi^2}\frac{V_I^2}{R} = \frac{2}{\pi^2}\frac{V_I^2}{R}. \tag{3.26}$$

Figure 3.4 shows plots of $P_I/(V_I^2/R)$, $P_O/(V_I^2/R)$, and $P_D/(V_I^2/R)$ as a function of V_m/V_I.

3.2.4 Efficiency of Class B Amplifier

The drain efficiency of the Class B RF power amplifier is

$$\eta_D = \frac{P_O}{P_I} = \frac{\pi}{4}\frac{V_m}{V_I} = \frac{\pi}{4}\frac{(V_I - V_{DSmin})}{V_I} = \frac{\pi}{4}\left(1 - \frac{V_{DSmin}}{V_I}\right). \tag{3.27}$$

For $V_m = V_I$,

$$\eta_D = \frac{P_{Omax}}{P_I} = \frac{\pi}{4} \approx 78.54\,\%. \tag{3.28}$$

The drain efficiency η_D as a function of V_m/V_I is depicted in Figure 3.5.

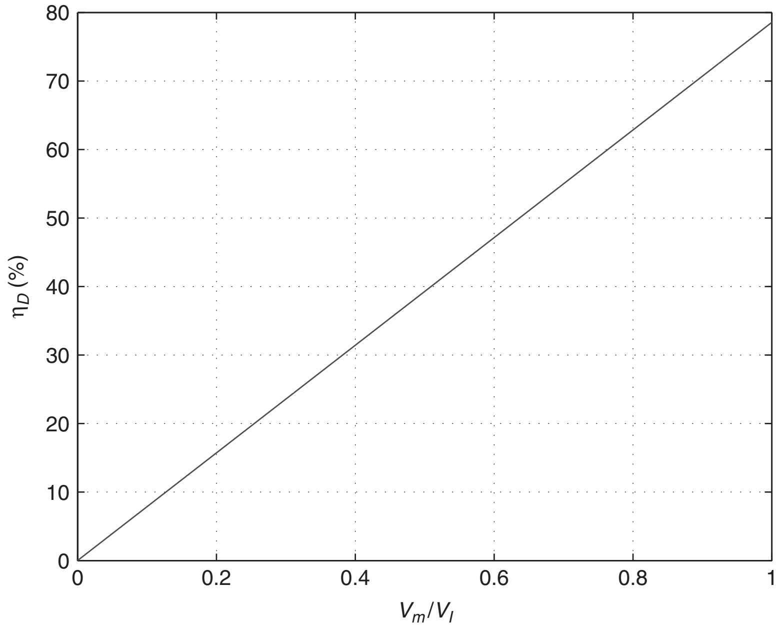

Figure 3.5 Drain efficiency η_D as a function of V_m/V_I for the Class B RF power amplifier.

The maximum drain current is

$$I_{DM} = \pi I_I = 2I_m \tag{3.29}$$

and the maximum drain-to-source voltage is

$$V_{DSM} = 2V_I. \tag{3.30}$$

The output power capability is

$$c_p = \frac{P_O}{V_{DSM} I_{DM}} = \frac{\eta_D P_I}{V_{DSM} I_{DM}} = \eta_D \frac{I_I}{I_{DM}} \frac{V_I}{V_{DSM}} = \frac{\pi}{4} \times \frac{1}{\pi} \times \frac{1}{2} = \frac{1}{8} = 0.125. \tag{3.31}$$

It is interesting to note that the value of c_p of the Class B amplifier is the same as that of the Class A amplifier.

Example 3.1

Design a Class B power amplifier to deliver power of 20 W at $f = 2.4$ GHz. The bandwidth is $BW = 480$ MHz. The power supply voltage is $V_I = 24$ V and $V_{DSmin} = 2$ V.

Solution. The maximum amplitude of the output voltage is

$$V_m = V_I - V_{DSmin} = 24 - 2 = 22\,\text{V}. \tag{3.32}$$

The load resistance is

$$R = \frac{V_m^2}{2P_O} = \frac{22^2}{2 \times 20} = 12.1\,\Omega. \tag{3.33}$$

Pick $R = 12\,\Omega$. The amplitude the fundamental component of the drain current and the output current is given by

$$I_m = \frac{V_m}{R} = \frac{22}{12} = 1.833\,\text{A}. \tag{3.34}$$

The dc supply current is

$$I_I = \frac{2}{\pi}I_m = \frac{2}{\pi} \times 1.833 = 1.167\,\text{A}. \tag{3.35}$$

The maximum drain current is

$$I_{DM} = \pi I_I = \pi \times 1.167 = 3.666\,\text{A} \tag{3.36}$$

and the maximum drain-to-source voltage is

$$V_{DSM} = 2V_I = 2 \times 24 = 48\,\text{V}. \tag{3.37}$$

The dc supply power is

$$P_I = I_I V_I = 1.167 \times 24 = 28\,\text{W}. \tag{3.38}$$

The drain power dissipation is

$$P_D = P_I - P_O = 28 - 20 = 8\,\text{W}. \tag{3.39}$$

The drain efficiency is

$$\eta_D = \frac{P_O}{P_I} = \frac{20}{28} = 71.43\,\%. \tag{3.40}$$

The loaded quality factor is

$$Q_L = \frac{f}{BW} = \frac{2.4}{0.48} = 5. \tag{3.41}$$

The reactances of the resonant circuit components are

$$X_L = X_C = \frac{R}{Q_L} = \frac{12}{5} = 2.4\,\Omega \tag{3.42}$$

yielding

$$L = \frac{X_L}{\omega} = \frac{2.4}{2\pi \times 2.4 \times 10^9} = 0.1592\,\text{nH} \tag{3.43}$$

and

$$C = \frac{1}{\omega X_C} = \frac{1}{2\pi \times 2.4 \times 10^9 \times 2.4} = 27.6\,\text{pF}. \tag{3.44}$$

3.3 Class AB and C RF Power Amplifiers

3.3.1 Waveforms of Class AB and C RF Power Amplifiers

The circuit of the Class C power amplifier is the same as that of the Class B RF power amplifier. The operating point of the transistor is located in the cutoff region. The dc component of the gate-to-source voltage V_{GS} is less than the transistor threshold voltage V_t. Therefore, the conduction angle of the drain current 2θ is less than 180°. Voltage and current waveforms in the Class C power amplifier are shown in Figure 3.6. The only difference is the conduction angle of the drain current, which is determined by the operating

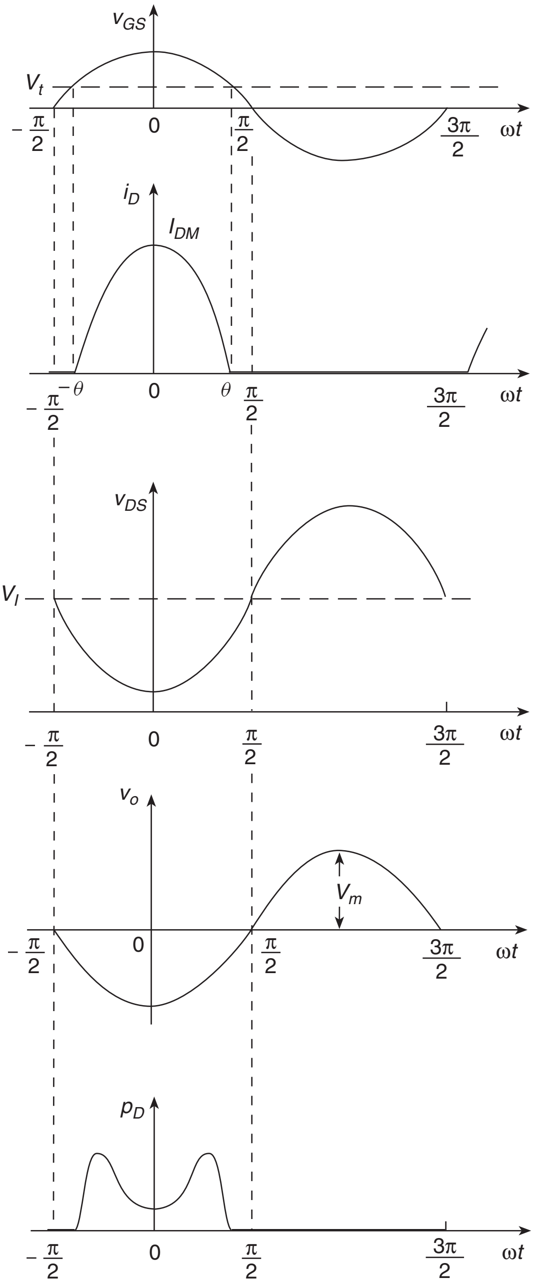

Figure 3.6 Waveforms in Class C RF power amplifier.

point. The drain current waveform for any conduction angle θ, i.e., for Class AB, Class B, and Class C, is given by

$$i_D = \begin{cases} I_{DM}\dfrac{\cos\omega t - \cos\theta}{1-\cos\theta} & \text{for} \quad -\theta < \omega t \le \theta \\ 0 & \text{for} \quad \theta < \omega t \le 2\pi - \theta. \end{cases} \tag{3.45}$$

The drain current waveform is an even function of ωt and satisfies the condition: $i_D(\omega t) = i_D(-\omega t)$. The drain current waveform can be expanded into Fourier series

$$i_D(\omega t) = I_{DM}\left[\alpha_0 + \sum_{n=1}^{\infty} \alpha_n \cos n\omega t\right]. \tag{3.46}$$

The dc component of the drain current is

$$\begin{aligned} I_I &= \frac{1}{2\pi}\int_{-\theta}^{\theta} i_D \, d(\omega t) = \frac{1}{\pi}\int_{0}^{\theta} i_D \, d(\omega t) = \frac{I_{DM}}{\pi}\int_{0}^{\theta} \frac{\cos\omega t - \cos\theta}{1-\cos\theta} d(\omega t) \\ &= I_{DM}\frac{\sin\theta - \theta\cos\theta}{\pi(1-\cos\theta)} = \alpha_0 I_{DM} \end{aligned} \tag{3.47}$$

where

$$\alpha_0 = \frac{I_I}{I_{DM}} = \frac{\sin\theta - \theta\cos\theta}{\pi(1-\cos\theta)}. \tag{3.48}$$

The amplitude of the fundamental component of the drain current is given by

$$\begin{aligned} I_m &= \frac{1}{\pi}\int_{-\theta}^{\theta} i_D \cos\omega t \, d(\omega t) = \frac{2}{\pi}\int_{0}^{\theta} i_D \cos\omega t \, d(\omega t) = \frac{2}{\pi}\int_{0}^{\theta} \frac{\cos\omega t - \cos\theta}{1-\cos\theta}\cos\omega t \, d(\omega t) \\ &= I_{DM}\frac{\theta - \sin\theta\cos\theta}{\pi(1-\cos\theta)} = \alpha_1 I_{DM} \end{aligned} \tag{3.49}$$

where

$$\alpha_1 = \frac{I_m}{I_{DM}} = \frac{\theta - \sin\theta\cos\theta}{\pi(1-\cos\theta)}. \tag{3.50}$$

The amplitude of the n-th harmonic is

$$\begin{aligned} I_{m(n)} &= \frac{1}{\pi}\int_{-\theta}^{\theta} i_D \cos n\omega t \, d(\omega t) = \frac{2}{\pi}\int_{0}^{\theta} i_D \cos n\omega t \, d(\omega t) \\ &= \frac{2}{\pi}\int_{0}^{\theta} \frac{\cos\omega t - \cos\theta}{1-\cos\theta}\cos n\omega t \, d(\omega t) = I_{DM}\frac{2}{\pi}\frac{\sin n\theta\cos\theta - n\cos n\theta\sin\theta}{n(n^2-1)(1-\cos\theta)} = \alpha_n I_{DM} \end{aligned} \tag{3.51}$$

where

$$\alpha_n = \frac{I_{m(n)}}{I_{DM}} = \frac{2}{\pi}\frac{\sin n\theta\cos\theta - n\cos n\theta\sin\theta}{n(n^2-1)(1-\cos\theta)} \quad \text{for} \quad n = 2, 3, 4, \ldots. \tag{3.52}$$

Figure 3.7 shows the Fourier coefficients α_n as a function of the conduction angle θ.

The ratio of the amplitude of the fundamental component to the dc component of the drain current is given by

$$\gamma_1 = \frac{I_m}{I_I} = \frac{\alpha_1}{\alpha_0} = \frac{\theta - \sin\theta\cos\theta}{\sin\theta - \theta\cos\theta}. \tag{3.53}$$

The ratio I_m/I_I as a function of conduction angle θ of the drain current is shown in Figure 3.8.

The drain current waveform is given by

$$i_D = I_I\frac{\pi(\cos\omega t - \cos\theta)}{\sin\theta - \theta\cos\theta} \quad \text{for} \quad -\theta \le \omega t \le \theta. \tag{3.54}$$

The drain-to-source waveform at $f = f_0$ is

$$v_{DS} = V_I - V_m\cos\omega t = V_I\left(1 - \frac{V_m}{V_I}\right)\cos\omega t. \tag{3.55}$$

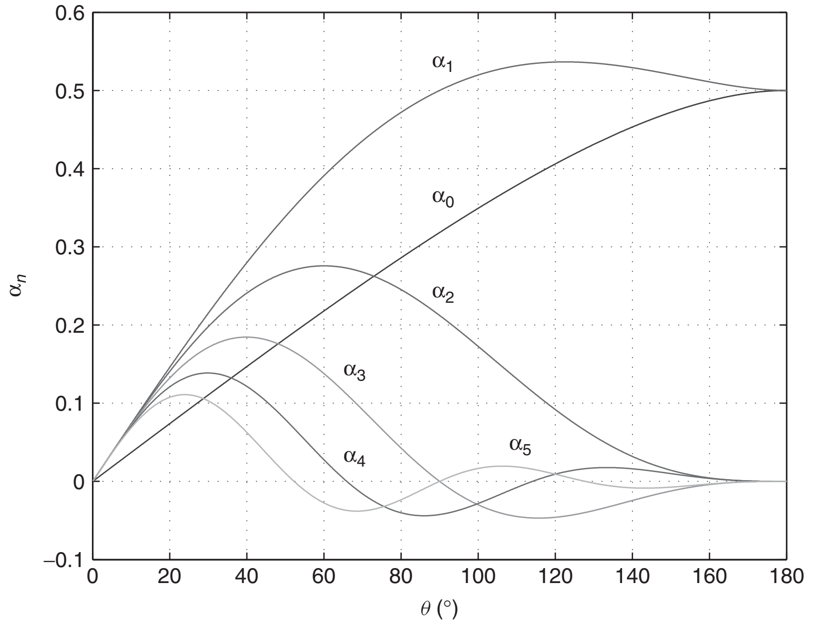

Figure 3.7 Fourier coefficients α_n of the drain current i_D as a function of the conduction angle θ for the Class AB, B, and C RF power amplifiers.

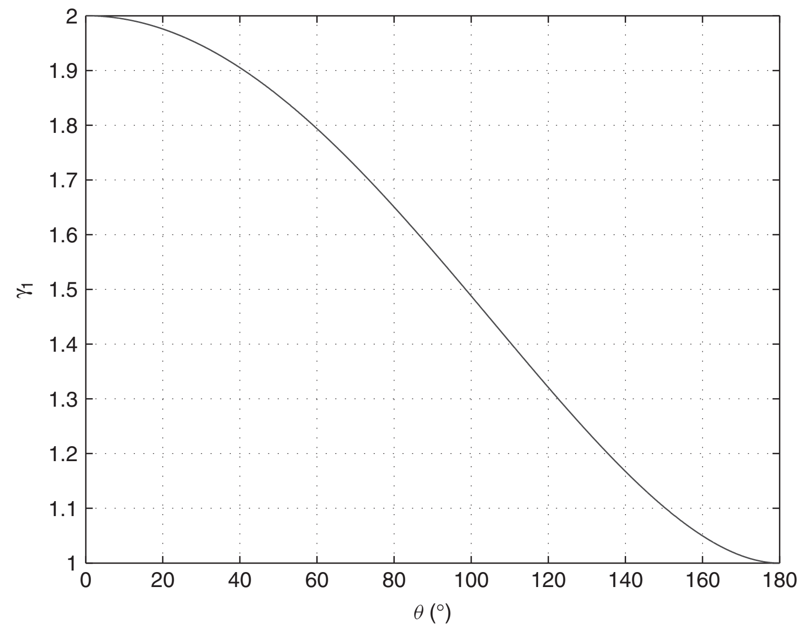

Figure 3.8 Ratio of the fundamental component to the dc component I_m/I_I of the drain current i_D as a function of conduction angle θ for the Class AB, B, and C RF power amplifiers.

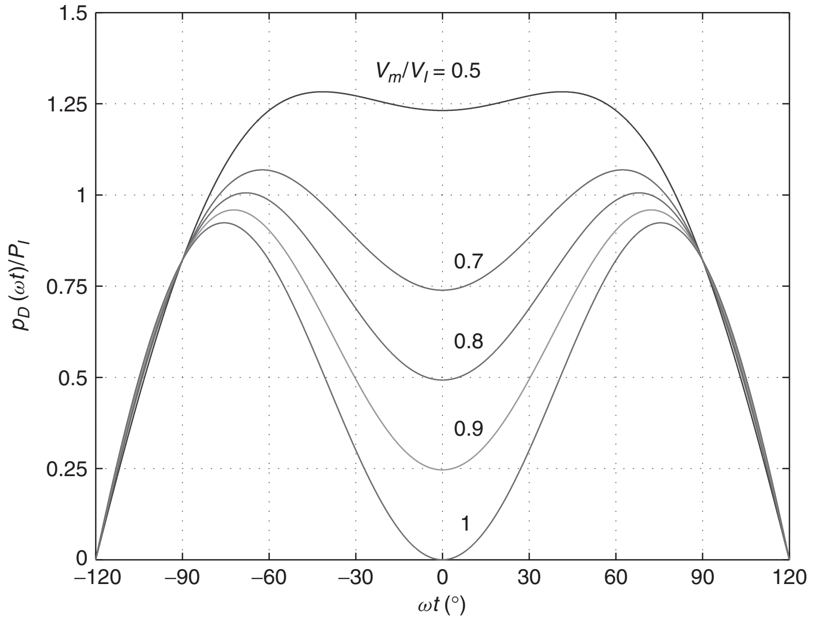

Figure 3.9 Normalized drain power loss $p_D(\omega t)/P_I$ at conduction angle $\theta = 120^\circ$ and at $f = f_0$ for the Class AB RF power amplifier.

The waveform of the normalized drain power loss at $f = f_0$ is

$$\frac{p_D(\omega t)}{P_I} = \frac{\pi(\cos\omega t - \cos\theta)}{\sin\theta - \theta\cos\theta}\left(1 - \frac{V_m}{V_I}\cos\omega t\right) \quad \text{for} \quad -\theta \le \omega t \le \theta. \tag{3.56}$$

The waveforms of the normalized drain power loss for $\theta = 120^\circ$, 60° and $\theta = 45^\circ$ at $f = f_0$ are shown in Figures 3.9, 3.10, and 3.11, respectively. As the conduction angle θ decreases, the peak values of $p_D(\omega t)/P_I$ increase.

The waveform of the normalized drain power loss at f not equal to f_0 is

$$\frac{p_D(\omega t)}{P_I} = \frac{\pi(\cos\omega t - \cos\theta)}{\sin\theta - \theta\cos\theta}\left[1 - \frac{V_m}{V_I}\cos(\omega t + \phi)\right] \quad \text{for} \quad -\theta \le \omega t \le \theta. \tag{3.57}$$

The waveforms of the normalized drain power loss for $\theta = 60^\circ$ at $\phi = 15^\circ$ are shown in Figure 3.12.

3.3.2 Power of the Class AB, B, and C Amplifiers

The dc supply power is

$$P_I = I_I V_I = \alpha_0 I_{DM} V_I. \tag{3.58}$$

The output power is

$$P_O = \frac{1}{2} I_m V_m = \frac{1}{2}\alpha_1 I_{DM} V_m. \tag{3.59}$$

The power dissipated in the transistor is

$$P_D = P_I - P_O = \alpha_0 I_{DM} V_I - \frac{1}{2}\alpha_1 I_{DM} V_m. \tag{3.60}$$

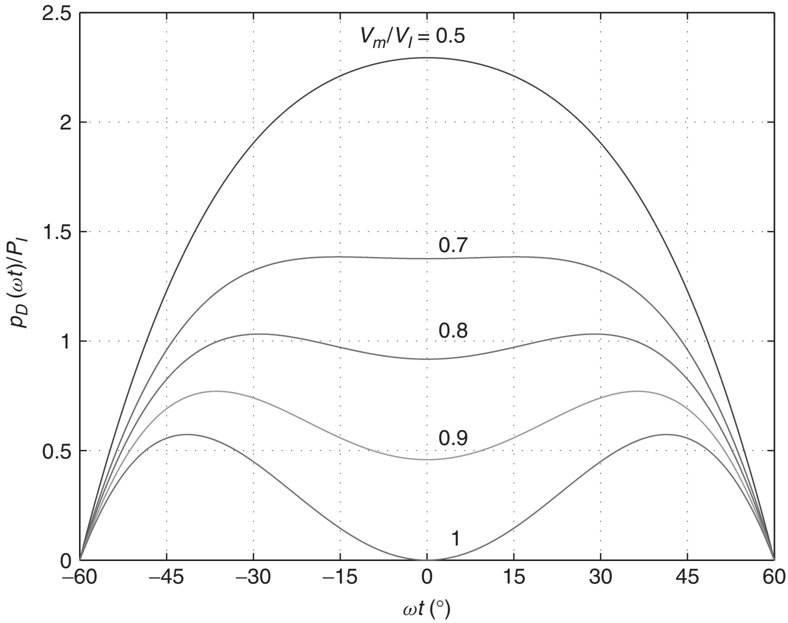

Figure 3.10 Normalized drain power loss $p_D(\omega t)/P_I$ at conduction angle $\theta = 60^\circ$ and at $f = f_0$ for the Class C RF power amplifier.

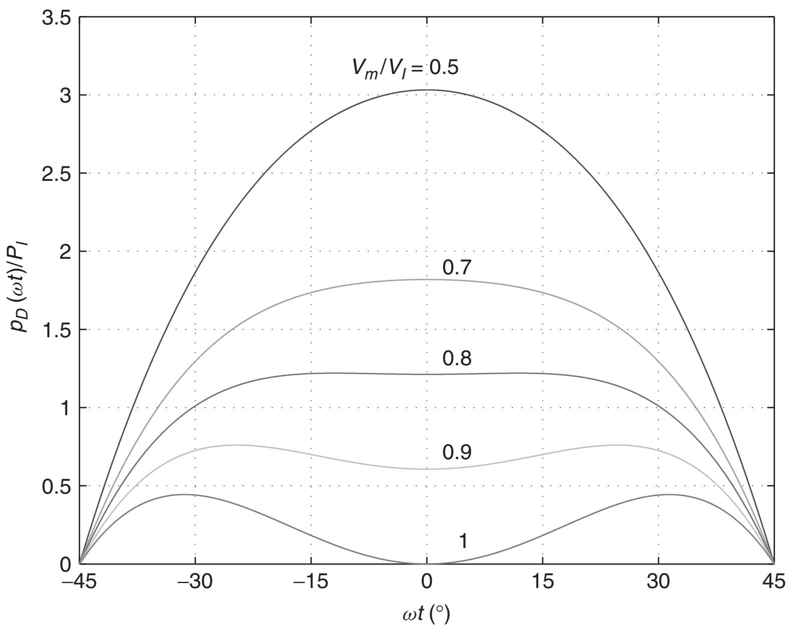

Figure 3.11 Normalized drain power loss $p_D(\omega t)/P_I$ at conduction angle $\theta = 45^\circ$ for the Class C RF power amplifier.

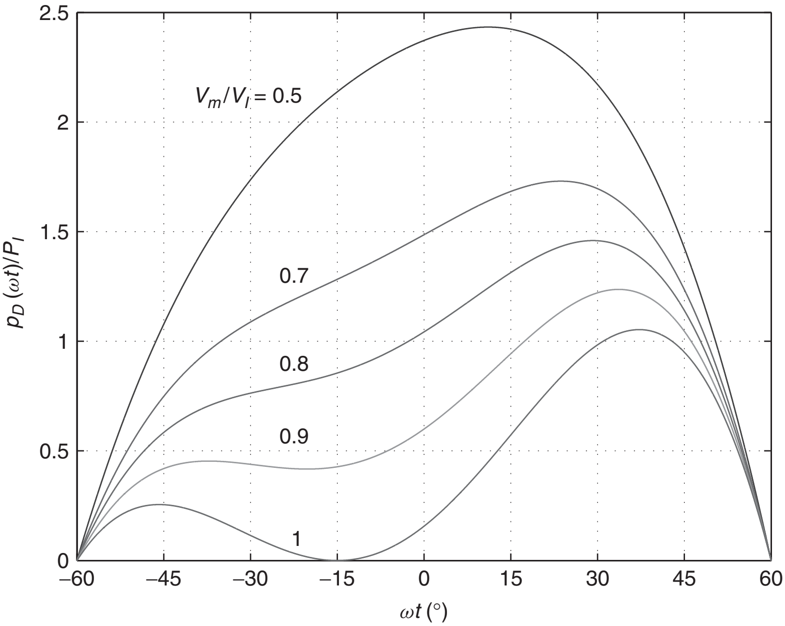

Figure 3.12 Normalized drain power loss $p_D(\omega t)/P_I$ at conduction angle $\theta = 60^\circ$ and at $\phi = 15^\circ$ for the Class C RF power amplifier.

The ratio of the fundamental component of the drain-to-source voltage V_m to the dc supply voltage V_I is defined as

$$\xi_1 = \frac{V_m}{V_I}. \tag{3.61}$$

3.3.3 Efficiency of the Class AB, B, and C Amplifiers

The efficiency of the Class AB, B, and C amplifiers is given by

$$\eta_D = \frac{P_O}{P_I} = \frac{1}{2}\left(\frac{I_m}{I_I}\right)\left(\frac{V_m}{V_I}\right) = \frac{1}{2}\gamma_1\xi_1 = \frac{1}{2}\frac{\alpha_1}{\alpha_0}\frac{V_m}{V_I} = \frac{1}{2}\left(\frac{V_m}{V_I}\right)\frac{\theta - \sin\theta\cos\theta}{\sin\theta - \theta\cos\theta}$$

$$= \frac{\theta - \sin\theta\cos\theta}{2(\sin\theta - \theta\cos\theta)}\left(1 - \frac{V_{DSmin}}{V_I}\right). \tag{3.62}$$

The drain efficiency η_D as a function of the conduction angle θ at selected values of V_m/V_I is illustrated in Figure 3.13. As the conduction angle θ decreases from 180° to 0°, the drain efficiency η_D increases from 50 % to 100 % at $V_m = V_I$.

The maximum drain-to-source voltage is

$$V_{DSM} = V_I + V_m = 2V_I. \tag{3.63}$$

The maximum drain current is

$$I_{DM} = \frac{I_m}{\alpha_1}. \tag{3.64}$$

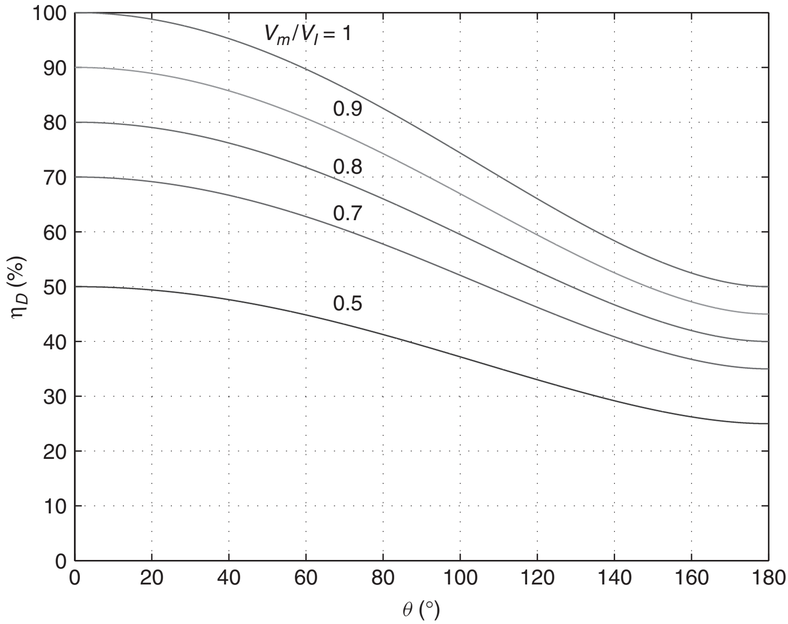

Figure 3.13 Drain efficiency η_D as a function of conduction angle θ at various values of V_m/V_I for the Class C RF power amplifier.

Thus, the maximum output power capability is

$$c_p = \frac{P_O}{V_{DSM} I_{DM}} = \frac{\frac{1}{2} V_m I_m}{2 V_I I_{DM}} = \frac{I_m}{4 I_{DM}} = \frac{\alpha_1}{4} = \frac{\theta - \sin\theta\cos\theta}{4\pi(1-\cos\theta)}. \tag{3.65}$$

The output power capability c_p as a function of V_m/V_I is shown in Figure 3.14. As the conduction angle θ approaches zero, c_p also approaches zero.

3.3.4 Parameters of Class AB Amplifier at $\theta = 120°$

For $90° \le \theta \le 180°$, we obtain the Class AB power amplifier. The Fourier coefficients of the drain current at the typical conduction angle $\theta = 120°$ are

$$\alpha_0 = \frac{I_I}{I_{DM}} = \frac{3\sqrt{3} + 2\pi}{9\pi} \approx 0.406 \tag{3.66}$$

and

$$\alpha_1 = \frac{I_m}{I_{DM}} = \frac{3\sqrt{3} + 8\pi}{18\pi} \approx 0.5363. \tag{3.67}$$

The ratio of the fundamental component to the dc component of the drain current is given by

$$\gamma_1 = \frac{I_m}{I_I} = \frac{\alpha_1}{\alpha_0} = \frac{3\sqrt{3} + 8\pi}{2(3\sqrt{3} + 2\pi)} = 1.321. \tag{3.68}$$

The drain efficiency at $\theta = 120°$ is

$$\eta_D = \frac{P_O}{P_I} = \frac{3\sqrt{3} + 8\pi}{4(3\sqrt{3} + 2\pi)} \left(1 - \frac{V_{DSmin}}{V_I}\right) \approx 0.6605 \left(1 - \frac{V_{DSmin}}{V_I}\right). \tag{3.69}$$

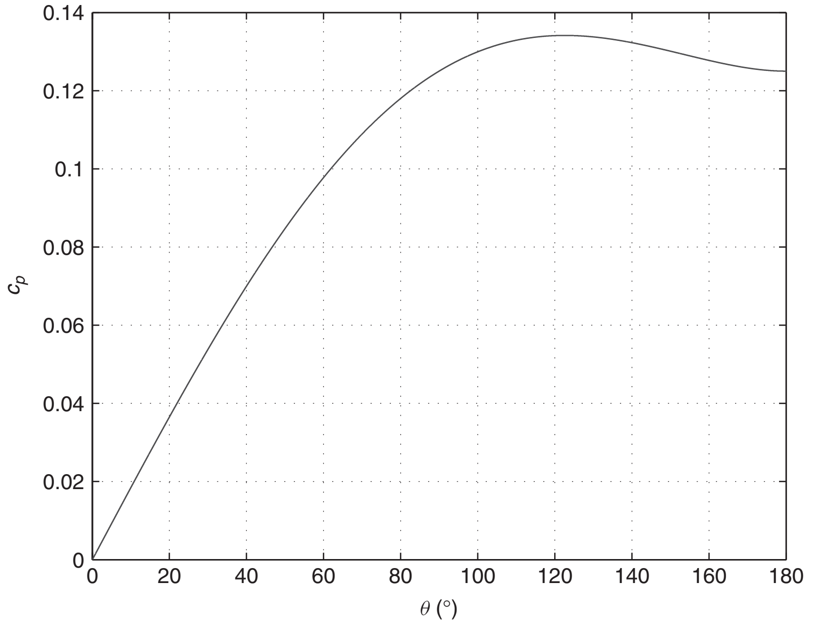

Figure 3.14 Output power capability c_p as a function of conduction angle θ for the Class C RF power amplifier.

The output power capability at $\theta = 120^\circ$ is

$$c_p = \frac{P_O}{V_{DSM} I_{DM}} = \frac{3\sqrt{3} + 8\pi}{72\pi} \approx 0.13408. \quad (3.70)$$

Example 3.2

Design a Class AB power amplifier to deliver power of 12 W at $f = 5$ GHz. The bandwidth is $BW = 500$ MHz and the conduction angle is $\theta = 120^\circ$. The power supply voltage is $V_I = 24$ V and $V_{DSmin} = 1$ V.

Solution. The maximum amplitude of the output voltage is

$$V_m = V_I - V_{DSmin} = 24 - 1 = 23\,\text{V}. \quad (3.71)$$

The load resistance is

$$R = \frac{V_m^2}{2P_O} = \frac{23^2}{2 \times 12} = 22\,\Omega. \quad (3.72)$$

The amplitude of the output current is

$$I_m = \frac{V_m}{R} = \frac{23}{22} = 1.04545\,\text{A}. \quad (3.73)$$

The dc supply current is

$$I_I = \frac{I_m}{\gamma_1} = \frac{1.04545}{1.321} = 1.098\,\text{A}. \quad (3.74)$$

For $\theta = 120^\circ$, the maximum drain current is

$$I_{DM} = \frac{I_I}{\alpha_0} = \frac{1.4508}{0.406} = 3.573\,\text{A}. \tag{3.75}$$

The maximum drain-to-source voltage is

$$V_{DSM} = 2V_I = 2 \times 24 = 48\,\text{V}. \tag{3.76}$$

The dc supply power is

$$P_I = I_I V_I = 1.098 \times 24 = 26.352\,\text{W}. \tag{3.77}$$

The drain power dissipation is

$$P_D = P_I - P_O = 26.352 - 12 = 16.9056\,\text{W}. \tag{3.78}$$

The drain efficiency is

$$\eta_D = \frac{P_O}{P_I} = \frac{12}{26.352} = 45.52\,\%. \tag{3.79}$$

The loaded quality factor is

$$Q_L = \frac{f}{BW} = \frac{5}{0.5} = 10. \tag{3.80}$$

The reactances of the resonant circuit components are

$$X_L = X_C = \frac{R}{Q_L} = \frac{22}{10} = 2.2\,\Omega \tag{3.81}$$

yielding

$$L = \frac{X_L}{\omega} = \frac{2.2}{2\pi \times 5 \times 10^9} = 0.07\,\text{nH} \tag{3.82}$$

and

$$C = \frac{1}{\omega X_C} = \frac{1}{2\pi \times 5 \times 10^9 \times 2.2} = 14.47\,\text{pF}. \tag{3.83}$$

3.3.5 Parameters of Class C Amplifier at $\theta = 60^\circ$

For $\theta < 90^\circ$, we obtain the Class C amplifier. The Fourier coefficients of the drain current at the typical conduction angle $\theta = 60^\circ$ are

$$\alpha_0 = \frac{I_I}{I_{DM}} = \frac{\sqrt{3}}{\pi} - \frac{1}{3} \approx 0.218 \tag{3.84}$$

and

$$\alpha_1 = \frac{I_m}{I_{DM}} = \frac{2}{3} - \frac{\sqrt{3}}{2\pi} \approx 0.391. \tag{3.85}$$

The ratio of the fundamental component to the dc component of the drain current is given by

$$\gamma_1 = \frac{I_m}{I_I} = \frac{\alpha_1}{\alpha_0} = \frac{4\pi - 3\sqrt{3}}{2(3\sqrt{3} - \pi)} = 1.7936. \tag{3.86}$$

The drain efficiency at $\theta = 60^\circ$ is

$$\eta_D = \frac{P_O}{P_I} = \frac{4\pi - 3\sqrt{3}}{4(3\sqrt{3} - \pi)}\left(1 - \frac{V_{DSmin}}{V_I}\right) \approx 0.8968\left(1 - \frac{V_{DSmin}}{V_I}\right). \tag{3.87}$$

The output power capability at $\theta = 60^\circ$ is

$$c_p = \frac{P_O}{V_{DSM} I_{DM}} = \frac{1}{6} - \frac{\sqrt{3}}{8\pi} \approx 0.09777. \tag{3.88}$$

Example 3.3

Design a Class C power amplifier to deliver power of 6 W at $f = 2.4$ GHz. The bandwidth is $BW = 240$ MHz and the conduction angle is $\theta = 60^\circ$. The power supply voltage is $V_I = 12$ V and $V_{DSmin} = 1$ V.

Solution. The maximum amplitude of the output voltage is

$$V_m = V_I - V_{DSmin} = 12 - 1 = 11 \text{ V}. \tag{3.89}$$

The load resistance is

$$R = \frac{V_m^2}{2P_O} = \frac{11^2}{2 \times 6} = 10\,\Omega. \tag{3.90}$$

The amplitude of the output current is

$$I_m = \frac{V_m}{R} = \frac{11}{10} = 1.1 \text{ A}. \tag{3.91}$$

The dc supply current is

$$I_I = \frac{I_m}{\gamma_1} = \frac{1.1}{1.7936} = 0.6133 \text{ A}. \tag{3.92}$$

For $\theta = 60^\circ$, the maximum drain current is

$$I_{DM} = \frac{I_I}{\alpha_0} = \frac{0.6133}{0.218} = 2.813 \text{ A}. \tag{3.93}$$

The maximum drain-to-source voltage is

$$V_{DSM} = 2V_I = 2 \times 12 = 24 \text{ V}. \tag{3.94}$$

The dc supply power is

$$P_I = I_I V_I = 0.6133 \times 12 = 7.3596 \text{ W}. \tag{3.95}$$

The drain power dissipation is

$$P_D = P_I - P_O = 7.3596 - 6 = 1.3596 \text{ W}. \tag{3.96}$$

The drain efficiency is

$$\eta_D = \frac{P_O}{P_I} = \frac{6}{7.3596} = 81.52\,\%. \tag{3.97}$$

The loaded quality factor is

$$Q_L = \frac{f}{BW} = \frac{2.4}{0.24} = 10. \tag{3.98}$$

The reactances of the resonant circuit components are

$$X_L = X_C = \frac{R}{Q_L} = \frac{10}{10} = 1\,\Omega \tag{3.99}$$

yielding

$$L = \frac{X_L}{\omega} = \frac{1}{2\pi \times 2.4 \times 10^9} = 0.0663 \text{ nH} \tag{3.100}$$

and

$$C = \frac{1}{\omega X_C} = \frac{1}{2\pi \times 2.4 \times 10^9 \times 1} = 66.63\,\text{pF}. \tag{3.101}$$

3.3.6 Parameters of Class C Amplifier at $\theta = 45^\circ$

The Fourier coefficients of the drain current at the conduction angle $\theta = 45^\circ$ are

$$\alpha_0 = \frac{I_I}{I_{DM}} = \frac{\sqrt{2}(4-\pi)}{4\pi(2-\sqrt{2})} \approx 0.16491 \tag{3.102}$$

and

$$\alpha_1 = \frac{I_m}{I_{DM}} = \frac{\pi - 2}{2\pi(2-\sqrt{2})} \approx 0.31016. \tag{3.103}$$

The ratio I_m/I_I is

$$\gamma_1 = \frac{I_m}{I_I} = \frac{\alpha_1}{\alpha_0} = \frac{\sqrt{2}(\pi-2)}{4-\pi} = 1.8808. \tag{3.104}$$

The drain efficiency at $\theta = 45^\circ$ is

$$\eta_D = \frac{P_O}{P_I} = \frac{\pi - 2}{\sqrt{2}(4-\pi)}\left(1 - \frac{V_{DSmin}}{V_I}\right) \approx 0.940378\left(1 - \frac{V_{DSmin}}{V_I}\right). \tag{3.105}$$

The output power capability at $\theta = 60^\circ$ is

$$c_p = \frac{P_O}{V_{DSM} I_{DM}} = \frac{\pi - 2}{8\pi(2-\sqrt{2})} \approx 0.07754. \tag{3.106}$$

Various coefficients for Class AB, B, and C power amplifiers are given in Table 3.1.

Table 3.1 Coefficients for Class AB, B, and C amplifiers.

θ	α_0	α_1	γ_1	η_D	c_p
10°	0.0370	0.0738	1.9939	0.9967	0.01845
20°	0.0739	0.1461	1.9756	0.9879	0.03651
30°	0.1106	0.2152	1.9460	0.9730	0.05381
40°	0.1469	0.2799	1.9051	0.9526	0.06998
45°	0.1649	0.3102	1.8808	0.9404	0.0775
50°	0.1828	0.3388	1.8540	0.9270	0.08471
60°	0.2180	0.3910	1.7936	0.8968	0.09775
70°	0.2525	0.4356	1.7253	0.8627	0.10889
80°	0.2860	0.4720	1.6505	0.8226	0.11800
90°	0.3183	0.5000	1.5708	0.7854	0.12500
100°	0.3493	0.5197	1.4880	0.7440	0.12993
110°	0.3786	0.5316	1.4040	0.7020	0.13290
120°	0.4060	0.5363	1.3210	0.6605	0.13409
130°	0.4310	0.5350	1.2414	0.6207	0.13376
140°	0.4532	0.5292	1.1675	0.5838	0.13289
150°	0.4720	0.5204	1.1025	0.5512	0.1301
160°	0.4868	0.5110	1.0498	0.5249	0.12775
170°	0.4965	0.5033	1.0137	0.5069	0.12582
180°	0.5000	0.5000	1.0000	0.5000	0.12500

Example 3.4

Design a Class C power amplifier to deliver power of 1 W at $f = 2.4$ GHz. The bandwidth is $BW = 240$ MHz and the conduction angle is $\theta = 45^\circ$. The power supply voltage is $V_I = 5$ V and $V_{DSmin} = 0.2$ V.

Solution. The maximum amplitude of the output voltage is

$$V_m = V_I - V_{DSmin} = 5 - 0.2 = 4.8\,\text{V}. \tag{3.107}$$

The load resistance is

$$R = \frac{V_m^2}{2P_O} = \frac{4.8^2}{2 \times 1} = 11.52\,\Omega. \tag{3.108}$$

The amplitude of output current is

$$I_m = \frac{V_m}{R} = \frac{4.8}{11.52} = 0.416\,\text{A}. \tag{3.109}$$

The dc supply current is

$$I_I = \frac{I_m}{\gamma_1} = \frac{0.416}{1.808} = 0.2211\,\text{A}. \tag{3.110}$$

For $\theta = 45^\circ$, the maximum drain current is

$$I_{DM} = \frac{I_I}{\alpha_0} = \frac{0.2211}{0.1613} = 1.3707\,\text{A}. \tag{3.111}$$

The maximum drain-to-source voltage is

$$V_{DSM} = 2V_I = 2 \times 5 = 10\,\text{V}. \tag{3.112}$$

The dc supply power is

$$P_I = I_I V_I = 0.2211 \times 5 = 1.1055\,\text{W}. \tag{3.113}$$

The drain power dissipation is

$$P_D = P_I - P_O = 1.1055 - 1 = 0.1055\,\text{W}. \tag{3.114}$$

The drain efficiency is

$$\eta_D = \frac{P_O}{P_I} = \frac{1}{1.1055} = 90.45\,\%. \tag{3.115}$$

The loaded quality factor is

$$Q_L = \frac{f}{BW} = \frac{2.4}{0.24} = 10. \tag{3.116}$$

The reactances of the resonant circuit components are

$$X_L = X_C = \frac{R}{Q_L} = \frac{11.52}{100} = 1.152\,\Omega \tag{3.117}$$

yielding

$$L = \frac{X_L}{\omega} = \frac{1.152}{2\pi \times 2.4 \times 10^9} = 0.07639\,\text{nH} \tag{3.118}$$

and

$$C = \frac{1}{\omega X_C} = \frac{1}{2\pi \times 2.4 \times 10^9 \times 1.152} = 57.564\,\text{pF}. \tag{3.119}$$

3.4 Push-pull Complementary Class AB, B, and C RF Power Amplifiers

3.4.1 Circuit

A circuit of push-pull CMOS Class AB, B, or C RF power amplifiers is shown in Figure 3.15. It consists of complementary pair of transistors (NMOS and PMOS), a parallel-resonant circuit, and coupling capacitor C_C. The transistors should have matched characteristics and are operated as voltage-dependent current sources. Since complementary transistors are used, the circuit is called a *complementary push-pull amplifier* or a *complementary-symmetry push-pull amplifier*. If MOSFETs are used, the circuit is called a *CMOS push-pull power amplifier*. The circuit may also employ complementary bipolar junction transistors (CBJT): an *npn* transistor and a *pnp* transistor. A Class B push-pull amplifier uses one transistor to amplify the positive portion of the input voltage and another transistor to amplify the negative portion of the input voltage. The coupling capacitor C_C blocks the dc voltage from the load. It also maintains the dc voltage $V_I/2$ and supplies the PMOS transistor when the NMOS transistor is not conducting. Also, two supply voltages V_I can be connected to

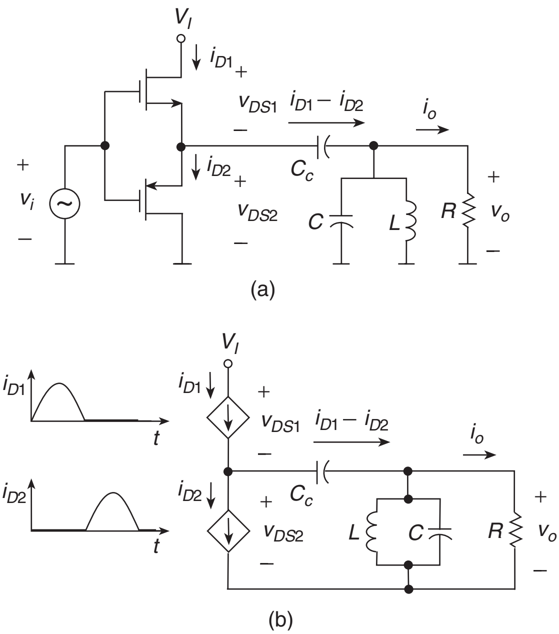

Figure 3.15 Circuit of push-pull CMOS Class AB, B, and C RF power amplifiers.

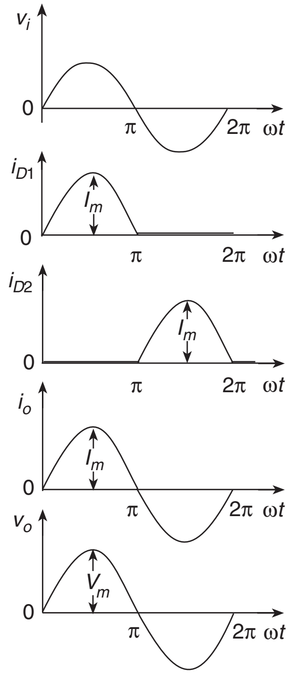

Figure 3.16 Current and voltage waveforms in push-pull CMOS Class B RF power amplifier.

the drains of both transistors. Current and voltage waveforms for the push-pull CMOS Class B RF power amplifier are shown in Figure 3.16. Similar waveforms can be drawn for Class AB and C amplifiers.

3.4.2 Even Harmonic Cancellation in Push-pull Amplifiers

Let us assume that both transistors are identical. The drain current of the upper MOSFET can be expanded into a Fourier series

$$i_{D1} = I_D + i_{d1} + i_{d2} + i_{d3} + \cdots = I_D + I_{dm1}\cos\omega t + I_{dm2}\cos 2\omega t + I_{dm3}\cos 3\omega t + \cdots \tag{3.120}$$

The drain current of the lower MOSFET is shifted in phase with respect of the drain current of the upper MOSFET by 180° and can be expanded into a Fourier series

$$\begin{aligned} i_{D2} &= i_{D1}(\omega t - 180^\circ) \\ &= I_D + I_{dm1}\cos(\omega t - 180^\circ) + I_{dm2}\cos 2(\omega t - 180^\circ) + I_{dm3}\cos 3(\omega t - 180^\circ) + \cdots. \\ &= I_D - I_{dm1}\cos\omega t + I_{dm2}\cos 2\omega t - I_{dm3}\cos 3\omega t + \cdots. \end{aligned} \tag{3.121}$$

Hence, the load current flowing through the coupling capacitor C_B is

$$i_L = i_{D1} - i_{D2} = 2I_{dm1}\cos\omega t + 3I_{dm3}\cos 3\omega t + \cdots. \tag{3.122}$$

Thus, cancellation of all even harmonics of the load current takes place in push-pull power amplifiers, reducing distortion of the output voltage and the THD. Only odd harmonics must be filtered out by the parallel-resonant circuit, which is a bandpass filter. The same property holds true for all push-pull amplifiers operating in all classes.

3.4.3 Power Relationships

Let us consider the Class B RF power amplifier. The output voltage is

$$v_o = V_m \cos \omega t \tag{3.123}$$

and the output current is

$$i_o = I_m \cos \omega t \tag{3.124}$$

and

$$I_m = \frac{V_m}{R} = 2I_{dm} \tag{3.125}$$

where $I_{dm} = I_{dm1}$ is the peak value of the drain current. The ac output power is

$$P_O = \frac{V_m I_m}{2} = \frac{V_m^2}{2R} = \frac{R I_m^2}{2}. \tag{3.126}$$

The dc supply power is equal to the average value of the drain current of the upper transistor

$$I_I = \frac{1}{2\pi} \int_{-\pi/2}^{\pi/2} i_{D1} \, d(\omega t) = \frac{I_m}{\pi} = \frac{V_m}{\pi R}. \tag{3.127}$$

The dc resistance is

$$R_{DC} = \frac{V_I}{I_I} = \frac{\pi V_I}{V_m} R. \tag{3.128}$$

For $V_m = 0$, $R_{DC} = \infty$. For $V_m = V_I$, the dc resistance is

$$R_{DSmin} = \frac{V_I}{I_{Imax}} = \pi R. \tag{3.129}$$

The dc supply power is

$$P_I = V_I I_I = \frac{V_I I_m}{\pi} = \frac{V_I V_m}{\pi R}. \tag{3.130}$$

The drain power dissipated in both transistors is

$$P_D = P_I - P_O = \frac{V_I V_m}{\pi R} - \frac{V_m^2}{2R}. \tag{3.131}$$

Setting the derivative of P_D to zero,

$$\frac{dP_D}{dV_m} = \frac{V_I}{\pi R} - \frac{V_m}{R} = 0 \tag{3.132}$$

we obtain

$$V_{m(cr)} = \frac{V_I}{\pi}. \tag{3.133}$$

Hence, the maximum power dissipation in both transistors is

$$P_{Dmax} = \frac{V_I^2}{2\pi^2 R}. \tag{3.134}$$

The drain efficiency is

$$\eta_D = \frac{P_O}{P_I} = \frac{\pi}{4}\frac{V_m}{V_I}. \tag{3.135}$$

For $V_m = V_I$,

$$\eta_{Dmax} = \frac{\pi}{4} = 78.5\,\%. \tag{3.136}$$

For $V_{m(max)} = V_I - V_{DSmin}$,

$$\eta_D = \frac{\pi}{4}\frac{V_{m(max)}}{V_I} = \frac{\pi}{4}\left(1 - \frac{V_{DSmin}}{V_I}\right). \tag{3.137}$$

3.4.4 Device Stresses

The transistor current and voltage stresses are

$$I_{DM} = I_m \tag{3.138}$$

and

$$V_{DSM} = 2V_I. \tag{3.139}$$

The amplitudes of the currents through the resonant inductance and capacitance are

$$I_{Lm} = I_{Cm} = Q_L I_m. \tag{3.140}$$

The output-power capability is

$$c_p = \frac{P_O}{2I_{DM}V_{DSM}} = \frac{1}{4}\left(\frac{I_m}{I_{DM}}\right)\left(\frac{V_m}{V_{DSM}}\right) = \frac{1}{4} \times 1 \times \frac{1}{2} = \frac{1}{8}. \tag{3.141}$$

Example 3.5

Design a CMOS Class B RF power amplifier to deliver power of 50 W at $f = 1.8$ GHz. The load resistance is $R_L = 50\,\Omega$. The bandwidth is $BW = 180$ MHz. The power supply voltage is $V_I = 48$ V and $V_{DSmin} = 1$ V.

Solution. Assuming the efficiency of the resonant circuit $\eta_r = 0.95$, the drain power is

$$P_{DS} = \frac{P_O}{\eta_r} = \frac{50}{0.95} = 52.632\,\text{W}. \tag{3.142}$$

The amplitude of the drain-to-source voltage is

$$V_{dm} = V_I - V_{DSmin} = 48 - 1 = 47\,\text{V}. \tag{3.143}$$

The load resistance is

$$R = \frac{V_{dm}^2}{2P_{DS}} = \frac{47^2}{2 \times 52.632} = 20.985\Omega. \tag{3.144}$$

The amplitude of the fundamental component of the drain current is

$$I_{dm} = \frac{V_{dm}}{R} = \frac{47}{20.985} = 2.2396\,\text{A} \tag{3.145}$$

$$I_I = \frac{2}{\pi}I_{dm} = \frac{2}{\pi} \times 2.2396 = 1.4257\text{A}. \tag{3.146}$$

The dc supply power is

$$P_I = V_I I_I = 48 \times 1.4257 = 68.43\,\mathrm{W}. \tag{3.147}$$

The total efficiency is

$$\eta = \frac{P_O}{P_I} = \frac{50}{68.43} = 73.06\,\%. \tag{3.148}$$

The maximum power loss in each transistor is

$$P_{Dmax} = \frac{V_I^2}{\pi^2 R} = \frac{48^2}{\pi^2 \times 20.985} = 11.124\,\mathrm{W}. \tag{3.149}$$

The maximum drain current is

$$I_{DM} = I_{dm} = 2.2396\,\mathrm{A}. \tag{3.150}$$

The maximum drain-to-source voltage is

$$V_{DSM} = 2V_I = 2 \times 48 = 96\,\mathrm{A}. \tag{3.151}$$

The loaded-quality factor is

$$Q_L = \frac{f_c}{BW} = \frac{1800}{180} = 10. \tag{3.152}$$

The resonant inductance is

$$L = \frac{R_L}{\omega Q_L} = \frac{20.985}{2\pi \times 1.8 \times 10^9 \times 10} = 0.1855\,\mathrm{nH}. \tag{3.153}$$

The resonant capacitance is

$$C = \frac{Q_L}{\omega R_L} = \frac{10}{2\pi \times 1.8 \times 10^9 \times 20.985} = 42.1\,\mathrm{pF}. \tag{3.154}$$

3.5 Transformer-coupled Class B Push-pull Amplifier

3.5.1 Waveforms

A circuit of the transformer-coupled push-pull Class AB, B, and C RF power amplifiers is shown in Figure 3.17. Waveforms are depicted in Figure 3.18. The output current is

$$i_o = I_m \sin \omega t = nI_{dm} \sin \omega t \tag{3.155}$$

where I_m is the amplitude of the output current, I_{dm} is the peak drain current and n is the transformer turns ratio, equal to the ratio of number of turns of one primary to the number of turns of the secondary

$$n = \frac{I_m}{I_{dm}} = \frac{V_{dm}}{V_m}. \tag{3.156}$$

The output voltage is

$$v_o = V_m \sin \omega t = \frac{V_{dm}}{n} \sin \omega t \tag{3.157}$$

where the amplitude of the output current is

$$V_m = I_m R_L. \tag{3.158}$$

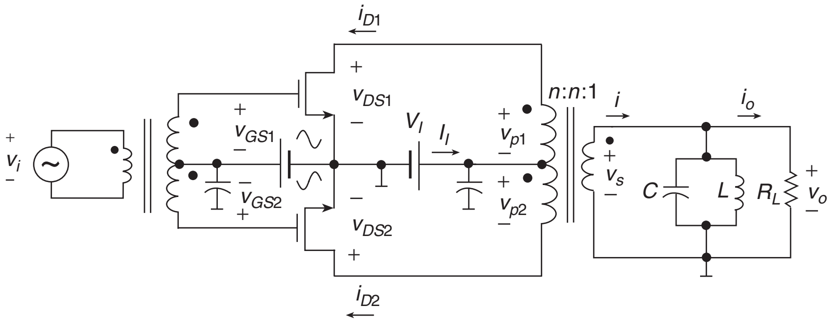

Figure 3.17 Circuit of transformer-coupled push-pull Class AB, B, and C RF power amplifiers.

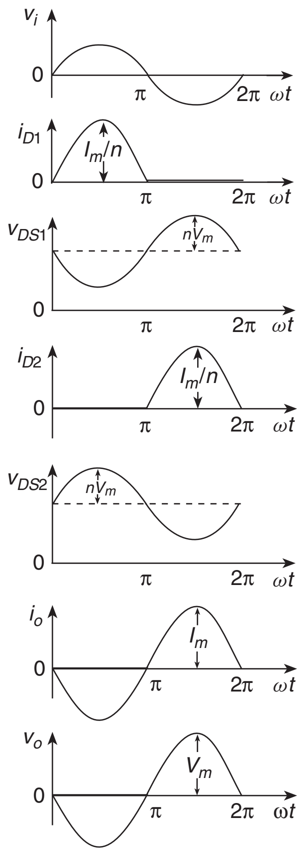

Figure 3.18 Waveforms in transformer-coupled push-pull Class AB, B, and C RF power amplifiers.

The resistance seen by each transistor across each primary winding with the other primary winding open is

$$R = n^2 R_L. \tag{3.159}$$

When the drive voltage is positive, transistor Q_1 is ON and transistor Q_2 is OFF. The waveforms are

$$i_{D1} = I_{dm}\sin\omega t = \frac{I_m}{n}\sin\omega t \quad \text{for} \quad 0 < \omega t \le \pi \tag{3.160}$$

$$i_{D2} = 0 \tag{3.161}$$

$$v_{p1} = -i_{D1}R = -i_{D1}n^2R_L = v_{p2} = -RI_{dm}\sin\omega t = -\frac{I_m}{n}R\sin\omega t$$
$$= -nI_mR_L\sin\omega t \tag{3.162}$$

and

$$v_{DS1} = V_I + v_{p1} = V_I - i_{D1}R = V_I - i_{D1}n^2R_L = V_I - I_{dm}R\sin\omega t$$
$$= V_I - \frac{I_m}{n}R\sin\omega t = V_I - nI_mR_L\sin\omega t. \tag{3.163}$$

When the drive voltage is negative, transistor Q_1 is OFF and transistor Q_2 is ON,

$$i_{D1} = 0 \tag{3.164}$$

$$i_{D2} = -I_{dm}\sin\omega t = -\frac{I_m}{n}\sin\omega t \quad \text{for} \quad \pi < \omega t \le 2\pi \tag{3.165}$$

$$v_{p2} = i_{D2}R = i_{D2}n^2R_L = v_{p2} = -I_{dm}R\sin\omega t = -\frac{I_m}{n}R\sin\omega t$$
$$= -nI_mR_L\sin\omega t \tag{3.166}$$

and

$$v_{DS2} = V_I - v_{p2} = V_I - i_{D2}R = V_I - i_{D2}n^2R_L = V_I + I_{dm}R\sin\omega t = V_I - \frac{I_m}{n}R_L\sin\omega t$$
$$= V_I + nI_mR_L\sin\omega t. \tag{3.167}$$

The voltage between the drains of the MOSFETs is

$$v_{D1D2} = v_{p1} + v_{p2} = -n^2R_L(i_{D1} - i_{D2}) = -2nR_LI_m\sin\omega t. \tag{3.168}$$

Ideally, all even harmonics cancel out as shown in Section 3.4.1. The voltage across the secondary winding is

$$v_s = \frac{v_{D1D2}}{2n} = -\frac{nR_L}{2}(i_{D1} - i_{D2}) = -R_LI_m\sin\omega t. \tag{3.169}$$

The current through the dc voltage source V_I is a full-wave rectified sinusoid given by

$$i_I = i_{D1} + i_{D2} = I_{dm}|\sin\omega t| = \frac{I_m}{n}|\sin\omega t|. \tag{3.170}$$

The dc supply current is

$$I_I = \frac{1}{2\pi}\int_0^{2\pi} I_{dm}|\sin\omega t|\,\mathrm{d}(\omega t) = \frac{1}{2\pi}\int_0^{2\pi}\frac{I_m}{n}|\sin\omega t|\,\mathrm{d}(\omega t) = \frac{2}{\pi}\frac{I_m}{n} = \frac{2}{\pi}\frac{V_m}{nR_L}. \tag{3.171}$$

3.5.2 Power Relationships

The dc supply power is given by

$$P_I = V_I I_I = \frac{2}{\pi}\frac{V_I V_m}{nR_L}. \tag{3.172}$$

The output power is

$$P_O = \frac{V_m^2}{2R_L}. \tag{3.173}$$

The drain power dissipation is

$$P_D = P_I - P_O = \frac{2}{\pi}\frac{V_I V_m}{nR_L} - \frac{V_m^2}{2R_L}. \tag{3.174}$$

The derivative of the power P_D with respect of the output voltage amplitude V_m is

$$\frac{\mathrm{d}P_D}{\mathrm{d}V_m} = \frac{2}{\pi}\frac{V_I}{nR_L} - \frac{V_m}{R_L} = 0. \tag{3.175}$$

resulting in the critical value of V_m at which the maximum value of the drain power loss P_D occurs

$$V_{m(cr)} = \frac{2}{\pi}\frac{V_I}{n}. \tag{3.176}$$

The maximum drain power loss in both transistors is

$$P_{Dmax} = \frac{2}{\pi^2}\frac{V_I^2}{n^2 R_L} = \frac{2}{\pi^2}\frac{V_I^2}{R}. \tag{3.177}$$

The amplitude of the drain-to-source voltage is

$$V_{dm} = \frac{V_m}{n} = \frac{2}{\pi}V_I. \tag{3.178}$$

The drain efficiency is

$$\eta_D = \frac{P_O}{P_I} = \frac{\pi}{4}\frac{V_{dm}}{V_I} = \frac{\pi}{4}\frac{nV_m}{V_I}. \tag{3.179}$$

For $V_{dm} = nV_m = V_I$, the dc supply power is

$$P_I = V_I I_I = \frac{2}{\pi}\frac{V_I^2}{n^2 R_L} = \frac{2}{\pi}\frac{V_I^2}{R}. \tag{3.180}$$

the output power is

$$P_O = \frac{V_m^2}{2R_L} = \frac{n^2 V_I^2}{2R_L}. \tag{3.181}$$

and the drain efficiency is

$$\eta_{Dmax} = \frac{\pi}{4} = 78.5\,\%. \tag{3.182}$$

3.5.3 Device Stresses

The MOSFET current and voltage stresses are

$$I_{DM} = I_{dm} = \frac{I_m}{n} \tag{3.183}$$

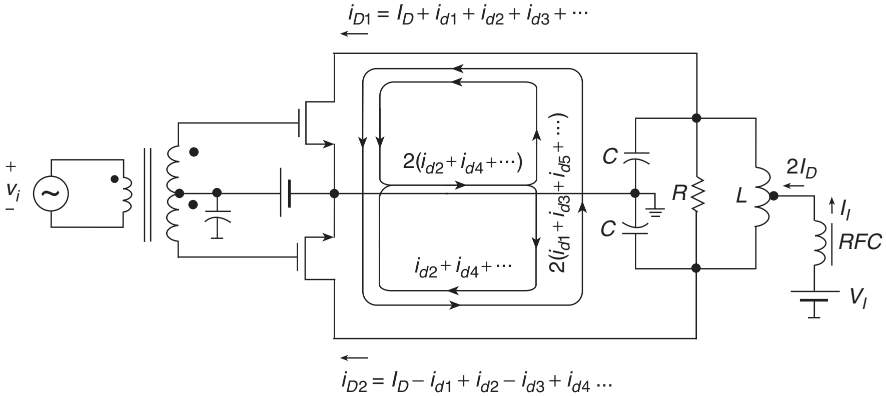

Figure 3.19 A circuit of push-pull Class AB, B, and C RF power amplifiers with tapped capacitor.

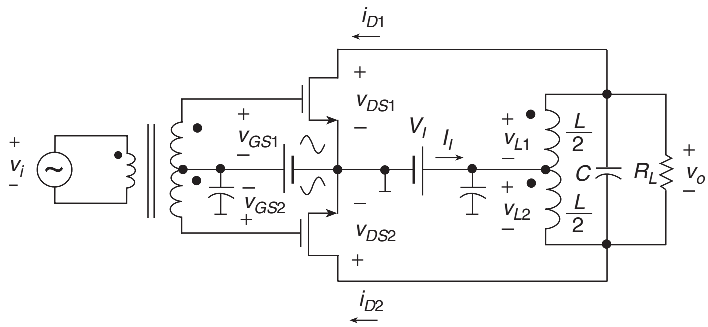

Figure 3.20 A circuit of push-pull Class AB, B, and C RF power amplifiers with tapped inductor.

and

$$V_{DSM} = 2V_I. \tag{3.184}$$

The output-power capability is

$$c_p = \frac{P_{Omax}}{2I_{DM}V_{DSM}} = \frac{P_{Omax}}{2I_{DM}V_{DSM}} = \frac{1}{4}\left(\frac{I_{dm}}{I_{SM}}\right)\left(\frac{V_m}{V_{DSM}}\right) = \frac{1}{4} \times 1 \times \frac{1}{2} = \frac{1}{8}. \tag{3.185}$$

A circuit of the push-pull Class AB, B, and C RF power amplifiers with tapped capacitor is depicted in Figure 3.19. This figure shows the distribution of the various components through the circuit and the cancellation of even harmonics in the load network. A circuit of the push-pull Class AB, B, and C RF power amplifiers with tapped inductor is shown in Figure 3.20. All equations for these two amplifiers can be obtained by setting $n = 1$ in the equations for the push-pull amplifier shown in Figure 3.17.

Example 3.6

Design a transformer-coupled Class B power amplifier to deliver power of 25 W at $f = 2.4$ GHz. The load resistance is $R_L = 50\,\Omega$. The bandwidth is $BW = 240$ MHz. The power supply voltage is $V_I = 28$ V and $V_{DSmin} = 1$ V.

Solution. Assuming the efficiency of the resonant circuit $\eta_r = 0.94$, the drain power is

$$P_{DS} = \frac{P_O}{\eta_r} = \frac{25}{0.94} = 26.596\,\text{W}. \tag{3.186}$$

The resistance seen by each transistor across one part of the primary winding is

$$R = \frac{\pi^2}{2}\frac{V_I^2}{P_{DS}} = \frac{\pi^2}{2} \times \frac{28^2}{26.596} = 145.469\,\Omega. \tag{3.187}$$

The transformer turns ratio is

$$n = \sqrt{\frac{R}{R_L}} = \sqrt{\frac{145.469}{50}} = 1.7 \approx \frac{7}{4}. \tag{3.188}$$

The maximum value of the drain-to-source voltage is

$$V_{dm} = \pi V_I = \pi \times 28 = 87.965\,\text{V}. \tag{3.189}$$

The amplitude of the drain current is

$$I_{dm} = \frac{V_{dm}}{R} = \frac{87.965}{145.469} = 0.605\,\text{A}. \tag{3.190}$$

The amplitude of the output voltage is

$$V_m = \frac{V_{dm}}{n} = \frac{87.965}{1.7} = 51.74\,\text{V}. \tag{3.191}$$

The amplitude of the output current is

$$I_m = \frac{V_m}{R_L} = \frac{51.74}{50} = 1.0348\,\text{A}. \tag{3.192}$$

The dc supply current is

$$I_I = \frac{\pi}{2} I_{dm} = \frac{\pi}{2} \times 0.605 = 0.95\,\text{A}. \tag{3.193}$$

The dc supply power is

$$P_I = V_I I_I = 28 \times 0.95 = 26.6\,\text{W}. \tag{3.194}$$

The total efficiency is

$$\eta = \frac{P_O}{P_I} = \frac{25}{26.6} = 93.98\,\%. \tag{3.195}$$

The loaded-quality factor is

$$Q_L = \frac{f_c}{BW} = \frac{2.4}{0.24} = 10. \tag{3.196}$$

The resonant inductance is

$$L = \frac{R_L}{\omega Q_L} = \frac{50}{2\pi \times 2.4 \times 10^9 \times 10} = 0.3316\,\text{nH}. \tag{3.197}$$

The resonant capacitance is

$$C = \frac{Q_L}{\omega R_L} = \frac{10}{2\pi \times 2.4 \times 10^9 \times 50} = 13.26\,\text{pF}. \tag{3.198}$$

3.6 Class AB, B, and C Amplifiers of Variable-envelope Signals

The Class AB, B, and C amplifier can be used to amplify variable-envelope signals, such as AM signals. The ac component of the AM gate-to-source voltage is

$$v_{gs(AM)} = V_{gsm}(1 + m_{in} \cos \omega_m t) \cos \omega_c t. \tag{3.199}$$

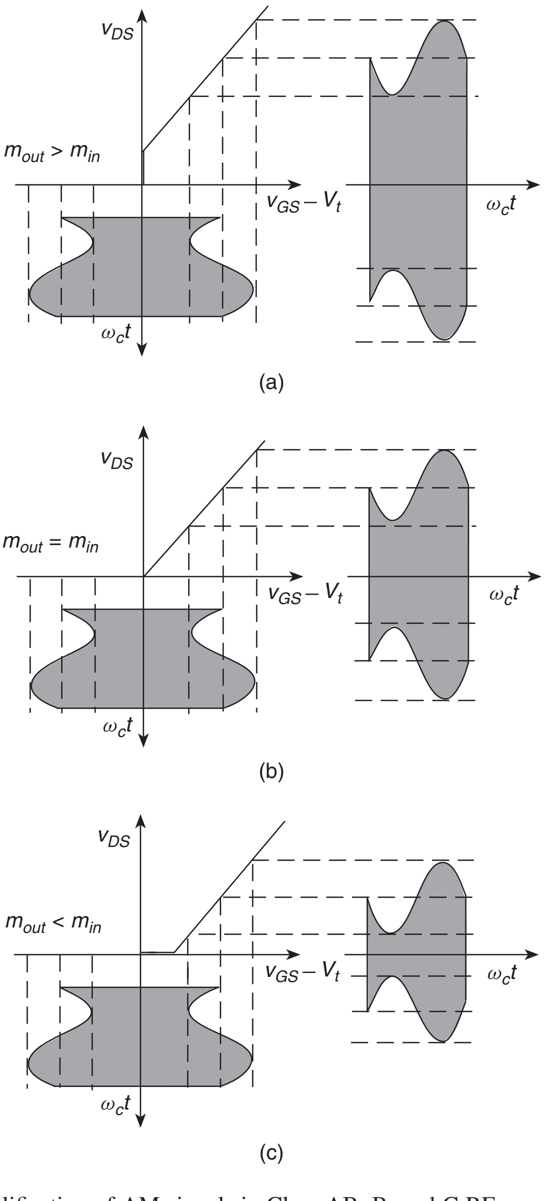

Figure 3.21 Amplification of AM signals in Class AB, B, and C RF power amplifiers. (a) Class AB amplifier ($m_{out} > m_{in}$). (b) Class B linear RF power amplifier ($m_{out} = m_{in}$). (c) Class C amplifier ($m_{out} < m_{in}$).

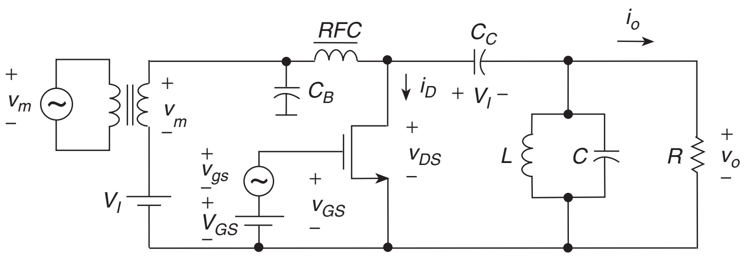

Figure 3.22 Drain AM signals in Class AB, B, and C RF power amplifiers.

The overall AM gate-to-source voltage is

$$v_{GS} = V_{GS} + v_{gs(AM)} = V_{GS} + V_{gsm}(1 + m_{in}\cos\omega_m t)\cos\omega_c t. \tag{3.200}$$

The AM drain current waveform of a power MOSFET is

$$\begin{aligned} i_D &= \frac{1}{2}C_{ox}Wv_{sat}(v_{GS} - V_t) \\ &= \frac{1}{2}C_{ox}Wv_{sat}[V_{GS} - V_t + V_{gsm}(1 + m_{in}\cos\omega_m t)\cos\omega_c t] \quad \text{for} \quad v_{GS} > V_t. \end{aligned} \tag{3.201}$$

Assuming that the impedance of the parallel-resonant circuit Z is equal to R (i.e., $Z \approx R$), we obtain the AM drain-to-source voltage

$$\begin{aligned} v_{DS} &\approx Ri_D \\ &= \frac{1}{2}C_{ox}Wv_{sat}R[V_{GS} - V_t + V_{gsm}(1 + m_{in}\cos\omega_m t)\cos\omega_c t] \quad \text{for} \quad v_{GS} > V_t. \end{aligned} \tag{3.202}$$

The choice of the class of operation, i.e., the operating point Q, has an important effect on nonlinear distortion of variable-envelope signals in RF power amplifiers in which transistors are operated as voltage-dependent current sources. The amplification of AM signals in Class AB, B, and C amplifiers is shown in Figure 3.21. Figure 3.21(a) illustrates the amplification of an AM signal in the Class AB amplifier, which produces an AM output voltage with $m_{out} > m_{in}$. The output voltage exhibits shallower modulation than the input signal. Figure 3.21(b) illustrates the amplification of an AM signal in the Class B amplifier, where $m_{out} = m_{in}$. The Class B amplifier behaves like a linear RF power amplifier. Its characteristic $v_{DS} = f(v_{GS} - V_t)$ is nearly linear and starts at the origin. Figure 3.21(c) illustrates the amplification of an AM signal in the Class C amplifier, which produces an AM output signal with $m_{out} < m_{in}$. The output voltage exhibits deeper modulation than the input signal. Class AB and C amplifiers can be used for amplifying AM signals with a small modulation index m.

Figure 3.22 shows a circuit of Class AB, B, and C power amplifiers with AM modulation. In these amplifiers, an AM signal is generated by placing the modulating voltage source v_m in series with the drain dc supply voltage source V_I of a Class C RF power amplifier that is driven into the ohmic (triode) region of the MOSFET. The gate-to-source voltage v_{GS} has a constant amplitude V_{gsm} and its frequency is equal to the carrier frequency f_c. The dc gate-to-source voltage V_{GS} is fixed. Therefore, the conduction angle θ of the drain current i_D is also fixed. These circuits require an audio-frequency transformer.

Class AB, B, and C power amplifiers can be used to amplify constant-envelope signals, such as FM and PM signals.

3.7 Summary

- The Class B RF power amplifier consists of a transistor, parallel resonant circuit, RF choke, and blocking capacitor.
- The transistor in the Class B power amplifier is operated as a dependent-current source.
- The conduction angle of the drain or collector current in the Class B power amplifier is 180°.
- The drain efficiency of the Class B power amplifier η_D is high. Ideally, $\eta_D = 78.5\,\%$.
- The conduction angle of the drain or collector current in the Class C power amplifier is less than 180°.
- The transistor is operated as a dependent-current source in the Class C power amplifier.
- The drain efficiency of the Class AB and C power amplifiers η_D increases from $50\,\%$ to $100\,\%$ as the conduction angle θ decreases from 180° to 0° at $V_m = V_I$.
- The drain efficiency of the Class C power amplifier is $89.68\,\%$ at $\theta = 60^\circ$ and $V_m = V_I$.
- The drain efficiency of the Class C power amplifier is $94.04\,\%$ at $\theta = 45^\circ$ and $V_m = V_I$.
- Class B and C RF power amplifiers are narrowband circuits because the resonant circuit acts like a bandpass filter.
- Class B and C RF power amplifiers are linear (or semilinear) because the amplitude of the drain current is proportional to the amplitude of the gate-to-source voltage.
- Class B and C RF power amplifiers are used as medium and high-power narrowband linear power amplifiers.
- A push-pull Class B power amplifier uses one transistor to amplify the positive portion of the input voltage and another transistor to amplify the negative portion of the input voltage. One transistor is 'pushing' the current into the load and the other transistor is 'pulling' the current from the load.
- In the push-pull topology, even harmonics are cancelled in the load, reducing distortion of the output voltage and the THD.
- The transformer in the transformer-coupled push-pull power amplifier performs the impedance matching function.

3.8 References

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3.9 Review Questions

3.1 List the components of the Class B RF power amplifier.

3.2 What is the transistor mode of operation in the Class B RF power amplifier?

3.3 What is the location of the operating point of the Class B RF power amplifier?

3.4 How high is the drain efficiency of the Class B RF power amplifier?

3.5 What is the mode of operation of the transistor in the Class C RF power amplifier?

3.6 What is the location of the operating point of the Class C RF power amplifier?

3.7 What is the drain efficiency of the Class C RF power amplifier at $\theta = 60^\circ$ and $\theta = 45^\circ$?

3.8 Are the drain current and output voltage dependent on the gate-to-source voltage in Class B and C amplifiers?

3.9 Explain the principle of operation of the CMOS push-pull Class B RF power amplifier.

3.10 Explain the principle of operation of the transformer-coupled push-pull Class B RF power amplifier.

3.11 What kind of harmonics is present in the load in push-pull amplifiers?

3.12 How does the transformer contribute to the impedance matching in power amplifiers?

3.10 Problems

3.1 Design a Class B RF power amplifier to meet the following specifications: $V_I = 3.3$ V, $P_O = 1$ W, $V_{DSmin} = 0.2$ V, $BW = 240$ MHz, and $f = 2.4$ GHz.

3.2 Design a Class AB RF power amplifier to meet the following specifications: $V_I = 48$ V, $P_O = 22$ W, $V_{DSmin} = 1$ V, $\theta = 120^\circ$, $BW = 90$ MHz, and $f = 0.9$ GHz.

3.3 Design a Class C RF power amplifier to meet the following specifications: $V_I = 3.3$ V, $P_O = 0.25$ W, $V_{DSmin} = 0.2$ V, $\theta = 60^\circ$, $BW = 240$ MHz, and $f = 2.4$ GHz.

3.4 Design a Class C RF power amplifier to meet the following specifications: $V_I = 12$ V, $P_O = 6$ W, $V_{DSmin} = 1$ V, $\theta = 45^\circ$, $BW = 240$ MHz, and $f = 2.4$ GHz.

4

Class D RF Power Amplifiers

4.1 Introduction

Class D RF resonant power amplifiers [1–18], also called Class D dc-ac resonant power inverters, were was invented in 1959 by Baxandall [1] and have been widely used in various applications [2–12] to convert dc energy into ac energy. Examples of applications of resonant amplifiers are radio transmitters, dc-dc resonant converters, solid-state electronic ballasts for fluorescent lamps, high-frequency electric heating applied in induction welding, surface hardening, soldering and annealing, induction sealing for tamper-proof packaging, fiber-optics production, and dielectric heating for plastic welding. In Class D amplifiers, transistors are operated as switches. Class D amplifiers can be classified into two groups:

- Class D voltage-switching (or voltage-source) amplifiers;
- Class D current-switching (or current-source) amplifiers.

Class D voltage-switching amplifiers are fed by a dc voltage source. They employ (1) a series-resonant circuit or (2) a resonant circuit that is derived from the series-resonant circuit. If the loaded quality factor is sufficiently high, the current through the resonant circuit is sinusoidal and the current through the switches is a half sine wave. The voltage waveforms across the switches are square waves.

In contrast, Class D current-switching amplifiers are fed by a dc current source in the form of an RF choke and a dc voltage source. These amplifiers contain a parallel-resonant circuit or a resonant circuit that is derived from the parallel-resonant circuit. The voltage across the resonant circuit is sinusoidal for high values of the loaded quality factor. The voltage across the switches is a half-wave sinusoid, and the current through the switches is square wave.

One main advantage of Class D voltage-switching amplifiers is the low voltage across each transistor, which is equal to the supply voltage. This makes these amplifiers suitable for high-voltage applications. For example, a 220 V or 277 V rectified line voltage is used to supply the Class D amplifiers. In addition, low-voltage MOSFETs can also be used. Such

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MOSFETs have a low on-resistance, reducing the conduction losses and operating junction temperature. This yields high efficiency. The MOSFET's on-resistance r_{DS} increases considerably with increasing junction temperature. This causes the conduction loss $r_{DS}I_{rms}^2$ to increase, where I_{rms} is the rms value of the drain current. Typically, r_{DS} doubles as the temperature rises by $100\,^\circ$C (for example, from 25 to $125\,^\circ$C), doubling the conduction loss. The MOSFET's on-resistance r_{DS} increases with temperature T because both the mobility of electrons $\mu_n \approx K_1/T^{2.5}$ and the mobility of holes $\mu_p \approx K_2/T^{2.7}$ decrease with T for $100\,\text{K} \le T \le 400\,\text{K}$, where K_1 and K_2 are constants. In many applications, the output power or the output voltage can be controlled by varying the operating frequency f (FM control) or phase shift (phase control).

In this chapter, we study Class D half-bridge and full-bridge series-resonant amplifiers. The design procedure of the Class D amplifier is illustrated with detailed examples.

4.2 Circuit Description

Figure 4.1 shows the circuit of a Class D voltage-switching (voltage-source) RF power amplifier with a pulse transformer driver. The circuit consists of two n-channel MOSFETs, a series resonant circuit, and a driver. It is difficult to drive the upper MOSFET. A high-side gate driver is required. A pulse transformer can be used to drive the MOSFETs. The noninverting output of the transformer drives the upper MOSFET and the inverting output the transformer drives the lower MOSFET. A pump-charge IC driver can also be used. A circuit of the Class D voltage-switching RF power amplifier with two power supplies V_I and $-V_I$ is depicted in Figure 4.2 [14].

Figure 4.3 shows a circuit of the Class D CMOS RF power amplifier, in which a PMOS MOSFET Q_P and an NMOS MOSFET Q_N are used as switching devices. This circuit can be integrated for high-frequency applications, such as RF transmitters for wireless communications. The CMOS Class D amplifier requires only one driver. However, cross-conduction of both transistors during the MOSFETs transitions may cause spikes in the drain currents. Nonoverlapping gate-to-source voltages may reduce this problem, but the driver will become more complex [16, 17]. The peak-to-peak value of the gate-to-source drive voltage v_G is equal or close to the dc supply voltage V_I, like in CMOS digital gates. Therefore, this circuit is appropriate only for low values of the dc supply voltage V_I, usually below 20 V. At high values of the dc supply voltage V_I, the gate-to-source voltage should also be high, which may cause voltage breakdown of the gate.

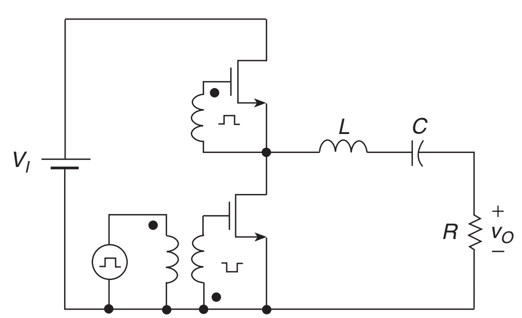

Figure 4.1 Class D half-bridge voltage-switching (voltage-source) RF power amplifier with a series-resonant circuit and a pulse transformer driver.

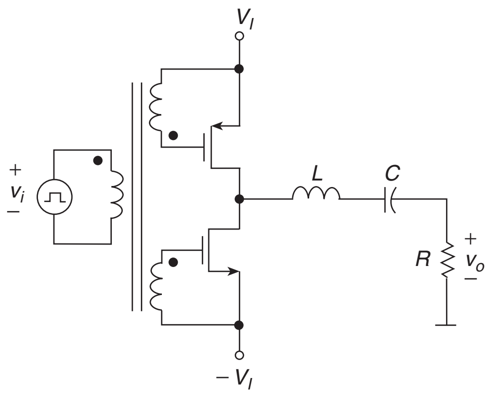

Figure 4.2 Class D half-bridge voltage-switching (voltage-source) RF power amplifier with a series-resonant circuit and a pulse transformer driver.

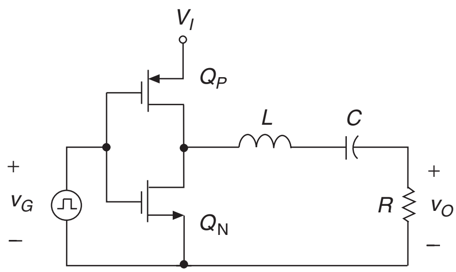

Figure 4.3 Class D CMOS half-bridge voltage-switching (voltage-source) RF power amplifier with a series-resonant circuit.

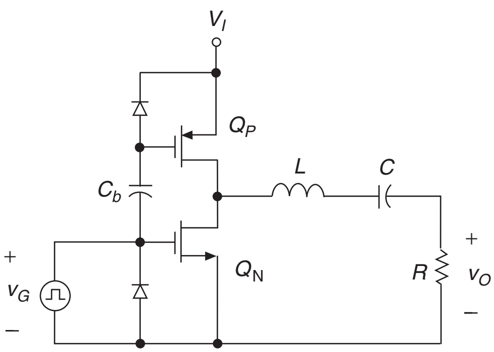

Figure 4.4 Class D half-bridge RF power amplifier with a voltage mirror driver [5].

Figure 4.4 shows the circuit of a Class D RF power amplifier with a voltage mirror driver or voltage level shifter [5]. The dc supply voltage V_I can be much higher than the peak-to-peak voltage of the gate-to-source voltage. Therefore, this circuit can be used in high voltage applications.

A circuit of the Class D half-bridge RF power amplifier [1–12] with a series-resonant circuit and a pulse transformer driver is shown in Figure 4.5(a). It consists of two bidirectional

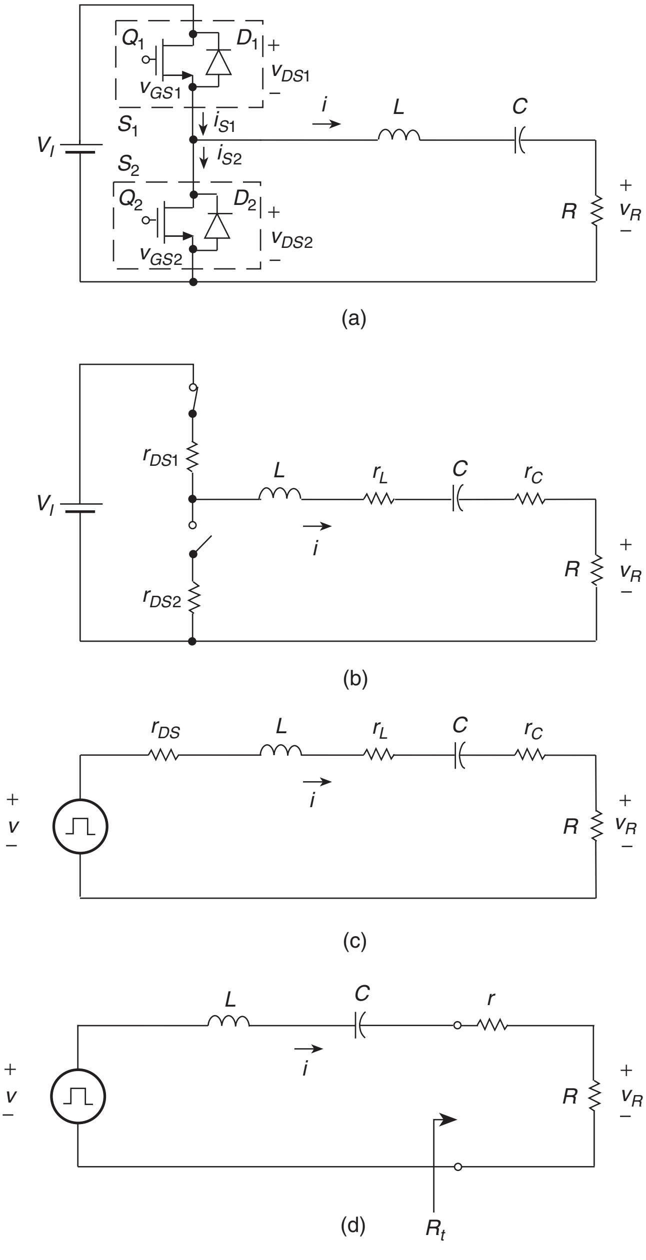

Figure 4.5 Class D half-bridge RF power amplifier with a series-resonant circuit. (a) Circuit. (b)–(d) Equivalent circuits.

switches S_1 and S_2 and a series-resonant circuit L-C-R. Each switch consists of a transistor and an antiparallel diode. The MOSFET's intrinsic body-drain *pn* junction diode may be used as an antiparallel diode in case of an inductive load, which will be discussed shortly. The switch can conduct either positive or negative current. However, it can only accept voltages higher than the negative value of a forward diode voltage $-V_{Don} \approx -1$ V. A positive or negative switch current can flow through the transistor if the transistor is ON. If the transistor is OFF, the switch can conduct only negative current that flows through the diode.

The transistors are driven by nonoverlapping rectangular-wave voltages v_{GS1} and v_{GS2} with a dead time at an operating frequency $f = 1/T$. Switches S_1 and S_2 are alternately ON and OFF with a duty cycle of 50 % or slightly less. The dead time is the time interval, when both the switching devices are OFF. Input resistance R is an ac load to which the ac power is to be delivered. If the amplifier is part of a dc-dc resonant converter, R represents an input resistance of the rectifier.

There has been increasing interest in designing RF power amplifiers in digital technology in the age of a system-on-chip (SoC). There is a trend to integrate a complete transceiver together with the digital baseband subsystem on a single chip. There are two main issues in designing power amplifiers in submicron CMOS technology: oxide breakdown and hot carrier effects. Both these problems became worse as the technology scales down. The oxide breakdown is a catastrophic effect and sets a limit on the maximum signal swing on the MOSFET drain. The hot carrier effect reduces the reliability. It increases the threshold voltage and consequently degrades the performance of the transistors. Switching-mode RF power amplifiers offer high efficiency and can be used for constant-envelope modulated signals, such as Gaussian minimum shift keying (GMSK) used in Global Systems for Mobile (GSM) Communications.

Integrated MOSFETs may be used at low output power levels and discrete power MOSFETs at high output power levels. In integrated MOSFETs, the source and the drain are on the same side of the chip surface, causing horizontal current flow from drain to source when the device is ON. When the device is OFF, the depletion region of the reverse-biased drain-to-body pn junction diode spreads into the lightly doped short channel, resulting in a low punch-through breakdown voltage between the drain and the source. The breakdown voltage is proportional to the channel length L, whereas the maximum drain current is inversely proportional to the channel length L. If an integrated MOSFET is designed to have a high breakdown voltage, its channel length must be increased, which reduces the device aspect ratio W/L and the maximum drain current decreases.

In integrated MOSFETs, two contradictory requirements are imposed on the region between the drain and the source: a short channel to achieve a high drain current when the device is ON and a long channel to achieve a high breakdown voltage when the device is OFF.

The i_D-v_{DS} characteristics of a MOSFET in the ohmic region are given by

$$i_D = \mu_n C_{ox}\left(\frac{W}{L}\right)\left[(v_{GS} - V_t)v_{DS} - \frac{v_{DS}^2}{2}\right] \quad \text{for} \quad v_{GS} > V_t \quad \text{and} \quad v_{DS} < v_{GS} - V_t. \tag{4.1}$$

The large-signal channel resistance of a MOSFET in the ohmic region is given by

$$r_{DS} = \frac{v_{DS}}{i_D} = \frac{1}{\mu_n C_{ox}\left(\frac{W}{L}\right)(v_{GS} - V_t - v_{DS}/2)} \quad \text{for} \quad v_{DS} < v_{GS} - V_t \tag{4.2}$$

which simplifies to the form

$$r_{DS} \approx \frac{1}{\mu_n C_{ox}\left(\frac{W}{L}\right)(v_{GS} - V_t)} \quad \text{for} \quad v_{DS} \ll 2(v_{GS} - V_t). \tag{4.3}$$

The drain-to-source resistance of discrete power MOSFETs consists of the channel resistance, accumulation region resistance, neck region resistance, and drift region resistance [18]. The drift resistance is dominant in high voltage MOSFETs.

Equivalent circuits of the Class D amplifier are shown in Figure 4.5(b)–(d). In Figure 4.5(b), the MOSFETs are modeled by switches whose on-resistances are r_{DS1} and r_{DS2}. Resistance r_L is the equivalent series resistance (ESR) of the physical inductor L and

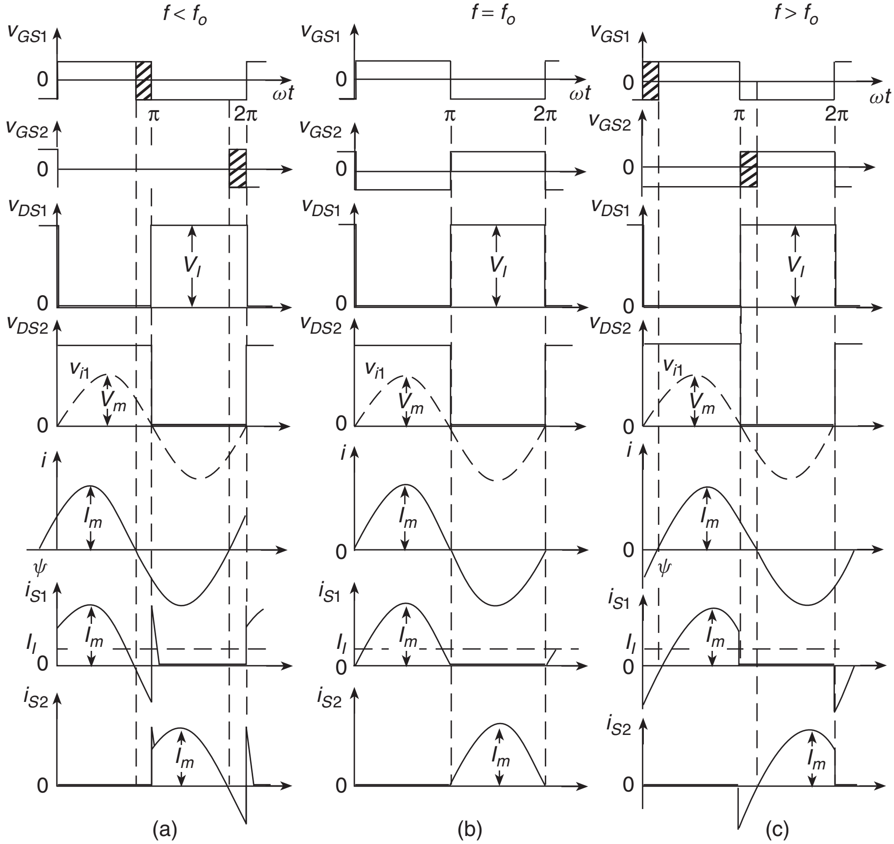

Figure 4.6 Waveforms in the Class D half-bridge voltage-switching RF power amplifier. (a) For $f < f_o$. (b) For $f = f_o$. (c) For $f > f_o$.

resistance r_C is the equivalent series resistance of the physical capacitor C. In Figure 4.5(c), $r_{DS} \approx (r_{DS1} + r_{DS2})/2$ represents the average equivalent on-resistance of the MOSFETs. In Figure 4.5(d), the total parasitic resistance is represented by

$$r = r_{DS} + r_L + r_C, \tag{4.4}$$

which yields an overall resistance in the series-resonant circuit

$$R_t = R + r \approx R + r_{DS} + r_L + r_C. \tag{4.5}$$

4.3 Principle of Operation

The principle of operation of the Class D amplifier is explained by the waveforms sketched in Figure 4.6. The voltage at the input of the series-resonant circuit is a square wave of magnitude V_I. If the loaded quality factor $Q_L = \sqrt{L/C}/R$ of the resonant circuit is high enough (for example, $Q_L \geq 2.5$), the current i through this circuit is nearly a sine wave. Only at $f = f_o$, the MOSFETs turn on and off at zero current, resulting in zero switching losses and an increase in efficiency. In this case, the antiparallel diode never conducts.

In many applications, the operating frequency f is not equal to the resonant frequency $f_o = 1/(2\pi\sqrt{LC})$ because the output power or the output voltage is often controlled by varying the operating frequency f (FM control). Figure 4.6(a), (b), and (c) shows the waveforms for $f < f_o, f = f_o$, and $f > f_o$, respectively. The tolerance of the gate-to-source voltage turn-on time is indicated by the shaded areas. Each transistor should be turned off for $f < f_o$ or turned on for $f > f_o$ in the time interval during which the switch current is negative. During this time interval, the switch current can flow through the antiparallel diode. To prevent cross-conduction (also called shoot-through current), the waveforms of the drive voltages v_{GS1} and v_{GS2} should be non-overlapping and have a sufficient dead time (not shown in Figure 4.6). At turn-off, MOSFETs have a delay time and bipolar junction transistors (BJTs) have a storage time. If the dead time of the gate-to-source voltages of the two transistors is too short, one transistor remains ON while the other turns off. Consequently, both the transistors are ON at the same time and the voltage power supply V_I is short-circuited by the transistor on-resistances r_{DS1} and r_{DS2}. For this reason, cross-conduction current pulses of magnitude $I_{pk} = V_I/(r_{DS1} + r_{DS2})$ flow through the transistors. For example, if $V_I = 200\,\mathrm{V}$ and $r_{DS1} = r_{DS2} = 0.5\,\Omega$, $I_{pk} = 200\,\mathrm{A}$. The excessive current stress may cause immediate failure of the devices. The dead time should not be too long, which will be discussed in Sections 4.3.1 and 4.3.2. The maximum dead time increases as f/f_o increases for $f > f_o$ or decreases for $f < f_o$. This is because the time interval during which the switch current is negative becomes longer. The shortest dead time must be at $f = f_o$. There are commercial IC drivers available, which have an adjustable dead time, for example, TI 2525.

4.3.1 Operation Below Resonance

For $f < f_o$, the series-resonant circuit represents a capacitive load. This means that the current through the resonant circuit i leads the fundamental component v_{i1} of the voltage v_{DS2} by the phase angle $|\psi|$, where $\psi < 0$. Therefore, the switch current i_{S1} is positive after the switch turns on at $\omega t = 0$ and i_{S1} is negative before the switch turns off at $\omega t = \pi$. The conduction sequence of the semiconductor devices is Q_1-D_1-Q_2-D_2. Notice that the current in the resonant circuit is diverted from the diode of one switch to the transistor of the other switch (Figure 4.5). This causes a lot of problems, which is explained shortly. Consider the time, when the switch S_2 is turned on, as shown in Figure 4.5. Prior to this transition, the current i flows through the antiparallel diode D_1 of the switch S_1. When transistor Q_2 is turned on by the drive voltage v_{GS2}, v_{DS2} is decreased, causing v_{DS1} to increase. Therefore, the diode D_1 turns off and the current i is diverted from D_1 to Q_2. There are three undesired effects, when the MOSFET turns on:

(1) reverse recovery of the antiparallel diode of the opposite switch;

(2) discharging the transistor output capacitance;

(3) Miller's effect.

The most severe drawback for operating below resonance is the diode reverse-recovery stress when the diode turns off. The MOSFET's intrinsic body-drain diode is a minority carrier device. Each diode turns off at a very large $\mathrm{d}v/\mathrm{d}t$, resulting in a large $\mathrm{d}i/\mathrm{d}t$. It generates a large reverse-recovery current spike (turned upside down). This spike flows through the other transistor because it cannot flow through the resonant circuit. The resonant inductor L does not permit an abrupt current change. Consequently, the spikes occur in the switch current waveform at both turn-on and turn-off transitions of the switch. The magnitude of these spikes can be much (for example, 10 times) higher than the magnitude of the steady-state switch current. High current spikes may destroy the transistors and always cause a considerable increase in switching losses and noise. During a part of the reverse-recovery interval, the diode voltage increases from -1 V to V_I and both the diode current and voltage are simultaneously high, causing a high reverse-recovery power loss.

The turn-off failure of the power MOSFETs may be caused by the *second breakdown* of the parasitic bipolar transistor. This parasitic bipolar transistor is an integral part of a power MOSFET structure. The body region serves as the base of the parasitic BJT, the source as the BJT emitter, and drain as the BJT collector. If the body-drain diode is forward biased just prior to the sudden application of drain voltage, the turn-on process of the parasitic bipolar transistor may be initiated by the reverse-recovery current of the antiparallel diode. The second breakdown of the parasitic BJT may destroy the power MOSFET structure. The second breakdown voltage is usually one-half of the voltage at which the device fails if the diode is forward biased. This voltage is denoted as V_{DSS} by manufacturers. For the reasons given above, operation at $f < f_o$ should be avoided if power MOSFETs are to be used as switches. For example, the current spikes can be reduced by adding Schottky antiparallel diodes. Silicon Schottky diodes have low breakdown voltages, typically below 100 V. However, silicon-carbide Schottky diodes have high breakdown voltages. Since the forward voltage of the Schottky diode is lower than that of the *pn* junction body diode, most of the negative switch current flows through the Schottky diode, reducing the reverse-recovery current of the *pn* junction body diode. Another circuit arrangement is to connect a diode in series with the MOSFET and an ultrafast diode in parallel with the series combination of the MOSFET and diode. This arrangement does not allow the intrinsic diode to conduct and to store the excess minority charge. However, the higher number of components, the additional cost, and the voltage drop across the series diode (which reduces the efficiency) are undesirable. Also, the peak voltages of the transistor and the series diode may become much higher than V_I. Transistors with a higher permissible voltage and a higher on-resistance should be used. High voltage MOSFETs have high on-resistance. However, high on-resistance increases conduction loss. This method may reduce, but cannot eliminate the spikes. Snubbers should be used to slow down the switching process, and reverse-recovery spikes can be reduced by connecting a small inductance in series with each power MOSFET.

For $f < f_o$, the turn-off switching loss is zero, but the turn-on switching loss is not zero. The transistors are turned on at a high voltage, equal to V_I. When the transistor is turned on, its output capacitance is discharged, causing switching loss. Suppose that the upper MOSFET is initially ON and the output capacitance C_{out} of the upper transistor is initially discharged. When the upper transistor is turned off, the energy drawn from the dc input voltage source V_I to charge the output capacitance C_{out} from 0 to V_I is given by

$$W_I = \int_0^T V_I i_{Cout}\,\mathrm{d}t = V_I \int_0^T i_{Cout}\,\mathrm{d}t = V_I Q \tag{4.6}$$

where i_{Cout} is the charging current of the output capacitance and Q is the charge transferred from the source V_I to the capacitor. This charge equals the integral of the current i_{Cout} over the charging time interval, which is usually much shorter than the time period T of the operating frequency f. Equation (4.6) holds true for both linear and nonlinear capacitances. If the transistor output capacitance is assumed to be linear, $Q = C_{out} V_I$ and (4.6) becomes

$$W_I = C_{out} V_I^2. \tag{4.7}$$

The energy stored in the linear output capacitance at the voltage V_I is

$$W_C = \frac{1}{2} C_{out} V_I^2. \tag{4.8}$$

The charging current flows through a resistance that consists of the on-resistance of the bottom MOSFET and lead resistances. The energy dissipated in this resistance is

$$W_R = W_I - W_C = \frac{1}{2} C_{out} V_I^2 \tag{4.9}$$

which is the same amount of energy as that stored in the capacitance. Note that charging a linear capacitance from a dc voltage source through a resistance requires twice the energy stored in the capacitance. When the upper MOSFET is turned on, its output capacitance is discharged through the on-resistance of the upper MOSFET, dissipating energy in that transistor. Thus, the energy dissipated during charging and discharging of the transistor output capacitance is

$$W_{sw} = C_{out} V_I^2. \tag{4.10}$$

Accordingly, the turn-on switching loss per transistor is

$$P_{ton} = \frac{W_C}{T} = \frac{1}{2} f C_{out} V_I^2. \tag{4.11}$$

The total power loss associated with the charging and discharging of the transistor output capacitance of each MOSFET is

$$P_{sw} = \frac{W_{sw}}{T} = f C_{out} V_I^2. \tag{4.12}$$

The charging and discharging process of the output capacitance of the bottom transistor is similar. In reality, the drain-source *pn* step junction capacitance is nonlinear. An analysis of turn-on switching loss with a nonlinear transistor output capacitance is given in Section 4.8.2.

Another effect that should be considered at turn-on of the MOSFET is Miller's effect. Since the gate-source voltage increases and the drain-source voltage decreases during the turn-on transition, Miller's effect is significant, by increasing the transistor input capacitance, the gate drive charge, and drive power requirements, reducing the turn-on switching speed.

The advantage of operating below resonance is that the transistors are turned off at nearly zero voltage, resulting in *zero turn-off switching loss*. For example, the drain-source voltage v_{DS1} is held at nearly -1 V by the antiparallel diode D_1 when i_{S1} is negative. During this time interval, transistor Q_1 is turned off by drive voltage v_{GS1}. The drain-source voltage v_{DS1} is almost zero and the drain current is low during the MOSFET turn-off, yielding zero turn-off switching loss in the MOSFET. Since v_{DS1} is constant, Miller's effect is absent during turn-off, the transistor input capacitance is not increased by Miller's effect, the gate drive requirement is reduced, and the turn-off switching speed is enhanced. In summary, for $f < f_o$, there is a turn-on switching loss in the transistor and a turn-off (reverse-recovery) switching loss in the diode. The transistor turn-off and the diode turn-on are lossless.

As mentioned earlier, the drive voltages v_{GS1} and v_{GS2} are nonoverlapping and have a dead time. However, this dead time should not be made too long. If the transistor Q_1 is turned off too early when the switch current i_{S1} is positive, diode D_1 cannot conduct and diode D_2 turns on, decreasing v_{DS2} to -0.7 V and increasing v_{DS1} to V_I. When the current through D_2 reaches zero, diode D_1 turns on, v_{DS1} decreases to -0.7 V, and v_{DS2} increases to V_I. These two additional transitions of each MOSFET voltage would result in switching losses. Note that only the turn-on transition of each switch is *forced* and is directly controlled by the driver, while the turn-off transition is caused by the turn-on of the opposite transistor (i.e., it is automatic).

In very high power applications, thyristors with antiparallel diodes can be used as switches in Class D amplifiers with a series-resonant circuit topologies. The advantage of such an arrangement is that, for operation below resonance, thyristors are turned off naturally when the switch current crosses zero. Thyristors, however, require more complicated and powerful drive circuitry, and their operating frequency range is limited to 20 kHz. Such low frequencies make the size and weight of the resonant components large, increasing the conduction losses.

4.3.2 Operation Above Resonance

For $f > f_o$, the series-resonant circuit represents an inductive load and the current i lags behind the voltage v_{i1} by the phase angle ψ, where $\psi > 0$. Hence, the switch current is negative after turn-on (for part of the switch 'on' interval) and positive before turn-off. The conduction sequence of the semiconductor devices is D_1-Q_1-D_2-Q_2. Consider the turn-off of switch S_1. When transistor Q_1 is turned off by the drive voltage v_{GS1}, v_{DS1} increases, causing v_{DS2} to decrease. As v_{DS2} reaches -0.7 V, D_2 turns on and the current i is diverted from transistor Q_1 to diode D_2. Thus, the turn-off switch transition is *forced* by the driver, while the turn-on transition is caused by the turn-off transition of the opposite transistor, not by the driver. Only the turn-off transition is directly controllable.

The transistors are turned on at *zero voltage*. In fact, there is a small negative voltage of the antiparallel diode, but this voltage is negligible in comparison to the input voltage V_I. For example, transistor Q_2 is turned on by v_2 when i_{S2} is negative. Voltage v_{DS2} is maintained at nearly -1 V by the antiparallel diode D_2 during the transistor turn-on transition. Therefore, the turn-on switching loss is eliminated, Miller's effect is absent, transistor input capacitance is not increased by Miller's effect, the gate drive power is low, and the turn-on switching speed is high. The diodes turn on at a very low $\mathrm{d}i/\mathrm{d}t$. The diode reverse-recovery current is a fraction of a sine wave and becomes a part of the switch current when the switch current is positive. Therefore, the antiparallel diodes can be slow, and the MOSFET's body-drain diodes are sufficiently fast as long as the reverse-recovery time is less than one-half of the cycle. The diode voltage is kept at a low voltage of the order of 1 V by the transistor in on-state during the reverse-recovery interval, reducing the diode reverse-recovery power loss. The transistor can be turned on not only when the switch current is negative but also when the switch current is positive and the diode is still conducting because of the reverse recovery current. Therefore, the range of the on-duty cycle of the gate-source voltages and the dead time can be larger. If the dead time is too long, the current will be diverted from the recovered diode D_2 to diode D_1 of the opposite transistor until transistor Q_2 is turned on, causing extra transitions of both switch voltages, current spikes, and switching losses.

For $f > f_o$, the turn-on switching loss is zero, but there is a turn-off loss in the transistor. Both the switch voltage and current waveforms overlap during turn-off, causing a turn-off switching loss. Also, Miller's effect is considerable, increasing the transistor input capacitance, the gate drive requirements, and reducing the turn-off speed. An approximate analysis of the turn-off switching loss is given in Section 4.8.3. In summary, for $f > f_o$, there is a turn-off switching loss in the transistor, while the turn-on transitions of the transistor and the diode are lossless. The turn-off switching loss can be eliminated by adding a shunt capacitor to one of the transistors and using a dead time in the drive voltages.

4.4 Topologies of Class D Voltage-source RF Power Amplifiers

Figure 4.7 shows a Class D voltage-switching amplifier with various resonant circuits. These resonant circuits are derived from a series-resonant circuit. In Figure 4.7(b), C_c is a large coupling capacitor, which can also be connected in series with the resonant inductor. The resonant frequency (that is, the boundary between the capacitive and inductive load) for the circuits of Figure 4.7(b)–(g) depends on the load. The resonant circuit shown in Figure 4.7(b) is employed in a parallel resonant converter. The circuit of Figure 4.7(d) is used in a series-parallel resonant converter. The circuit of Figure 4.7(e) is used in a

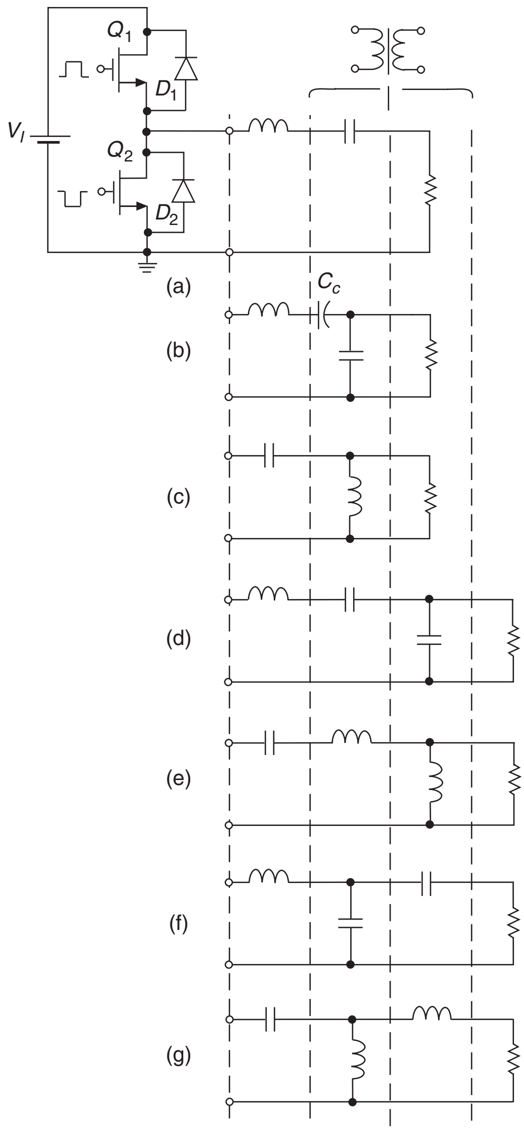

Figure 4.7 Class D voltage-switching amplifier with various resonant circuits.

CLL resonant converter. Resonant circuits of Figure 4.7(a), (f), and (g) supply a sinusoidal output current. Therefore, they are compatible with current-driven rectifiers. The amplifiers of Figures 4.7(b)–(e) produce a sinusoidal voltage output and are compatible with voltage-driven rectifiers. A high-frequency transformer can be inserted in places indicated in Figure 4.7.

Half-bridge topologies of the Class D voltage-switching amplifier are depicted in Figure 4.8. They are equivalent for ac components to the basic topology of Figure 4.5. Figure 4.8(a) shows a half-bridge amplifier with two dc voltage sources. The bottom voltage source $V_I/2$ acts as a short circuit for the current through the resonant circuit, resulting in the circuit of Figure 4.8(b). A drawback of this circuit is that the load current flows through the internal resistances of the dc voltage sources, reducing efficiency. In Figure 4.8(c), blocking capacitors $C_B/2$ act as short circuits for the ac component. The dc voltage across each of them is $V_I/2$, but the ac power is dissipated in the ESRs of the capacitors. An equivalent circuit for the amplifier of Figure 4.8(c) is shown in Figure 4.8(d). This is a useful circuit if the dc power supply contains a voltage doubler. The voltage stress across the filter capacitors is lower than that in the basic circuit of Figure 4.5. In Figure 4.8(e), the resonant capacitor is split into two halves, which are connected in parallel for the ac

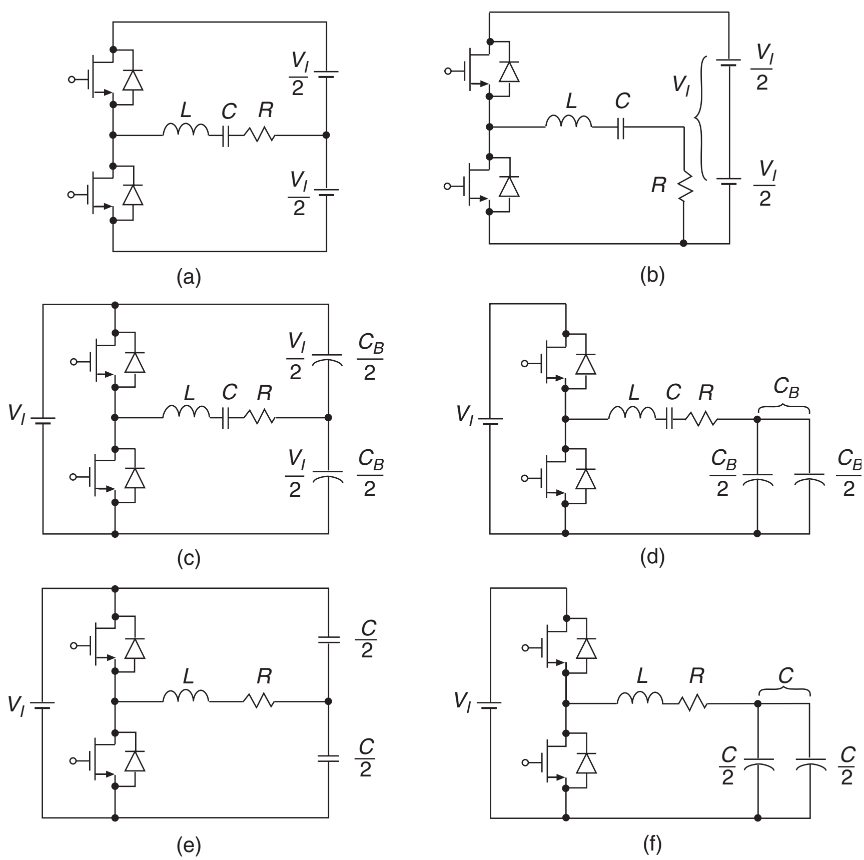

Figure 4.8 Half-bridge topologies of the Class D voltage-switching amplifier. (a) With two dc voltage sources. (b) Equivalent circuit of amplifier of Figure 4.8(a). (c) With two filter capacitors. (d) Equivalent circuit of amplifier of Figure 4.8(c). (e) With a resonant capacitor split into two halfs. (f) Equivalent circuit of amplifier of Figure 4.8(e).

component. This is possible because the dc input voltage source V_I acts as a short circuit for the ac component of the upper capacitor. The disadvantage of all transformerless versions of the half-bridge amplifier of Figure 4.8 is that the load resistance R is not grounded.

4.5 Analysis

4.5.1 Assumptions

The analysis of the Class D amplifier of Figure 4.5 is based on the equivalent circuit of Figure 4.5(d) and the following assumptions:

(1) The transistor and the diode form a resistive switch whose on-resistance is linear, the parasitic capacitances of the switch are neglected, and the switching times are zero.

(2) The elements of the series-resonant circuit are passive, linear, time invariant, and do not have parasitic reactive components.

(3) The loaded quality factor Q_L of the series-resonant circuit is high enough so that the current i through the resonant circuit is sinusoidal.

4.5.2 Series-resonant Circuit

The parameters of the series-resonant circuit are defined as follows:

- the resonant frequency

$$\omega_o = \frac{1}{\sqrt{LC}}; \tag{4.13}$$

- the characteristic impedance

$$Z_o = \sqrt{\frac{L}{C}} = \omega_o L = \frac{1}{\omega_o C}; \tag{4.14}$$

- the loaded quality factor

$$Q_L = \frac{\omega_o L}{R + r} = \frac{1}{\omega_o C (R + r)} = \frac{Z_o}{R + r} = \frac{\sqrt{\frac{L}{C}}}{R + r}; \tag{4.15}$$

- the unloaded quality factor

$$Q_o = \frac{\omega_o L}{r} = \frac{1}{\omega_o C r} = \frac{Z_o}{r} \tag{4.16}$$

where

$$r = r_{DS} + r_L + r_C \tag{4.17}$$

and

$$R_t = R + r. \tag{4.18}$$

The series-resonant circuit acts like a second-order bandpass filter. The bandwidth of this circuit is

$$BW = \frac{f_o}{Q_L}. \tag{4.19}$$

The loaded quality factor is defined as

$$Q_L \equiv 2\pi \frac{\text{Total average magnetic and electric energy stored at resonant frequency } f_o}{\text{Energy dissipated and delivered to load at resonant frequency } f_o}$$
$$= 2\pi \frac{\text{Peak magnetic energy at } f_o}{\text{Energy lost in one cycle at } f_o} = 2\pi \frac{\text{Peak electric energy at } f_o}{\text{Energy lost in one cycle at } f_o}$$
$$= 2\pi \frac{W_s}{T_o P_{Rt}} = 2\pi \frac{f_o W_s}{P_{Rt}} = \frac{\omega_o W_s}{P_O + P_r} = \frac{Q}{P_O + P_r} \tag{4.20}$$

where W_s is the total energy stored in the resonant circuit at the resonant frequency $f_o = 1/T_o$, and $Q = \omega_o W_s$ is the reactive power of inductor L or capacitor C at the resonant frequency f_o.

The current waveform through the inductor L is given by

$$i_L(\omega t) = I_m \sin(\omega t - \psi) \tag{4.21}$$

resulting in the instantaneous energy stored in the inductor

$$w_L(\omega t) = \frac{1}{2} L I_m^2 \sin^2(\omega t - \psi). \tag{4.22}$$

The voltage waveform across the capacitor C is given by

$$v_C(\omega t) = V_{Cm} \cos(\omega t - \psi) \tag{4.23}$$

resulting in the instantaneous energy stored in the capacitor

$$w_C(\omega t) = \frac{1}{2} C V_{Cm}^2 \cos^2(\omega t - \psi). \tag{4.24}$$

The total energy stored in the resonant circuit at any frequency is given by

$$w_s(\omega t) = w_L(\omega t) + w_C(\omega t) = \frac{1}{2}[L I_m^2 \sin^2(\omega t - \psi) + C V_{Cm}^2 \cos^2(\omega t - \psi)]. \tag{4.25}$$

Since $V_{Cm} = X_C I_m = I_m/(\omega C)$, the total energy stored in the resonant circuit at any frequency can be expressed as

$$w_s(\omega t) = \frac{1}{2} L I_m^2 [\sin^2(\omega t - \psi) + \frac{1}{\omega^2 LC} \cos^2(\omega t - \psi)]$$
$$= \frac{1}{2} L I_m^2 \left[\sin^2(\omega t - \psi) + \frac{1}{\left(\frac{\omega}{\omega_o}\right)^2} \cos^2(\omega t - \psi) \right]. \tag{4.26}$$

For $\omega = \omega_o$, the total energy stored in the resonant circuit W_s is equal to the peak magnetic energy stored in the inductor L

$$w_s(\omega t) = \frac{1}{2} L I_m^2 [\sin^2(\omega t - \psi) + \cos^2(\omega t - \psi)] = \frac{1}{2} L I_m^2 = W_s = W_{Lmax}. \tag{4.27}$$

Because $I_m = V_{Cm}/X_C = \omega C V_{Cm}$, the total energy stored in the resonant circuit at any frequency becomes

$$w_s(\omega t) = \frac{1}{2} C V_{Cm}^2 [\omega^2 LC \sin^2(\omega t - \psi) + \cos^2(\omega t - \psi)]$$
$$= \frac{1}{2} C V_{Cm}^2 \left[\left(\frac{\omega}{\omega_o}\right)^2 \sin^2(\omega t - \psi) + \cos^2(\omega t - \psi) \right]. \tag{4.28}$$

For $\omega = \omega_o$, the total energy stored in the resonant circuit W_s is equal to the peak electric energy stored in the capacitor C

$$w_s(\omega t) = \frac{1}{2}CV_{Cm}^2[\sin^2(\omega t - \psi) + \cos^2(\omega t - \psi)] = W_s = W_{Cmax}. \tag{4.29}$$

For steady-state operation at the resonant frequency f_o, the total instantaneous energy stored in the resonant circuit is constant and equal to the maximum energy stored in the inductor

$$W_s = W_{Lmax} = \frac{1}{2}LI_m^2 \tag{4.30}$$

or, using (4.13), in the capacitor

$$W_s = W_{Cmax} = \frac{1}{2}CV_{Cm}^2 = \frac{1}{2}C\frac{I_m^2}{(\omega_o C)^2} = \frac{1}{2}\frac{I_m^2}{(C\omega_o^2)} = \frac{1}{2}LI_m^2. \tag{4.31}$$

Substitution of (4.30) and (4.31) into (4.20) produces

$$Q_L = \frac{\omega_o LI_m^2}{2P_{Rt}} = \frac{\omega_o CV_{Cm}^2}{2P_{Rt}}. \tag{4.32}$$

The reactive power of the inductor at f_o is $Q = (1/2)V_{Lm}I_m = (1/2)\omega_o LI_m^2$ and of the capacitor is $Q = (1/2)I_m V_{Cm} = (1/2)\omega_o CV_{Cm}^2$. Thus, the quality factor can be defined as the ratio of the reactive power of the inductor or the capacitor to the real power dissipated in the form of heat in all resistances at the resonant frequency f_o. The total power dissipated in $R_t = R + r$ is

$$P_{Rt} = \frac{1}{2}R_t I_m^2 = \frac{1}{2}(R + r)I_m^2. \tag{4.33}$$

Substitution of (4.30) or (4.31), and (4.33) into (4.20) gives

$$Q_L = \frac{\omega_o L}{R + r} = \frac{1}{\omega_o C(R + r)}. \tag{4.34}$$

The average magnetic energy stored in the resonant circuit is

$$W_{L(AV)} = \frac{1}{2\pi}\int_0^{2\pi} w_L \, d(\omega_o t) = \frac{1}{2}LI_m^2 \times \frac{1}{2\pi}\int_0^{2\pi} \sin^2(\omega_o t)\, d(\omega t) = \frac{1}{4}LI_m^2. \tag{4.35}$$

Similarly, the average electric energy stored in the resonant circuit is

$$W_{C(AV)} = \frac{1}{2\pi}\int_0^{2\pi} w_C \, d(\omega_o t) = \frac{1}{2}CV_{Cm}^2 \times \frac{1}{2\pi}\int_0^{2\pi} \cos^2(\omega_o t)\, d(\omega t) = \frac{1}{4}CV_{Cm}^2. \tag{4.36}$$

Hence, the total average energy stored in the resonant circuit is

$$W_s = W_{L(AV)} + W_{C(AV)} = \frac{1}{4}LI_m^2 + \frac{1}{4}CV_{Cm}^2 = \frac{1}{2}LI_m^2 = \frac{1}{2}CV_{Cm}^2. \tag{4.37}$$

This leads to the loaded quality factor.

For $R = 0$,

$$P_r = \frac{1}{2}rI_m^2 \tag{4.38}$$

and the unloaded quality factor is defined as

$$Q_o \equiv \frac{\omega_o W_s}{P_r} = \frac{\omega_o L}{r} = \frac{\omega_o L}{r_{DS} + r_L + r_C} = \frac{1}{\omega_o Cr} = \frac{1}{\omega_o C(r_{DS} + r_L + r_C)}. \tag{4.39}$$

Similarly, the quality factor of the inductor is

$$Q_{Lo} \equiv \frac{\omega_o W_s}{P_{rL}} = \frac{\omega_o L}{r_L} \tag{4.40}$$

and the capacitor is

$$Q_{Co} \equiv \frac{\omega_o W_s}{P_{rC}} = \frac{1}{\omega_o C r_C}. \tag{4.41}$$

4.5.3 Input Impedance of Series-resonant Circuit

The input impedance of the series-resonant circuit is

$$\begin{aligned}\mathbf{Z} &= R + r + j\left(\omega L - \frac{1}{\omega C}\right) = (R + r)\left[1 + jQ_L\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)\right] \\ &= Z_o\left[\frac{R + r}{Z_o} + j\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)\right] = |Z|\,e^{j\psi} = R + r + jX \end{aligned}\tag{4.42}$$

where

$$\begin{aligned}|Z| &= (R + r)\sqrt{1 + Q_L^2\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)^2} = Z_o\sqrt{\left(\frac{(R + r)}{Z_o}\right)^2 + \left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)^2} \\ &= Z_o\sqrt{\frac{1}{Q_L^2} + \left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)^2}\end{aligned}\tag{4.43}$$

$$\psi = \arctan\left[Q_L\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)\right] \tag{4.44}$$

$$R_t = R + r = |Z|\cos\psi \tag{4.45}$$

$$X = |Z|\sin\psi. \tag{4.46}$$

The reactance of the resonant circuit becomes zero at the resonant frequency f_o. From (4.44),

$$\cos\psi = \frac{1}{\sqrt{1 + Q_L^2\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)^2}}. \tag{4.47}$$

Figure 4.9 shows a three-dimensional representation of $|Z|/Z_o$ as a function of the normalized frequency f/f_o and the loaded quality factor Q_L. Plots of $|Z|/Z_o$ and ψ as a function of f/f_o at fixed values of Q_L are shown in Figures 4.10 and 4.11.

For $f < f_o$, ψ is less than zero, which means that the resonant circuit represents a capacitive load to the switching part of the amplifier. For $f > f_o$, ψ is greater than zero, which indicates that the resonant circuit represents an inductive load. The magnitude of impedance $|Z|$ is usually normalized with respect to R_t, but it is not a good normalization if R changes, and therefore $R_t = R + r$ is variable at fixed resonant components L and C.

4.5.4 Currents, Voltages, and Powers

When S_1 is ON and S_2 is OFF, $v = V_I$. When S_1 is OFF and S_2 is ON, $v = 0$. Referring to Figure 4.5(d), the input voltage of the series-resonant circuit is a square wave

$$v \approx v_{DS2} = \begin{cases} V_I, & \text{for} \quad 0 < \omega t \leq \pi \\ 0, & \text{for} \quad \pi < \omega t \leq 2\pi. \end{cases} \tag{4.48}$$

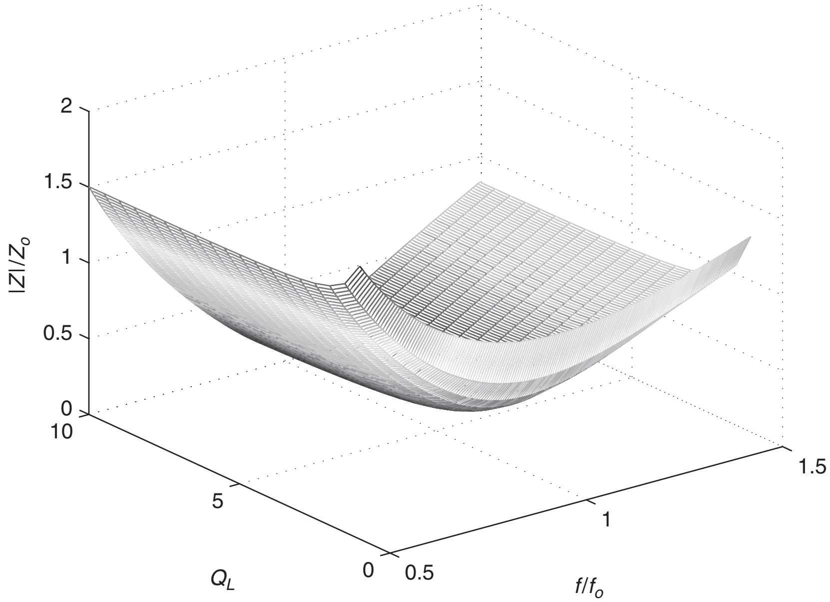

Figure 4.9 Plots of $|Z|/Z_o$ as a function of f/f_o and Q_L.

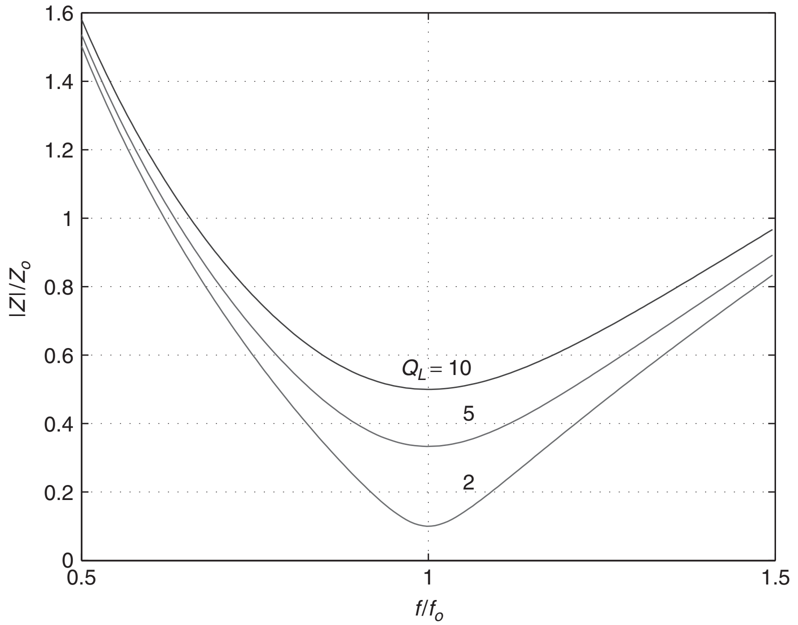

Figure 4.10 Modulus of the normalized input impedance $|Z|/Z_o$ as a function of f/f_o at constant values of Q_L.

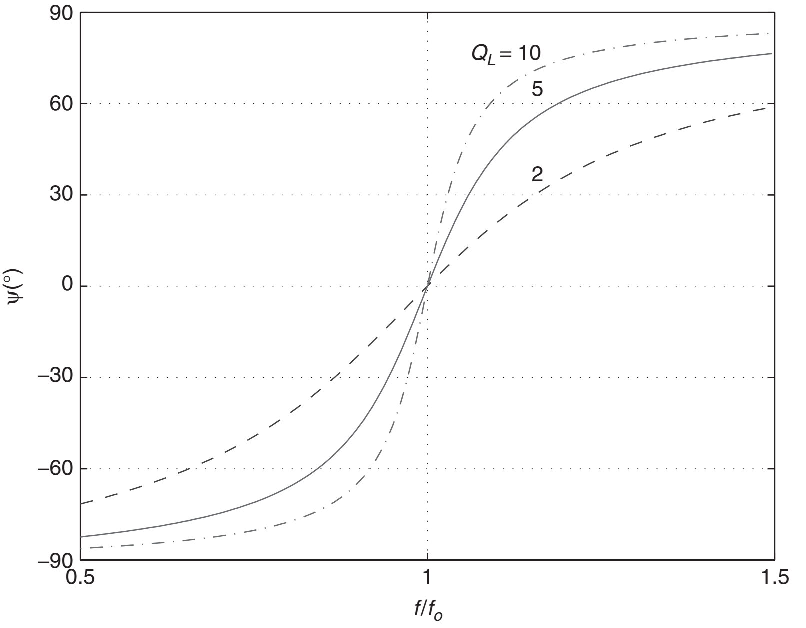

Figure 4.11 Phase of the input impedance ψ as a function of f/f_o at constant values of Q_L.

This voltage can be expressed by Fourier series

$$v \approx v_{DS2} = \frac{V_I}{2} + \frac{2V_I}{\pi}\sum_{n=1}^{\infty}\frac{1-(-1)^n}{2n}\sin(n\omega t) = V_I\left\{\frac{1}{2} + \sum_{k=1}^{\infty}\frac{\sin[(2k-1)\omega t]}{2k-1}\right\}$$
$$= V_I\left(\frac{1}{2} + \frac{2}{\pi}\sin\omega t + \frac{2}{3\pi}\sin 3\omega t + \frac{2}{5\pi}\sin 5\omega t + \cdots\right). \quad (4.49)$$

The fundamental component of voltage v is

$$v_{i1} = V_m \sin\omega t \quad (4.50)$$

where its amplitude is given by

$$V_m = \frac{2V_I}{\pi} \approx 0.637V_I. \quad (4.51)$$

This leads to the rms value of voltage v_{i1}

$$V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{\sqrt{2}V_I}{\pi} \approx 0.45V_I. \quad (4.52)$$

The series-resonant circuit acts like a bandpass filter. For frequencies lower than the resonant frequency f_o, the series-resonant circuit represents a high capacitive impedance. On the other hand, for frequencies higher than the resonant frequency f_o, the series-resonant circuit represents a high inductive impedance. If the operating frequency f is close to the resonant frequency f_o, the impedance of the resonant circuit is very high for higher harmonics, and therefore the current through the resonant circuit is approximately sinusoidal and equal to the fundamental component

$$i = I_m \sin(\omega t - \psi) \quad (4.53)$$

where from (4.46), (4.47), and (4.51)

$$I_m = \frac{V_m}{|Z|} = \frac{2V_I}{\pi|Z|} = \frac{2V_I \cos\psi}{\pi R_t} = \frac{2V_I}{\pi R_t\sqrt{1 + Q_L^2\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)^2}}$$

$$= \frac{2V_I}{\pi Z_o\sqrt{\left(\frac{R_t}{Z_o}\right)^2 + \left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)^2}}. \tag{4.54}$$

Figure 4.12 shows a three-dimensional representation of $I_m/(V_I/Z_o)$ as a function of f/f_o and R_t/Z_o. Plots of $I_m/(V_I/Z_o)$ as a function of f/f_o at fixed values of Q_L are depicted in Figure 4.13. It can be seen that high values of $I_m/(V_I/Z_o)$ occur at the resonant frequency f_o and at low total resistance R_t. At $f = f_o$, the amplitude of the current through the resonant circuit and the transistors becomes

$$I_{mr} = \frac{2V_I}{\pi R_t}. \tag{4.55}$$

The output voltage is also sinusoidal

$$v_o = iR = V_m \sin(\omega t - \psi). \tag{4.56}$$

The input current of the amplifier i_I equals the current through the switch S_1 and is given by

$$i_I = i_{S1} = \begin{cases} I_m \sin(\omega t - \psi), & \text{for} \quad 0 < \omega t \le \pi \\ 0, & \text{for} \quad \pi < \omega t \le 2\pi. \end{cases} \tag{4.57}$$

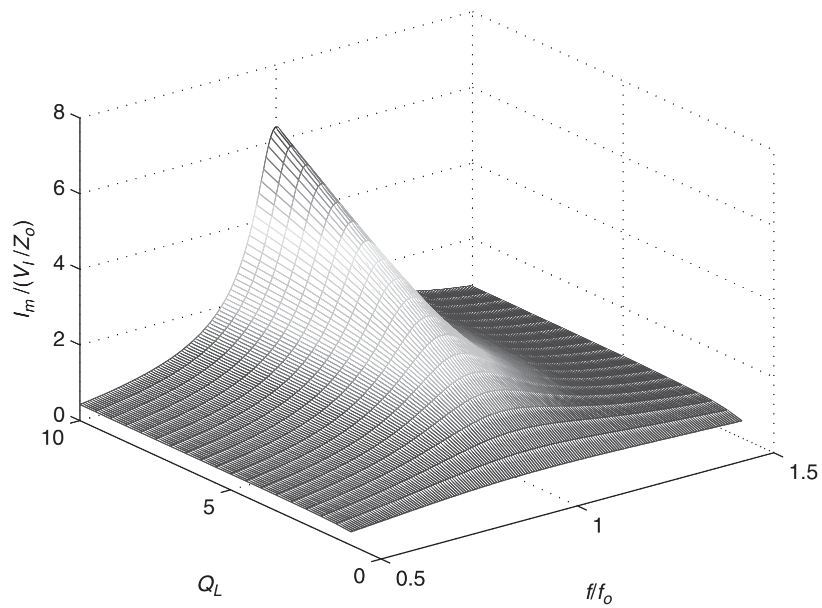

Figure 4.12 Normalized amplitude $I_m/(V_I/Z_o)$ of the current through the resonant circuit as a function of f/f_o and Q_L.

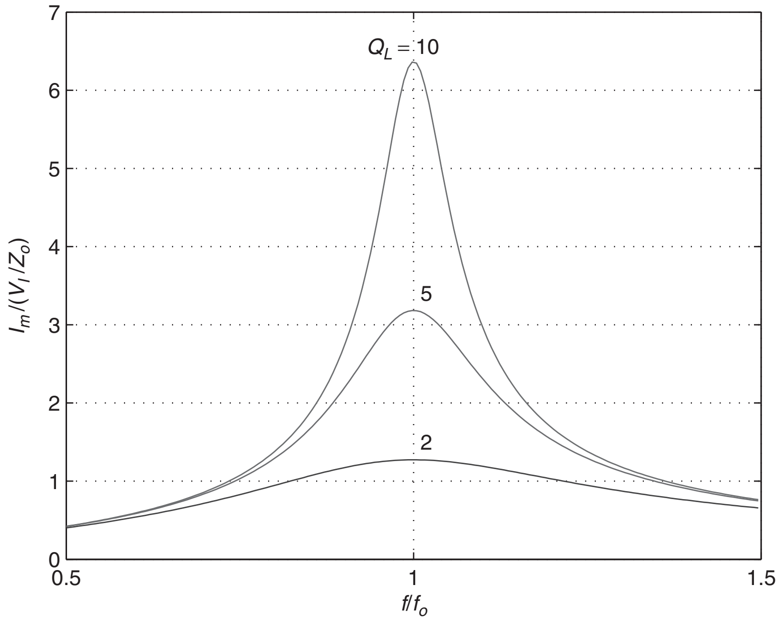

Figure 4.13 Normalized amplitude $I_m/(V_I/Z_o)$ of the current in the resonant circuit as a function of f/f_o at fixed values of Q_L.

Hence, from (4.43), (4.46), and (4.51), one obtains the dc component of the input current

$$I_I = \frac{1}{2\pi}\int_0^{2\pi} i_{S1}\,\mathrm{d}(\omega t) = \frac{I_m}{2\pi}\int_0^{\pi} \sin(\omega t - \psi)\,\mathrm{d}(\omega t) = \frac{I_m \cos\psi}{\pi} = \frac{V_m\cos\psi}{\pi|Z|}$$
$$= \frac{2V_I\cos^2\psi}{\pi^2 R_t} = \frac{2V_I R_t}{\pi^2|Z|^2} = \frac{I_m}{\pi\sqrt{1+Q_L^2\left(\frac{\omega}{\omega_o}-\frac{\omega_o}{\omega}\right)^2}}$$
$$= \frac{2V_I}{\pi^2 R_t\left[1+Q_L^2\left(\frac{\omega}{\omega_o}-\frac{\omega_o}{\omega}\right)^2\right]}. \tag{4.58}$$

At $f = f_o$,

$$I_I = \frac{I_m}{\pi} = \frac{2V_I}{\pi^2 R_t} \approx \frac{V_I}{5R_t}. \tag{4.59}$$

The dc input power can be expressed as

$$P_I = I_I V_I = \frac{2V_I^2\cos^2\psi}{\pi^2 R_t} = \frac{2V_I^2}{\pi^2 R_t[1+Q_L^2\left(\frac{\omega}{\omega_o}-\frac{\omega_o}{\omega}\right)^2]}$$
$$= \frac{2V_I^2 R_t}{\pi^2 Z_o^2\left[\left(\frac{R_t}{Z_o}\right)^2+\left(\frac{\omega}{\omega_o}-\frac{\omega_o}{\omega}\right)^2\right]}. \tag{4.60}$$

At $f = f_o$,

$$P_I = \frac{2V_I^2}{\pi^2 R_t} \approx \frac{V_I^2}{5R_t}. \tag{4.61}$$

Using (4.54), one arrives at the output power

$$P_O = \frac{I_m^2 R}{2} = \frac{2V_I^2 R \cos^2\psi}{\pi^2 R_t^2} = \frac{2V_I^2 R}{\pi^2 R_t^2\left[1 + Q_L^2\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)^2\right]}$$
$$= \frac{2V_I^2 R}{\pi^2 Z_o^2\left[\left(\frac{R_t}{Z_o}\right)^2 + \left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)^2\right]}. \tag{4.62}$$

At $f = f_o$,

$$P_O = \frac{2V_I^2 R}{\pi^2 R_t^2} \approx \frac{2V_I^2}{\pi^2 R}. \tag{4.63}$$

Figure 4.14 depicts $P_O/(V_I^2 R_t/Z_o^2)$ as a function of f/f_o and Q_L. The normalized output power $P_O/(V_I^2 R_t/Z_o^2)$ is plotted as a function of f/f_o at different values of Q_L in Figure 4.15. The maximum output power occurs at the resonant frequency f_o and at low total resistance R_t.

4.5.5 Current and Voltage Stresses

The peak voltage across each switch is equal to the dc input voltage

$$V_{SM} = V_I. \tag{4.64}$$

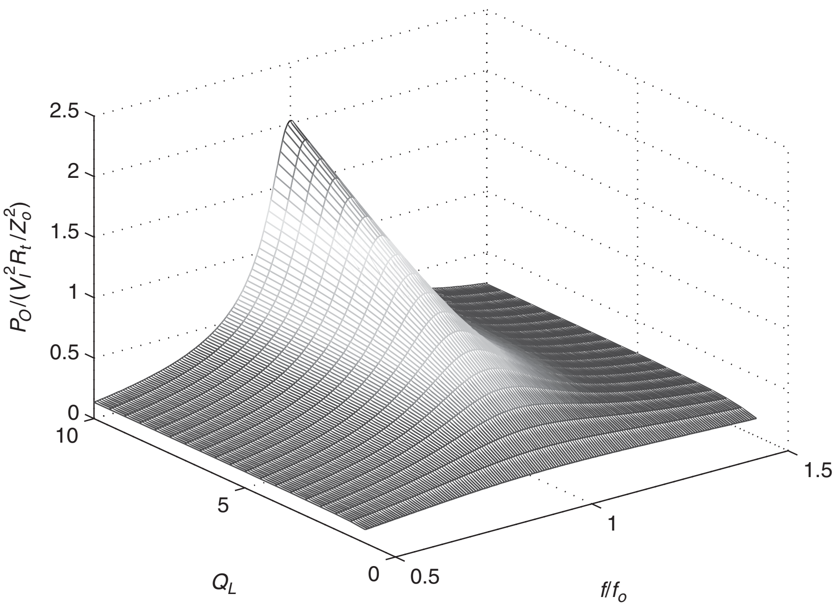

Figure 4.14 Normalized output power $P_O/(V_I^2 R_t/Z_o^2)$ as a function of f/f_o and Q_L.

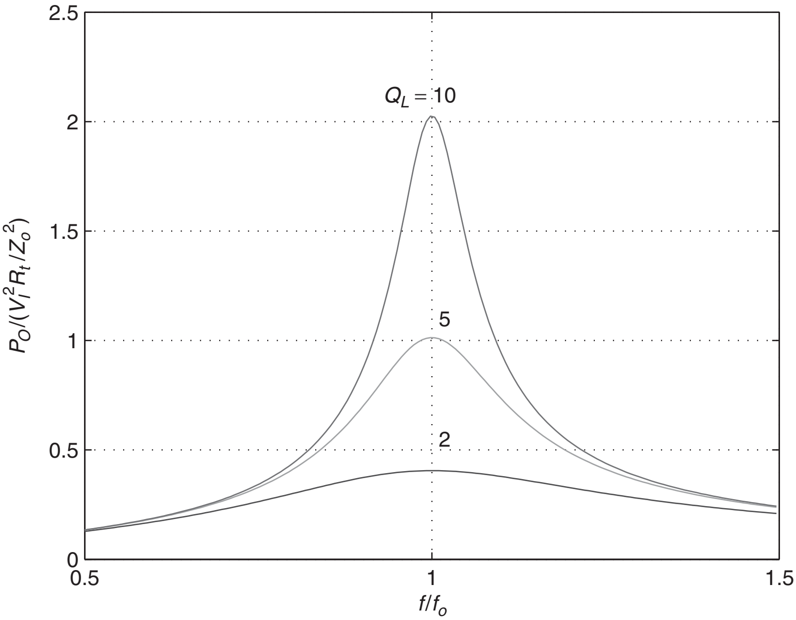

Figure 4.15 Normalized output power $P_O/(V_I^2 R_t/Z_o^2)$ as a function of f/f_o at fixed values of Q_L.

The maximum value of the switch peak currents and the maximum amplitude of the current through the resonant circuit occurs at $f = f_o$. Hence, from (4.54)

$$I_{SM} = I_{mr} = \frac{V_m}{R_t} = \frac{2V_I}{\pi R_t}. \tag{4.65}$$

The ratio of the input voltage of the resonant circuit to the voltage across the capacitor C is the voltage transfer function of the second-order low-pass filter. The amplitude of the voltage across the capacitor C is obtained from (4.54)

$$V_{Cm} = \frac{I_m}{\omega C} = \frac{2V_I}{\pi\left(\frac{\omega}{\omega_o}\right)\sqrt{\left(\frac{R_t}{Z_o}\right)^2 + \left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)^2}}. \tag{4.66}$$

A three-dimensional representation of V_{Cm}/V_I is shown in Figure 4.16. Figure 4.17 shows plots of V_{Cm}/V_I as a function of f/f_o at fixed values of Q_L.

The ratio of the input voltage of the resonant circuit to the voltage across the inductor L is the transfer function of the second-order high-pass filter. The amplitude of the voltage across the inductor L is expressed as

$$V_{Lm} = \omega L I_m = \frac{2V_I\left(\frac{\omega}{\omega_o}\right)}{\pi\sqrt{\left(\frac{R_t}{Z_o}\right)^2 + \left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)^2}}. \tag{4.67}$$

Figure 4.18 shows V_{Lm}/V_I as a function of f/f_o and Q_L. Plots of V_{Lm}/V_I as a function of f/f_o at constant values of Q_L are displayed in Figure 4.19. At $f = f_o$,

$$V_{Cm(max)} = V_{Lm(max)} = Z_o I_{mr} = Q_L V_m = \frac{2V_I Q_L}{\pi}. \tag{4.68}$$

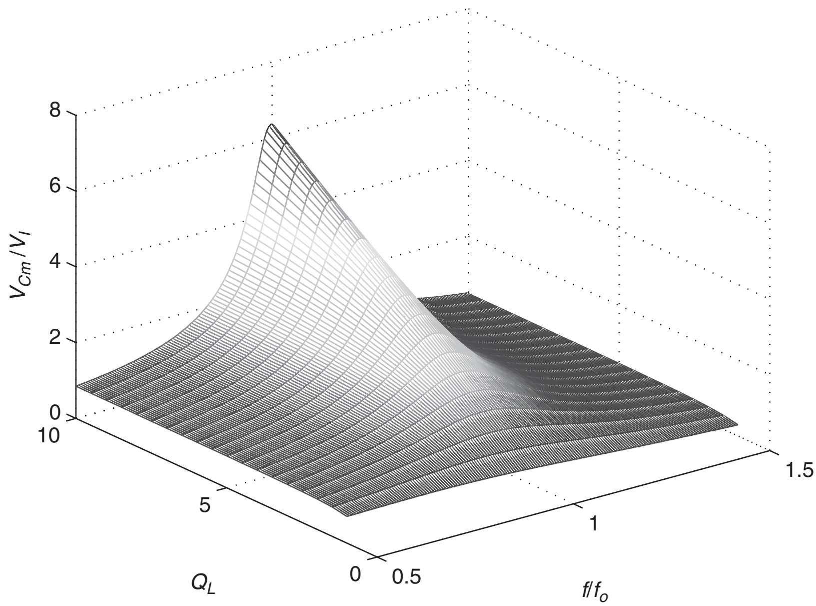

Figure 4.16 Normalized amplitude V_{Cm}/V_I of the voltage across the resonant capacitor C as a function of f/f_o and Q_L.

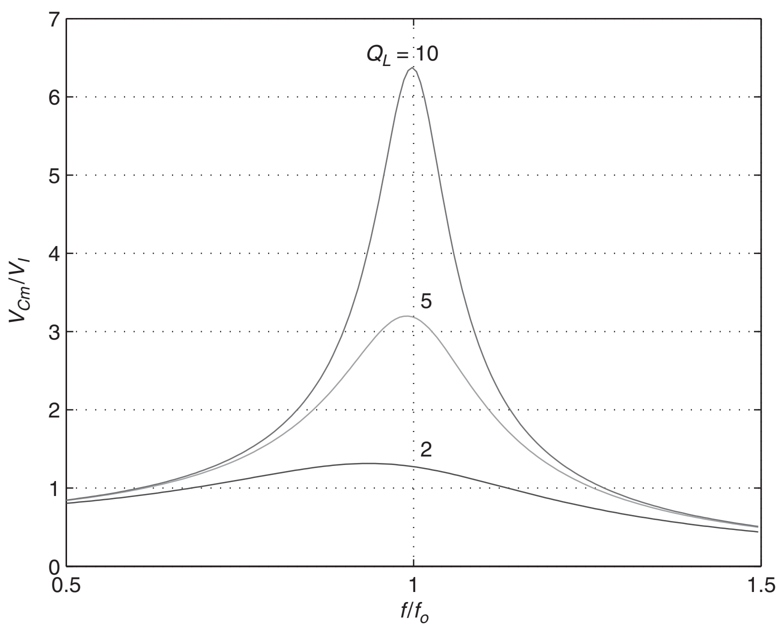

Figure 4.17 Normalized amplitude V_{Cm}/V_I of the voltage across the resonant capacitor C as a function of f/f_o at fixed values of Q_L.

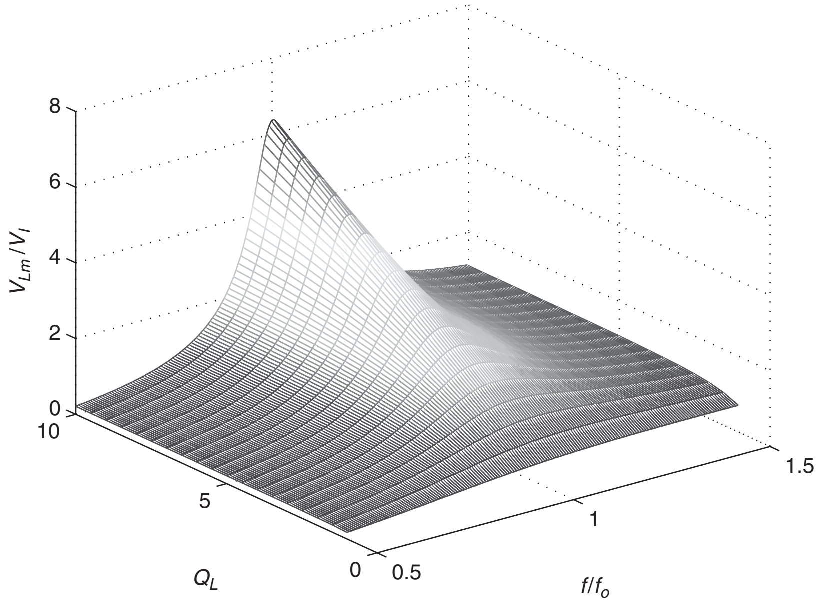

Figure 4.18 Normalized amplitude V_{Lm}/V_I of the voltage across the resonant inductor L as a function of f/f_o and Q_L.

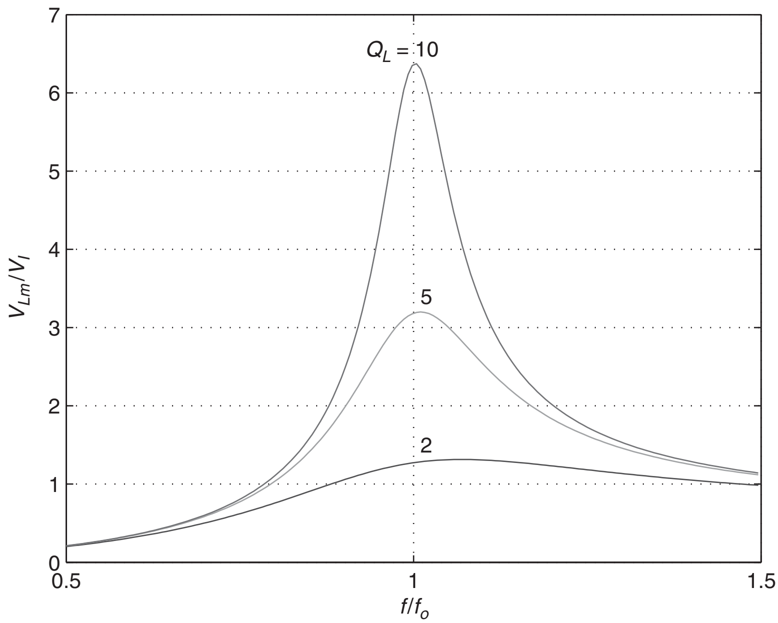

Figure 4.19 Normalized amplitude V_{Lm}/V_I of the voltage across the resonant inductor L as a function of f/f_o at fixed values of Q_L.

The maximum voltage stress of the resonant components occurs at the resonant frequency $f \approx f_o$, at a maximum dc input voltage $V_I = V_{Imax}$, and at a maximum loaded quality factor Q_L. Actually, the maximum value of V_{Lm} occurs slightly above f_o and the maximum value of V_{Cm} occurs slightly below f_o. However, this effect is negligible for practical purposes. At the resonant frequency $f = f_o$, the amplitudes of the voltages across the inductor and capacitor are Q_L times higher than the amplitude V_m of the fundamental component of the voltage at the input of the resonant circuit, which is equal to the amplitude of the output voltage V_{om}.

The output power capability is give by

$$c_p = \frac{P_O}{2I_{SM}V_{SM}} = \frac{1}{4}\left(\frac{I_m}{I_{SM}}\right)\left(\frac{V_m}{V_{SM}}\right) = \frac{1}{4} \times 1 \times \left(\frac{2}{\pi}\right) = \frac{1}{2\pi} = 0.159. \tag{4.69}$$

4.5.6 Operation Under Short-circuit and Open-circuit Conditions

The Class D amplifier with a series-resonant circuit can operate safely with an open circuit at the output. However, it is prone to catastrophic failure if the output is short-circuited at f close to f_o. If $R = 0$, the amplitude of the current through the resonant circuit and the switches is

$$I_m = \frac{2V_I}{\pi r\sqrt{1 + \left(\frac{Z_o}{r}\right)^2\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)^2}}. \tag{4.70}$$

The maximum value of I_m occurs at $f = f_o$ and is given by

$$I_{SM} = I_{mr} = \frac{2V_I}{\pi r} \tag{4.71}$$

and the amplitudes of the voltages across the resonant components L and C are

$$V_{Cm} = V_{Lm} = \frac{I_{mr}}{\omega_o C} = \omega_o L I_{mr} = Z_o I_{mr} = \frac{2V_I Z_o}{\pi r} = \frac{2V_I Q_o}{\pi}. \tag{4.72}$$

For instance, if $V_I = 320\,\mathrm{V}$ and $r = 2\,\Omega$, $I_{SM} = I_{mr} = 102\,\mathrm{A}$ and $V_{Cm} = V_{Lm} = 80\,\mathrm{kV}$. Thus, the excessive current in the switches and the resonant circuit, as well as the excessive voltages across L and C, can lead to catastrophic failure of the amplifier.

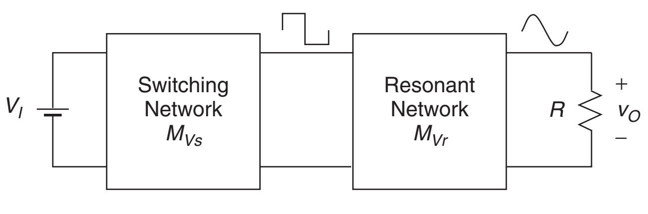

Figure 4.20 Block diagram of a Class D amplifier.

4.6 Voltage Transfer Function

The Class D amplifier can be functionally divided into two parts: the switching network and the resonant network. A block diagram of the Class D amplifier is shown in Figure 4.20. The switching part is composed of a dc input voltage source V_I and a set of switches. The switches are controlled to produce a square-wave voltage v. Since a resonant circuit forces a sinusoidal current, only the power of the fundamental component is transferred from the switching part to the resonant part. Therefore, it is sufficient to consider only the fundamental component of the voltage v given by (4.50). A voltage transfer function of the switching part can be defined as

$$M_{Vs} \equiv \frac{V_{rms}}{V_I} \tag{4.73}$$

where V_{rms} is the rms value of the fundamental component v_{i1} of the voltage v. The resonant network of the amplifier converts the square-wave voltage v into a sinusoidal current or voltage signal. Because the dc input source V_I and switches S_1 and S_2 form a nearly ideal ac voltage source, many resonant circuits can be connected in parallel. If the resonant circuit is loaded by a resistance R, a voltage transfer function of the resonant part is

$$\mathbf{M_{Vr}} \equiv \frac{\mathbf{V_{O(rms)}}}{\mathbf{V_{rms}}} = |M_{Vr}|\, e^{j\varphi} \tag{4.74}$$

where $\mathbf{V_{rms}}$ is the phasor of voltage v_{i1} and $\mathbf{V_{O(rms)}}$ is the phasor of the sinusoidal output voltage v_O across R. The modulus of $\mathbf{M_{Vr}}$ is

$$|M_{Vr}| = \frac{V_{O(rms)}}{V_{rms}} \tag{4.75}$$

where $V_{O(rms)}$ is the rms value of v_O. A voltage transfer function of the entire amplifier is defined as a product of (4.73) and (4.75)

$$M_{VI} = M_{Vs}|M_{Vr}| = \frac{V_{O(rms)}}{V_I}. \tag{4.76}$$

Let us consider the half-bridge circuit. Using (4.52), one arrives at the voltage transfer function from the input of the amplifier to the input of the series-resonant circuit

$$M_{Vs} = \frac{\sqrt{2}}{\pi} = 0.45. \tag{4.77}$$

The ac-to-ac voltage transfer function of the series-resonant circuit is

$$\mathbf{M_{Vr}} = \frac{\mathbf{V_{O(rms)}}}{\mathbf{V_{rms}}} = \frac{R}{R + r + j\left(\omega L - \dfrac{1}{\omega C}\right)} = \frac{\eta_{Ir}}{1 + jQ_L\left(\dfrac{\omega}{\omega_o} - \dfrac{\omega_o}{\omega}\right)} = |M_{Vr}|\, e^{j\varphi} \tag{4.78}$$

from which

$$|M_{Vr}| = \frac{V_{O(rms)}}{V_{rms}} = \frac{\eta_{Ir}}{\sqrt{1 + Q_L^2\left(\dfrac{\omega}{\omega_o} - \dfrac{\omega_o}{\omega}\right)^2}} \tag{4.79}$$

$$\varphi = -\arctan\left[Q_L\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)\right] \tag{4.80}$$

where $\eta_{Ir} = R/(R + r) = R/R_t$ is the efficiency of the amplifier, by taking into account the conduction losses only (see Section 4.8.1). Figure 4.21 illustrates (4.79) in a three-dimensional space. Figure 4.22 shows the voltage transfer function $|M_{Vr}|$ as a function of f/f_o at fixed values of Q_L for $\eta_{Ir} = 0.95$.

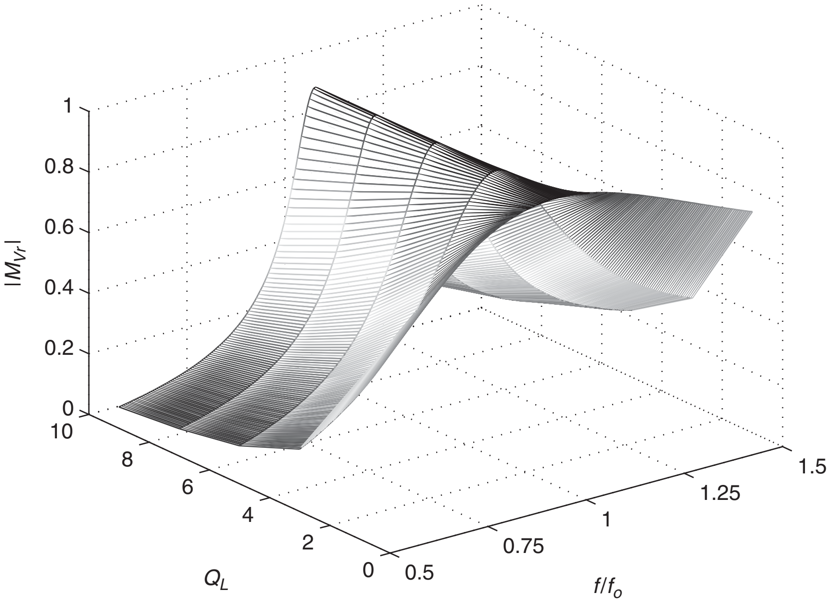

Figure 4.21 Three-dimensional representation of $|M_{Vr}|$ as a function of f/f_o and Q_L for $\eta_{Ir} = 0.95$.

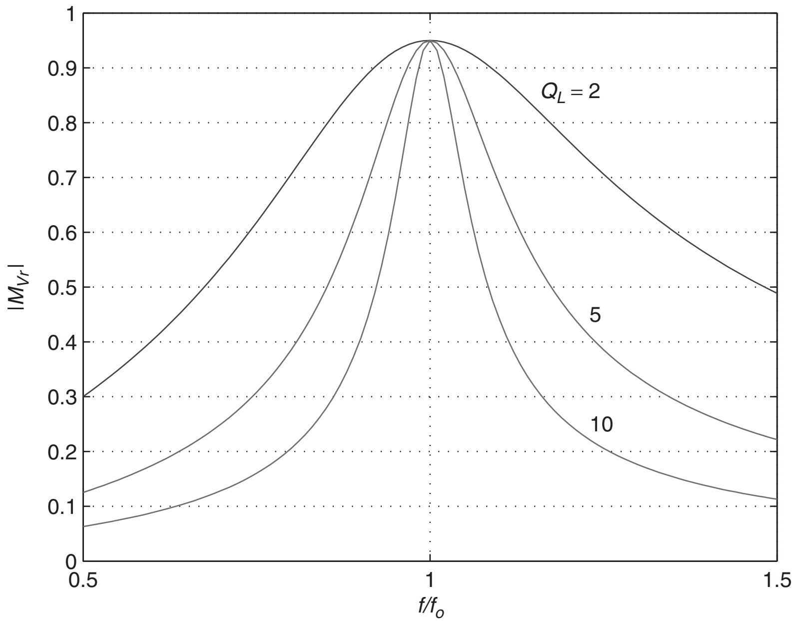

Figure 4.22 Transfer function $|M_{Vr}|$ as a function of f/f_o at fixed values of Q_L for $\eta_{Ir} = 0.95$.

Rearrangement of (4.79) yields

$$\frac{f}{f_o} = \frac{\sqrt{1 - (|M_{Vr}|/\eta_{Ir})^2 + 4Q_L^2(|M_{Vr}|/\eta_{Ir})^2} - \sqrt{1 - (|M_{Vr}|/\eta_{Ir})^2}}{2Q_L(|M_{Vr}|/\eta_{Ir})} \quad \text{for} \quad \frac{f}{f_o} \leq 1 \tag{4.81}$$

and

$$\frac{f}{f_o} = \frac{\sqrt{1 - (|M_{Vr}|/\eta_{Ir})^2 + 4Q_L^2(|M_{Vr}|/\eta_{Ir})^2} + \sqrt{1 - (|M_{Vr}|/\eta_{Ir})^2}}{2Q_L(|M_{Vr}|/\eta_{Ir})} \quad \text{for} \quad \frac{f}{f_o} \geq 1. \tag{4.82}$$

Comparison of (4.54) and (4.79) yields

$$I_m = \frac{2V_I|M_{Vr}|}{\pi R_t \eta_{Ir}}. \tag{4.83}$$

At constant values of $|M_{Vr}|$, the amplitude of the current through the series-resonant circuit I_m is inversely proportional to the resistance R_t.

Combining (4.77) and (4.79) gives the magnitude of the dc-to-ac voltage transfer function for the Class D series-resonant amplifier

$$|M_{VI}| = \frac{V_{O(rms)}}{V_I} = \frac{V_{O(rms)}}{V_{rms}} \frac{V_{rms}}{V_I} = M_{Vs}|M_{Vr}| = \frac{\sqrt{2}\eta_{Ir}}{\pi\sqrt{1 + Q_L^2\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)^2}}. \tag{4.84}$$

The maximum value of M_{VI} occurs at $f/f_o = 1$ and $M_{VImax} = \sqrt{2}\eta_{Ir}/\pi = 0.45\eta_{Ir}$. Thus, the values of M_{VI} range from 0 to $0.45\eta_{Ir}$.

4.7 Bandwidth of Class D Amplifier

Equation (4.79) represents the magnitude of the voltage transfer function of a bandpass filter. The 3-dB bandwidth satisfies the following condition:

$$\frac{1}{\sqrt{1 + Q_L^2\left(\frac{f}{f_o} - \frac{f_o}{f}\right)^2}} = \frac{1}{\sqrt{2}}. \tag{4.85}$$

Hence,

$$Q_L^2\left(\frac{f}{f_o} - \frac{f_o}{f}\right)^2 = 1 \tag{4.86}$$

resulting in two second-order equations. The first equation is

$$Q_L\left(\frac{f}{f_o} - \frac{f_o}{f}\right) = 1 \tag{4.87}$$

yielding the quadratic

$$\left(\frac{f}{f_o}\right)^2 - \frac{1}{Q_L}\left(\frac{f}{f_o}\right) - 1 = 0 \tag{4.88}$$

whose positive solution gives the upper 3-dB frequency

$$f_H = f_o\left[\sqrt{1 + \frac{1}{4Q_L^2}} + \frac{1}{2Q_L}\right] \approx f_o\left(1 + \frac{1}{2Q_L}\right). \tag{4.89}$$

Similarly, the second equation is

$$Q_L\left(\frac{f}{f_o}-\frac{f_o}{f}\right)=-1 \tag{4.90}$$

yielding the quadratic

$$\left(\frac{f}{f_o}\right)^2+\frac{1}{Q_L}\left(\frac{f}{f_o}\right)-1=0 \tag{4.91}$$

whose positive solution gives the lower 3-dB frequency

$$f_L=f_o\left[\sqrt{1+\frac{1}{4Q_L^2}}-\frac{1}{2Q_L}\right]\approx f_o\left(1-\frac{1}{2Q_L}\right). \tag{4.92}$$

Thus, the 3-dB bandwidth is obtained as

$$BW=\Delta f=f_H-f_L=\frac{f_o}{Q_L}. \tag{4.93}$$

As the loaded quality factor Q_L decreases, the bandwidth BW increases.

4.8 Efficiency of Half-bridge Class D Power Amplifier

4.8.1 Conduction Losses

The conduction loss for the power MOSFET is

$$P_{rDS}=\frac{r_{DS}I_m^2}{4}, \tag{4.94}$$

for the resonant inductor is

$$P_{rL}=\frac{r_L I_m^2}{2}, \tag{4.95}$$

and for the resonant capacitor is

$$P_{rC}=\frac{r_C I_m^2}{2}. \tag{4.96}$$

Hence, the conduction power loss in both the transistors and the resonant circuit is

$$P_r=2P_{rDS}+P_{rL}+P_{rC}=\frac{(r_{DS}+r_L+r_C)I_m^2}{2}=\frac{rI_m^2}{2}. \tag{4.97}$$

The output power is

$$P_O=\frac{I_m^2 R}{2}. \tag{4.98}$$

Neglecting the switching losses and the gate-drive losses and using (4.60) and (4.62), one obtains the efficiency of the amplifier determined by the conduction losses only

$$\eta_{Ir}=\frac{P_O}{P_I}=\frac{P_O}{P_O+P_r}=\frac{R}{R+r}=\frac{1}{1+\frac{r}{R}}=1-\frac{r}{R+r}$$
$$=1-\left(\frac{\omega_o L}{R+r}\right)\left(\frac{r}{\omega_o L}\right)=1-\frac{Q_L}{Q_o}. \tag{4.99}$$

Note that in order to achieve a high efficiency, the ratio of the load resistance R to the parasitic resistance r must be high. The efficiency is high, when Q_L is low and Q_o is high. For example, for $Q_L = 5$ and $Q_o = 200$, the efficiency is $\eta_{Ir} = 1 - 5/200 = 0.975$.

The turn-on loss for operation below resonance is given in the next section and the turn-off loss for operation above resonance is given in Section 4.8.3. Expressions for the efficiency for the two cases are given in those sections.

4.8.2 Turn-on Switching Loss

For operation below resonance, the turn-off switching loss is zero; however, there is a turn-on switching loss. This loss is associated with the charging and discharging of the output capacitances of the MOSFETs. The diode junction capacitance is

$$C_j(v_D) = \frac{C_{j0}}{\left(1 - \frac{v_D}{V_B}\right)^m} = \frac{C_{j0}V_B^m}{(V_B - v_D)^m}, \quad \text{for} \quad v_D \le V_B \tag{4.100}$$

where C_{j0} is the junction capacitance at $v_D = 0$ and m is the grading coefficient; $m = 1/2$ for step junctions and m = 1/3 for graded junctions. The barrier potential is

$$V_B = V_T ln\left(\frac{N_A N_D}{n_i^2}\right) \tag{4.101}$$

where n_i is the intrinsic carrier density ($1.5 \times 10^{10}\,\text{cm}^{-3}$ for silicon at $25\,^\circ$C), N_A is the acceptor concentration, and N_D is the donor concentration. The thermal voltage is

$$V_T = \frac{kT}{q} = \frac{T}{11\,609} \quad \text{(V)} \tag{4.102}$$

where $k = 1.38 \times 10^{-23}$ J/K is Boltzmann's constant, $q = 1.602 \times 10^{-19}$ C is the charge per electron, and T is the absolute temperature in K. For p^+n diodes, a typical value of the acceptor concentration is $N_A = 10^{16}\,\text{cm}^{-3}$, and a typical value of the donor concentration is $N_D = 10^{14}\,\text{cm}^{-3}$, which gives $V_B = 0.57$ V. The zero-voltage junction capacitance is given by

$$C_{j0} = A\sqrt{\frac{\varepsilon_r \varepsilon_o q}{2V_B\left(\frac{1}{N_D} + \frac{1}{N_A}\right)}} \approx A\sqrt{\frac{\varepsilon_r \varepsilon_o q N_D}{2V_B}}, \quad \text{for} \quad N_D \ll N_A \tag{4.103}$$

where A is the junction area in cm^2, $\varepsilon_r = 11.7$ for silicon, and $\varepsilon_o = 8.85 \times 10^{-14}$ (F/cm). Hence, $C_{j0}/A = 3.1234 \times 10^{-16}\sqrt{N_D}$ (F/cm^2). For instance, if $N_D = 10^{14}\,\text{cm}^{-3}$, $C_{j0}/A \approx 3$ nF/cm^2. Typical values of C_{j0} are of the order of 1 nF for power diodes.

The MOSFET's drain-source capacitance C_{ds} is the capacitance of the body-drain pn step junction diode. Setting $v_D = -v_{DS}$ and $m = 1/2$, one obtains from (4.100)

$$C_{ds}(v_{DS}) = \frac{C_{j0}}{\sqrt{1 + \frac{v_{DS}}{V_B}}} = C_{j0}\sqrt{\frac{V_B}{v_{DS} + V_B}}, \quad \text{for} \quad v_{DS} \ge -V_B. \tag{4.104}$$

Hence,

$$\frac{C_{ds1}}{C_{ds2}} = \sqrt{\frac{v_{DS2} + V_B}{v_{DS1} + V_B}} \approx \sqrt{\frac{v_{DS2}}{v_{DS1}}} \tag{4.105}$$

where C_{ds1} is the drain-source capacitance at v_{DS1} and C_{ds2} is the drain-source capacitance at v_{DS2}. Manufacturers of power MOSFETs usually specify the capacitances $C_{oss} = C_{gd} + C_{ds}$ and $C_{rss} = C_{gd}$ at $V_{DS} = 25\,\mathrm{V}$, $V_{GS} = 0\,\mathrm{V}$, and $f = 1\,\mathrm{MHz}$. Thus, the drain-source capacitance at $V_{DS} = 25\,\mathrm{V}$ can be found as $C_{ds(25V)} = C_{oss} - C_{rss}$. The interterminal capacitances of MOSFETs are essentially independent of frequency. From (4.105), the drain-source capacitance at the dc voltage V_I is

$$C_{ds(V_I)} = C_{ds(25V)}\sqrt{\frac{25 + V_B}{V_I + V_B}} \approx \frac{5C_{ds(25V)}}{\sqrt{V_I}}\ \text{(F)}. \tag{4.106}$$

The drain-source capacitance at $v_{DS} = 0$ is

$$C_{j0} = C_{ds(25V)}\sqrt{\frac{25}{V_B} + 1} \approx 6.7C_{ds(25V)} \tag{4.107}$$

for $V_B = 0.57\,\mathrm{V}$. Also,

$$C_{ds}(v_{DS}) = C_{ds(V_I)}\sqrt{\frac{V_I + V_B}{v_{DS} + V_B}} \approx C_{ds(V_I)}\sqrt{\frac{V_I}{v_{DS}}}. \tag{4.108}$$

Using (4.104) and $\mathrm{d}Q_j = C_{ds}\mathrm{d}v_{DS}$, the charge stored in the drain-source junction capacitance at v_{DS} can be found as

$$\begin{aligned} Q_j(v_{DS}) &= \int_{-V_B}^{v_{DS}} \mathrm{d}Q_j = \int_{-V_B}^{v_{DS}} C_{ds}(v_{DS})\,\mathrm{d}v_{DS} \\ &= C_{j0}\sqrt{V_B}\int_{-V_B}^{v_{DS}} \frac{\mathrm{d}v_{DS}}{\sqrt{v_{DS} + V_B}} = 2C_{j0}\sqrt{V_B(v_{DS} + V_B)} \\ &= 2C_{j0}V_B\sqrt{1 + \frac{v_{DS}}{V_B}} = 2(v_{DS} + V_B)C_{ds}(v_{DS}) \approx 2v_{DS}C_{ds}(v_{DS}) \end{aligned} \tag{4.109}$$

which, by substituting (4.106) at $v_{DS} = V_I$, simplifies to

$$Q_j(V_I) = 2V_I C_{ds(V_I)} = 10C_{ds(25V)}\sqrt{V_I}\text{(C)}. \tag{4.110}$$

Hence, the energy transferred from the dc input source V_I to the output capacitance of the upper MOSFET after the upper transistor is turned off is

$$\begin{aligned} W_I &= \int_{-V_B}^{V_I} vi\,\mathrm{d}t = V_I\int_{-V_B}^{V_I} i\,\mathrm{d}t = V_I Q_j(V_I) \\ &= 2V_I^2 C_{ds(V_I)} = 10\sqrt{V_I^3}C_{ds(25V)}\text{(W)}. \end{aligned} \tag{4.111}$$

Using $\mathrm{d}W_j = (1/2)Q_j\,\mathrm{d}v_{DS}$ and (4.109), the energy stored in the drain-source junction capacitance C_{ds} at v_{DS} is

$$\begin{aligned} W_j(v_{DS}) &= \frac{1}{2}\int_{-V_0}^{v_{DS}} Q_j\,\mathrm{d}v_{DS} = C_{j0}\sqrt{V}_B\int_{-V_0}^{v_{DS}} \sqrt{v_{DS} + V_B}\,\mathrm{d}v_{DS} \\ &= \frac{2}{3}C_{j0}\sqrt{V_B}(v_{DS} + V_B)^{\frac{3}{2}} = \frac{2}{3}C_{ds}(v_{DS})(v_{DS} + V_B)^2 \approx \frac{2}{3}C_{ds}(v_{DS})v_{DS}^2. \end{aligned} \tag{4.112}$$

Hence, from (4.106) the energy stored in the drain-source junction capacitance at $v_{DS} = V_I$ is

$$W_j(V_I) = \frac{2}{3}C_{ds(V_I)}V_I^2 = \frac{10}{3}C_{ds(25V)}\sqrt{V_I^3}\text{(J)}. \tag{4.113}$$

This energy is lost as heat when the transistor turns on and the capacitor is discharged through r_{DS}, resulting in the turn-on switching power loss per transistor

$$P_{tron} = \frac{W_j(V_I)}{T} = fW_j(V_I) = \frac{2}{3}fC_{j0}\sqrt{V_B}(V_I + V_B)^{\frac{3}{2}}$$
$$= \frac{2}{3}fC_{ds(V_I)}V_I^2 = \frac{10}{3}fC_{ds(25V)}\sqrt{V_I^3}\,(\mathrm{W}). \qquad (4.114)$$

Figure 4.23 shows C_{ds}, Q_j, and W_j as functions of v_{DS} given by (4.104), (4.109), and (4.112).

Using (4.111) and (4.113), one arrives at the energy lost in the resistances of the charging path during the charging process of the capacitance C_{ds}

$$W_{char}(V_I) = W_I(V_I) - W_j(V_I) = \frac{4}{3}C_{ds(V_I)}V_I^2 = \frac{20}{3}C_{ds(25V)}\sqrt{V_I^3}\,(\mathrm{J}) \qquad (4.115)$$

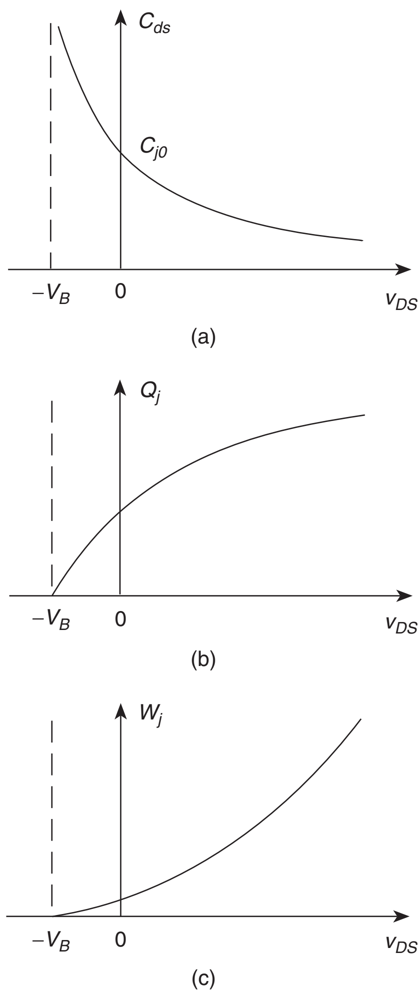

Figure 4.23 Plots of C_{ds}, Q_j, and W_j versus v_{DS}. (a) C_{ds} versus v_{DS}. (b) Q_j versus v_{DS}. (c) W_j versus v_{DS}.

and the corresponding power associated with the charging of the capacitance C_{ds} is

$$P_{char} = \frac{W_{char}(V_I)}{T} = fW_{char}(V_I) = \frac{4}{3}fC_{ds(V_I)}V_I^2 = \frac{20}{3}fC_{ds(25V)}\sqrt{V_I^3}(\mathrm{W}). \tag{4.116}$$

From (4.111), one arrives at the total switching power loss per transistor

$$P_{sw} = \frac{W(V_I)}{T} = fW_I(V_I) = 2fC_{j0}V_I\sqrt{V_B(V_I + V_B)}$$

$$= 2fC_{ds(V_I)}V_I^2 = 10fC_{ds(25V)}\sqrt{V_I^3}(\mathrm{W}). \tag{4.117}$$

The switching loss associated with charging and discharging an equivalent linear capacitance C_{eq} is $P_{sw} = fC_{eq}V_I^2$. Hence, from (4.117)

$$C_{eq} = 2C_{ds(V_I)} = \frac{10C_{ds(25V)}}{\sqrt{V_I}}. \tag{4.118}$$

Example 4.1

For MTP5N40 MOSFETs, the data sheets give that $C_{oss} = 300\,\mathrm{pF}$ and $C_{rss} = 80\,\mathrm{pF}$ at $V_{DS} = 25\,\mathrm{V}$ and $V_{GS} = 0\,\mathrm{V}$. These MOSFETs are to be used in a Class D half-bridge series-resonant amplifier that is operated at frequency $f = 100\,\mathrm{kHz}$ and fed by dc voltage source $V_I = 350\,\mathrm{V}$. Calculate the drain-source capacitance at the dc supply voltage V_I, the drain-source capacitance at $v_{DS} = 0$, the charge stored in the drain-source junction capacitance at V_I, the energy transferred from the dc input source V_I to the output capacitance of the MOSFET during turn-on transition, the energy stored in the drain-source junction capacitance C_{ds} at V_I, the turn-on switching power loss, and the total switching power loss per transistor in the amplifier operating below resonance. Assume $V_B = 0.57\,\mathrm{V}$.

Solution. Using data sheets,

$$C_{ds(25V)} = C_{oss} - C_{rss} = 300 - 80 = 220\,\mathrm{pF}. \tag{4.119}$$

From (4.106), one obtains the drain-source capacitance at the dc supply voltage $V_I = 350\,\mathrm{V}$

$$C_{ds(V_I)} = \frac{5C_{ds(25V)}}{\sqrt{V_I}} = \frac{5 \times 220 \times 10^{-12}}{\sqrt{350}} = 58.79 \approx 59\,\mathrm{pF}. \tag{4.120}$$

Equation (4.107) gives the drain-source capacitance at $v_{DS} = 0$

$$C_{j0} = 6.7C_{ds(25V)} = 6.7 \times 220 \times 10^{-12} = 1474\,\mathrm{pF}. \tag{4.121}$$

The charge stored in the drain-source junction capacitance at $V_I = 350\,\mathrm{V}$ is obtained from (4.110)

$$Q_j(V_I) = 2V_I C_{ds(V_I)} = 2 \times 350 \times 59 \times 10^{-12} = 41.3\,\mathrm{nC}. \tag{4.122}$$

The energy transferred from the input voltage source V_I to the amplifier is calculated from (4.111) as

$$W_I(V_I) = V_I Q_j(V_I) = 350 \times 41.153 \times 10^{-9} = 14.4\,\mu\mathrm{J} \tag{4.123}$$

and the energy stored in the drain-source junction capacitance C_{ds} at V_I is calculated from (4.113) as

$$W_j(V_I) = \frac{10}{3}C_{ds(25V)}\sqrt{V_I^3} = \frac{10}{3} \times 220 \times 10^{-12}\sqrt{350^3} = 4.8\,\mu\mathrm{J}. \tag{4.124}$$

Using (4.116), the power associated with charging the capacitance C_{ds} is calculated as

$$P_{char} = \frac{20}{3} f C_{ds(25V)} \sqrt{V_I^3} = \frac{20}{3} \times 10^5 \times 220 \times 10^{-12} \sqrt{350^3} = 0.96\,\text{W}. \tag{4.125}$$

From (4.114), the turn-on switching power loss per transistor for operating below resonance is

$$P_{tron} = \frac{10}{3} f C_{ds(25V)} \sqrt{V_I^3} = \frac{10}{3} \times 10^5 \times 220 \times 10^{-12} \sqrt{350^3} = 0.48\,\text{W}. \tag{4.126}$$

Using (4.117), one arrives at the total switching power loss per transistor for operating below resonance

$$P_{sw} = 10 f C_{ds(25V)} \sqrt{V_I^3}(\text{W}) = 10 \times 10^5 \times 220 \times 10^{-12} \sqrt{350^3} = 1.44\,\text{W}. \tag{4.127}$$

Note that $P_{tron} = \frac{1}{3} P_{sw}$ and $P_{char} = \frac{2}{3} P_{sw}$. The equivalent linear capacitance is $C_{eq} = 2C_{ds(V_I)} = 2 \times 59 = 118\,\text{pF}$.

The overall power dissipation in the Class D amplifier is

$$P_T = P_r + 2P_{sw} + 2P_G = \frac{rI_m^2}{2} + 20 f C_{ds(25V)} \sqrt{V_I^3} + 2 f Q_g V_{GSpp}. \tag{4.128}$$

Hence, the efficiency of the half-bridge amplifier for operating below resonance is

$$\eta_I = \frac{P_O}{P_O + P_T} = \frac{P_O}{P_O + P_r + 2P_{sw} + 2P_G}. \tag{4.129}$$

4.8.3 Turn-off Switching Loss

For operation above resonance, the turn-on switching loss is zero, but there is a turn-off switching loss. The switch current and voltage waveforms during turn-off for $f > f_o$ are shown in Figure 4.24. These waveforms were observed in various Class D experimental circuits. Notice that the voltage v_{DS2} increases slowly at its lower values and much faster at its higher values. This is because the MOSFET output capacitance is highly nonlinear, and it is much higher at low voltage v_{DS2} than at high voltage v_{DS2}. The current that charges this capacitance is approximately constant. The drain-to-source voltage v_{DS2} during voltage rise time t_r can be approximated by a parabolic function

$$v_{DS2} = a(\omega t)^2. \tag{4.130}$$

Because $v_{DS2}(\omega t_r) = V_I$, one obtains

$$a = \frac{V_I}{(\omega t_r)^2}. \tag{4.131}$$

Hence, (4.130) becomes

$$v_{DS2} = \frac{V_I(\omega t)^2}{(\omega t_r)^2}. \tag{4.132}$$

The switch current during rise time t_r is a small portion of a sinusoid and can be approximated by a constant

$$i_{S2} = I_{OFF}. \tag{4.133}$$

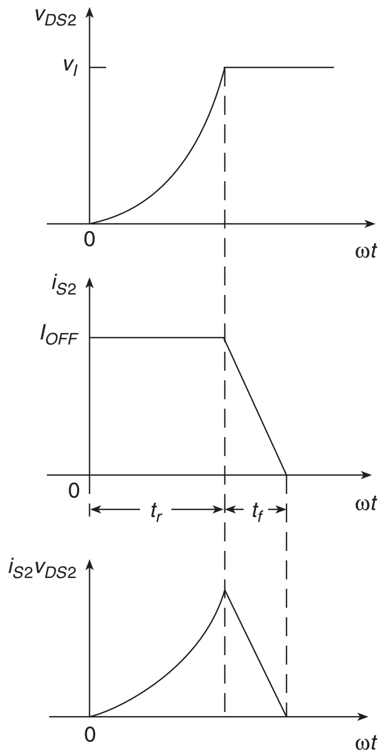

Figure 4.24 Waveforms of v_{DS2}, i_{D2}, and $i_{S2}v_{DS2}$ during turn-off for $f > f_o$.

The average value of the power loss associated with the voltage rise time t_r is

$$P_{tr} = \frac{1}{2\pi}\int_0^{2\pi} i_{S2}v_{DS2}\,\mathrm{d}(\omega t) = \frac{V_I I_{OFF}}{2\pi(\omega t_r)^2}\int_0^{\omega t_r}(\omega t)^2\,\mathrm{d}(\omega t)$$
$$= \frac{\omega t_r V_I I_{OFF}}{6\pi} = \frac{f t_r V_I I_{OFF}}{3} = \frac{t_r V_I I_{OFF}}{3T}. \tag{4.134}$$

The switch current during fall time t_f can be approximated by a ramp function

$$i_{S2} = I_{OFF}\left(1 - \frac{\omega t}{\omega t_f}\right) \tag{4.135}$$

and the drain-to-source voltage is

$$v_{DS2} = V_I \tag{4.136}$$

which yields the average value of the power loss associated with the fall time t_f of the current of the semiconductor device

$$P_{tf} = \frac{1}{2\pi}\int_0^{2\pi} i_{S2}v_{DS2}\,\mathrm{d}(\omega t) = \frac{V_I I_{OFF}}{2\pi}\int_0^{\omega t_f}\left(1 - \frac{\omega t}{\omega t_f}\right)\mathrm{d}(\omega t)$$
$$= \frac{\omega t_f V_I I_{OFF}}{4\pi} = \frac{f t_f V_I I_{OFF}}{2} = \frac{t_f V_I I_{OFF}}{2T}. \tag{4.137}$$

Hence, the turn-off switching loss is

$$P_{toff} = P_{tr} + P_{tf} = fV_I I_{OFF}\left(\frac{t_r}{3} + \frac{t_f}{2}\right). \tag{4.138}$$

Usually t_r is much longer than t_f. The overall power dissipation in the Class D half-bridge amplifier is

$$P_T = P_r + 2P_{toff} + 2P_G = \frac{rI_m^2}{2} + fV_I I_{OFF}\left(\frac{2t_r}{3} + t_f\right) + 2fQ_g V_{GSpp}. \tag{4.139}$$

Hence, the efficiency of the amplifier for operation above resonance is

$$\eta_I = \frac{P_O}{P_O + P_T} = \frac{P_O}{P_O + P_r + 2P_{toff} + 2P_G}. \tag{4.140}$$

4.9 Design Example

A design procedure of the Class D voltage-switching power amplifier with a series-resonant circuit is illustrated by means of an example.

Example 4.2

Design a Class D half-bridge amplifier of Figure 4.5 that meets the following specifications: $V_I = 100\,\text{V}$, $P_O = 50\,\text{W}$, and $f = 110\,\text{kHz}$. Assume $Q_L = 5.5$, $\psi = 30^\circ$ (that is, $\cos^2\psi = 0.75$), and the efficiency $\eta_{Ir} = 90\,\%$. The converter employs IRF621 MOSFETs (International Rectifier) with $r_{DS} = 0.5\,\Omega$, $C_{ds(25V)} = 110\,\text{pF}$, and $Q_g = 11\,\text{nC}$. Check the initial assumption about η_{Ir} using $Q_{Lo} = 300$ and $Q_{Co} = 1200$. Estimate switching losses and gate-drive power loss assuming $V_{GSpp} = 15\,\text{V}$.

Solution. From (4.99), the dc input power of the amplifier is

$$P_I = \frac{P_O}{\eta_{Ir}} = \frac{50}{0.9} = 55.56\,\text{W}. \tag{4.141}$$

Using (4.60), the overall resistance of the amplifier can be calculated as

$$R_t = \frac{2V_I^2}{\pi^2 P_I}\cos^2\psi = \frac{2 \times 100^2}{\pi^2 \times 55.56} \times 0.75 = 27.35\,\Omega. \tag{4.142}$$

Relationships (4.99) and (4.18) give the load resistance

$$R = \eta_{Ir} R_t = 0.9 \times 27.35 = 24.62\,\Omega \tag{4.143}$$

and the maximum total parasitic resistance of the amplifier

$$r = R_t - R = 27.35 - 24.62 = 2.73\,\Omega. \tag{4.144}$$

The dc supply current is obtained from (4.60)

$$I_I = \frac{P_I}{V_I} = \frac{55.56}{100} = 0.556\,\text{A}. \tag{4.145}$$

The peak value of the switch current is

$$I_m = \sqrt{\frac{2P_O}{R}} = \sqrt{\frac{2 \times 50}{24.62}} = 2.02\,\text{A} \tag{4.146}$$

and from (4.64) the peak value of the switch voltage is equal to the input voltage

$$V_{SM} = V_I = 100\,\text{V}. \tag{4.147}$$

Using (4.44), one arrives at the ratio f/f_o at full load

$$\frac{f}{f_o} = \frac{1}{2}\left(\frac{\tan\psi}{Q_L} + \sqrt{\frac{\tan^2\psi}{Q_L^2} + 4}\right) = \frac{1}{2}\left[\frac{\tan(30^\circ)}{5.5} + \sqrt{\frac{\tan^2(30^\circ)}{5.5^2} + 4}\right] = 1.054 \tag{4.148}$$

from which

$$f_o = \frac{f}{(f/f_o)} = \frac{110 \times 10^3}{1.054} = 104.4\,\text{kHz}. \tag{4.149}$$

The values of the reactive components of the resonant circuit are calculated from (4.15) as

$$L = \frac{Q_L R_t}{\omega_o} = \frac{5.5 \times 27.35}{2\pi \times 104.4 \times 10^3} = 229.3\,\mu\text{H} \tag{4.150}$$

$$C = \frac{1}{\omega_o Q_L R_t} = \frac{1}{2\pi \times 104.4 \times 10^3 \times 5.5 \times 27.35} = 10\,\text{nF}. \tag{4.151}$$

From (4.14),

$$Z_o = \sqrt{\frac{L}{C}} = \sqrt{\frac{229.3 \times 10^{-6}}{10 \times 10^{-9}}} = 151.427\,\Omega. \tag{4.152}$$

The maximum voltage stresses for the resonant components can be approximated using (4.68)

$$V_{Cm(max)} = V_{Lm(max)} = \frac{2V_I Q_L}{\pi} = \frac{2 \times 100 \times 5.5}{\pi} = 350\,\text{V}. \tag{4.153}$$

Once the values of the resonant components are known, the parasitic resistance of the amplifier can be recalculated. From (4.40) and (4.41),

$$r_L = \frac{\omega L}{Q_{Lo}} = \frac{2\pi \times 110 \times 10^3 \times 229.3 \times 10^{-6}}{300} = 0.53\,\Omega \tag{4.154}$$

and

$$r_C = \frac{1}{\omega C Q_{Co}} = \frac{1}{2\pi \times 110 \times 10^3 \times 10 \times 10^{-9} \times 1200} = 0.12\,\Omega. \tag{4.155}$$

Thus, the parasitic resistance is

$$r = r_{DS} + r_L + r_C = 0.5 + 0.53 + 0.12 = 1.15\,\Omega. \tag{4.156}$$

From (4.94), the conduction loss in each MOSFET is

$$P_{rDS} = \frac{r_{DS} I_m^2}{4} = \frac{0.5 \times 2.02^2}{4} = 0.51\,\text{W}. \tag{4.157}$$

Using (4.95), the conduction loss in the resonant inductor L is

$$P_{rL} = \frac{r_L I_m^2}{2} = \frac{0.53 \times 2.02^2}{2} = 1.08\,\text{W}. \tag{4.158}$$

From (4.96), the conduction loss in the resonant capacitor C is

$$P_{rC} = \frac{r_C I_m^2}{2} = \frac{0.12 \times 2.02^2}{2} = 0.245\,\text{W}. \tag{4.159}$$

Hence, one obtains the overall conduction loss

$$P_r = 2P_{rDS} + P_{rL} + P_{rC} = 2 \times 0.51 + 1.08 + 0.245 = 2.345\,\text{W}. \tag{4.160}$$

The efficiency η_{Ir} associated with the conduction losses only at full power is

$$\eta_{Ir} = \frac{P_O}{P_O + P_r} = \frac{50}{50 + 2.345} = 95.52\,\%. \tag{4.161}$$

Assuming the peak-to-peak gate-source voltage $V_{GSpp} = 15\,\text{V}$, the gate-drive power loss in both MOSFETs is

$$2P_G = 2fQ_g V_{GSpp} = 2 \times 110 \times 10^3 \times 11 \times 10^{-9} \times 15 = 0.036\,\text{W}. \tag{4.162}$$

The sum of the conduction losses and the gate-drive power loss is

$$P_{LS} = P_r + 2P_G = 2.245 + 0.036 = 2.38\,\text{W}. \tag{4.163}$$

The turn-on conduction loss is zero because the amplifier is operated above resonance. The efficiency of the amplifier associated with the conduction loss and the gate-drive power at full power is

$$\eta_I = \frac{P_O}{P_O + P_{LS}} = \frac{50}{50 + 2.34} = 95.45\,\%. \tag{4.164}$$

4.10 Class D RF Power Amplifier with Amplitude Modulation

A circuit of the Class D power amplifier with amplitude modulation is shown in Figure 4.25. The modulating voltage source v_m is connected in series with the dc supply voltage V_I. Amplitude modulation can be accomplished in the Class D amplifier because the amplitude of the output voltage is directly proportional to the dc supply voltage V_I. Voltage waveforms that explain the AM modulation process are depicted in Figure 4.26. The modulating voltage waveform is given by

$$v_m = V_m \sin \omega_m t. \tag{4.165}$$

The AM voltage waveform across the bottom switch is

$$v = v_{DS2} = (V_I + V_m \sin \omega_m t) \left\{ \frac{1}{2} + \frac{2}{\pi} \sum_{k=1}^{\infty} \frac{\sin[(2k-1)\omega_c t]}{2k-1} \right\}. \tag{4.166}$$

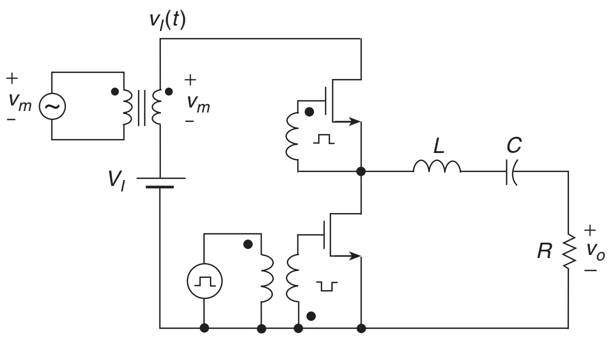

Figure 4.25 Class D RF power amplifier with amplitude modulation.

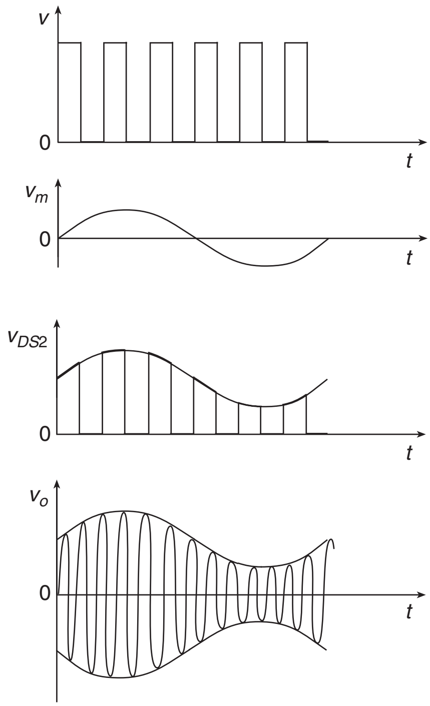

Figure 4.26 Waveforms in Class D RF power amplifier with amplitude modulation.

Since

$$\sin x \sin y = \frac{1}{2}[\cos(x - y) - \cos(x + y)] \tag{4.167}$$

we obtain the AM output voltage

$$\begin{aligned} v_o &= (V_I + V_m \sin \omega_c t)\left(\frac{2}{\pi} \sin \omega_c t\right) = \frac{2}{\pi} V_I \left(1 + \frac{V_m}{V_I} \sin \omega_c t\right) \sin \omega_c t \\ &= V_c\,(1 + m \sin \omega_c t) \sin \omega_c t = V_c \sin \omega_c t + \frac{mV_c}{2} \cos(\omega_c - \omega_m)t - \frac{mV_c}{2} \cos(\omega_c + \omega_m)t \end{aligned} \tag{4.168}$$

where the amplitude of the carrier is

$$V_c = \frac{2}{\pi} V_I. \tag{4.169}$$

4.11 Transformer-coupled Push-pull Class D Voltage-switching RF Power Amplifier

4.11.1 Waveforms

A push-pull Class D voltage-switching (voltage-source) RF power amplifier is shown in Figure 4.27. Transistors are switched on and off alternately. Current and voltage waveforms

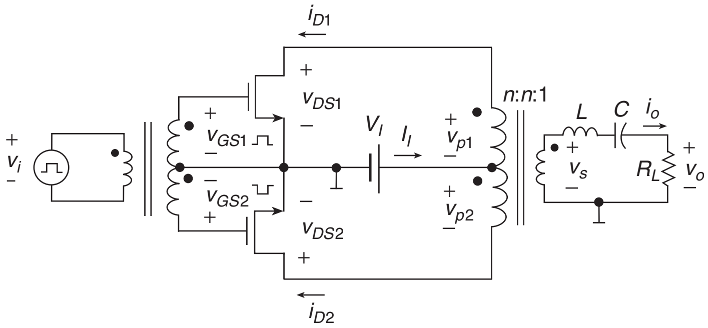

Figure 4.27 Push-pull Class D voltage-switching (voltage-source) RF power amplifier.

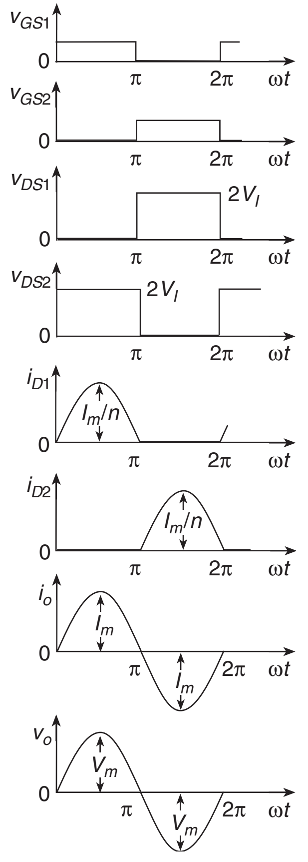

Figure 4.28 Waveforms in push-pull Class D voltage-switching (voltage-source) RF power amplifier.

are depicted in Figure 4.28. The drain-to-source voltages v_{DS1} and v_{DS2} are square waves and the drain currents i_{D1} and i_{D2} are half-sine waves.

For the operating frequency equal to the resonant frequency, the series-resonant circuit forces a sinusoidal output current

$$i_o = I_m \sin \omega t \tag{4.170}$$

resulting in a sinusoidal output voltage

$$v_o = V_m \sin \omega t \tag{4.171}$$

where

$$V_m = R_L I_m . \tag{4.172}$$

The resistance seen by each transistor across each primary winding with the other primary winding open is given by

$$R = n^2 R_L \tag{4.173}$$

where n is the transformer turn ratio of one primary winding to the secondary winding.

For $0 < \omega t \le \pi$, the drive voltage is positive and the transistor Q_1 is ON and the transistor Q_2 is OFF. The current and voltage waveforms are

$$v_{p2} = V_I = v_{p1} \tag{4.174}$$

$$v_{DS1} = V_I + v_{p1} = V_I + V_I = 2V_I \tag{4.175}$$

$$v_s = \frac{v_{p1}}{n} = \frac{v_{p2}}{n} = \frac{V_I}{n} \tag{4.176}$$

and

$$i_{D1} = I_{dm} \sin \omega t = \frac{I_m}{n} \sin \omega t \quad \text{for} \quad 0 < \omega t \le \pi \tag{4.177}$$

and

$$i_{D2} = 0 \tag{4.178}$$

where I_{dm} is the peak drain current.

For $\pi < \omega t \le 2\pi$, the drive voltage is negative and transistor Q_1 is OFF and transistor Q_2 is ON. The waveforms for this time interval are

$$v_{p1} = -V_I = v_{p1} \tag{4.179}$$

$$v_{DS2} = V_I - v_{p2} = V_I - (-V_I) = 2V_I \tag{4.180}$$

$$v_s = \frac{v_{p1}}{n} = \frac{v_{p2}}{n} = -\frac{V_I}{n} \tag{4.181}$$

$$i_{D1} = 0 \tag{4.182}$$

and

$$i_{D2} = -I_{dm} \sin \omega t = -\frac{I_m}{n} \sin \omega t \quad \text{for} \quad \pi < \omega t \le 2\pi . \tag{4.183}$$

The amplitude of the fundamental component of the drain-to-source voltage is

$$V_{dm} = \frac{4}{\pi} V_I . \tag{4.184}$$

The amplitude of the output voltage is

$$V_m = \frac{V_{dm}}{n} = \frac{4}{\pi} \frac{V_I}{n} . \tag{4.185}$$

The current through the dc voltage source V_I is a full-wave rectified sinusoid

$$i_I = i_{D1} + i_{D2} = I_{dm} |\sin \omega t| = \frac{I_m}{n} |\sin \omega t| . \tag{4.186}$$

The dc supply current is

$$I_I = \frac{1}{2\pi}\int_0^{2\pi}\frac{I_m}{n}|\sin\omega t|\,\mathrm{d}(\omega t) = \frac{2}{\pi}\frac{I_m}{n} = \frac{2}{\pi}I_{dm} = \frac{2}{\pi}\frac{V_m}{n^2R_L} = \frac{8}{\pi^2}\frac{V_I}{n^2R_L}. \tag{4.187}$$

The dc resistance presented by the amplifier to the dc supply source V_I is

$$R_{DC} = \frac{V_I}{I_I} = \frac{\pi^2}{8}n^2R_L = \frac{V_I}{I_I} = \frac{\pi^2}{8}R. \tag{4.188}$$

4.11.2 Power

The output power is

$$P_O = \frac{V_m^2}{2R_L} = \frac{8}{\pi^2}\frac{V_I^2}{n^2R_L} = \frac{8}{\pi^2}\frac{V_I^2}{R}. \tag{4.189}$$

The dc supply power is

$$P_I = V_II_I = \frac{8}{\pi^2}\frac{V_I^2}{n^2R_L}. \tag{4.190}$$

Neglecting conduction losses in the MOSFET on-resistances and switching losses, the drain efficiency is

$$\eta_D = \frac{P_O}{P_I} = 1. \tag{4.191}$$

4.11.3 Current and Voltage Stresses

The MOSFET current and voltage stresses are

$$I_{SM} = \frac{I_m}{n} \tag{4.192}$$

and

$$V_{SM} = 2V_I = \frac{\pi n}{2}V_m. \tag{4.193}$$

The output-power capability is

$$c_p = \frac{P_{Omax}}{2I_{DM}V_{DSM}} = \frac{1}{4}\left(\frac{I_M}{I_{DM}}\right)\left(\frac{V_m}{V_{DSM}}\right) = \frac{1}{4}\times n\times\frac{2}{2\pi} = \frac{1}{2\pi} = 0.159. \tag{4.194}$$

4.11.4 Efficiency

The rms value of the drain current is

$$I_{Srms} = \sqrt{\frac{1}{2\pi}\int_\pi^{2\pi}\frac{I_m^2}{n^2}\mathrm{d}(\omega t)} = \frac{I_m}{n\sqrt{2}} = \frac{I_{dm}}{\sqrt{2}}. \tag{4.195}$$

Hence, the conduction power loss in the MOSFET on-resistance r_{DS} is

$$P_{rDS} = r_{DS}I_{Srms}^2 = \frac{r_{DS}I_{dm}^2}{2} = \frac{r_{DS}I_m^2}{2n^2} = \frac{r_{DS}}{4n^2R_L}P_O. \tag{4.196}$$

The drain efficiency is

$$\eta_D = \frac{P_O}{P_O + 2P_{rDS}} = \frac{1}{1 + \dfrac{2P_{rDS}}{P_O}} = \frac{1}{1 + \dfrac{r_{DS}}{n^2 R_L}}. \tag{4.197}$$

The power loss in the ESR of the resonant inductor r_L is

$$P_{rL} = \frac{r_L I_m^2 Q_L^2}{2} = \frac{r_L Q_L^2}{R_L} P_O \tag{4.198}$$

and the power loss in the ESR of the resonant capacitor r_C is

$$P_{rC} = \frac{r_C I_m^2 Q_L^2}{2} = \frac{r_C Q_L^2}{R_L} P_O. \tag{4.199}$$

The total power loss is

$$P_{Loss} = 2P_{rDS} + P_{rL} + P_{rC} = P_O \left[\frac{r_{DS}}{n^2 R_L} + \frac{Q_L^2 (r_L + r_C)}{R_L} \right]. \tag{4.200}$$

Hence, the overall efficiency is

$$\eta = \frac{P_O}{P_I} = \frac{P_O}{P_O + P_{Loss}} = \frac{1}{1 + \dfrac{r_{DS}}{n^2 R_L} + \dfrac{Q_L^2 (r_L + r_C)}{R_L}}. \tag{4.201}$$

Example 4.3

Design a push-pull Class D voltage-switching RF power amplifier to meet the following specifications: $V_I = 28\,\text{V}$, $P_O = 50\,\text{W}$, $R_L = 50\,\Omega$, $BW = 240\,\text{MHz}$, and $f = 2.4\,\text{GHz}$. Neglect switching losses.

Solution. Assuming the efficiency of the resonant circuits $\eta_r = 0.96$, the drain power is

$$P_{DS} = \frac{P_O}{\eta_r} = \frac{50}{0.96} = 52.083\,\text{W}. \tag{4.202}$$

Assume the minimum value of the drain-to-source voltage $V_{DSmin} = 1\,\text{V}$. The resistance seen across one part of the primary winding is

$$R = \frac{8}{\pi^2} \frac{(V_I - V_{DSmin})^2}{P_{DS}} = \frac{8}{\pi^2} \frac{(28 - 1)^2}{52.083} = 11.345\,\text{W}. \tag{4.203}$$

The transformer turns ratio is

$$n = \sqrt{\frac{R}{R_L}} = \sqrt{\frac{11.345}{50}} = 0.476. \tag{4.204}$$

Pick $n = \frac{1}{2}$.

The amplitude of the fundamental component of the drain-to-source voltage is

$$V_{dm} = \frac{4}{\pi} V_I = \frac{4}{\pi} \times 28 = 35.65\,\text{V}. \tag{4.205}$$

The amplitude of the fundamental component of the drain current is

$$I_{dm} = \frac{V_{dm}}{R} = \frac{35.65}{11.345} = 3.142\,\text{A}. \tag{4.206}$$

The dc supply voltage is

$$I_I = \frac{2}{\pi} I_{dm} = \frac{2}{\pi} \times 3.142 = 2\,\text{V}. \tag{4.207}$$

The dc supply power is

$$P_I = V_I I_I = 28 \times 2 = 56\,\text{W}. \tag{4.208}$$

The rms value of each MOSFET is

$$I_{Srms} = \frac{I_{dm}}{\sqrt{2}} = \frac{3.142}{\sqrt{2}} = 2.222\,\text{A}. \tag{4.209}$$

Assuming $r_{DS} = 0.2\,\Omega$, we obtain the conduction loss in each MOSFET

$$P_{rDS} = r_{DS} I_{Srms}^2 = 0.2 \times 2.222^2 = 0.9875\,\text{W}. \tag{4.210}$$

The drain efficiency is

$$\eta_D = \frac{P_O}{P_O + 2P_{rDS}} = \frac{50}{50 + 2 \times 0.9875} = 96.2\,\%. \tag{4.211}$$

The maximum drain current is

$$I_{SM} = I_{dm} = 3.142\,\text{A}. \tag{4.212}$$

The maximum drain-to-source voltage is

$$V_{SM} = 2V_I = 2 \times 28 = 56\,\text{V}. \tag{4.213}$$

The loaded-quality factor is

$$Q_L = \frac{f_c}{BW} = \frac{2400}{240} = 10. \tag{4.214}$$

The resonant inductance is

$$L = \frac{Q_L R_L}{\omega_c} = \frac{10 \times 11.345}{2\pi \times 2.4 \times 10^9} = 7.523\,\text{nH}. \tag{4.215}$$

The resonant capacitance is

$$C = \frac{1}{\omega_c Q_L R_L} = \frac{1}{2\pi \times 2.4 \times 10^9 \times 10 \times 11.345} = 0.585\,\text{pF}. \tag{4.216}$$

4.12 Class D Full-bridge RF Power Amplifier

4.12.1 Currents, Voltages, and Powers

The circuit of a Class D full-bridge with a series-resonant amplifier is shown in Figure 4.29. It consists of four controllable switches and a series-resonant circuit. Current and voltage waveforms in the amplifier are shown in Figure 4.30. Notice that the voltage at the input of the resonant circuit is twice as high as that of the half-bridge amplifier. The average resistance of the on-resistances of power MOSFETs is $r_S = (r_{DS1} + r_{DS2} + r_{DS3} + r_{DS4})/4 \approx 2r_{DS}$. The total parasitic resistance is represented by

$$r \approx 2r_{DS} + r_L + r_C \tag{4.217}$$

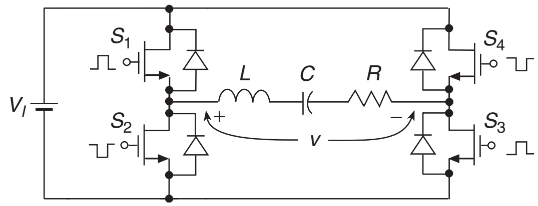

Figure 4.29 Full-bridge Class D power amplifier with a series-resonant circuit.

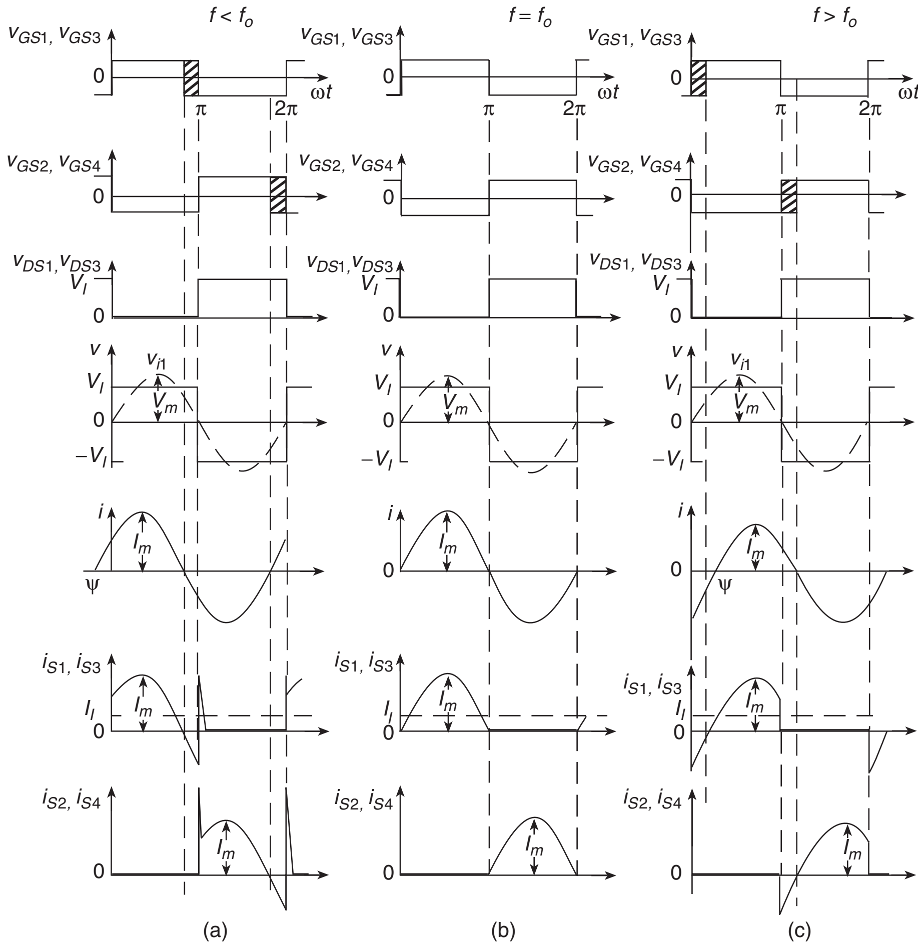

Figure 4.30 Waveforms in the Class D full-bridge power amplifier. (a) For $f < f_o$. (b) For $f = f_o$. (c) For $f > f_o$.

which yields the overall resistance

$$R_t = R + r \approx R + 2r_{DS} + r_L + r_C. \tag{4.218}$$

When the switches S_1 and S_3 are ON and the switches S_2 and S_4 are OFF, $v = V_I$. When the switches S_1 and S_3 are OFF and the switches S_2 and S_4 are ON, $v = -V_I$. Referring to

Figure 4.30, the input voltage of the series-resonant circuit is a square wave described by

$$v = \begin{cases} V_I, & \text{for} \quad 0 < \omega t \le \pi \\ -V_I, & \text{for} \quad \pi < \omega t \le 2\pi. \end{cases} \tag{4.219}$$

The Fourier expansion of this voltage is

$$\begin{aligned} v &= \frac{4V_I}{\pi} \sum_{n=1}^{\infty} \frac{1-(-1)^n}{2n} \sin n\omega t = \frac{4V_I}{\pi} \sum_{k=1}^{\infty} \frac{\sin[(2k-1)\omega t]}{2k-1} \\ &= V_I \left(\frac{4}{\pi} \sin \omega t + \frac{4}{3\pi} \sin 3\omega t + \frac{4}{5\pi} \sin 5\omega t + \cdots \right). \end{aligned} \tag{4.220}$$

The fundamental component of voltage v is

$$v_{i1} = V_m \sin \omega t \tag{4.221}$$

where its amplitude is given by

$$V_m = \frac{4V_I}{\pi} \approx 1.273 V_I. \tag{4.222}$$

Hence, one obtains the rms value of v_{i1}

$$V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{2\sqrt{2}V_I}{\pi} \approx 0.9V_I. \tag{4.223}$$

The current through the switches S_1 and S_3 is

$$i_{S1} = i_{S3} = \begin{cases} I_m \sin(\omega t - \psi), & \text{for} \quad 0 < \omega t \le \pi \\ 0, & \text{for} \quad \pi < \omega t \le 2\pi \end{cases} \tag{4.224}$$

and the current through the switches S_2 and S_4 is

$$i_{S2} = i_{S4} = \begin{cases} 0, & \text{for} \quad 0 < \omega t \le \pi \\ -I_m \sin(\omega t - \psi), & \text{for} \quad \pi < \omega t \le 2\pi. \end{cases} \tag{4.225}$$

The input current of the amplifier is

$$i_I = i_{S1} + i_{S4}. \tag{4.226}$$

The frequency of the input current is twice the operating frequency. Hence, from (4.43), (4.46), and (4.222), one obtains the dc component of the input current

$$\begin{aligned} I_I &= \frac{1}{\pi} \int_0^{\pi} i_{S1} d(\omega t) = \frac{I_m}{\pi} \int_0^{\pi} \sin(\omega t - \psi) d(\omega t) = \frac{2I_m \cos\psi}{\pi} = \frac{2V_m \cos\psi}{\pi Z} = \frac{8V_I \cos\psi}{\pi^2 Z} \\ &= \frac{8V_I \cos^2\psi}{\pi^2 R_t} = \frac{8V_I R_t}{\pi^2 Z^2} = \frac{2I_m}{\pi \sqrt{1 + Q_L^2 \left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega} \right)^2}} \\ &= \frac{8V_I}{\pi^2 R_t \left[1 + Q_L^2 \left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega} \right)^2 \right]}. \end{aligned} \tag{4.227}$$

At $f = f_o$,

$$I_I = \frac{2I_m}{\pi} = \frac{8V_I}{\pi^2 R_t}. \tag{4.228}$$

The dc input power is

$$P_I = I_I V_I = \frac{8V_I^2 \cos^2\psi}{\pi^2 R_t} = \frac{8V_I^2}{\pi^2 R_t \left[1 + Q_L^2 \left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)^2\right]}$$

$$= \frac{8V_I^2 R_t}{\pi^2 Z_o^2 \left[\left(\frac{R_t}{Z_o}\right)^2 + \left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)^2\right]}. \quad (4.229)$$

At $f = f_o$,

$$P_I = \frac{8V_I^2}{\pi^2 R_t}. \quad (4.230)$$

The current through the series-resonant circuit is given by (4.53). From (4.46), (4.47), and (4.222), its amplitude can be found as

$$I_m = \frac{V_m}{Z} = \frac{4V_I}{\pi Z} = \frac{4V_I \cos\psi}{\pi R_t} = \frac{4V_I}{\pi R_t \sqrt{1 + Q_L^2 \left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)^2}}$$

$$= \frac{4V_I}{\pi Z_o \sqrt{\left(\frac{R_t}{Z_o}\right)^2 + \left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)^2}}. \quad (4.231)$$

At $f = f_o$,

$$I_{SM} = I_m(f_o) = \frac{4V_I}{\pi R_t}. \quad (4.232)$$

The voltage stress of each switch is

$$V_{SM} = V_I. \quad (4.233)$$

The output power is obtained from (4.231)

$$P_O = \frac{I_m^2 R}{2} = \frac{8V_I^2 R \cos^2\psi}{\pi^2 R_t^2} = \frac{8V_I^2 R}{\pi^2 R_t^2 \left[1 + Q_L^2 \left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)^2\right]}$$

$$= \frac{8V_I^2 R}{\pi^2 Z_o^2 \left[\left(\frac{R_t}{Z_o}\right)^2 + \left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)^2\right]}. \quad (4.234)$$

At $f = f_o$,

$$P_O = \frac{8V_I^2 R}{\pi^2 R_t^2} \approx \frac{8V_I^2}{\pi^2 R}. \quad (4.235)$$

From (4.231), one obtains the amplitude of the voltage across the capacitor C

$$V_{Cm} = \frac{I_m}{\omega C} = \frac{4V_I}{\pi \left(\frac{\omega}{\omega_o}\right) \sqrt{\left(\frac{R_t}{Z_o}\right)^2 + \left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)^2}}. \quad (4.236)$$

Similarly, the amplitude of the voltage across the inductor L is

$$V_{Lm} = \omega L I_m = \frac{4V_I\left(\frac{\omega}{\omega_o}\right)}{\pi\sqrt{\left(\frac{R_t}{Z_o}\right)^2 + \left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)^2}}. \tag{4.237}$$

At $f = f_o$,

$$V_{Cm} = V_{Lm} = Z_o I_{mr} = Q_L V_m = \frac{4V_I Q_L}{\pi}. \tag{4.238}$$

4.12.2 Efficiency of Full-bridge Class D RF Power Amplifier

The conduction losses in each transistor, resonant inductor, and resonant capacitor are given by (4.94), (4.95), and (4.96), respectively. The conduction power loss in the four transistors and the resonant circuit is

$$P_r = \frac{rI_m^2}{2} = \frac{(2r_{DS} + r_L + r_C)I_m^2}{2}. \tag{4.239}$$

The turn-on switching loss per transistor for operation below resonance P_{sw} is given by (4.117). The overall power dissipation in the Class D amplifier for operation below resonance is

$$P_T = P_r + 4P_{sw} + 4P_G = \frac{rI_m^2}{2} + 40C_{ds(25V)}\sqrt{V_I^3} + 4fQ_g V_{GSpp}. \tag{4.240}$$

Hence, the efficiency of the full-bridge amplifier operating below resonance is

$$\eta_I = \frac{P_O}{P_O + P_T} = \frac{P_O}{P_O + P_r + 4P_{sw} + 4P_G}. \tag{4.241}$$

The turn-off switching loss per transistor operating above resonance P_{toff} is given by (4.138). The overall power dissipation in the amplifier operating above resonance is

$$P_T = P_r + 4P_{toff} + 4P_G = \frac{rI_m^2}{2} + fV_I I_{OFF}\left(\frac{4t_r}{3} + 2t_f\right) + 4fQ_g V_{GSpp} \tag{4.242}$$

resulting in the efficiency of the Class D full-bridge series-resonant amplifier operating above resonance

$$\eta_I = \frac{P_O}{P_O + P_T} = \frac{P_O}{P_O + P_r + 4P_{toff} + 4P_G}. \tag{4.243}$$

4.12.3 Operation Under Short-circuit and Open-circuit Conditions

The Class D amplifier with a series-resonant circuit can operate safely with an open circuit at the output. However, it is prone to catastrophic failure if the output is short-circuited at f close to f_o. If $R = 0$, the amplitude of the current through the resonant circuit and the switches is

$$I_m = \frac{4V_I}{\pi r\sqrt{1 + \left(\frac{Z_o}{r}\right)^2\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)^2}}. \tag{4.244}$$

The maximum value of I_m occurs at $f = f_o$ and is given by

$$I_{mr} = \frac{4V_I}{\pi r}. \tag{4.245}$$

The amplitudes of the voltages across the resonant components L and C are

$$V_{Cm} = V_{Lm} = \frac{I_{mr}}{\omega_o C} = \omega_o L I_{mr} = Z_o I_{mr} = \frac{4V_I Z_o}{\pi r} = \frac{4V_I Q_o}{\pi}. \tag{4.246}$$

4.12.4 Voltage Transfer Function

Using (4.223), one obtains the voltage transfer function from the input of the amplifier to the input of the series-resonant circuit

$$M_{Vs} = \frac{V_{rms}}{V_I} = \frac{2\sqrt{2}}{\pi} = 0.9. \tag{4.247}$$

The product of (4.79) and (4.247) yields the magnitude of the dc-to-ac voltage transfer function for the Class D full-bridge series-resonant amplifier

$$M_{VI} = \frac{V_{O(rms)}}{V_I} = \frac{V_{O(rms)}}{V_{rms}}\frac{V_{rms}}{V_I} = M_{Vs}M_{Vr} = \frac{2\sqrt{2}\eta_{Ir}}{\pi\sqrt{1 + Q_L^2\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)^2}}. \tag{4.248}$$

The maximum value of M_{VI} occurs at $f/f_o = 1$ and equals $M_{VImax} = 2\sqrt{2}\eta_{Ir}/\pi = 0.9\eta_{Ir}$. Thus, the values of M_{VI} range from 0 to $0.9\eta_{Ir}$.

Example 4.4

Design a Class D full-bridge amplifier of Figure 4.29 to meet the following specifications: $V_I = 270\,\text{V}$, $P_O = 500\,\text{W}$, and $f = 110\,\text{kHz}$. Assume $Q_L = 5.3$, $\psi = 30^\circ$ (that is, $\cos^2\psi = 0.75$) and the efficiency $\eta_{Ir} = 94\,\%$. Neglect switching losses.

Solution. The input power of the amplifier is

$$P_I = \frac{P_O}{\eta_{Ir}} = \frac{500}{0.94} = 531.9\,\text{W}. \tag{4.249}$$

The overall resistance of the amplifier can be obtained from (4.234)

$$R_t = \frac{8V_I^2}{\pi^2 P_I}\cos^2\psi = \frac{8 \times 270^2}{\pi^2 \times 531.9} \times 0.75 = 83.3\,\Omega. \tag{4.250}$$

Following the design procedure of Example 4.2, one obtains:

$$R = \eta_{Ir}R_t = 0.94 \times 83.3 = 78.3\,\Omega \tag{4.251}$$

$$r = R_t - R = 83.3 - 78.3 = 5\,\Omega \tag{4.252}$$

$$I_I = \frac{P_I}{V_I} = \frac{531.9}{270} = 1.97\,\text{A} \tag{4.253}$$

$$I_m = \sqrt{\frac{2P_{R(rms)}}{R}} = \sqrt{\frac{2 \times 500}{78.3}} = 3.574\,\text{A} \tag{4.254}$$

$$\frac{f}{f_o} = \frac{1}{2}\left(\frac{\tan\psi}{Q_L} + \sqrt{\frac{\tan^2\psi}{Q_L^2} + 4}\right) = \frac{1}{2}\left[\frac{\tan(30^\circ)}{5.3} + \sqrt{\frac{\tan^2(30^\circ)}{5.3^2} + 4}\right] = 1.054 \tag{4.255}$$

$$f_o = \frac{f}{(f/f_o)} = \frac{110 \times 10^3}{1.054} = 104.2\,\text{kHz} \tag{4.256}$$

$$L = \frac{Q_L R_t}{\omega_o} = \frac{5.3 \times 83.3}{2\pi \times 104.2 \times 10^3} = 674\,\mu\text{H} \tag{4.257}$$

$$C = \frac{1}{\omega_o Q_L R_t} = \frac{1}{2\pi \times 104.2 \times 10^3 \times 5.3 \times 83.3} = 3.46\,\text{nF} \tag{4.258}$$

$$Z_o = \sqrt{\frac{L}{C}} = \sqrt{\frac{674 \times 10^{-6}}{3.46 \times 10^{-9}}} = 441.4\,\Omega. \tag{4.259}$$

From (4.246), the maximum voltage stresses for the resonant components are

$$V_{Cm} = V_{Lm} = \frac{4V_I Q_L}{\pi} = \frac{4 \times 270 \times 5.3}{\pi} = 1822\,\text{V}. \tag{4.260}$$

Referring to (4.233), the peak value of the switch voltage is equal to the input voltage

$$V_{SM} = V_I = 270\,\text{V}. \tag{4.261}$$

4.13 Phase Control of Full-bridge Class D Power Amplifier

Figure 4.31 shows a full-bridge Class D power amplifier with phase control. The output voltage, output current, and output power can be controlled by varying the phase shift $\Delta\phi$ between the gate-to-source voltages of the left leg and the right leg, while maintaining a constant operating frequency f. The operating frequency f can be equal to the resonant frequency f_o. Constant-frequency operation is preferred in most applications. Equivalent circuits of the amplifier are shown in Figure 4.32. Figure 4.33 shows the waveforms of the gate-to-source voltages and the voltage across the resonant circuit.

Figure 4.32(a) shows the equivalent circuit, when S_1 and S_3 are ON and S_2 and S_4 are OFF, producing the voltage across the resonant circuit $v = 0$. Figure 4.32(b) shows the equivalent circuit, when S_1 and S_4 are ON and S_2 and S_3 are OFF. In this case, the voltage across the resonant circuit is $v = V_I$. Figure 4.32(c) shows the equivalent circuit, when S_1 and S_3 are OFF and S_2 and S_4 are ON, generating the voltage across the resonant circuit $v = 0$. Figure 4.32(d) shows the equivalent circuit, when S_1 and S_4 are OFF and S_2 and S_3 are ON. In this case, the voltage across the resonant circuit is $v = -V_I$. The duty cycle corresponding to the width of the positive pulse or the negative pulse of the voltage across the resonant circuit v is given by

$$D = \frac{\pi - \Delta\phi}{2\pi} = \frac{1}{2} - \frac{\Delta\phi}{2\pi}. \tag{4.262}$$

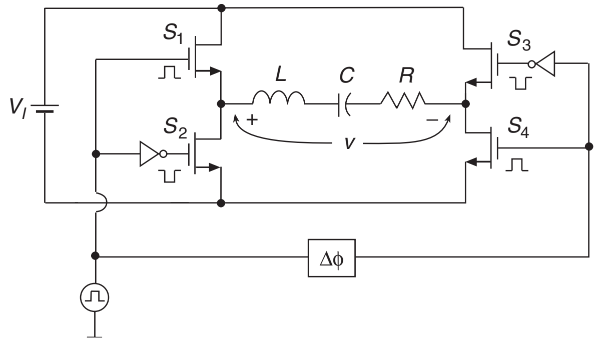

Figure 4.31 Full-bridge Class D power amplifier with phase control.

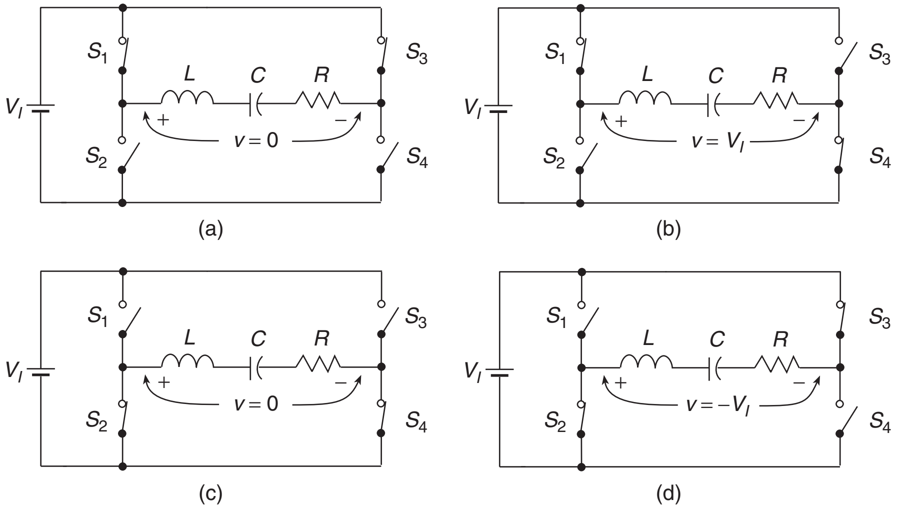

Figure 4.32 Equivalent circuits of full-bridge Class D power amplifier with phase control. (a) S_1 and S_3 ON, while S_2 and S_4 OFF. (b) S_1 and S_4 ON, while S_2 and S_3 OFF. (c) S_2 and S_4 ON, while S_1 and S_3 OFF. (d) S_2 and S_3 ON, while S_1 and S_4 OFF.

When the phase shift $\Delta\phi$ increases from 0 to π, the duty cycle D decreases from 50% to zero. The phase shift $\Delta\phi$ in terms of the duty cycle D is expressed by

$$\Delta\phi = \pi - 2\pi D = 2\pi\left(\frac{1}{2} - D\right) \quad \text{for} \quad 0 \le D \le 0.5. \tag{4.263}$$

The even harmonics of the voltage v are zero. The amplitudes of the odd harmonics of the voltage v are

$$V_{m(n)} = \frac{4V_I}{\pi n}\sin(n\pi D) \quad \text{for} \quad 0 \le D \le 0.5 \quad \text{and} \quad n = 1, 3, 5, \ldots. \tag{4.264}$$

Higher harmonics are attenuated by the resonant circuit, which acts as a bandpass filter. The amplitude of the fundamental component V_m of the voltage v is given by

$$V_m = \frac{4V_I}{\pi}\sin(\pi D) = \frac{4V_I}{\pi}\cos\left(\frac{\Delta\phi}{2}\right) \quad \text{for} \quad 0 \le D \le 0.5. \tag{4.265}$$

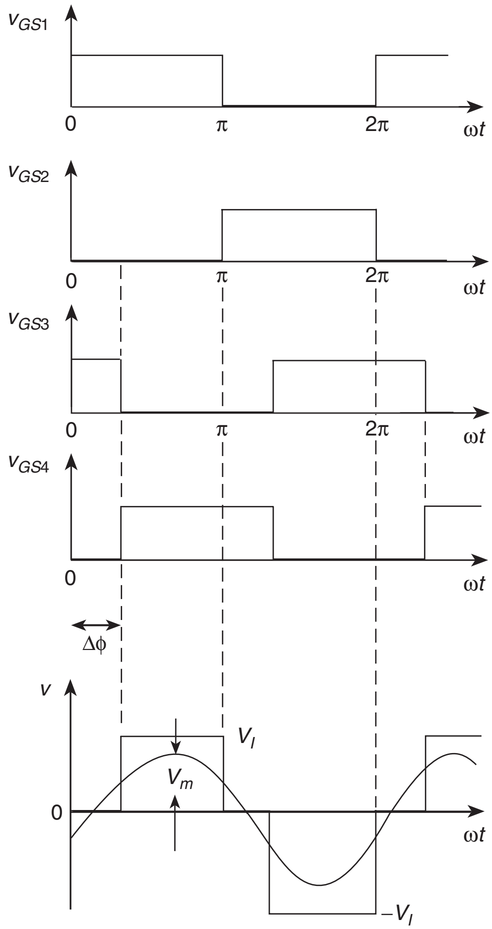

Figure 4.33 Waveforms of full-bridge Class D power amplifier with phase control.

Thus, the amplitude of the fundamental component V_m of the voltage v decreases from $V_{m(max)} = 4V_I/\pi$ to zero as D decreases from 0.5 to 0, or $\Delta\phi$ increases from 0 to π. Therefore, the output current, the output voltage, and the output power can be controlled by varying the phase shift $\Delta\phi$.

4.14 Class D Current-switching RF Power Amplifier

4.14.1 Circuit and Waveforms

The Class D current-switching (current-source) RF power amplifier of Figure 4.34(a) was introduced in [12]. This circuit is dual to the Class D voltage-switching (voltage-source) RF power amplifier. An equivalent circuit of the amplifier is shown in Figure 4.34(b). Voltage and current waveforms, which explain the principle of operation of the amplifier,

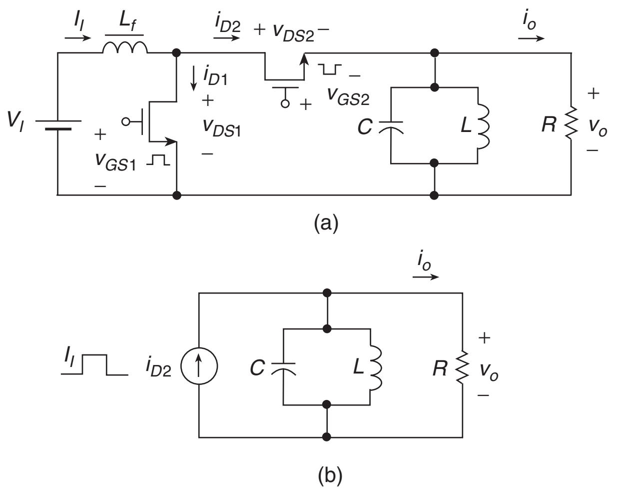

Figure 4.34 Class D current-switching (current-source) RF power amplifier [12]. (a) Circuit. (b) Equivalent circuit.

are depicted in Figure 4.35. If can be seen the MOSFETs are operated under ZVS conditions. The dc voltage source V_I and the RF choke RFC form a dc current source I_I. When the MOSFET switch S_1 is OFF and the MOSFET switch S_2 is ON, the current flowing into the load network is

$$i_{D2} = I_I. \tag{4.266}$$

When the MOSFET switch S_1 is ON and the MOSFET switch S_2 is OFF, the current flowing into the load network is

$$i_{D2} = 0. \tag{4.267}$$

Thus, dc voltage source V_I, RF choke RFC, and two MOSFET switches form a square-wave current-source whose lower value is zero and the upper value is I_I. The parallel-resonant circuit behaves like a bandpass filter and filters out all harmonics and only the fundamental component flows to the load resistance R. When the operating frequency is equal to the resonant frequency $f_0 = 1/(2\pi\sqrt{LC})$, the parallel-resonant circuit forces a sinusoidal voltage

$$v_o = V_m \sin \omega t \tag{4.268}$$

yielding a sinusoidal output current

$$i_o = I_m \sin \omega t \tag{4.269}$$

where the amplitude of the output current is

$$I_m = \frac{V_m}{R}. \tag{4.270}$$

The dc input voltage is

$$V_I = \frac{1}{2\pi}\int_0^{2\pi} v_{DS1}\, \mathrm{d}(\omega t) = \frac{1}{2\pi}\int_0^{2\pi} V_m \sin \omega t\, \mathrm{d}(\omega t) = \frac{V_m}{\pi} = \frac{I_m R}{\pi}. \tag{4.271}$$

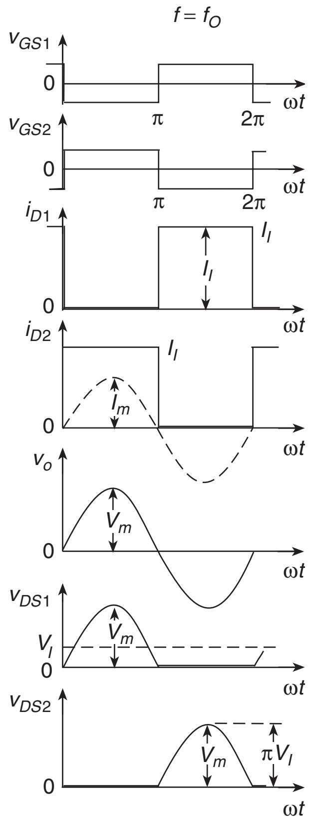

Figure 4.35 Waveforms in Class D current-switching (current-source) RF power amplifier.

The amplitude of the output current is

$$I_m = \frac{2}{\pi} I_I. \quad (4.272)$$

Hence,

$$I_I = \frac{\pi}{2} I_m = \frac{\pi}{2} \frac{V_m}{R} = \frac{\pi^2}{2} \frac{V_I}{R}. \quad (4.273)$$

The dc resistance is

$$R_{DC} = \frac{V_I}{I_I} = \frac{2}{\pi^2} \frac{V_m}{I_m} = \frac{2}{\pi^2} R = 0.2026R. \quad (4.274)$$

4.14.2 Power

The output power is

$$P_O = \frac{V_m^2}{2R} = \frac{\pi^2}{2} \frac{V_I^2}{R}. \quad (4.275)$$

The dc supply power is

$$P_I = V_I I_I = \frac{\pi^2}{2} \frac{V_I^2}{R}. \quad (4.276)$$

Hence, the drain efficiency for the idealized amplifier is

$$\eta_D = \frac{P_O}{P_I} = 1. \tag{4.277}$$

4.14.3 Voltage and Current Stresses

The transistor current and voltage stresses are

$$V_{DSM} = \pi V_m \tag{4.278}$$

and

$$I_{DM} = I_I. \tag{4.279}$$

The peak currents in the resonant inductor L and capacitor are

$$I_{Lm} = I_{Cm} = Q_L I_m \tag{4.280}$$

where the loaded-quality factor is

$$Q_L = \frac{R}{\omega_0 L} = \omega_0 CR = \frac{R}{Z_o} \tag{4.281}$$

and the characteristic impedance of the resonant circuit is

$$Z_o = \sqrt{\frac{L}{C}} = \omega_0 L = \frac{1}{\omega_0 C}. \tag{4.282}$$

The output power capability is

$$c_p = \frac{P_O}{2I_{DM} V_{DSM}} = \frac{1}{2}\left(\frac{I_I}{I_{DM}}\right)\left(\frac{V_I}{V_{DSM}}\right) = \frac{1}{2\pi} = 0.159. \tag{4.283}$$

4.14.4 Efficiency

The rms value of the MOSFET current is

$$I_{Srms} = \sqrt{\frac{1}{2\pi}\int_{\pi}^{2\pi} I_I^2 \, \mathrm{d}(\omega t)} = \frac{I_I}{\sqrt{2}}. \tag{4.284}$$

Hence, the conduction power loss in the MOSFET on-resistance r_{DS} is

$$P_{rDS} = r_{DS} I_{Srms}^2 = \frac{r_{DS} I_I^2}{2} = \frac{\pi^2}{4}\frac{r_{DS}}{R} P_O. \tag{4.285}$$

The drain efficiency is

$$\eta_D = \frac{P_O}{P_O + 2P_{rDS}} = \frac{1}{1 + \dfrac{2P_{rDS}}{P_O}} = \frac{1}{1 + \dfrac{\pi^2}{2}\dfrac{r_{DS}}{R}}. \tag{4.286}$$

The power loss in the ESR od the RF choke r_{RFC} is

$$P_{RFC} = r_{RFC} I_I^2 = \frac{\pi^2 r_{RFC}}{4} I_m^2 = \frac{\pi^2}{2}\frac{r_{RFC}}{R} P_O. \tag{4.287}$$

The power loss in the ESR of the resonant inductor r_L is

$$P_{rL} = \frac{r_L I_m^2 Q_L^2}{2} = \frac{r_L Q_L^2}{R} P_O \tag{4.288}$$

and the power loss in the ESR of the resonant capacitor r_C is

$$P_{rC} = \frac{r_C I_m^2 Q_L^2}{2} = \frac{r_C Q_L^2}{R} P_O. \tag{4.289}$$

The total power loss is

$$P_{Loss} = 2P_{rDS} + P_{RFC} + P_{rL} + P_{rC} = P_O \left[\frac{\pi^2 r_{DS}}{2R} + \frac{\pi^2 r_{RFC}}{2R} + \frac{Q_L^2(r_L + r_C)}{R} \right]. \tag{4.290}$$

Hence, the overall efficiency is

$$\eta = \frac{P_O}{P_I} = \frac{P_O}{P_O + P_{Loss}} = \frac{1}{1 + \dfrac{\pi^2}{2} \dfrac{r_{DS} + r_{RFC}}{R} + \dfrac{Q_L^2(r_L + r_C)}{R}}. \tag{4.291}$$

Example 4.5

Design a push-pull Class D current-switching RF power amplifier to meet the following specifications: $V_I = 12\,\text{V}$, $P_O = 10\,\text{W}$, $BW = 240\,\text{MHz}$, and $f_c = 2.4\,\text{GHz}$.

Solution. Assuming the efficiency of the resonant circuit is $\eta_r = 0.94$, the drain power is

$$P_{DS} = \frac{P_O}{\eta_r} = \frac{10}{0.94} = 10.638\,\text{W}. \tag{4.292}$$

The load resistance is

$$R = \frac{\pi^2}{2} \frac{V_I^2}{P_{DS}} = \frac{\pi^2}{2} \frac{12^2}{10.638} = 66.799\,\text{W}. \tag{4.293}$$

The peak value of the drain-to-source voltage is

$$V_m = \pi V_I = \pi \times 12 = 37.7\,\text{V}. \tag{4.294}$$

The amplitude of the fundamental component of the drain current is

$$I_m = \frac{V_m}{R} = \frac{37.7}{66.799} = 0.564\,\text{A}. \tag{4.295}$$

The dc supply power is

$$I_I = \frac{\pi}{2} I_m = \frac{\pi}{2} \times 0.564 = 0.8859\,\text{A}. \tag{4.296}$$

The dc supply power is

$$P_I = V_I I_I = 12 \times 0.8859 = 10.631\,\text{W}. \tag{4.297}$$

The rms value of the switch current is

$$I_{Srms} = \frac{I_I}{\sqrt{2}} = \frac{0.8859}{\sqrt{2}} = 0.626\,\text{A}. \tag{4.298}$$

Assuming $r_{DS} = 0.1\,\Omega$, the conduction power loss of each MOSFET is

$$P_{rDS} = r_{DS} I_{Srms}^2 = 0.1 \times 0.626^2 = 0.0392\,\text{W}. \tag{4.299}$$

The drain efficiency is

$$\eta_D = \frac{P_O}{P_O + 2P_{rDS}} = \frac{10}{10 + 2 \times 0.0392} = 99.22\,\%. \tag{4.300}$$

The loaded-quality factor is

$$Q_L = \frac{f_c}{BW} = \frac{2400}{240} = 10. \tag{4.301}$$

The resonant inductance is

$$L = \frac{R}{\omega_c Q_L} = \frac{66.799}{2\pi \times 2.4 \times 10^9 \times 10} = 0.443\,\text{nH}. \tag{4.302}$$

The resonant capacitance is

$$C = \frac{Q_L}{\omega_c R} = \frac{10}{2\pi \times 2.4 \times 10^9 \times 66.799} = 9.93\,\text{pF}. \tag{4.303}$$

4.15 Transformer-coupled Push-pull Class D Current-switching RF Power Amplifier

4.15.1 Waveforms

A circuit of a push-pull Class D current-switching power amplifier is shown in Figure 4.36. It is the dual of the push-pull voltage-switching Class D amplifier. The impedance matching of the load to the transistors is achieved by the transformer. The magnetizing inductance on the transformer secondary side is absorbed into the resonant inductance L.

Current and voltage waveforms are shown in Figure 4.37. The transistors turn on at zero voltage if the switching frequency is equal to the resonant frequency of the parallel-resonant circuit. The output current is

$$i_o = I_m \sin \omega t \tag{4.304}$$

and the output voltage is

$$v_o = V_m \sin \omega t \tag{4.305}$$

where

$$V_m = R_L I_m. \tag{4.306}$$

When the drive voltage is positive, transistor Q_1 is ON and transistor Q_2 is OFF. The drain currents are

$$i_{D1} = 0 \quad \text{for} \quad 0 < \omega t \leq \pi \tag{4.307}$$

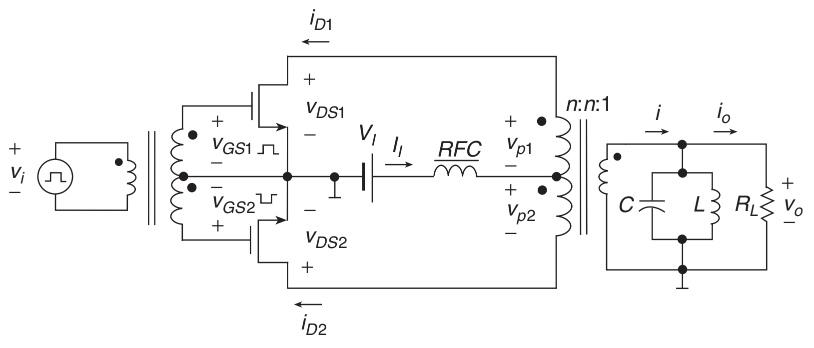

Figure 4.36 Push-pull Class D current-switching (current-source) RF power amplifier.

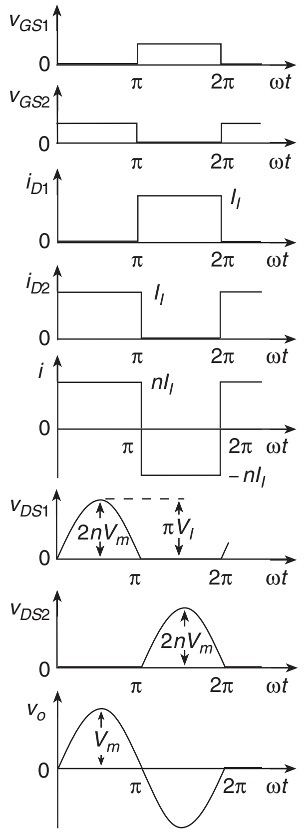

Figure 4.37 Waveforms in push-pull Class D current-switching (current-source) RF power amplifier.

and

$$i_{D2} = I_I \quad \text{for} \quad 0 < \omega t \le \pi. \tag{4.308}$$

The transformer output current is

$$i = ni_{D1} = nI_I \quad \text{for} \quad 0 < \omega t \le \pi. \tag{4.309}$$

The voltage across the upper half of the primary winding is

$$v_{p1} = -nv_o = nV_m \sin \omega t = \frac{V_{dm}}{2} \sin \omega t. \tag{4.310}$$

The drain-to-source voltages are

$$v_{DS1} = v_{p1} + v_{p2} = 2v_{p1} = V_{dm} \sin \omega t = 2nV_m \sin \omega t \quad \text{for} \quad 0 < \omega t \le \pi \tag{4.311}$$

and

$$v_{DS2} = 0 \quad \text{for} \quad 0 < \omega t \le \pi \tag{4.312}$$

where the peak value of the drain-to-source voltage is

$$V_{dm} = 2nV_m. \tag{4.313}$$

When the drive voltage is negative, transistor Q_1 is OFF and transistor Q_2 is ON. The drain currents are

$$i_{D1} = I_I \quad \text{for} \quad \pi < \omega t \le 2\pi \tag{4.314}$$

and

$$i_{D2} = 0 \quad \text{for} \quad \pi < \omega t \le 2\pi. \tag{4.315}$$

The transformer output current is

$$i = -ni_{D2} = -nI_I \quad \text{for} \quad \pi < \omega t \le 2\pi. \tag{4.316}$$

The voltage across the lower half of the primary winding is

$$v_{p2} = nv_o = -nV_m \sin\omega t = -\frac{V_{dm}}{2}\sin\omega t \quad \text{for} \quad \pi < \omega t \le 2\pi. \tag{4.317}$$

The drain-to-source voltages are

$$v_{DS1} = 0 \quad \text{for} \quad \pi < \omega t \le 2\pi \tag{4.318}$$

and

$$v_{DS2} = v_{p1} = -V_{dm}\sin\omega t = -2nV_m \sin\omega t \quad \text{for} \quad \pi < \omega t \le 2\pi. \tag{4.319}$$

The output current of the transformer is

$$i = \frac{4nV_I}{\pi}\sum_{k=1}^{\infty}\frac{\sin[(2k-1)\omega t]}{2k-1}. \tag{4.320}$$

The parallel-resonant circuit presents a very low reactance for current harmonics and an infinite reactance for the fundamental frequency component. Therefore, only the fundamental component flows through the load resistance, resulting in sinusoidal output current and voltage waveforms.

The amplitude of the fundamental component of the drain current is

$$I_{dm} = \frac{2}{\pi}I_I. \tag{4.321}$$

The amplitude of the output current is

$$I_m = \frac{4}{\pi}nI_I = 2nI_{dm}. \tag{4.322}$$

The dc supply voltage is

$$V_I = \frac{V_{dm}}{\pi} = \frac{2nV_m}{\pi}. \tag{4.323}$$

yielding

$$V_m = \frac{\pi}{2n}V_I. \tag{4.324}$$

The resistance seen by each MOSFET is

$$R = n^2 R_L. \tag{4.325}$$

4.15.2 Power

The dc supply current is

$$I_I = \frac{\pi}{2}I_{dm} = \frac{\pi}{4}\frac{I_m}{n} = \frac{\pi}{4n}\frac{V_m}{R_L} = \frac{\pi^2}{8}\frac{V_I}{n^2 R_L} = \frac{\pi^2}{8}\frac{V_I}{R}. \tag{4.326}$$

The dc resistance seen by the dc power supply is

$$R_{DC} = \frac{V_I}{I_I} = \frac{8}{\pi^2}R. \tag{4.327}$$

The output power is

$$P_O = \frac{V_m^2}{2R_L} = \frac{\pi^2}{8}\frac{V_I^2}{n^2 R_L} = \frac{\pi^2}{8}\frac{V_I^2}{R}. \tag{4.328}$$

The dc supply current is

$$P_I = V_I I_I = \frac{\pi^2}{8}\frac{V_I^2}{n^2 R_L} = \frac{\pi^2}{8}\frac{V_I^2}{R}. \tag{4.329}$$

Ideally, the drain efficiency is

$$\eta_D = \frac{P_O}{P_I} = 1. \tag{4.330}$$

4.15.3 Device Stresses

The current and voltage stresses of the MOSFETs are

$$I_{DM} = I_I \tag{4.331}$$

and

$$V_{DSM} = \pi V_I. \tag{4.332}$$

The output-power capability is

$$c_p = \frac{P_O}{2I_{DM}V_{DSM}} = \frac{P_I}{2I_{DM}V_{DSM}} = \frac{I_I V_I}{2I_{DM}V_{DSM}} = \frac{1}{2\pi} = 0.159. \tag{4.333}$$

4.15.4 Efficiency

The rms drain current is

$$I_{Srms} = \sqrt{\frac{1}{2\pi}\int_{\pi}^{2\pi} I_I^2 \, d(\omega t)} = \frac{I_I}{\sqrt{2}}. \tag{4.334}$$

Hence, the conduction power loss in the MOSFET on-resistance r_{DS} is

$$P_{rDS} = r_{DS} I_{Srms}^2 = \frac{r_{DS} I_I^2}{2} = \frac{\pi^2}{4}\frac{r_{DS}}{n^2 R_L} P_O. \tag{4.335}$$

The drain efficiency is

$$\eta_D = \frac{P_O}{P_O + 2P_{rDS}} = \frac{1}{1 + \frac{2P_{rDS}}{P_O}} = \frac{1}{1 + \frac{\pi^2}{2}\frac{r_{DS}}{n^2 R_L}}. \tag{4.336}$$

The power loss in the ESR of the RF choke r_{RFC} is

$$P_{RFC} = r_{RFC} I_I^2 = \frac{\pi^2 r_{RFC}}{4} I_m^2 = \frac{\pi^2}{2}\frac{r_{RFC}}{n^2 R_L} P_O. \tag{4.337}$$

The power loss in the ESR of the resonant inductor r_L is

$$P_{rL} = \frac{r_L I_m^2 Q_L^2}{2} = \frac{r_L Q_L^2}{R_L} P_O \tag{4.338}$$

and the power loss in the ESR of the resonant capacitor r_C is

$$P_{rC} = \frac{r_C I_m^2 Q_L^2}{2} = \frac{r_C Q_L^2}{R_L} P_O. \tag{4.339}$$

The total power loss is

$$P_{Loss} = 2P_{rDS} + P_{RFC} + P_{rL} + P_{rC} = P_O \left[\frac{\pi^2 (r_{DS} + r_{RFC})}{2n^2 R_L} + \frac{Q_L^2 (r_L + r_C)}{R_L} \right]. \tag{4.340}$$

Hence, the overall efficiency is

$$\eta = \frac{P_O}{P_I} = \frac{P_O}{P_O + P_{Loss}} = \frac{1}{1 + \frac{\pi^2}{2} \frac{r_{DS} + r_{RFC}}{n^2 R_L} + \frac{Q_L^2 (r_L + r_C)}{R_L}}. \tag{4.341}$$

Example 4.6

Design a transformer-coupled push-pull Class D current-switching RF power amplifier to meet the following specifications: $V_I = 12\,\text{V}$, $P_O = 12\,\text{W}$, $BW = 180\,\text{MHz}$, $R_L = 50\,\Omega$, and $f_c = 1.8\,\text{GHz}$.

Solution. Assuming the efficiency of the resonant circuit is $\eta_r = 0.93$, the drain power power is

$$P_{DS} = \frac{P_O}{\eta_r} = \frac{12}{0.93} = 12.903\,\text{W}. \tag{4.342}$$

The resistance seen across one part of the primary winding is

$$R = \frac{\pi^2}{8} \frac{V_I^2}{P_{DS}} = \frac{\pi^2}{8} \frac{12^2}{12.903} = 13.768\,\Omega. \tag{4.343}$$

The transformer turns ratio is

$$n = \sqrt{\frac{R}{R_L}} = \sqrt{\frac{13.768}{50}} = 0.5247. \tag{4.344}$$

Pick $n = \frac{1}{2}$. The resistance seen by each MOSFET is

$$R = n^2 R_L = \left(\frac{1}{2}\right)^2 \times 50 = 12.5\,\Omega. \tag{4.345}$$

The peak value of the drain-to-source voltage is

$$V_{dm} = \pi V_I = \pi \times 12 = 37.7\,\text{V}. \tag{4.346}$$

The amplitude of the output voltage is

$$V_m = \frac{\pi}{2n} V_I = \frac{\pi}{2 \times 0.5} \times 12 = 37.7\,\text{V}. \tag{4.347}$$

The dc resistance seen by the dc power supply is

$$R_{DC} = \frac{V_I}{I_I} = \frac{8}{\pi^2} R = \frac{8}{\pi^2} \times 13.768 = 11.16\,\Omega. \tag{4.348}$$

The amplitude of the fundamental component of the drain current is

$$I_{dm} = \frac{V_{dm}}{R} = \frac{37.7}{12.5} = 3.016\,\text{A}. \tag{4.349}$$

The dc supply current is

$$I_I = \frac{\pi^2}{8}\frac{V_I}{R} = \frac{\pi^2}{8}\frac{12}{12.5} = 1.184\,\text{A}. \tag{4.350}$$

The amplitude of the output current is

$$I_m = \frac{V_m}{R} = \frac{37.7}{12.5} = 3.016\,\text{A}. \tag{4.351}$$

The output power is

$$P_O = \frac{V_m^2}{2R_L} = \frac{37.7^2}{2 \times 50} = 14.21\,\text{W}. \tag{4.352}$$

The dc supply power is

$$P_I = V_I I_I = 12 \times 1.184 = 14.208\,\text{W}. \tag{4.353}$$

The rms value of the switch current is

$$I_{Srms} = \frac{I_I}{\sqrt{2}} = \frac{1.184}{\sqrt{2}} = 0.8372\,\text{A}. \tag{4.354}$$

Assuming $r_{DS} = 0.1\,\Omega$, the conduction power loss in each MOSFET is

$$P_{rDS} = r_{DS} I_{Srms}^2 = 0.1 \times 0.8372^2 = 0.07\,\text{W}. \tag{4.355}$$

The drain efficiency is

$$\eta_D = \frac{P_O}{P_O + 2P_{rDS}} = \frac{1}{1 + \frac{\pi^2}{4n^2}\frac{r_{DS}}{R}} = \frac{1}{1 + \frac{\pi^2}{4 \times 0.5^2}\frac{0.1}{12.5}} = 92.79\,\%. \tag{4.356}$$

Assuming the resistance $r_{RFC} = 0.11\,\Omega$, we have

$$P_{RFC} = r_{RFC} I^2 = 0.11 \times 1.184^2 = 0.154\,\Omega. \tag{4.357}$$

The loaded-quality factor is

$$Q_L = \frac{f_c}{BW} = \frac{1800}{180} = 10. \tag{4.358}$$

The resonant inductance is

$$L = \frac{R_L}{\omega_c Q_L} = \frac{50}{2\pi \times 1.8 \times 10^9 \times 10} = 0.442\,\text{nH}. \tag{4.359}$$

The resonant capacitance is

$$C = \frac{Q_L}{\omega_c R_L} = \frac{10}{2\pi \times 1.8 \times 10^9 \times 50} = 17.68\,\text{pF}. \tag{4.360}$$

Assuming $r_L = 0.009\,\Omega$ and $r_C = 0.01\,\Omega$, we get

$$P_{rL} = \frac{r_L I_m^2 Q_L^2}{2} = \frac{0.009 \times 3.016^2 \times 10^2}{2} = 4.09\,\text{W} \tag{4.361}$$

$$P_{rC} = \frac{r_C I_m^2 Q_L^2}{2} = \frac{0.01 \times 3.016^2 \times 10^2}{2} = 4.548\,\text{W}. \tag{4.362}$$

Hence, the total power loss is

$$P_{Loss} = 2P_{rDS} + P_{RFC} + P_{rL} + P_{rC} = 2 \times 0.07 + 0.154 + 4.09 + 4.548 = 8.932\,\text{W}. \tag{4.363}$$

The overall efficiency is

$$\eta = \frac{P_O}{P_O + P_{Loss}} = \frac{14.21}{14.21 + 8.932} = 61.4\,\%. \tag{4.364}$$

4.16 Bridge Class D Current-switching RF Power Amplifier

A circuit of a bridge Class D current-switching (current-source) RF power amplifier is shown in Figure 4.38(a). When one transistor is ON, the other transistor is OFF. Current and voltage waveforms are shown in Figure 4.39. Figure 4.38(b) shows an equivalent circuit when transistor Q_1 is OFF and transistor Q_2 is ON. For the switching frequency equal to the resonant frequency of the parallel-resonant circuit, the transistors turns on at zero voltage, reducing switching losses. The main disadvantage of the circuit is that the currents of both chokes flow through one of the transistors at all times, causing high conduction loss.

The current and voltage waveforms are given by

$$i = I_I \quad \text{for} \quad 0 < \omega t \leq \pi \tag{4.365}$$

$$i_{D1} = 0 \quad \text{for} \quad 0 < \omega t \leq \pi \tag{4.366}$$

$$i_{D2} = 2I_I \quad \text{for} \quad 0 < \omega t \leq \pi \tag{4.367}$$

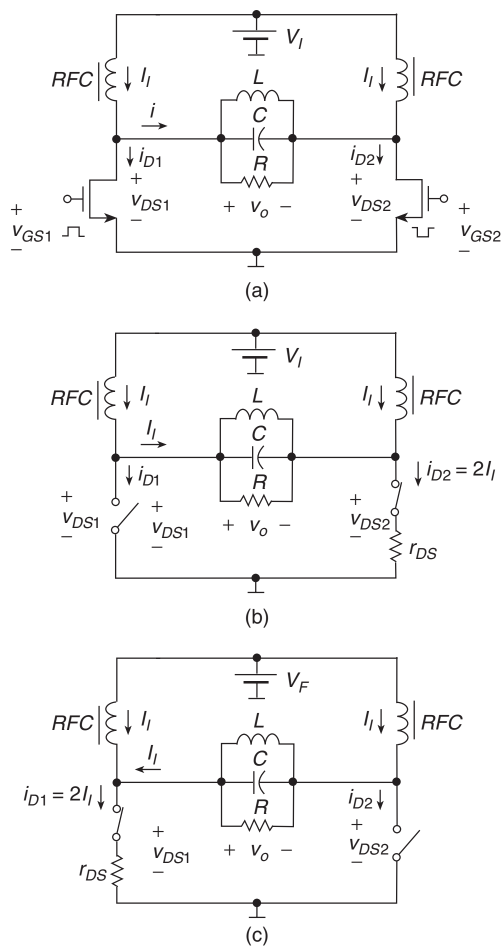

Figure 4.38 Bridge Class D current-switching (current-source) RF power amplifier. (a) Circuit. (b) Equivalent circuit when transistor Q_1 is OFF and transistor Q_2 is ON. (c) Equivalent circuit when transistor Q_1 is ON and transistor Q_2 is OFF.

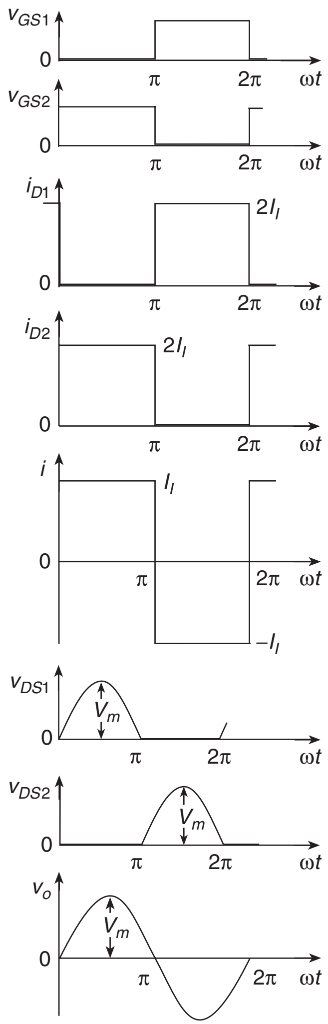

Figure 4.39 Waveforms in bridge Class D current-switching (current-source) RF power amplifier.

and

$$v_{DS1} = V_m \sin \omega t \quad \text{for} \quad 0 < \omega t \leq \pi. \tag{4.368}$$

Figure 4.38(c) shows an equivalent circuit when transistor Q_1 is ON and transistor Q_2 is OFF. The waveforms are as follows:

$$i = -I_I \quad \text{for} \quad \pi < \omega t \leq 2\pi \tag{4.369}$$

$$i_{D1} = 2I_I \quad \text{for} \quad \pi < \omega t \leq 2\pi \tag{4.370}$$

$$i_{D2} = 0 \quad \text{for} \quad \pi < \omega t \leq 2\pi. \tag{4.371}$$

and

$$v_{DS2} = -V_m \sin \omega t \quad \text{for} \quad \pi < \omega t \leq 2\pi. \tag{4.372}$$

The output voltage is

$$v_o = V_m \sin \omega t \tag{4.373}$$

where

$$V_m = RI_m = \pi V_I. \tag{4.374}$$

The output current is

$$i_o = I_m \sin \omega t \tag{4.375}$$

where the amplitude of the output current is

$$I_m = \frac{4}{\pi} I_I \tag{4.376}$$

resulting in

$$I_I = \frac{\pi}{4} I_m = \frac{\pi}{4} \frac{V_m}{R} = \frac{\pi^2}{4} \frac{V_I}{R}. \tag{4.377}$$

The dc resistance is

$$R_{DC} = \frac{V_I}{2I_I} = \frac{2}{\pi^2} R. \tag{4.378}$$

The dc supply power is

$$P_I = 2I_I V_I = \frac{\pi^2}{2} \frac{V_I^2}{R}. \tag{4.379}$$

The output power is

$$P_O = \frac{V_m^2}{2R} = \frac{RI_m^2}{2} = \frac{\pi^2}{2} \frac{V_I^2}{R}. \tag{4.380}$$

The rms value of the switch current is

$$I_{Srms} = \sqrt{\frac{1}{2\pi} \int_0^{2\pi} i_{D2}^2 \, d(\omega t)} = \sqrt{\frac{1}{2\pi} \int_0^{\pi} (2I_I)^2 \, d(\omega t)} = \sqrt{2} I_I. \tag{4.381}$$

Hence, the conduction loss in each transistor is

$$P_{rDS} = r_{DS} I_{Srms}^2 = 2r_{DS} I_I^2 = \frac{\pi^2}{8} r_{DS} I_m^2 = \frac{\pi^2}{4} \frac{r_{DS}}{R} P_O. \tag{4.382}$$

The drain efficiency is

$$\eta_D = \frac{P_O}{P_O + 2P_{rDS}} = \frac{1}{1 + \frac{\pi^2}{2} \frac{r_{DS}}{R}}. \tag{4.383}$$

The power loss in each RFC choke resistance r_{RFC} is

$$P_{RFC} = r_{RFC} I_I^2 = r_{RFC} \frac{\pi^2}{16} \frac{V_m^2}{R} = r_{RFC} \frac{\pi^2}{16} \frac{V_m^2}{R} = \frac{\pi^2}{8} \frac{r_{RFC}}{R} P_O. \tag{4.384}$$

The power loss in the ESR of the resonant inductor r_L is

$$P_{rL} = \frac{r_L I_m^2 Q_L^2}{2} = \frac{r_L Q_L^2}{R_L} P_O \tag{4.385}$$

and the power loss in the ESR of the resonant capacitor r_C is

$$P_{rC} = \frac{r_C I_m^2 Q_L^2}{2} = \frac{r_C Q_L^2}{R_L} P_O. \tag{4.386}$$

The total power loss is

$$P_{Loss} = 2P_{rDS} + 2P_{RFC} + P_{rL} + P_{rC} = P_O \left[\frac{\pi^2 r_{DS}}{2R} + \frac{\pi^2 r_{RFC}}{4R} + \frac{Q_L^2 (r_L + r_C)}{R} \right]. \tag{4.387}$$

Hence, the overall efficiency is

$$\eta = \frac{P_O}{P_I} = \frac{P_O}{P_O + P_{Loss}} = \frac{1}{1 + \dfrac{\pi^2}{2}\dfrac{r_{DS}}{R} + \dfrac{\pi^2}{4}\dfrac{r_{RFC}}{R} + \dfrac{Q_L^2(r_L + r_C)}{R}}. \tag{4.388}$$

Example 4.7

Design a bridge Class D current-switching RF power amplifier to meet the following specifications: $V_I = 5\,\text{V}$, $P_O = 6\,\text{W}$, $BW = 240\,\text{MHz}$, and $f_c = 2.4\,\text{GHz}$.

Solution. Assuming the efficiency of the resonant circuit is $\eta_r = 0.92$, the drain power is

$$P_{DS} = \frac{P_O}{\eta_r} = \frac{6}{0.92} = 6.522\,\text{W}. \tag{4.389}$$

The load resistance is

$$R = \frac{\pi^2}{2}\frac{V_I^2}{P_{DS}} = \frac{\pi^2}{2}\frac{5^2}{6.522} = 18.916\,\Omega. \tag{4.390}$$

The amplitude of the output voltage and the peak value of the drain-to-source voltage is

$$V_m = \pi V_I = \pi \times 5 = 15.708\,\text{V}. \tag{4.391}$$

The dc resistance seen by the dc power supply is

$$R_{DC} = \frac{V_I}{2I_I} = \frac{2}{\pi^2}R = \frac{2}{\pi^2} \times 18.916 = 3.833\,\Omega. \tag{4.392}$$

The amplitude of the output current is

$$I_m = \frac{V_m}{R} = \frac{15.708}{18.916} = 0.83\,\text{A}. \tag{4.393}$$

The dc supply current is

$$I_I = \frac{\pi^2}{4}\frac{V_I}{R} = \frac{\pi^2}{4}\frac{5}{18.916} = 0.652\,\text{A}. \tag{4.394}$$

The dc supply power is

$$P_I = 2I_I V_I = 2 \times 0.652 \times 5 = 6.522\,\text{W}. \tag{4.395}$$

Assuming $r_{DS} = 0.1\,\Omega$, the conduction power loss in each MOSFET is

$$P_{rDS} = r_{DS} I_I^2 = 0.1 \times 0.652^2 = 0.0425\,\text{W}. \tag{4.396}$$

The drain efficiency is

$$\eta_D = \frac{P_O}{P_O + 2P_{rDS}} = \frac{1}{1 + \dfrac{\pi^2}{2}\dfrac{r_{DS}}{R}} = \frac{1}{1 + \dfrac{\pi^2}{2}\dfrac{0.1}{18.916}}$$
$$= 97.46\,\%. \tag{4.397}$$

Assuming the resistance $r_{RFC} = 0.12\,\Omega$,

$$P_{rRRFC} = r_{RFC} I_I^2 = 0.12 \times 0.652^2 = 0.051\,\Omega. \tag{4.398}$$

The loaded-quality factor is

$$Q_L = \frac{f_c}{BW} = \frac{2400}{240} = 10. \tag{4.399}$$

The resonant inductance is

$$L = \frac{R_L}{\omega_c Q_L} = \frac{18.916}{2\pi \times 2.4 \times 10^9 \times 10} = 0.1254\,\text{nH}. \tag{4.400}$$

The resonant capacitance is

$$C = \frac{Q_L}{\omega_c R_L} = \frac{10}{2\pi \times 2.4 \times 10^9 \times 18.916} = 35.06\,\text{pF}. \tag{4.401}$$

Assuming $r_L = 0.08\,\Omega$ and $r_C = 0.05\,\Omega$, we get

$$P_{rL} = \frac{r_L I_m^2 Q_L^2}{2} = \frac{0.08 \times 0.83^2 \times 10^2}{2} = 2.7556\,\text{W} \tag{4.402}$$

$$P_{rC} = \frac{r_C I_m^2 Q_L^2}{2} = \frac{0.05 \times 0.83^2 \times 10^2}{2} = 1.722\,\text{W}. \tag{4.403}$$

Hence, the total power loss is

$$\begin{aligned} P_{Loss} &= 2P_{rDS} + 2P_{RFC} + P_{rL} + P_{rC} \\ &= 2 \times 0.0425 + 2 \times 0.051 + 2.7556 + 1.722 = 4.6646\,\text{W}. \end{aligned} \tag{4.404}$$

The overall efficiency is

$$\eta = \frac{P_O}{P_O + P_{Loss}} = \frac{5}{5 + 4.6646} = 51.74\,\%. \tag{4.405}$$

4.17 Summary

- The maximum voltage across the switches in both Class D half-bridge and full-bridge amplifiers is low and equal to the dc input voltage V_I.
- Operation with a capacitive load (that is, below resonance) is not recommended. The antiparallel diodes turn off at a high di/dt. If the MOSFET's body-drain *pn* junction diode (or any *pn* junction diode) is used as an antiparallel diode, it generates high reverse-recovery current spikes. These spikes occur in the switch current waveforms at both the switch turn-on and turn-off, and may destroy the transistor. The reverse-recovery spikes may initiate the turn-on of the parasitic BJT in the MOSFET structure and may cause the MOSFET to fail due to the second breakdown of parasitic BJT. The current spikes can be reduced by adding a Schottky antiparallel diode (if V_I is below 100 V), or a series diode and an antiparallel diode.
- For operation below resonance, the transistors are turned on at a high voltage equal to V_I and the transistor output capacitance is short-circuited by a low transistor on-resistance, dissipating the energy stored in that capacitance. Therefore, the turn-on switching loss is high, Miller's effect is significant, the transistor input capacitance is high, the gate drive power is high, and the turn-on transition speed is reduced.
- Operation with an inductive load (that is, above resonance) is preferred. The antiparallel diodes turn off at a low di/dt. Therefore, the MOSFET's body-drain *pn* junction diodes can be used as antiparallel diodes because these diodes do not generate reverse-recovery current spikes and are sufficiently fast.

- For operation above resonance, the transistors turn on at zero voltage. For this reason, the turn-on switching loss is reduced, Miller's effect is absent, the transistor input capacitance is low, the gate drive power is low, and turn-on speed is high. However, the turn-off is lossy.
- The efficiency is high at light loads because R/r increases when R increases [see Equation (4.99)].
- The amplifier can operate safely with an open circuit at the output.
- There is a risk of catastrophic failure if the output is short-circuited when the operating frequency f approaches the resonant frequency f_o.
- The input voltage of the resonant circuit in the Class D full-bridge amplifier is a square wave whose low level is $-V_I$ and whose high level is V_I. The peak-to-peak voltage across the resonant circuit in the full-bridge amplifier is twice as high as that of the half-bridge amplifier. Therefore, the output power of the full-bridge amplifier is four times higher than that in the half-bridge amplifier at the same load resistance R, at the same dc supply voltage V_I, and at the same ratio f/f_o.
- The dc voltage source V_I and the switches form an ideal square-wave voltage source, and therefore many loads can be connected between the two switches and ground, and can be operated without mutual interactions.
- Not only MOSFETs, but other power switches can be used, such as MESFETs, BJTs, thyristors, MOS-controlled thyristors (MCTs), gate turn-off thyristors (GTOs), and isolated-gate bipolar transistors (IGBTs).

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4.19 Review Questions

4.1 Draw the inductive reactance X_L, capacitive reactance X_C, and total reactance $X_L - X_C$ versus frequency for the series-resonant circuit. What occurs at the resonance frequency?

4.2 What is the voltage across the switches in Class D half-bridge and full-bridge amplifier?

4.3 What is the frequency range in which a series-resonant circuit represents a capacitive load to the switching part of the Class D series-resonant circuit amplifiers?

4.4 What are the disadvantages of operation of the Class D series-resonant circuit amplifier with a capacitive load?

4.5 Is the turn-on switching loss of the power MOSFETs zero below resonance?

4.6 Is the turn-off switching loss of the power MOSFETs zero below resonance?

4.7 Is Miller's effect present at turn-on or turn-off below resonance?

4.8 What is the influence of zero-voltage switching on Miller's effect?

4.9 What is the frequency range in which a series-resonant circuit represents an inductive load to the switching part of the amplifier?

4.10 What are the merits of operation of the Class D amplifier with an inductive load?

4.11 Is the turn-on switching loss of the power MOSFETs zero above resonance?

4.12 Is the turn-off switching loss of the power MOSFETs zero above resonance?

4.13 What is the voltage stress of the resonant capacitor and inductor in half-bridge and full-bridge amplifiers?

4.14 What are the worst conditions for the voltage stresses of resonant components?

4.15 What happens when the output of the amplifier is short-circuited?

4.16 Is the part-load efficiency of the Class D amplifier with a series-resonant circuit high?

4.17 What kind of switching takes place in Class D current-switching power amplifiers when the operating frequency is equal to the resonant frequency?

4.18 What is the role of the transformer in impedance matching in push-pull power amplifiers?

4.20 Problems

4.1 Design a Class D RF power amplifier to meet the following specifications: $V_I = 3.3\,\text{V}$, $P_O = 1\,\text{W}$, $f = f_o = 1\,\text{GHz}$, $r_{DS} = 0.1\,\Omega$, $Q_L = 7$, $Q_{Lo} = 200$, and $Q_{Co} = 1000$.

4.2 A series-resonant circuit consists of an inductor $L = 84\,\mu\text{H}$ and a capacitor $C = 300\,\text{pF}$. The ESRs of these components at the resonant frequency are $r_L = 1.4\,\Omega$ and $r_C = 50\,\text{m}\Omega$, respectively. The load resistance is $R = 200\,\Omega$. The resonant circuit is driven by a sinusoidal voltage source whose amplitude is $V_m = 100\,\text{V}$. Find the resonant frequency f_o, characteristic impedance Z_o, loaded quality factor Q_L, unloaded quality factor Q_o, quality factor of the inductor Q_{Lo}, and quality factor of the capacitor Q_{Co}.

4.3 For the resonant circuit given in Problem 4.2, find the reactive power of the inductor Q and the total real power P_O.

4.4 For the resonant circuit given in Problem 4.2, find the voltage and current stresses for the resonant inductor and the resonant capacitor. Calculate also the reactive power of the resonant components.

4.5 Find the efficiency for the resonant circuit given in Problem 4.2. Is the efficiency dependent on the operating frequency?

4.6 Write general expressions for the instantaneous energy stored in the resonant inductor $w_L(t)$ and in the resonant capacitor $w_C(t)$, as well as the total instantaneous energy stored in the resonant circuit $w_t(t)$. Sketch these waveforms for $f = f_o$. Explain briefly how the energy is transferred between the resonant components.

4.7 A Class D half-bridge amplifier is supplied by a dc voltage source of 350–400 V. Find the voltage stresses of the switches. Repeat the same problem for the Class D full-bridge amplifier.

4.8 A series-resonant circuit, which consists of a resistance $R = 25\,\Omega$, inductance $L = 100\,\mu\text{H}$, and capacitance $C = 4.7\,\text{nF}$, is driven by a sinusoidal voltage source $v = 100 \sin \omega t$ (V). The operating frequency can be changed over a wide range. Calculate exactly the maximum voltage stresses for the resonant components. Compare the results with voltages across the inductance and the capacitance at the resonant frequency.

4.9 Design a Class D half-bridge series-resonant amplifier that delivers power $P_O = 30\,\text{W}$ to the load resistance. The amplifier is supplied from input voltage source $V_I = 180\,\text{V}$. It is required that the operating frequency is $f = 210\,\text{kHz}$. Neglect switching and drive-power losses.

4.10 Design a full-bridge Class D power amplifier with the following specifications: $V_I = 100\,\text{V}$, $P_O = 80\,\text{W}$, $f = f_o = 500\,\text{kHz}$, and $Q_L = 5$.

5

Class E RF Zero-voltage-switching RF Power Amplifier

5.1 Introduction

There are two types of Class E power amplifiers [1–69], also called Class E dc-ac inverters: (1) Class E zero-voltage-switching (ZVS) power amplifiers, which are the subject of this chapter, and (2) Class E zero-current-switching (ZCS) power amplifiers. In Class E amplifiers, the transistor is operated as a switch. Class E ZVS power amplifiers [1–35] are the most efficient amplifiers known so far. The current and voltage waveforms of the switch are displaced with respect to time, yielding a very low power dissipation in the transistor. In particular, the switch turns on at zero voltage if the component values of the resonant circuit are properly chosen. Since the switch current and voltage waveforms do not overlap during the switching time intervals, switching losses are virtually zero, yielding high efficiency.

We shall start by presenting a simple qualitative description of the operation of the Class E ZVS amplifier. Though simple, this description provides considerable insight into the performance of the amplifier as a basic power cell. Further, we shall quickly move to the quantitative description of the amplifier. Finally, we will present matching resonant circuits and give a design procedure for the amplifier. By the end of this chapter, the reader will be able to perform rapid first-order analysis as well as design a single-stage Class E ZVS amplifier.

5.2 Circuit Description

The basic circuit of the Class E ZVS power amplifier is shown in Figure 5.1(a). It consists of power MOSFET operating as a switch, L-C-R series-resonant circuit, shunt capacitor C_1,

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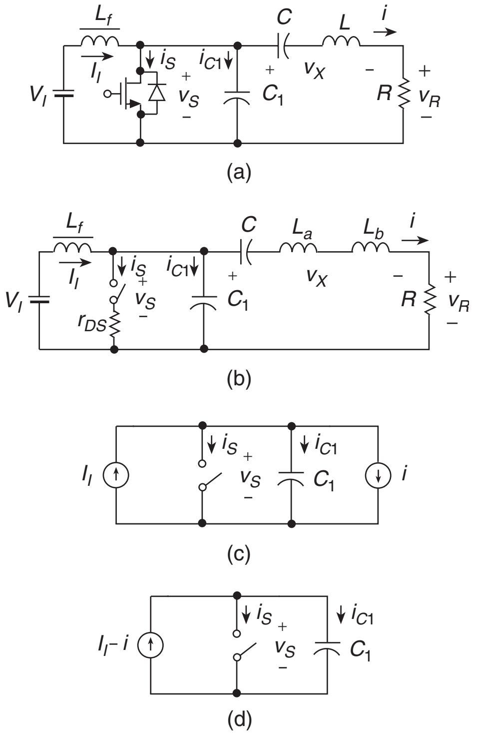

Figure 5.1 Class E zero-voltage-switching RF power amplifier. (a) Circuit. (b) Equivalent circuit for operation above resonance. (c) Equivalent circuit with the dc voltage source V_I and the RF choke L_f replaced by a dc current source I_I and the series-resonant circuit replaced by an ac current source i. (d) Equivalent circuit with the two current sources combined into one current source $I_I - i$.

and choke inductor L_f. The switch turns on and off at the operating frequency $f = \omega/(2\pi)$ determined by a driver. The transistor output capacitance, the choke parasitic capacitance, and stray capacitances are included in the shunt capacitance C_1. For high operating frequencies, all of the capacitance C_1 can be supplied by the overall shunt parasitic capacitance. The resistor R is an ac load. The choke inductance L_f is assumed to be high enough so that the ac current ripple on the dc supply current I_I can be neglected. A small inductance with a large current ripple is also possible [41].

When the switch is ON, the resonant circuit consists of L, C, and R because the capacitance C_1 is short-circuited by the switch. However, when the switch is OFF, the resonant circuit consists of C_1, L, C, and R connected in series. Because C_1 and C are connected in series, the equivalent capacitance

$$C_{eq} = \frac{CC_1}{C + C_1} \tag{5.1}$$

is lower than C and C_1. The load network is characterized by two resonant frequencies and two loaded quality factors. When the switch is ON,

$$f_{o1} = \frac{1}{2\pi\sqrt{LC}} \tag{5.2}$$

and

$$Q_{L1} = \frac{\omega_{o1} L}{R} = \frac{1}{\omega_{o1} CR}. \tag{5.3}$$

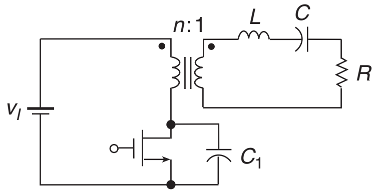

Figure 5.2 Class E zero-voltage-switching RF power amplifier with a transformer.

When the switch is OFF,

$$f_{o2} = \frac{1}{2\pi\sqrt{\frac{LCC_1}{C + C_1}}} \tag{5.4}$$

and

$$Q_{L2} = \frac{\omega_{o2} L}{R} = \frac{1}{\frac{\omega_{o2} RCC_1}{C + C_1}}. \tag{5.5}$$

The ratio of the two resonant frequencies is

$$\frac{f_{o1}}{f_{o2}} = \frac{Q_{L1}}{Q_{L2}} = \sqrt{\frac{C_1}{C_1 + C}}. \tag{5.6}$$

An equivalent circuit of the amplifier for operation above resonance is shown in Figure 5.1(b). In Figure 5.1(c), the dc source V_I and RF choke L_f are replaced by a dc current source I_I and the series-resonant circuit is replaced by an ac current source i. Figure 5.1(d) shows an equivalent circuit of the Class E amplifier with the two current sources combined into one current source $I_I - i$.

If the operating frequency f is greater than the resonant frequency f_{o1}, the L-C-R series-resonant circuit represents an inductive load at the operating frequency f. Therefore, the inductance L can be divided into two inductances, L_a and L_b, connected in series such that $L = L_a + L_b$ and L_a resonates with C at the operating frequency f, that is,

$$\omega = \frac{1}{\sqrt{L_a C}}. \tag{5.7}$$

The loaded quality factor is defined at the operating frequency as

$$Q_L = \frac{\omega L}{R} = \frac{\omega(L_a + L_b)}{R} = \frac{1}{\omega CR} + \frac{\omega L_b}{R}. \tag{5.8}$$

A transformer version of the Class E amplifier is shown in Figure 5.2. The transformer leakage inductance can be absorbed into the inductance L.

5.3 Circuit Operation

Circuits with *hard-switching* operation of semiconductor components, such as PWM power converters and digital gates, suffer from switching losses. The voltage waveform in these circuits decreases abruptly from a high value, often equal to the dc supply voltage V_I, to nearly zero, when a switching device turns on. The energy stored in the transistor output capacitance and load capacitance C just before the turn-on transitions (assuming that these capacitances are linear) is given by

$$W = \frac{1}{2}CV_I^2 \tag{5.9}$$

where V_I is the dc supply voltage. When the transistor is turned on, the current is circulating through the transistor on-resistance r_{DS}, and all the stored energy is lost in the on-resistance r_{DS} as heat. This energy is independent of the transistor on-resistance r_{DS}. The switching power loss of the transistor is given by

$$P_{sw} = \frac{1}{2}fCV_I^2. \tag{5.10}$$

The switching losses can be avoided if the voltage across the transistor v_S is zero, when the transistor turns on

$$v_S(t_{turn-on}) = 0. \tag{5.11}$$

Then the charge stored in the transistor output capacitance is zero, and the energy stored in this capacitance is zero. The main idea of the Class E RF power amplifier is that the transistor turns on as a switch at zero voltage, resulting in zero switching loss and high efficiency. The Class E power amplifier in its basic form contains a single switch. The switch turns on at zero voltage (ZVS), and the switch may also turn on at zero derivative (ZDS). In general, this type of operation is called *soft-switching*.

Figure 5.3 shows the current and voltage waveforms in the Class E ZVS amplifier for three cases: (1) $dv_S(\omega t)/d(\omega t) = 0$; (2) $dv_S(\omega t)/d(\omega t) < 0$; and (3) $dv_S(\omega t)/d(\omega t) > 0$ at $\omega t = 2\pi$ when the switch turns on. In all three cases, the voltage v_S across the switch and the shunt capacitance C_1 is zero when the switch turns on. Therefore, the energy stored in the shunt capacitance C_1 is zero when the switch turns on, yielding zero turn-on switching loss. Thus, the *ZVS condition* is expressed as

$$v_S(2\pi) = 0. \tag{5.12}$$

The choke inductor L_f forces a dc current I_I. To achieve zero-voltage switching turn-on of the switch, the operating frequency $f = \omega/(2\pi)$ should be greater than the resonant frequency $f_{o1} = 1/(2\pi\sqrt{LC})$, that is, $f > f_{o1}$. However, the operating frequency is usually lower than $f_{o2} = 1/(2\pi\sqrt{LC_{eq}})$, that is, $f < f_{o2}$. The shape of the waveform of current i depends on the loaded quality factor. If Q_L is high (i.e., $Q_L \geq 2.5$), the shape of the waveform of current i is approximately sinusoidal. If Q_L is low, the shape of the waveform of the current i becomes close to an exponential function [22]. The combination of the choke inductor L_f and the L-C-R series-resonant circuit acts as a current source whose current is I_I-i. When the switch is ON, the current I_I-i flows through the switch. When the switch is OFF, the current I_I-i flows through the capacitor C_1, producing the voltage across the shunt capacitor C_1 and the switch. Therefore, the shunt capacitor C_1 shapes the voltage across the switch.

5.4 ZVS and ZDS Operation of Class E Amplifier

When the transistor turns on at $\omega t = 2\pi$ in the Class E power amplifier, the *zero-voltage switching* (ZVS) condition and *zero-derivative switching* (ZDS) condition are satisfied

$$v_S(2\pi) = 0 \tag{5.13}$$

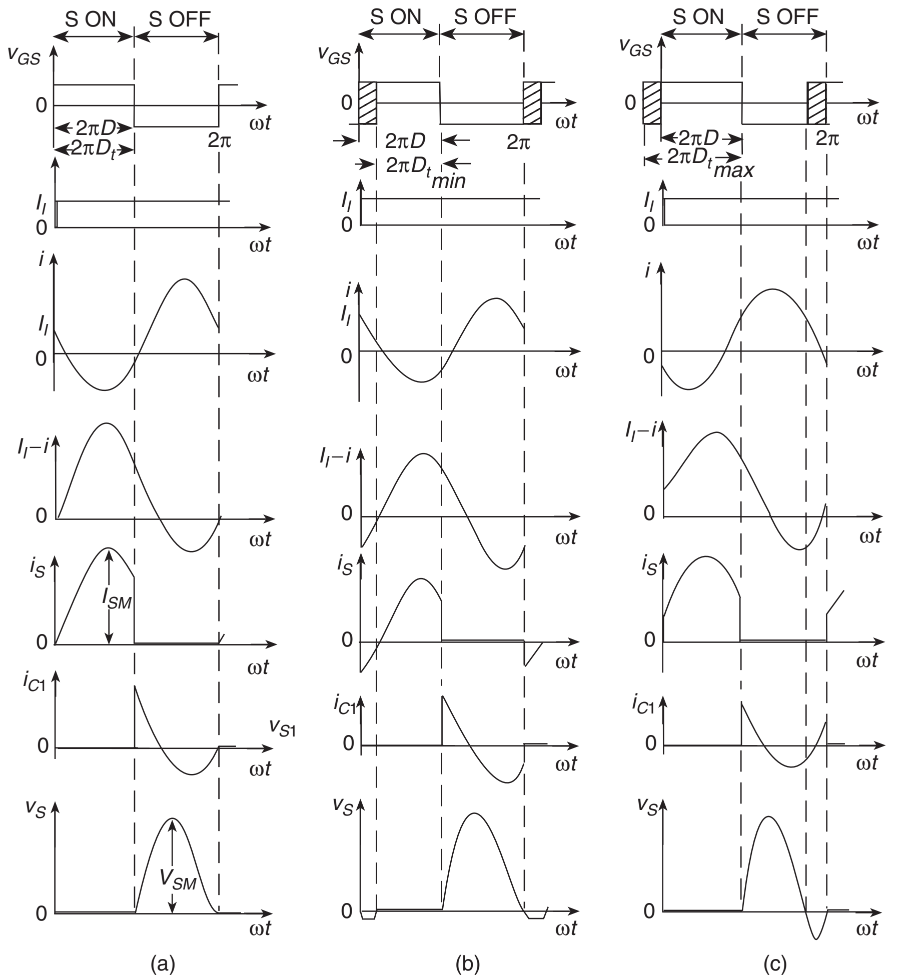

Figure 5.3 Waveforms in Class E zero-voltage-switching amplifier. (a) For optimum operation. (b) For suboptimum operation with $dv_S(\omega t)/d(\omega t) < 0$ at $\omega t = 2\pi$. (c) For suboptimum operation with $dv_S(\omega t)/d(\omega t) > 0$ at $\omega t = 2\pi$.

and

$$\left.\frac{dv_S(\omega t)}{d(\omega t)}\right|_{\omega t=2\pi} = 0. \tag{5.14}$$

Current and voltage waveforms for ZVS and ZDS operation are shown in Figure 5.3(a). Zero-voltage switching implies that the energy stored in the shunt capacitance C_1 is zero when the transistor turns on, yielding zero turn-on switching loss. Because the derivative of v_S is zero at the time when the switch turns on, the switch current i_S increases gradually from zero after the switch is closed. The operation for which both the ZVS and ZDS conditions are satisfied simultaneously is called the *nominal operation* or *optimum operation*. Both the switch voltage and the switch current waveforms are positive for the optimum operation. Therefore, there is no need to add any diode to the switch.

Relationships among C_1, L_b, R, f, and D must be satisfied to achieve optimum operation [22]. Therefore, optimum operation is achieved only for the optimum load resistance

$R = R_{opt}$. In addition, the operating frequency f for optimum operation must be located between two resonant frequencies

$$f_{o1} < f < f_{o2}. \tag{5.15}$$

If $R > R_{opt}$, the amplitude I_m of the current i through the L-C-R series-resonant circuit is lower than that required for the optimum operation, the voltage drop across the shunt capacitor C_1 decreases. Also, the switch voltage v_S is greater than zero at turn-on. On the other hand, if $R < R_{opt}$, the amplitude I_m is higher than that required for optimum operation, the voltage drop across the shunt capacitor C_1 increases, and the switch voltage v_S is less than zero at turn-on. In both cases, assuming a linear capacitance C_1, the energy stored in C_1 just before turn-on of the switch is $W(2\pi-) = \frac{1}{2}C_1 v_S^2(2\pi-)$. This energy is dissipated in the transistor as heat after the switch is turned on, resulting in a turn-on switching loss. To obtain the ZVS operation at a wider load range, an antiparallel or a series diode can be added to the transistor. This improvement ensures that the switch automatically turns on at zero voltage for $R \leq R_{opt}$.

5.5 Suboptimum Operation

In many applications, the load resistance varies over a certain range. The turn-on of the switch at zero voltage can be achieved for suboptimum operation for $0 \leq R \leq R_{opt}$. For suboptimum operation, $v_S(2\pi) = 0$ and either $\mathrm{d}v_S(\omega t)/\mathrm{d}(\omega t) < 0$ or $\mathrm{d}v_S(\omega t)/\mathrm{d}(\omega t) > 0$. Figure 5.3(b) shows the current and voltage waveforms for the case when $v_S(2\pi) = 0$ and $\mathrm{d}v_S(\omega t)/\mathrm{d}(\omega t) < 0$ at $\omega t = 2\pi$. Power MOSFETs are bidirectional switches because their current can flow in both directions, but their voltage can only be greater than -0.7 V. When the switch voltage reaches -0.7 V, the antiparallel diode turns on and therefore the switch automatically turns on. The diode accelerates the time at which the switch turns on. This time is no longer determined by the gate-to-source voltage. Since the switch turns on at zero voltage, the turn-on switching loss is zero, yielding high efficiency. Such an operation can be achieved for $0 \leq R \leq R_{opt}$. In addition, if $R < R_{opt}$, the operating frequency f and the transistor ON switch duty cycle D_t can vary in bounded ranges. When the switch current is negative, the antiparallel diode is ON, but the transistor can either be ON or OFF. Therefore, the transistor ON switch duty cycle D_t is less than or equal to the ON switch duty cycle of the entire switch D. When the switch current is positive, the diode is OFF and the transistor must be ON. Hence, the range of D_t is $D_{t\ min} \leq D_t \leq D$, as indicated in Figure 5.3(b) by the shaded area.

Figure 5.3(c) depicts current and voltage waveforms for the case when $v_S(2\pi) = 0$ and $\mathrm{d}v_S(\omega t)/\mathrm{d}(\omega t) > 0$ at $\omega t = 2\pi$. Notice that the switch current i_S is always positive, but the switch voltage v_S has positive and negative values. Therefore, a unidirectional switch for current and a bidirectional switch for voltage is needed. Such a switch can be obtained by adding a diode in series with a MOSFET. When the switch voltage v_S is negative the diode is OFF and supports the switch voltage, regardless of the state of the MOSFET. The MOSFET is turned on during the time interval when the switch voltage is negative. Once the switch voltage reaches 0.7 V with a positive derivative, the diode turns on, turning the entire switch on. The series diode delays the time at which the switch turns on. The range of D_t is $D \leq D_t \leq D_{tmax}$, as shown in Figure 5.3(c) by the shaded area. The disadvantages of the switch with a series diode are higher on-voltage and higher conduction loss. Another disadvantage is associated with the transistor output capacitance. When the switch voltage increases, the transistor output capacitance is charged through the series diode to the peak value of the switch voltage and then remains at this voltage until the transistor turns on (because the diode is OFF). At this time, the transistor output capacitance is discharged through the MOSFET on-resistance, dissipating the stored energy.

5.6 Analysis

5.6.1 Assumptions

The analysis of the Class E ZVS amplifier of Figure 5.1(a) is carried out under the following assumptions:

1. The transistor and the antiparallel diode form an ideal switch whose on-resistance is zero, off-resistance is infinity, and switching times are zero.
2. The choke inductance is high enough so that the ac component is much lower than the dc component of the input current.
3. The loaded quality factor Q_L of the LCR series-resonant circuit is high enough so that the current i through the resonant circuit is sinusoidal.
4. The duty ratio D is 0.5.

5.6.2 Current and Voltage Waveforms

The current through the series-resonant circuit is sinusoidal and given by

$$i = I_m \sin(\omega t + \phi) \tag{5.16}$$

where I_m is the amplitude and ϕ is the initial phase of the current i. According to Figure 5.1(a),

$$i_S + i_{C1} = I_I - i = I_I - I_m \sin(\omega t + \phi). \tag{5.17}$$

For the time interval $0 < \omega t \leq \pi$, the switch is ON and therefore $i_{C1} = 0$. Consequently, the current through the MOSFET is given by

$$i_S = \begin{cases} I_I - I_m \sin(\omega t + \phi), & \text{for} \quad 0 < \omega t \leq \pi, \\ 0, & \text{for} \quad \pi < \omega t \leq 2\pi. \end{cases} \tag{5.18}$$

For the time interval $\pi < \omega t \leq 2\pi$, the switch is OFF which implies that $i_S = 0$. Hence, the current through the shunt capacitor C_1 is given by

$$i_{C1} = \begin{cases} 0, & \text{for} \quad 0 < \omega t \leq \pi, \\ I_I - I_m \sin(\omega t + \phi), & \text{for} \quad \pi < \omega t \leq 2\pi. \end{cases} \tag{5.19}$$

The voltage across the shunt capacitor and the switch is

$$v_S = v_{C1} = \frac{1}{\omega C_1}\int_{\pi}^{\omega t} i_{C1}\, d(\omega t) = \frac{1}{\omega C_1}\int_{\pi}^{\omega t} [I_I - I_m \sin(\omega t + \phi)]\, d(\omega t)$$

$$= \begin{cases} 0, & \text{for} \quad 0 < \omega t \leq \pi, \\ \dfrac{1}{\omega C_1}\{I_I(\omega t - \pi) + I_m[\cos(\omega t + \phi) + \cos\phi]\}, & \text{for} \quad \pi < \omega t \leq 2\pi. \end{cases} \tag{5.20}$$

Substitution of the ZVS condition $v_S(2\pi) = 0$ into (5.20) produces a relationship among I_I, I_m, and ϕ:

$$I_m = -I_I \frac{\pi}{2\cos\phi}. \tag{5.21}$$

Substitution of (5.21) into (5.18) yields the switch current

$$\frac{i_S}{I_I} = \begin{cases} 1 + \dfrac{\pi}{2\cos\phi}\sin(\omega t + \phi), & \text{for} \quad 0 < \omega t \le \pi\ , \\ 0, & \text{for} \quad \pi < \omega t \le 2\pi. \end{cases} \tag{5.22}$$

Likewise, substituting (5.21) into (5.19), one obtains the current through the shunt capacitor C_1

$$\frac{i_{C1}}{I_I} = \begin{cases} 0, & \text{for} \quad 0 < \omega t \le \pi, \\ 1 + \dfrac{\pi}{2\cos\phi}\sin(\omega t + \phi), & \text{for} \quad \pi < \omega t \le 2\pi. \end{cases} \tag{5.23}$$

From (5.21), equation (5.20) becomes

$$v_S = \begin{cases} 0, & \text{for } 0 < \omega t \le \pi, \\ \dfrac{I_I}{\omega C_1}\left[\omega t - \dfrac{3\pi}{2} - \dfrac{\pi}{2\cos\phi}[\cos(\omega t + \phi)]\right], & \text{for } \pi < \omega t \le 2\pi. \end{cases} \tag{5.24}$$

Using the condition $dv_S/d(\omega t) = 0$ at $\omega t = 2\pi$, one obtains

$$\tan\phi = -\frac{2}{\pi} \tag{5.25}$$

from which

$$\phi = \pi - \arctan\left(\frac{2}{\pi}\right) = 2.5747\,\text{rad} = 147.52^\circ. \tag{5.26}$$

Using trigonometric relationships,

$$\sin\phi = \frac{2}{\sqrt{\pi^2 + 4}} \tag{5.27}$$

and

$$\cos\phi = -\frac{\pi}{\sqrt{\pi^2 + 4}}. \tag{5.28}$$

Substitution of (5.28) into (5.21) yields

$$I_m = \frac{\sqrt{\pi^2 + 4}}{2} I_I \approx 1.8621 I_I. \tag{5.29}$$

Hence,

$$\frac{i_S}{I_I} = \begin{cases} 1 - \dfrac{\sqrt{\pi^2 + 4}}{2}\sin(\omega t + \phi), & \text{for} \quad 0 < \omega t \le \pi, \\ 0, & \text{for} \quad \pi < \omega t \le 2\pi, \end{cases} \tag{5.30}$$

$$\frac{i_{C1}}{I_I} = \begin{cases} 0, & \text{for} \quad 0 < \omega t \le \pi, \\ 1 - \dfrac{\sqrt{\pi^2 + 4}}{2}\sin(\omega t + \phi), & \text{for} \quad \pi < \omega t \le 2\pi, \end{cases} \tag{5.31}$$

and

$$v_S = \begin{cases} 0, & \text{for } 0 < \omega t \le \pi, \\ \dfrac{I_I}{\omega C_1}\left(\omega t - \dfrac{3\pi}{2} - \dfrac{\pi}{2}\cos\omega t - \sin\omega t\right), & \text{for } \pi < \omega t \le 2\pi. \end{cases} \tag{5.32}$$

The dc component of the voltage across an ideal choke inductor is zero. From (5.24), the dc input voltage is found as

$$V_I = \frac{1}{2\pi}\int_{\pi}^{2\pi} v_S \, d(\omega t) = \frac{I_I}{2\pi\omega C_1}\int_{\pi}^{2\pi}\left(\omega t - \frac{3\pi}{2} - \frac{\pi}{2}\cos\omega t - \sin\omega t\right) d(\omega t) = \frac{I_I}{\pi\omega C_1}. \tag{5.33}$$

Rearrangement of this equation produces the dc input resistance of the Class E amplifier

$$R_{DC} \equiv \frac{V_I}{I_I} = \frac{1}{\pi\omega C_1}. \tag{5.34}$$

Hence,

$$I_m = \frac{\sqrt{\pi^2+4}}{2}\pi\omega C_1 V_I. \tag{5.35}$$

Using (5.32), one arrives at the normalized switch voltage waveform

$$\frac{v_S}{V_I} = \begin{cases} 0, & \text{for } 0 < \omega t \le 2\pi, \\ \pi\left(\omega t - \dfrac{3\pi}{2} - \dfrac{\pi}{2}\cos\omega t - \sin\omega t\right), & \text{for } \pi < \omega t \le 2\pi. \end{cases} \tag{5.36}$$

5.6.3 Current and Voltage Stresses

Differentiating (5.22),

$$\frac{di_S}{d(\omega t)} = -I_I\frac{\sqrt{\pi^2+4}}{2}\cos(\omega t + \phi) = 0, \tag{5.37}$$

one obtains the value of ωt at which the peak value of the switch current occurs

$$\omega t_{im} = \frac{3\pi}{2} - \phi = 270^\circ - 147.52^\circ = 122.48^\circ. \tag{5.38}$$

Substitution of this into (5.22) yields the switch peak current

$$I_{SM} = I_I\left(\frac{\sqrt{\pi^2+4}}{2} + 1\right) = 2.862 I_I. \tag{5.39}$$

Differentiating the switch voltage waveform v_S in (5.24)

$$\frac{dv_S}{d(\omega t)} = \pi V_I\left[1 + \frac{\pi}{2\cos\phi}\sin(\omega t + \phi)\right] = 0 \tag{5.40}$$

yields the trigonometric equation

$$\sin(\omega t_{vm} + \phi) = -\frac{2\cos\phi}{\pi} = \frac{2}{\sqrt{\pi^2+4}} = \sin\phi = \sin(\pi - \phi) = \sin(2\pi + \pi - \phi) \tag{5.41}$$

the solution of which gives the value of ωt at which the peak value of the switch voltage occurs

$$\omega t_{vm} = 3\pi - 2\phi = 3 \times 180^\circ - 2 \times 147.52^\circ = 244.96^\circ. \tag{5.42}$$

Hence,

$$V_{SM} = 2\pi(\pi - \phi)V_I = 2\pi(\pi - 2.5747)V_I = 3.562 V_I. \tag{5.43}$$

The amplitude of the voltage across the resonant capacitor C is

$$V_{Cm} = |X_C(f)|I_m = \frac{I_m}{\omega C} \tag{5.44}$$

and the amplitude of the voltage across the resonant inductor L is

$$V_{Lm} = X_L(f)I_m = \omega L I_m. \tag{5.45}$$

Neglecting the power losses, the ac output power P_O is equal to the dc input power $P_I = V_I I_I$. Hence, using I_{SM}/I_I and V_{SM}/V_I, one obtains the power-output capability

$$c_p \equiv \frac{P_O}{I_{SM}V_{SM}} = \frac{I_I V_I}{I_{SM}V_{SM}} = \frac{1}{\pi(\pi - \phi)(\sqrt{\pi^2+4}+2)} = \frac{1}{2.862} \times \frac{1}{3.562} = 0.0981. \tag{5.46}$$

5.6.4 Input Impedance of the Series Resonant Circuit

The current through the series-resonant circuit is sinusoidal. Consequently, higher harmonics of the input power are zero. Therefore, it is sufficient to consider the input impedance of the series-resonant circuit at the operating frequency f. Figure 5.4 shows an equivalent circuit of the series-resonant circuit above resonance at the operating frequency f. A phasor diagram of the voltages at the fundamental component is depicted in Figure 5.5. The voltage across the load resistance R is

$$v_R = iR = V_{Rm}\sin(\omega t + \phi) \tag{5.47}$$

where $V_{Rm} = RI_m$ is the amplitude of the output voltage and ϕ is the initial phase. The voltage v_X across the L-C components is nonsinusoidal, equal to $v_X = v_S - v_R$. The fundamental component of the voltage across the C-L_a circuit is zero because the reactance of this circuit at the operating frequency f is zero. The fundamental component of the voltage across the inductance L_b is

$$v_{Lb1} = V_{Lbm1}\cos(\omega t + \phi) \tag{5.48}$$

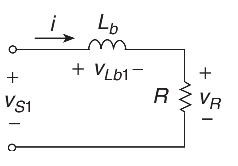

Figure 5.4 Equivalent circuit of the series-resonant circuit above resonance at the operating frequency f.

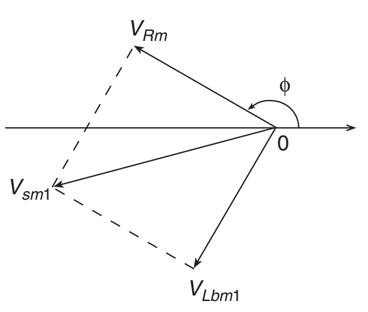

Figure 5.5 Phasor diagram of the voltages at the operating frequency f.

where $V_{Lbm1} = \omega L_b I_m$. The fundamental component of the switch voltage, which is the input voltage of the series-resonant circuit at the operating frequency, is given by

$$v_{s1} = v_R + v_{Lb1} = V_{Rm}\sin(\omega t + \phi) + V_{Lbm1}\cos(\omega t + \phi). \tag{5.49}$$

Using (5.24) and the Fourier trigonometric series formula,

$$\begin{aligned} V_{Rm} &= \frac{1}{\pi}\int_{\pi}^{2\pi} v_S \sin(\omega t + \phi)\, d(\omega t) \\ &= \frac{1}{\pi}\int_{\pi}^{2\pi} V_I \pi \left[\omega t - \frac{3\pi}{2} - \frac{\pi}{2\cos\phi}\cos(\omega t + \phi)\right]\sin(\omega t + \phi)\, d(\omega t) \\ &= \frac{4}{\sqrt{\pi^2+4}} V_I \approx 1.074 V_I. \end{aligned} \tag{5.50}$$

Substituting (5.24) into the Fourier formula and using (5.33), the amplitude of the fundamental component of the voltage across the input reactance of the series-resonant circuit (equal to the reactance of the inductance L_b) is obtained as

$$\begin{aligned} V_{Lbm1} = \omega L_b I_m &= \frac{1}{\pi}\int_{\pi}^{2\pi} v_S \cos(\omega t + \phi)\, d(\omega t) \\ &= \frac{1}{\pi}\int_{\pi}^{2\pi} V_I \pi \left[\omega t - \frac{3\pi}{2} - \frac{\pi}{2\cos\phi}\cos(\omega t + \phi)\right]\cos(\omega t + \phi)\, d(\omega t) \\ &= \frac{\pi(\pi^2-4)}{4\sqrt{\pi^2+4}} V_I \approx 1.2378 V_I. \end{aligned} \tag{5.51}$$

5.6.5 Output Power

From (5.50), one obtains the output power

$$P_O = \frac{V_{Rm}^2}{2R} = \frac{8}{\pi^2+4}\frac{V_I^2}{R} \approx 0.5768\frac{V_I^2}{R}. \tag{5.52}$$

5.6.6 Component Values

Combining (5.21), (5.33), and (5.50), we get

$$R = \frac{V_{Rm}}{I_m} = \frac{\dfrac{4}{\sqrt{\pi^2+4}}V_I}{\dfrac{\sqrt{\pi^2+4}}{2}\pi\omega C_1 V_I} = \frac{8}{\pi(\pi^2+4)\omega C_1} \tag{5.53}$$

resulting in

$$\omega C_1 R = \frac{8}{\pi(\pi^2+4)} \approx 0.1836. \tag{5.54}$$

Similarly, using (5.21), (5.33), and (5.51),

$$X_{Lb} = \omega L_b = \frac{V_{Lbm1}}{I_m} = \frac{\dfrac{\pi(\pi^2-4)}{4\sqrt{\pi^2+4}}V_I}{\dfrac{\sqrt{\pi^2+4}}{2}\pi\omega C_1 V_I} = \frac{\pi^2-4}{2(\pi^2+4)\omega C_1} \tag{5.55}$$

producing

$$\omega^2 L_b C_1 = \frac{\pi^2 - 4}{2(\pi^2 + 4)} \approx 0.2116. \tag{5.56}$$

The ratio of (5.56) to (5.54) is

$$\frac{\omega^2 L_b C_1}{\omega C_1 R} = \frac{\omega L_b}{R} = \frac{\pi(\pi^2 - 4)}{16} \approx 1.1525. \tag{5.57}$$

From (5.7), (5.8), and (5.57), the reactance of the resonant inductor is

$$\omega L = Q_L R \tag{5.58}$$

and the reactance of the resonant capacitor is

$$X_C = \frac{1}{\omega C} = \omega L_a = \omega(L - L_b) = Q_L R - \omega L_b = R\left(Q_L - \frac{\omega L_b}{R}\right)$$

$$= R\left[Q_L - \frac{\pi(\pi^2 - 4)}{16}\right] = R\left(Q_L - 1.1525\right). \tag{5.59}$$

The dc input resistance of the Class E amplifier is

$$R_{DC} = \frac{1}{\pi \omega C_1} = \frac{\pi^2 + 4}{8} R \approx 1.7337R. \tag{5.60}$$

5.7 Maximum Operating Frequency

At high frequencies, the transistor output capacitance C_o becomes higher than the shunt capacitance C_1 required to achieve the ZVS and ZDS operation. The maximum frequency occurs when $C_1 = C_o$. Assume that the transistor output capacitance C_o is linear. For $D = 0.5$, the maximum operating frequency f_{max} at which Class E ZVS and ZDS operation is achievable is determined by the condition

$$2\pi f_{max} C_o R = \frac{8}{\pi(\pi^2 + 4)} \tag{5.61}$$

yielding

$$f_{max} = \frac{4}{\pi^2(\pi^2 + 4) C_o R} \approx \frac{0.02922}{C_o R}. \tag{5.62}$$

Since

$$R = \frac{8}{\pi^2 + 4} \frac{V_I^2}{P_O} \tag{5.63}$$

we obtain the maximum operating frequency of the Class E amplifier that satisfies both the ZVS and ZDS conditions at $D = 0.5$

$$f_{max} = \frac{P_O}{2\pi^2 C_o V_I^2} \approx 0.05066 \frac{P_O}{C_o V_I^2}. \tag{5.64}$$

A higher maximum operating frequency than that given by (5.64) for $D = 0.5$ can be obtained when only the ZVS condition is satisfied, but the ZDS condition is not. The maximum operating frequency of the Class E amplifier under the ZVS and ZDS conditions increases as the duty cycle D decreases [68]. However, at low values of D (usually for $D < 0.2$), the efficiency of the amplifier decreases [55]. For $f > f_{max}$, a Class CE operation can be obtained, which offers a reasonably high efficiency [27].

5.8 Choke Inductance

When the switch is ON, the voltage across the choke inductor is

$$v_{Lf} = V_I \tag{5.65}$$

and the current through the choke inductance L_f is

$$i_{Lf} = \frac{1}{L_f}\int_0^t v_{Lf}\,dt + i_{Lf}(0) = \frac{1}{L_f}\int_0^t V_I\,dt + i_{Lf}(0) = \frac{V_I}{L_f}t + i_{Lf}(0). \tag{5.66}$$

Hence,

$$i_{Lf}\left(\frac{T}{2}\right) = \frac{V_I T}{2L_f} + i_{Lf}(0) = \frac{V_I}{2fL_f} + i_{Lf}(0). \tag{5.67}$$

The peak-to-peak current ripple through the choke inductor is

$$\Delta i_{Lf} = i_{Lf}\left(\frac{T}{2}\right) - i_{Lf}(0) = \frac{V_I}{2fL_f}. \tag{5.68}$$

Thus, the minimum choke inductance required for the maximum peak-to-peak current ripple is

$$\begin{aligned} L_{fmin} &= \frac{V_I}{2f\,\Delta i_{Lfmax}} = \frac{V_I}{2fI_I\dfrac{\Delta i_{Lfmax}}{I_I}} = \frac{R_{DC}}{2f\dfrac{\Delta i_{Lfmax}}{I_I}} = \frac{\pi^2+4}{16}\frac{R}{f\dfrac{\Delta i_{Lfmax}}{I_I}} \\ &\approx 0.86685\frac{R}{f\dfrac{\Delta i_{Lfmax}}{I_I}}. \end{aligned} \tag{5.69}$$

For $\Delta i_{Lfmax}/I_I = 0.1$,

$$L_{fmin} = 8.6685\frac{R}{f}. \tag{5.70}$$

An alternative method for determining L_{fmin} is by separating the low-frequency modulating voltage and the high-frequency carrier voltage in a Class E RF transmitter with amplitude modulation (AM). The minimum value of choke inductance L_{fmin} to obtain the peak-to-peak current ripple less than 10 % of dc current I_I is given by [17]

$$L_{fmin} = 2\left(\frac{\pi^2}{4}+1\right)\frac{R}{f} \approx \frac{7R}{f}. \tag{5.71}$$

5.9 Summary of Parameters at D = 0.5

The parameters of the Class E ZVS amplifier for the duty cycle $D = 0.5$ are as follows:

$$\frac{i_S}{I_I} = \begin{cases} \dfrac{\pi}{2}\sin\omega t - \cos\omega t + 1, & \text{for} \quad 0 < \omega t \le \pi, \\ 0, & \text{for} \quad \pi < \omega t \le 2\pi \end{cases} \tag{5.72}$$

$$\frac{v_S}{V_I} = \begin{cases} 0, & \text{for } 0 < \omega t \le \pi, \\ \pi\left(\omega t - \dfrac{3\pi}{2} - \dfrac{\pi}{2}\cos\omega t - \sin\omega t\right), & \text{for } \pi < \omega t \le 2\pi \end{cases} \tag{5.73}$$

$$\frac{i_{C1}}{I_I} = \begin{cases} 0, & \text{for} \quad 0 < \omega t \leq \pi, \\ \frac{\pi}{2} \sin \omega t - \cos \omega t + 1, & \text{for} \quad \pi < \omega t \leq 2\pi \end{cases} \tag{5.74}$$

$$\tan \phi = -\frac{2}{\pi} \tag{5.75}$$

$$\sin \phi = \frac{2}{\sqrt{\pi^2 + 4}} \tag{5.76}$$

$$\cos \phi = -\frac{\pi}{\sqrt{\pi^2 + 4}} \tag{5.77}$$

$$\phi = \pi - \arctan\left(\frac{2}{\pi}\right) = 2.5747\,\text{rad} = 147.52^\circ \tag{5.78}$$

$$R_{DC} \equiv \frac{V_I}{I_I} = \frac{1}{\pi \omega C_1} = \frac{\pi^2 + 4}{8} R = 1.7337R \tag{5.79}$$

$$\frac{I_{SM}}{I_I} = \frac{\sqrt{\pi^2 + 4}}{2} + 1 = 2.862 \tag{5.80}$$

$$\frac{V_{SM}}{V_I} = 2\pi(\pi - \phi) = 3.562 \tag{5.81}$$

$$c_p = \frac{I_I V_I}{I_{SM} V_{SM}} = \frac{1}{\pi(\pi - \phi)(2 + \sqrt{\pi^2 + 4})} = 0.0981 \tag{5.82}$$

$$\frac{I_m}{I_I} = \frac{\sqrt{\pi^2 + 4}}{2} = 1.8621 \tag{5.83}$$

$$\frac{V_{Rm}}{V_I} = \frac{4}{\sqrt{\pi^2 + 4}} = 1.074 \tag{5.84}$$

$$\frac{V_{Lbm1}}{V_I} = \frac{\pi(\pi^2 - 4)}{4\sqrt{\pi^2 + 4}} = 1.2378 \tag{5.85}$$

$$P_O = \frac{V_{Rm}^2}{2R} = \frac{8}{\pi^2 + 4} \frac{V_I^2}{R} = 0.5768 \frac{V_I^2}{R} \tag{5.86}$$

$$\omega C_1 R = \frac{8}{\pi(\pi^2 + 4)} = 0.1836 \tag{5.87}$$

$$\frac{\omega L_b}{R} = \frac{\pi(\pi^2 - 4)}{16} = 1.1525 \tag{5.88}$$

$$\omega^2 L_b C_1 = \frac{\pi^2 - 4}{2(\pi^2 + 4)} = 0.2116 \tag{5.89}$$

$$\frac{1}{\omega CR} = \left(Q_L - \frac{\omega L_b}{R}\right) = \left[Q_L - \frac{\pi(\pi^2 - 4)}{16}\right] \approx Q_L - 1.1525. \tag{5.90}$$

5.10 Efficiency

The power losses and the efficiency of the Class E amplifier will be considered for the duty cycle $D = 0.5$. The current through the input choke inductor L_f is nearly constant. Hence, from (5.83) the rms value of the inductor current is

$$I_{Lfrms} \approx I_I = \frac{2I_m}{\sqrt{\pi^2+4}}. \tag{5.91}$$

The amplifier efficiency is defined as $\eta_I = P_O/P_I$ and $P_O = RI_m^2/2$. From (5.86) and (5.91), the power loss in the dc ESR r_{Lf} of the choke inductor L_f is

$$P_{rLf} = r_{Lf} I_{Lfrms}^2 = \frac{4I_m^2 r_{Lf}}{(\pi^2+4)} = \frac{8r_{Lf}}{(\pi^2+4)R} P_O. \tag{5.92}$$

For the duty cycle $D = 0.5$, the rms value of the switch current is found from (5.72)

$$I_{Srms} = \sqrt{\frac{1}{2\pi}\int_0^{\pi} i_S^2 \, \mathrm{d}(\omega t)} = \frac{I_I\sqrt{\pi^2+28}}{4} = \frac{I_m}{2}\sqrt{\frac{\pi^2+28}{\pi^2+4}} \tag{5.93}$$

resulting in the switch conduction loss

$$P_{rDS} = r_{DS} I_{Srms}^2 = \frac{r_{DS} I_m^2(\pi^2+28)}{4(\pi^2+4)} = \frac{(\pi^2+28)r_{DS}}{2(\pi^2+4)R} P_O. \tag{5.94}$$

Using (5.74), the rms value of the current through the shunt capacitor C_1 is

$$I_{C1rms} = \sqrt{\frac{1}{2\pi}\int_{\pi}^{2\pi} i_{C1}^2 \, \mathrm{d}(\omega t)} = \frac{I_I\sqrt{\pi^2-4}}{4} = \frac{I_m}{2}\sqrt{\frac{\pi^2-4}{\pi^2+4}} \tag{5.95}$$

which leads to the power loss in the ESR r_{C1} of the shunt capacitor C_1

$$P_{rC1} = r_{C1} I_{C1rms}^2 = \frac{r_{C1} I_m^2(\pi^2-4)}{4(\pi^2+4)} = \frac{(\pi^2-4)r_{C1}}{2(\pi^2+4)R} P_O. \tag{5.96}$$

The power losses in the ESR r_L of the resonant inductor L is

$$P_{rL} = \frac{r_L I_m^2}{2} = \frac{r_L}{R} P_O \tag{5.97}$$

and in the ESR r_C of the resonant capacitor C is

$$P_{rC} = \frac{r_C I_m^2}{2} = \frac{r_C}{R} P_O. \tag{5.98}$$

The turn-on switching loss is zero if the ZVS condition is satisfied. The turn-off switching loss can be estimated as follows. Assume that the transistor current during the turn-off time t_f decreases linearly

$$i_S = 2I_I\left(1 - \frac{\omega t - \pi}{\omega t_f}\right), \quad \text{for} \quad \pi < \omega t \le \pi + \omega t_f. \tag{5.99}$$

The sinusoidal current through the resonant circuit does not change significantly during the fall time t_f and is $i \approx 2I_I$. Hence, the current through the shunt capacitor C_1 can be approximated by

$$i_{C1} \approx \frac{2I_I(\omega t - \pi)}{\omega t_f}, \quad \text{for} \quad \pi < \omega t \le \pi + \omega t_f \tag{5.100}$$

which gives the voltage across the shunt capacitor C_1 and the switch

$$v_S = \frac{1}{\omega C_1}\int_{\pi}^{\omega t} i_{C1} \, \mathrm{d}(\omega t) = \frac{I_I}{\omega C_1}\frac{(\omega t)^2 - 2\pi\omega t + \pi^2}{\omega t_f} = \frac{V_I\pi[(\omega t)^2 - 2\pi\omega t + \pi^2]}{\omega t_f}. \tag{5.101}$$

Thus, the average value of the power loss associated with the fall time t_f is

$$P_{tf} = \frac{1}{2\pi}\int_{\pi}^{\pi+\omega t_f} i_S v_S \, d(\omega t) = \frac{(\omega t_f)^2}{12}P_I \approx \frac{(\omega t_f)^2}{12}P_O. \tag{5.102}$$

From (5.92), (5.94), (5.96), (5.102), (5.97), and (5.98), one obtains the overall power loss

$$\begin{aligned} P_{LS} &= P_{rLf} + P_{rDS} + P_{rC1} + P_{rL} + P_{rC} + P_{tf} \\ &= P_O\left[\frac{8r_{Lf}}{(\pi^2+4)R} + \frac{(\pi^2+28)r_{DS}}{2(\pi^2+4)R} + \frac{r_{C1}(\pi^2-4)}{2(\pi^2+4)R} + \frac{r_L+r_C}{R} + \frac{(\omega t_f)^2}{12}\right]. \end{aligned} \tag{5.103}$$

The efficiency of the Class E amplifier is given by

$$\begin{aligned} \eta &\equiv \frac{P_O}{P_I} = \frac{P_O}{P_O+P_{LS}} = \frac{1}{1+\dfrac{P_{LS}}{P_O}} \\ &= \frac{1}{1+\frac{8r_{Lf}}{(\pi^2+4)R} + \frac{(\pi^2+28)r_{DS}}{2(\pi^2+4)R} + \frac{(\pi^2-4)r_{C1}}{2(\pi^2+4)R} + \frac{r_L+r_C}{R} + \frac{(\omega t_f)^2}{12}}. \end{aligned} \tag{5.104}$$

Figure 5.6 shows a plot of the efficiency η_I as a function of R.

The gate-drive power of each MOSFET that is required to charge and discharge a highly nonlinear MOSFET input capacitance is given by

$$P_G = fV_{GSm}Q_g \tag{5.105}$$

where V_{GSm} is the peak value of the gate-to-source voltage v_{GS} and Q_g is the gate charge at $v_{GS} = V_{GSm}$.

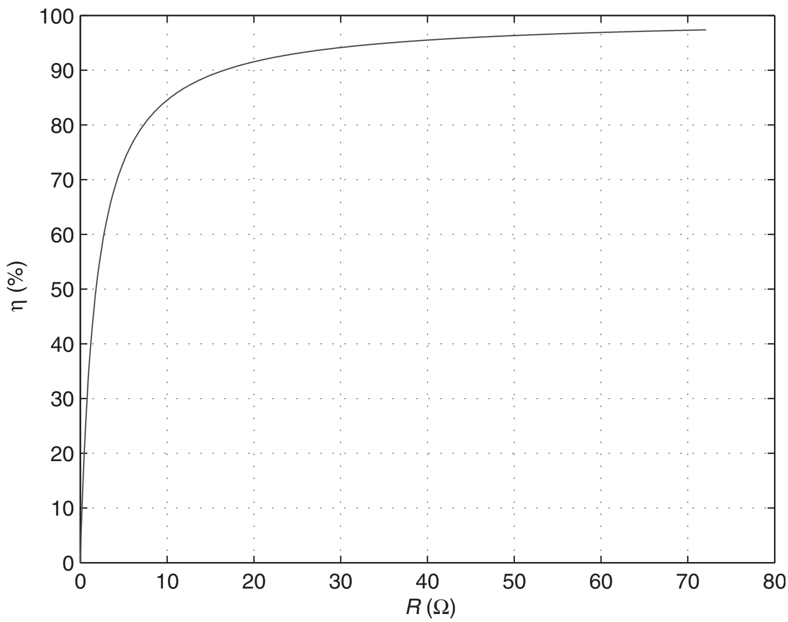

Figure 5.6 Efficiency of the Class E ZVS amplifier as a function of load resistance R for $r_{Lf} = 0.15\,\Omega$, $r_{DS} = 0.85\,\Omega$, $r_L = 0.5\,\Omega$, $r_C = 0.05\,\Omega$, $r_{C1} = 0.076\,\Omega$, and $t_f = 20\,\text{ns}$.

The power-added efficiency is

$$\eta_{PAE} = \frac{P_O - P_G}{P_I} = \frac{P_O - P_G}{P_O + P_{LS}}. \tag{5.106}$$

The power gain of the Class E ZVS amplifier at $D = 0.5$ is given by

$$k_p \equiv \frac{P_O}{P_G} = \frac{8}{\pi^2 + 4} \times \frac{V_I^2}{RfV_{GSm}Q_g}. \tag{5.107}$$

The maximum efficiency of the Class E amplifier is achieved under ZVS and ZDS conditions only when all the components are ideal with zero parasitic resistances. However, real Class E amplifiers are built using real components. The transistor on-resistance is nonzero, and the inductors and capacitors have non-zero ESRs. The maximum efficiency of the Class E amplifier with real components is obtained when the ZVS and ZDS conditions are not satisfied. The switch voltage when the transistor turns on $V_{turn-on}$ is positive, usually

$$V_{turn-on} = (0.03 - 0.15)V_I. \tag{5.108}$$

The derivative of the switch voltage when the switch turns on is usually slightly positive. Under these conditions, there is a non-zero switching power loss at the transistor turn-on time

$$P_{sw} = \frac{1}{2} f_s C_1 V_{turn-on}^2. \tag{5.109}$$

However, the shapes of the current waveforms and their rms values are modified so that the conduction losses in different components are reduced, yielding the maximum overall efficiency [69].

5.11 Design of Basic Class E Amplifier

The component values of the resonant circuit of the basic Class E amplifier shown in Figure 5.1(a) are obtained from (5.8), (5.86), (5.87), and (5.90) for optimum operation at $D = 0.5$:

$$R = \frac{8}{\pi^2 + 4}\frac{V_I^2}{P_O} \approx 0.5768\frac{V_I^2}{P_O} \tag{5.110}$$

$$X_{C1} = \frac{1}{\omega C_1} = \frac{\pi(\pi^2 + 4)R}{8} \approx 5.4466R \tag{5.111}$$

$$X_L = \omega L = Q_L R \tag{5.112}$$

$$X_C = \frac{1}{\omega C} = \left[Q_L - \frac{\pi(\pi^2 - 4)}{16}\right]R \approx (Q_L - 1.1525)R. \tag{5.113}$$

Suboptimum operation (i.e., for only ZVS operation) occurs for load resistance $R_{(sub)}$ lower than that given in (5.110), that is,

$$0 \leq R_{(sub)} < R. \tag{5.114}$$

The Class E ZVS amplifier with the basic resonant circuit shown in Figure 5.1(a) operates safely under short-circuit conditions.

The basic resonant circuit of Figure 5.1(a) does not have matching capability. In order to transfer a specified amount of power P_O at a specified dc voltage V_I, the load resistance R must be of the value determined by (5.110).

Example 5.1

Design a Class E ZVS amplifier of Figure 5.1(a) to satisfy the following specifications: $V_I = 100\,\text{V}$, $P_{Omax} = 80\,\text{W}$, and $f = 1.2\,\text{MHz}$. Assume $D = 0.5$.

Solution. It is sufficient to design the amplifier for the maximum power. Using (5.86), the full-load resistance is

$$R = \frac{8}{\pi^2 + 4}\frac{V_I^2}{P_O} = 0.5768 \times \frac{100^2}{80} = 72.1\,\Omega. \tag{5.115}$$

From (5.79), the dc resistance of the amplifier is

$$R_{DC} = \frac{\pi^2 + 4}{8}R = 1.7337 \times 72.1 = 125\,\Omega. \tag{5.116}$$

The amplitude of the output voltage is computed from (5.84)

$$V_{Rm} = \frac{4}{\sqrt{\pi^2 + 4}}V_I = 1.074 \times 100 = 107.4\,\text{V}. \tag{5.117}$$

The maximum voltage across the switch and the shunt capacitor C_1 can be calculated from (5.81) as

$$V_{SM} = V_{C1m} = 3.562V_I = 3.562 \times 100 = 356.2\,\text{V}. \tag{5.118}$$

From (5.79), the dc input current is

$$I_I = \frac{8}{\pi^2 + 4}\frac{V_I}{R} = 0.5768 \times \frac{100}{72.1} = 0.8\,\text{A}. \tag{5.119}$$

The maximum switch current obtained using (5.80) is

$$I_{SM} = \left(\frac{\sqrt{\pi^2 + 4}}{2} + 1\right)I_I = 2.862 \times 0.8 = 2.29\,\text{A}. \tag{5.120}$$

The amplitude of the current through the resonant circuit computed from (5.83) is

$$I_m = \frac{\sqrt{\pi^2 + 4}}{2}I_I = 1.8621 \times 0.8 = 1.49\,\text{A}. \tag{5.121}$$

Assuming $Q_L = 7$ and using (5.58), (5.87), and (5.90), the component values of the load network are:

$$L = \frac{Q_L R}{\omega} = \frac{7 \times 72.1}{2\pi \times 1.2 \times 10^6} = 66.9\,\mu\text{H} \tag{5.122}$$

$$C_1 = \frac{8}{\pi(\pi^2 + 4)\omega R} = \frac{8}{2\pi^2(\pi^2 + 4) \times 1.2 \times 10^6 \times 72.1} = 337.4\,\text{pF} \tag{5.123}$$

Let $C_o = 50\,\text{pF}$. Hence, the external shunt capacitance is

$$C_{1ext} = C_1 - C_o = 337.4 - 50 = 287.4\,\text{pF}. \tag{5.124}$$

Pick $C_{1ext} = 270\,\text{pF}$ and $18\,\text{pF}$ in parallel/400 V. Next,

$$C = \frac{1}{\omega R\left[Q_L - \dfrac{\pi(\pi^2 - 4)}{16}\right]} = \frac{1}{2\pi \times 1.2 \times 10^6 \times 72.1 \times (7 - 1.1525)} = 314.6\,\text{pF}. \tag{5.125}$$

The peak voltages across the resonant capacitor C and the inductor L are

$$V_{Cm} = \frac{I_m}{\omega C} = \frac{1.49}{2\pi \times 1.2 \times 10^6 \times 314.6 \times 10^{-12}} = 628.07\,\text{V} \tag{5.126}$$

and

$$V_{Lm} = \omega L I_m = 2\pi \times 1.2 \times 10^6 \times 66.9 \times 10^{-6} \times 1.49 = 751.57\,\text{V}. \tag{5.127}$$

Pick $C = 330\,\text{pF}/700\,\text{V}$.

It follows from (5.71) that in order to keep the current ripple in the choke inductor below 10 % of the full-load dc input current I_I, the value of the choke inductance must be greater than

$$L_f = 2\left(\frac{\pi^2}{4} + 1\right)\frac{R}{f} = \frac{7 \times 72.1}{1.2 \times 10^6} = 420.58\,\mu\text{H}. \tag{5.128}$$

The peak voltage across the RF choke is

$$V_{Lfm} = V_{SM} - V_I = 356.2 - 100 = 256.2\,\text{V}. \tag{5.129}$$

Assume that the dc ESR of the choke L_f is $r_{Lf} = 0.15\,\Omega$. Hence, from (5.92) the power loss in r_{Lf} is

$$P_{rLf} = r_{Lf} I_I^2 = 0.15 \times 0.8^2 = 0.096\,\text{W}. \tag{5.130}$$

From (5.93), the rms value of the switch current is

$$I_{Srms} = \frac{I_I\sqrt{\pi^2 + 28}}{4} = 0.8 \times 1.5385 = 1.231\,\text{A}. \tag{5.131}$$

Select an International Rectifier IRF840 power MOSFET, which has $V_{DSS} = 500\,\text{V}$, $I_{Dmax} = 8\,\text{A}$, $r_{DS} = 0.85\,\Omega$, $t_f = 20\,\text{ns}$, and $Q_g = 63\,\text{nC}$. The transistor conduction power loss is

$$P_{rDS} = r_{DS} I_{Srms}^2 = 0.85 \times 1.231^2 = 1.288\,\text{W}. \tag{5.132}$$

Using (5.95), one obtains the rms current through the shunt capacitance C_1

$$I_{C1rms} = \frac{I_I\sqrt{\pi^2 - 4}}{4} = 0.8 \times 0.6057 = 0.485\,\text{A}. \tag{5.133}$$

Assuming the ESR of the capacitor C_1 to be $r_{C1} = 76\,\text{m}\Omega$, one arrives at the conduction power loss in r_{C1}

$$P_{rC1} = r_{C1} I_{C1rms}^2 = 0.076 \times 0.485^2 = 0.018\,\text{W}. \tag{5.134}$$

Assume the ESRs of the resonant inductor L and the resonant capacitor C to be $r_L = 0.5\,\Omega$ and $r_C = 50\,\text{m}\Omega$ at $f = 1.2\,\text{MHz}$, the power losses in the resonant components are

$$P_{rL} = \frac{r_L I_m^2}{2} = \frac{0.5 \times 1.49^2}{2} = 0.555\,\text{W} \tag{5.135}$$

$$P_{rC} = \frac{r_C I_m^2}{2} = \frac{0.05 \times 1.49^2}{2} = 0.056\,\text{W}. \tag{5.136}$$

The drain current fall time is $t_f = 20\,\text{ns}$. Hence, $\omega t_f = 2\pi \times 1.2 \times 10^6 \times 20 \times 10^{-9} = 0.151\,\text{rad}$. From (5.102), the turn-off switching loss is

$$P_{tf} = \frac{(\omega t_f)^2 P_O}{12} = \frac{0.151^2 \times 80}{12} = 0.152\,\text{W}. \tag{5.137}$$

The power loss (excluding the gate-drive power) is

$$P_{LS} = P_{rLf} + P_{rDS} + P_{rC1} + P_{rL} + P_{rC} + P_{tf}$$
$$= 0.096 + 1.288 + 0.018 + 0.555 + 0.056 + 0.152$$
$$= 2.165\,\mathrm{W}. \quad (5.138)$$

The efficiency of the amplifier becomes

$$\eta = \frac{P_O}{P_O + P_{LS}} = \frac{80}{80 + 2.165} = 97.365\,\%. \quad (5.139)$$

Assuming $V_{GSm} = 8\,\mathrm{V}$, one obtains the gate-drive power

$$P_G = fV_{GSm}Q_g = 1.2 \times 10^6 \times 8 \times 63 \times 10^{-9} = 0.605\,\mathrm{W}. \quad (5.140)$$

The power-added efficiency (PAE) is

$$\eta_{PAE} = \frac{P_O - P_G}{P_I} = \frac{P_O - P_G}{P_O + P_{LS}} = \frac{80 - 0.605}{80 + 2.165} = 96.62\,\%. \quad (5.141)$$

The power gain of the amplifier is

$$k_p = \frac{P_O}{P_G} = \frac{80}{0.605} = 132.23. \quad (5.142)$$

The equivalent capacitance when the switch is OFF is

$$C_{eq} = \frac{CC_1}{C + C_1} = \frac{330 \times 337.4}{330 + 337.4} = 167\,\mathrm{pF}. \quad (5.143)$$

The resonant frequencies are

$$f_{o1} = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{66.9 \times 10^{-6} \times 330 \times 10^{-12}}} = 1.071\,\mathrm{MHz} \quad (5.144)$$

and

$$f_{o2} = \frac{1}{2\pi\sqrt{LC_{eq}}} = \frac{1}{2\pi\sqrt{66.9 \times 10^{-6} \times 167 \times 10^{-12}}} = 1.51\,\mathrm{MHz}. \quad (5.145)$$

Notice that the operating frequency $f = 1.2\,\mathrm{MHz}$ is between the resonant frequencies f_{o1} and f_{o2}.

5.12 Impedance Matching Resonant Circuits

The purpose of the impedance matching network is to convert the load resistance or impedance into the impedance required to produce the desired output power P_O at the specified supply voltage V_I and the operating frequency f. According to (5.110), V_I, P_O, and R are dependent quantities. In many applications, the load resistance is given and is different from that given in (5.110). Therefore, there is a need for a matching circuit that provides impedance transformation downwards or upwards. A block diagram of the class E amplifier with an impedance matching circuit is shown in Figure 5.7. Figure 5.8 shows various impedance matching resonant circuits. In the circuits shown in Figure 5.8(a) and (c), impedance transformation is accomplished by tapping the resonant capacitance C, and in the circuits shown in Figure 5.8(b) and (d) by tapping the resonant inductance L. All these circuits provide downward impedance transformation. The vertical inductance L_2 in Figure 5.8(b) and the vertical inductance L_1 in Figure 5.8(d) can be replaced by a step-up or step-down transformer, yielding additional impedance transformation. The topologies of these circuits are similar to the Greek letter π.

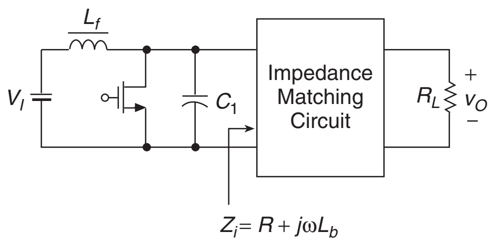

Figure 5.7 Block diagram of the Class E amplifier with impedance matching resonant circuit.

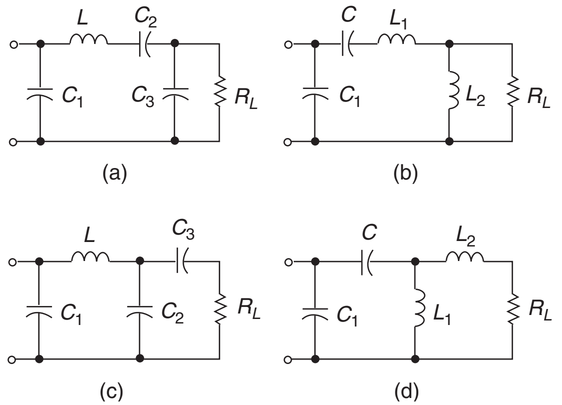

Figure 5.8 Matching resonant circuits. (a) Resonant circuit π1a. (b) Resonant circuit π2a. (c) Resonant circuit π1b. (d) Resonant circuit π2b.

5.12.1 Tapped Capacitor Downward Impedance Matching Resonant Circuit π 1a

Figure 5.9(a) shows the tapped capacitor matching circuit with downward impedance transformation. Its equivalent circuit is shown in Figure 5.9(b). Let us assume that the load resistance R_L is given. Using (5.110), the series equivalent resistance for optimum operation at $D = 0.5$ is given by

$$R = R_s = \frac{8}{\pi^2 + 4}\frac{V_I^2}{P_O} \approx 0.5768\frac{V_I^2}{P_O}. \tag{5.146}$$

The components C_1 and L are given by (5.111) and (5.112):

$$X_{C1} = \frac{1}{\omega C_1} = \frac{\pi(\pi^2 + 4)R}{8} \approx 5.4466R \tag{5.147}$$

and

$$X_L = \omega L = Q_L R. \tag{5.148}$$

The reactance factor of the R_L-C_3 and R_s-C_s equivalent single-port networks is

$$q = \frac{R_L}{X_{C_3}} = \frac{X_{Cs}}{R_s}. \tag{5.149}$$

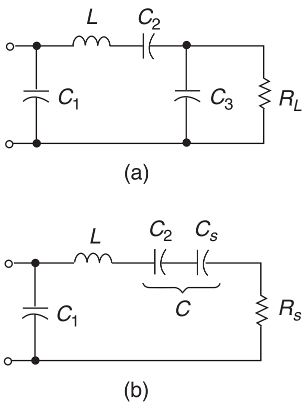

Figure 5.9 Tapped capacitor impedance matching resonant circuit π1a providing downward impedance transformation. (a) Matching circuit π1a. (b) Equivalent circuit of the matching circuit π1a.

Resistances R_s and R_L as well as the reactances X_{Cs} and X_{C3} are related by

$$R_s = R = \frac{R_L}{1+q^2} = \frac{R_L}{1+\left(\frac{R_L}{X_{C3}}\right)^2} \tag{5.150}$$

and

$$X_{Cs} = \frac{X_{C3}}{1+\frac{1}{q^2}} = \frac{X_{C3}}{1+(\frac{X_{C3}}{R_L})^2}. \tag{5.151}$$

Hence,

$$C_s = C_3\left(1+\frac{1}{q^2}\right) = C_3\left[1+\left(\frac{X_{C3}}{R_L}\right)^2\right]. \tag{5.152}$$

Rearrangement of (5.150) gives

$$q = \sqrt{\frac{R_L}{R_s}-1}. \tag{5.153}$$

From (5.149) and (5.153),

$$X_{C3} = \frac{1}{\omega C_3} = \frac{R_L}{q} = \frac{R_L}{\sqrt{\frac{R_L}{R_s}-1}}. \tag{5.154}$$

Substitution of (5.153) into (5.149) yields

$$X_{Cs} = qR_s = R_s\sqrt{\frac{R_L}{R_s}-1}. \tag{5.155}$$

Referring to Figure 5.9 and using (5.113) and (5.155), one arrives at

$$X_{C2} = \frac{1}{\omega C_2} = X_C - X_{Cs} = \left[Q_L - \frac{\pi(\pi^2-4)}{16}\right]R_s - qR_s$$

$$= R_s\left[Q_L - \frac{\pi(\pi^2-4)}{16} - \sqrt{\frac{R_L}{R_s}-1}\right]. \tag{5.156}$$

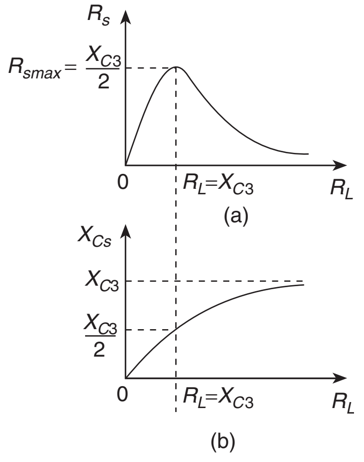

Figure 5.10 Series equivalent resistance R_s and reactance X_{Cs} as functions of load resistance R_L in the circuit π1a. (a) R_s versus R_L. (b) X_{Cs} versus R_L.

It follows from (5.154) that the circuit shown in Figure 5.8(a) can match the resistances that satisfy the inequality

$$R_s < R_L. \tag{5.157}$$

Suboptimum operation is obtained for

$$0 \le R_{s(sub)} < R_s \tag{5.158}$$

which corresponds to

$$R_L < R_{(sub)} < \infty. \tag{5.159}$$

Expressions (5.150) and (5.151) are illustrated in Figure 5.10. As R_L is increased from 0 to X_{C3}, R_s increases to $X_{C3}/2$, R_s reaches the maximum value $R_{smax} = X_{C3}/2$ at $R_L = X_{C3}$, and as R_L is increased from X_{C3} to ∞, R_s decreases from $X_{C3}/2$ to 0. Thus, the R_L-C_3 circuit acts as an *impedance inverter* [20] for $R_L > X_{C3}$. If the optimum operation occurs at $R_L = X_{C3}$, then $R_{smax} = X_{C3}/2$ and the amplifier operates under ZVS conditions at any load resistance R_L [28]. This is because $R_s \le R_{smax} = X_{C3}/2$ at any values of R_L. As R_L increases from 0 to ∞, X_{Cs} increases from 0 to X_{C3}, and C_s decreases from ∞ to C_3.

Example 5.2

Design a Class E ZVS amplifier of Figure 5.1(a) to satisfy the following specifications: $V_I = 100\,\text{V}$, $P_{Omax} = 80\,\text{W}$, $R_L = 150\,\Omega$, and $f = 1.2\,\text{MHz}$. Assume $D = 0.5$.

Solution. It is sufficient to design the amplifier for full power. The required series resistance is

$$R_s = R = \frac{8}{\pi^2 + 4}\frac{V_I^2}{P_O} = 0.5768 \times \frac{100^2}{80} = 72.1\,\Omega. \tag{5.160}$$

Thus, the amplifier requires a matching circuit. The reactance factor is

$$q = \sqrt{\frac{R_L}{R_s} - 1} = \sqrt{\frac{150}{72.1} - 1} = 1.039. \tag{5.161}$$

Hence,

$$X_{C3} = \frac{1}{\omega C_3} = \frac{R_L}{q} = \frac{150}{1.039} = 144.37\,\Omega \tag{5.162}$$

resulting in the capacitance

$$C_3 = \frac{1}{\omega X_{C3}} = \frac{1}{2\pi \times 1.2 \times 10^6 \times 144.37} = 919\,\text{pF}. \tag{5.163}$$

The amplitude of the voltage across the capacitor C_3 is the same as the amplitude of the voltage across the load resistance R_L. Thus,

$$V_{C3m} = V_{Rm} = \sqrt{2R_L P_O} = \sqrt{2 \times 150 \times 80} = 155\,\text{V}. \tag{5.164}$$

Pick $C_3 = 910\,\text{pF}/200\,\text{V}$. Let $Q_L = 7$. Hence,

$$X_{C2} = \frac{1}{\omega C_2} = X_C - X_{Cs} = R_s\left[Q_L - \frac{\pi(\pi^2 - 4)}{16} - q\right] = 72.1(7 - 1.1525 - 1.039)$$
$$= 346.7\,\Omega \tag{5.165}$$

yielding

$$C_2 = \frac{1}{\omega X_{C2}} = \frac{1}{2\pi \times 1.2 \times 10^6 \times 346.7} = 383\,\text{pF}. \tag{5.166}$$

The amplitude of the voltage across the capacitor C_2 is

$$V_{C2m} = X_{C2} I_m = 346.7 \times 1.49 = 517\,\text{V}. \tag{5.167}$$

Select $C_2 = 390\,\text{pF}/600\,\text{V}$. All other parameters are the same as those in Example 5.1.

5.12.2 Tapped Inductor Downward Impedance Matching Resonant Circuit π2a

Figure 5.11(a) shows the tapped inductor matching resonant circuit of Figure 5.8(b) that provides downward impedance transformation. Its equivalent circuit is depicted in Figure 5.11(b). The values of R_s, X_{C1}, and X_C for the resonant circuit can be calculated for

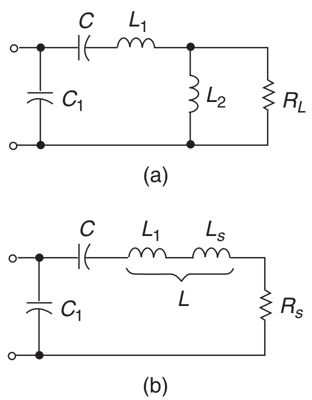

Figure 5.11 Tapped inductor downward impedance matching circuit π2a. (a) Matching circuit π2a. (b) The parallel impedance R_L-L_2 is converted into a series impedance R_s-L_s.

optimum operation at $D = 0.5$ from (5.146), (5.111), and (5.113), respectively. The series resistance is

$$R_s = R = \frac{8}{\pi^2 + 4}\frac{V_I^2}{P_O} \approx 0.5768\frac{V_I^2}{P_O} \tag{5.168}$$

$$X_{C1} = \frac{1}{\omega C_1} = \frac{\pi(\pi^2 + 4)R}{8} \approx 5.4466R \tag{5.169}$$

and

$$X_C = \frac{1}{\omega C} = \left[Q_L - \frac{\pi(\pi^2 - 4)}{16}\right]R \approx (Q_L - 1.1525)R. \tag{5.170}$$

The reactance factor of the R_L-L_2 and R_s-L_s equivalent single-port networks is

$$q = \frac{R_L}{X_{L2}} = \frac{X_{Ls}}{R_s}. \tag{5.171}$$

Resistances R_s and R_L as well as the reactances X_{Ls} and X_{L2} are related by

$$R_s = R = \frac{R_L}{1 + q^2} = \frac{R_L}{1 + \left(\frac{R_L}{X_{L2}}\right)^2} \tag{5.172}$$

and

$$X_{Ls} = \frac{X_{L2}}{1 + \frac{1}{q^2}} = \frac{X_{L2}}{1 + \left(\frac{X_{L2}}{R_L}\right)^2}. \tag{5.173}$$

Hence,

$$L_s = \frac{L_2}{1 + \frac{1}{q^2}} = \frac{L_2}{1 + \left(\frac{X_{L3}}{R_L}\right)^2}. \tag{5.174}$$

The reactance factor is

$$q = \sqrt{\frac{R_L}{R_s} - 1}. \tag{5.175}$$

Hence,

$$X_{L2} = \omega L_2 = \frac{R_L}{q} = \frac{R_L}{\sqrt{\frac{R_L}{R_s} - 1}}. \tag{5.176}$$

Since

$$Q_L = \frac{\omega L}{R_s} = \frac{X_L}{R_s} \tag{5.177}$$

and

$$X_{Ls} = qR_s \tag{5.178}$$

one obtains

$$X_{L1} = \omega L_1 = \omega(L - L_s) = X_L - X_{Ls} = (Q_L - q)R_s = \left(Q_L - \sqrt{\frac{R_L}{R_s} - 1}\right)R_s. \tag{5.179}$$

The range of resistances that can be matched by the circuit shown in Figure 5.8(b) is

$$R_s < R_L. \tag{5.180}$$

Suboptimum operation takes place for

$$0 \le R_{s(sub)} < R_s \tag{5.181}$$

and consequently for

$$R_L < R_{(sub)} < \infty. \tag{5.182}$$

It follows from (5.172) that when R_L is increased from 0 to X_{L2}, R_S increases from 0 to $R_{smax} = X_{L2}/2$, and when R_L is increased from X_{L2} to ∞, R_s decreases from $X_{L2}/2$ to 0. It is clear that the R_L-L_2 circuit behaves like an *impedance inverter* for $R_L > X_{L2}$. If the optimum operation occurs at $R_L = X_{L2}$, then $R_s = X_{L2}/2$ and the ZVS operation occurs at any load resistance [25, 30]. In this case, $R_s \le R_{smax} = X_{L2}/2$. As R_L increases from 0 to ∞, X_{Ls} increases from 0 to X_{L2}, and L_s increases from 0 to L_2.

The vertical inductance L_2 can be replaced by a step-up or step-down transformer, providing additional impedance transformation. The transformer leakage inductance can be absorbed into the horizontal inductance L_1. Figure 5.2 shows a Class E RF power amplifier with a transformer. The impedance transformation is increased by the square of the transformer turn ratio. The transformer leakage inductance L_{lp} is absorbed into the inductance L. The magnetizing inductance L_m is used as an inductor L_2 connected in parallel with the load reflected to the primary side of the transformer. The load of this circuit can be a rectifier. The transformer may be used for wireless power transmission. The circuit can be used to charge a battery installed inside a person's body.

Example 5.3

Design a Class E ZVS amplifier with the tapped inductor downward impedance matching circuit π2a shown in Figure 5.11(a) to satisfy the following specifications: $V_I = 100\,\text{V}$, $P_{Omax} = 80\,\text{W}$, $R_L = 150\,\Omega$, and $f = 1.2\,\text{MHz}$. Assume $D = 0.5$.

Solution. The series resistance R_s is

$$R_s = R = \frac{8}{\pi^2 + 4}\frac{V_I^2}{P_O} = 0.5768 \times \frac{100^2}{80} = 72.1\,\Omega. \tag{5.183}$$

A matching circuit is needed. The reactance factor is

$$q = \sqrt{\frac{R_L}{R_s} - 1} = \sqrt{\frac{150}{72.1} - 1} = 1.039. \tag{5.184}$$

The reactance X_{L2} is

$$X_{L2} = \omega L_2 = \frac{R_L}{q} = \frac{150}{1.039} = 144.37\,\Omega \tag{5.185}$$

resulting in the inductance

$$L_2 = \frac{X_{L2}}{\omega} = \frac{144.37}{2\pi \times 1.2 \times 10^6} = 19\,\mu\text{H}. \tag{5.186}$$

The amplitude of the voltage across the inductance L_2 is the same as the amplitude of the voltage across the load resistance R_L. Thus,

$$V_{L2m} = V_{Rm} = \sqrt{2R_L P_O} = \sqrt{2 \times 150 \times 80} = 155\,\text{V}. \tag{5.187}$$

Let $Q_L = 7$. Hence,

$$X_{L1} = \omega L_1 = X_L - X_{Ls} = R_s\,(Q_L - q) = 72.1(7 - 1.039) = 429.79\,\Omega \tag{5.188}$$

producing

$$L_1 = \frac{X_{L1}}{\omega} = \frac{429.79}{2\pi \times 1.2 \times 10^6} = 57\,\mu\mathrm{H}. \tag{5.189}$$

The amplitude of the voltage across the inductor L_1 is

$$V_{L1m} = X_{L1} I_m = 429.79 \times 1.49 = 640.387\,\mathrm{V}. \tag{5.190}$$

All other parameters are identical to those in Example 5.1.

5.12.3 Matching Resonant Circuit π 1b

Figure 5.12(a) shows the tapped capacitor downward impedance matching circuit. The values of R_s, X_{C1}, and X_L for the circuit shown in Figure 5.12(a) can be calculated for optimum operation at $D = 0.5$ from (5.146), (5.111), and (5.112), respectively. The series resistance R_s, and the reactances of the shunt capacitance C_1 and the resonant inductance L are

$$R_s = \frac{8}{\pi^2 + 4}\frac{V_I^2}{P_O} \approx 0.5768\frac{V_I^2}{P_O} \tag{5.191}$$

$$X_{C1} = \frac{1}{\omega C_1} = \frac{\pi(\pi^2 + 4)R}{8} \approx 5.4466R \tag{5.192}$$

and

$$X_L = Q_L R. \tag{5.193}$$

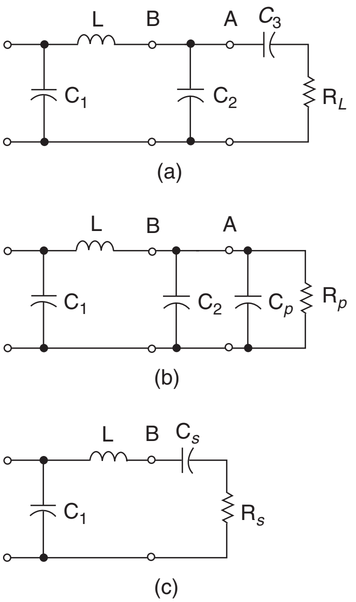

Figure 5.12 Tapped capacitor downward impedance matching circuit π 1b. (a) Matching circuit π 1b. (b) The series impedance R_L-C_3 is converted to a parallel impedance R_p-C_p. (c) The parallel impedance R_p-X_B is converted to the series impedance R_s-C_s.

The reactance factor of the series circuit R_L-C_3 located to the right of point A in Figure 5.12(a) is

$$q_A = \frac{X_{C3}}{R_L} = \frac{R_p}{X_{Cp}}. \tag{5.194}$$

The series circuit R_L-C_3 can be converted to the parallel circuit R_p-C_p as depicted in Figure 5.12(b), using the following equations:

$$R_p = R_L(1 + q_A^2) = R_L\left[1 + \left(\frac{X_{C3}}{R_L}\right)^2\right] \tag{5.195}$$

$$X_{Cp} = X_{C3}\left(1 + \frac{1}{q_A^2}\right) = X_{C3}\left[1 + \left(\frac{R_L}{X_{C3}}\right)^2\right] \tag{5.196}$$

and

$$C_p = \frac{C_3}{1 + \frac{1}{q_A^2}}. \tag{5.197}$$

The total capacitance to the right of point B is $C_B = C_2 + C_p$ and the total reactance to the right of point B is

$$\frac{1}{X_B} = \frac{1}{X_{C2}} + \frac{1}{X_{Cp}}. \tag{5.198}$$

The parallel impedance R_p-X_B can be converted to the series impedance R_s-C_s as shown in Figure 5.12(c). The conversion equations are

$$q_B = \frac{R_p}{X_B} = \frac{X_{Cs}}{R_s} \tag{5.199}$$

$$R_s = R = \frac{R_p}{1 + q_B^2} = \frac{R_p}{1 + \left(\frac{R_p}{X_B}\right)^2} \tag{5.200}$$

$$X_{Cs} = X_C = \frac{X_B}{1 + \frac{1}{q_B^2}} = \frac{X_B}{1 + \left(\frac{X_B}{R_p}\right)^2}. \tag{5.201}$$

On the other hand, the reactance of the capacitance C is

$$X_C = X_{Cs} = \left[Q_L - \frac{\pi(\pi^2 - 4)}{16}\right]R \approx (Q_L - 1.1525)R. \tag{5.202}$$

Hence, the reactance factor of the impedance to the right of point B is

$$q_B = \frac{X_{Cs}}{R_s} = \frac{X_C}{R_s} = Q_L - \frac{\pi(\pi^2 - 4)}{16} \approx Q_L - 1.1525. \tag{5.203}$$

The parallel resistance is

$$R_p = R\left(1 + q_B^2\right) = R\left\{1 + \left[Q_L - \frac{\pi(\pi^2 - 4)}{16}\right]^2\right\} = R[(Q_L - 1.1525)^2 + 1]. \tag{5.204}$$

The reactance factor of the impedance to the right of point A is

$$q_A = \sqrt{\frac{R_p}{R_L} - 1}. \tag{5.205}$$

Hence, the design equation for the reactance of the capacitor C_3 is

$$X_{C3} = \frac{1}{\omega C_3} = q_A R_L = R_L \sqrt{\frac{R[(Q_L - 1.1525)^2 + 1]}{R_L} - 1}. \tag{5.206}$$

The parallel reactance is

$$X_{Cp} = X_{C3}\left(1 + \frac{1}{q_A^2}\right) = \frac{R[(Q_L - 1.1525)^2 + 1]}{\sqrt{\frac{R[(Q_L - 1.1525)^2 + 1]}{R_L} - 1}}. \tag{5.207}$$

The parallel reactance is

$$X_B = \frac{R_p}{q_B} = \frac{R[Q_L - 1.1525)^2 + 1]}{Q_L - 1.1525}. \tag{5.208}$$

The reactance X_{C2} is given by

$$\frac{1}{X_{C2}} = \frac{1}{X_B} - \frac{1}{X_p}. \tag{5.209}$$

This gives the design equation for the reactance X_{C2} as

$$X_{C2} = \frac{1}{\omega C_2} = \frac{R[(Q_L - 1.1525)^2 + 1]}{Q_L - 1.1525 - \sqrt{\frac{R[(Q_L - 1.1525)^2 + 1]}{R_L} - 1}}. \tag{5.210}$$

The resistances that can be matched by the above-mentioned circuit are

$$\frac{R_L}{(Q_L - 1.1525)^2 + 1} < R_s < R_L. \tag{5.211}$$

Suboptimum operation takes place for

$$R_{s(sub)} < R_s \tag{5.212}$$

and therefore for

$$R_{s(sub)} < R_L. \tag{5.213}$$

Example 5.4

Design a Class E ZVS amplifier with tapped capacitor downward impedance matching circuit π1b depicted n Figure 5.12(a) to satisfy the following specifications: $V_I = 100\,\text{V}$, $P_{Omax} = 80\,\text{W}$, $R_L = 150\,\Omega$, and $f = 1.2\,\text{MHz}$. Assume $D = 0.5$.

Solution. Assuming $Q_L = 7$, the reactance of the capacitor C_3 is calculated as

$$X_{C3} = \frac{1}{\omega C_3} = R_L \sqrt{\frac{R[(Q_L - 1.1525)^2 + 1]}{R_L} - 1} = 150\sqrt{\frac{72.1[(7 - 1.1525)^2 + 1]}{150} - 1}$$
$$= 598.43\,\Omega \tag{5.214}$$

resulting in

$$C_3 = \frac{1}{\omega X_{C3}} = \frac{1}{2\pi \times 1.2 \times 10^6 \times 598.43} = 221.6285\,\text{pF}. \tag{5.215}$$

The amplitude of the current through the load resistance R_L and the capacitance C_3 is

$$I_{Rm} = \sqrt{\frac{2P_O}{R_L}} = \sqrt{\frac{2 \times 80}{150}} = 1.067\,\text{A}. \tag{5.216}$$

Hence, the amplitude of the voltage across the capacitance C_3 is

$$V_{C3m} = X_{C3}I_{Rm} = 598.43 \times 1.067 = 638.32\,\text{V}. \tag{5.217}$$

Pick $C_3 = 220\,\text{pF}/700\,\text{V}$.

The reactance of the capacitor C_2 is

$$X_{C2} = \frac{1}{\omega C_2} = \frac{R[(Q_L - 1.1525)^2 + 1]}{Q_L - 1.1525 - \sqrt{\frac{R[(Q_L - 1.1525)^2 + 1]}{R_L} - 1}}$$

$$= \frac{72.1[(7 - 1.1525)^2 + 1]}{7 - 1.1525 - \sqrt{\frac{72.1[(7 - 1.1525)^2 + 1]}{150} - 1}} = 1365.69\,\Omega \tag{5.218}$$

yielding

$$C_2 = \frac{1}{\omega X_{C2}} = \frac{1}{2\pi \times 1.2 \times 10^6 \times 1365.69} = 97.115\,\text{pF}. \tag{5.219}$$

The amplitude of the current through the capacitor C_2 is

$$V_{C2m} = \sqrt{V_{Rm}^2 + V_{C3m}^2} = \sqrt{160.05^2 + 638.32^2} = 658.08\,\text{V}. \tag{5.220}$$

Pick $C_2 = 100\,\text{pF}/700\,\text{V}$. All other parameters are the same as those in Example 5.1.

5.12.4 Matching Resonant Circuit π 2b

The values of R_s, X_{C1}, and X_C for the circuit shown in Figure 5.13(a) can be calculated for optimum operation at $D = 0.5$ from (5.146), (5.111), and (5.113), respectively. The series resistance R_s and the reactance of the shunt capacitance C_1 are

$$R_s = R = \frac{8}{\pi^2 + 4}\frac{V_I^2}{P_O} \approx 0.5768\frac{V_I^2}{P_O} \tag{5.221}$$

and

$$X_{C1} = \frac{1}{\omega C_1} = \left[Q_L - \frac{\pi(\pi^2 - 4)}{16}\right]R. \tag{5.222}$$

The reactance factor of the series circuit R_L-L_2 and the equivalent parallel circuit R_p-L_p is given by

$$q_A = \frac{X_{L2}}{R_L} = \frac{R_p}{X_{Lp}}. \tag{5.223}$$

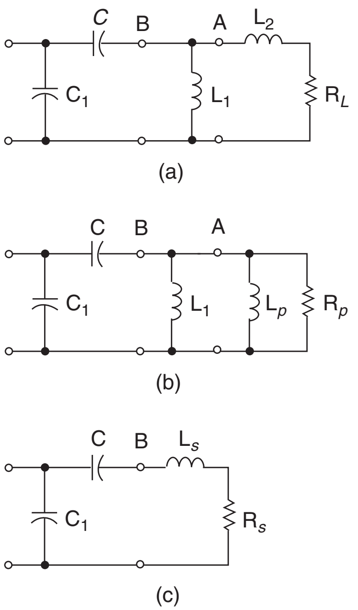

Figure 5.13 Tapped inductor downward impedance matching resonant circuit π2b. (a) Matching resonant circuit π2b. (b) Series impedance R_L-L_2 is converted into a parallel impedance R_p-L_p. (c) Parallel impedance R_p-L_p-L_1 is converted into a series impedance R_s-L_s.

The parallel components are

$$R_p = R_L\left(1 + q_A^2\right) = R_L\left[1 + \left(\frac{X_{L2}}{R_L}\right)^2\right] \tag{5.224}$$

$$X_p = X_{L2}\left(1 + \frac{1}{q_A^2}\right) = X_{L2}\left[1 + \left(\frac{R_L}{X_{L2}}\right)^2\right] \tag{5.225}$$

and

$$L_p = L_2\left(1 + \frac{1}{q_A^2}\right) = L_2\left[1 + \left(\frac{R_L}{X_{L2}}\right)^2\right]. \tag{5.226}$$

The reactance of the components L_1-L_p-R_p is

$$\frac{1}{X_B} = \frac{1}{X_{L1}} + \frac{1}{X_{Lp}}. \tag{5.227}$$

The series components R_s-L_s can be described by

$$Q_L = q_B = \frac{R_p}{X_p} = \frac{X_{Ls}}{R_s} = \frac{X_L}{R} \tag{5.228}$$

$$R_s = R = \frac{R_p}{1 + Q_L^2} = \frac{R_p}{1 + \left(\dfrac{R_p}{X_B}\right)^2} \tag{5.229}$$

and

$$X_L = X_s = \frac{X_B}{1 + \frac{1}{Q_L^2}} = \frac{X_B}{1 + \left(\frac{X_B}{R_p}\right)^2}. \tag{5.230}$$

The parallel resistance is

$$R_p = R_s(1 + q_B^2) = R(1 + Q_L^2). \tag{5.231}$$

Next,

$$q_A = \sqrt{\frac{R_p}{R_L} - 1} = \sqrt{\frac{R(Q_L^2 + 1)}{R_L} - 1}. \tag{5.232}$$

Hence,

$$X_{L2} = q_A R_L = R_L \sqrt{\frac{R(Q_L^2 + 1)}{R_L} - 1}. \tag{5.233}$$

The parallel reactance is

$$X_{Lp} = X_{L2}\left(1 + \frac{1}{q_A^2}\right) = \frac{R(Q_L^2 + 1)}{\sqrt{\frac{R(Q_L^2 + 1)}{R_L} - 1}}. \tag{5.234}$$

The parallel reactance is

$$X_B = \frac{R_p}{q_B} = \frac{R(Q_L^2 + 1)}{Q_L}. \tag{5.235}$$

Using

$$\frac{1}{X_{L1}} = \frac{1}{X_B} - \frac{1}{X_{LP}} \tag{5.236}$$

we obtain

$$X_{L1} = \omega L_1 = \frac{R(Q_L^2 + 1)}{Q_L - \sqrt{\frac{R(Q_L^2 + 1)}{R_L} - 1}}. \tag{5.237}$$

This circuit can match resistances in the range

$$\frac{R_L}{Q_L^2 + 1} < R_s < R_L. \tag{5.238}$$

Suboptimum operation takes place for

$$R_{s(sub)} < R_s \tag{5.239}$$

and therefore for

$$R_{s(sub)} < R_L. \tag{5.240}$$

Example 5.5

Design a Class E ZVS amplifier with tapped inductor downward impedance matching circuit π2b shown in Figure 5.13(a) to satisfy the following specifications: $V_I = 100\,\text{V}$, $P_{Omax} = 80\,\text{W}$, $R_L = 150\,\Omega$, and $f = 1.2\,\text{MHz}$. Assume $D = 0.5$.

Solution. Assume that $Q_L = 7$. The reactance of the inductance L_1 is

$$X_{L1} = \omega L_1 = \frac{R(Q_L^2+1)}{Q_L - \sqrt{\dfrac{R(Q_L^2+1)}{R_L} - 1}} = \frac{72.1(7^2+1)}{7 - \sqrt{\dfrac{72.1(7^2+1)}{150} - 1}} = 1638.1\,\Omega. \quad (5.241)$$

Hence, the inductance L_1 is

$$L_1 = \frac{X_{L1}}{\omega} = \frac{1638.1}{2\pi \times 1.2 \times 10^6} = 217.26\,\mu\text{H}. \quad (5.242)$$

The reactance of L_2 is

$$X_{L2} = R_L\sqrt{\frac{R(Q_L^2+1)}{R_L} - 1} = 150\sqrt{\frac{72.1(7^2+1)}{150} - 1} = 719.896\,\Omega. \quad (5.243)$$

Hence, the inductance L_2 is

$$L_2 = \frac{X_{L2}}{\omega} = \frac{719.896}{2\pi \times 1.2 \times 10^6} = 95.48\,\mu\text{H}. \quad (5.244)$$

All other parameters are identical to those of Example 5.1.

5.12.5 Quarter-wavelength Impedance Inverters

An impedance inverter can be realized using a quarter-wavelength transmission line inserted between the series-resonant circuit and a load impedance Z_L, as shown in Figure 5.14. The length of the transmission line is $l = \lambda/4$ or is an odd number of multiple of a quarter-wavelength $l = \lambda/[4(2n-1)]$. The input impedance of the $\lambda/4$ transmission line loaded with impedance Z_L is given by

$$Z_i = Z_o \frac{Z_L + jZ_o \tan\left(\dfrac{2\pi}{\lambda}\dfrac{\lambda}{4}\right)}{Z_o + jZ_L \tan\left(\dfrac{2\pi}{\lambda}\dfrac{\lambda}{4}\right)} = \frac{Z_o^2}{Z_L}. \quad (5.245)$$

Figure 5.15 shows two quarter-wavelength impedance inverters using π-configuration lumped-element resonant circuits. The characteristic impedance of both circuits is given by

$$Z_o = \omega L = \frac{1}{\omega C}. \quad (5.246)$$

The input impedance of the impedance inverter of Figure 5.15(a) is

$$Z_i = -jZ_o || \left(jZ_o + \frac{-jZ_o Z_L}{Z_L - jZ_o}\right) = jZ_o || \frac{Z_o^2}{Z_L - jZ_o} = \frac{Z_o^2}{Z_L}. \quad (5.247)$$

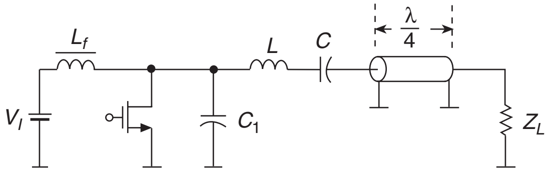

Figure 5.14 Class E RF power amplifier with a quarter-wavelength impedance inverter.

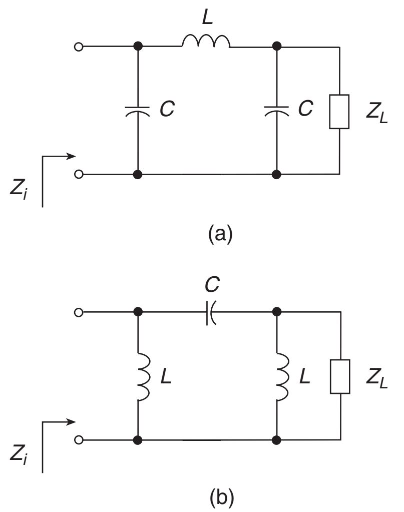

Figure 5.15 Quarter-wavelength impedance inverters using π networks consisting of lumped-element reactances. (a) Quarter-wavelength impedance inverter consisting of two capacitors and an inductor. (b) Quarter-wavelength impedance inverter consisting of two inductors and a capacitor.

Likewise, the input impedance of the impedance inverter of Figure 5.15(b) is

$$Z_i = jZ_o || \left(-jZ_o + \frac{jZ_o Z_L}{Z_L - jZ_o}\right) = jZ_o || \frac{Z_o^2}{Z_L + jZ_o} = \frac{Z_o^2}{Z_L}. \tag{5.248}$$

The expression for the input impedance Z_i of these circuits is the same as that for the input impedance of a quarter-wavelength impedance inverter using a transmission line. As $|Z_L|$ increases, $|Z_i|$ decreases.

If $Z_L = R_L$ and $Z_i = R$, then the required reactances of the quarter-wavelength matching circuits are given by

$$Z_o = X_L = \omega L = X_C = \frac{1}{\omega C} = \sqrt{RR_L}. \tag{5.249}$$

Example 5.6

Design a Class E ZVS power amplifier with a quarter-wavelength impedance inverter with the lumped-element parameters shown in Figure 5.15(a) to satisfy the following specifications: $V_I = 100\,\text{V}$, $P_{Omax} = 80\,\text{W}$, $R_L = 150\,\Omega$, and $f = 1.2\,\text{MHz}$. Assume $D = 0.5$.

Solution. The reactances of the quarter-wavelength impedance inverter are given by

$$Z_o = X_L = \omega L = X_C = \frac{1}{\omega C} = \sqrt{RR_L} = \sqrt{72.1 \times 150} = 103.995\,\Omega. \tag{5.250}$$

Hence, the inductance is

$$L = \frac{Z_o}{\omega} = \frac{103.995}{2\pi \times 1.2 \times 10^6} = 13.79\,\mu\text{H} \tag{5.251}$$

and the capacitances are

$$C = \frac{1}{\omega Z_o} = \frac{1}{2\pi \times 1.2 \times 10^6 \times 103.995} = 1.2753\,\text{nF}. \tag{5.252}$$

All other parameters are the same as those of Example 5.1.

Figure 5.16 shows two T-type quarter-wavelength transformers made up of lumped components. These transformers are introduced here. In the circuit of Figure 5.16(a), the capacitances C and C_t can be combined into a single capacitor. In the circuit of Figure 5.16(b), the inductances L and L_t can be combined into a single inductor. The characteristic impedance of both circuits is given by

$$Z_o = \omega L = \frac{1}{\omega C}. \tag{5.253}$$

The input impedance of the transformer shown in Figure 5.16(a) is

$$Z_i = -jZ_o + \frac{jZ_o(-jZ_o + Z_L)}{jZ_o - jZ_o + Z_L} = \frac{Z_o^2}{Z_L}. \tag{5.254}$$

The input impedance of the transformer shown in Figure 5.16(b) is

$$Z_i = jZ_o + \frac{(-jZ_o)(jZ_o + Z_L)}{-jZ_o + jZ_o + Z_L} = \frac{Z_o^2}{Z_L}. \tag{5.255}$$

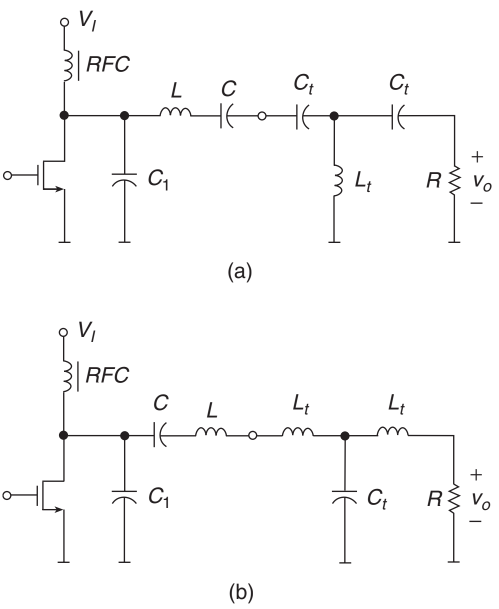

Figure 5.16 Quarter-wavelength impedance inverters using T-type networks consisting of lumped-element reactances. (a) Quarter-wavelength impedance inverter consisting of two capacitors and an inductor. (b) Quarter-wavelength impedance inverter consisting of two inductors and a capacitor.

5.13 Push-pull Class E ZVS RF Amplifier

A push-pull Class ZVS RF power amplifier with two RF chokes is depicted in Figure 5.17 [5]. The circuit with the two RF chokes can be simplified to the form depicted in Figure 5.18. This amplifier consists of two transistors, two shunt capacitors C_1, RF choke, center-tapped transformer, and series-resonant circuit driven by the secondary winding of the transformer. The transformer leakage inductance is absorbed into the resonant inductance L, and the transistor output capacitances are absorbed into shunt capacitances C_1.

Voltage and current waveforms, which explain the principle of operation of the amplifier of Figure 5.18, are depicted in Figure 5.19. The dc supply voltage source V_I is connected through an RF choke to the center tap of the primary winding of the output transformer. The switching transistors (MOSFETs) are driven on and off in opposite phase. The voltage across the secondary winding consists of positive and negative Class E pulses. The series-resonant circuit filters out all harmonics, and only the voltage of the fundamental component appears across the load resistance R_L. The odd harmonics ideally are zero in all push-pull amplifiers. The amplitude of the output voltage is twice the amplitude of the single-transistor Class E amplifier. As a result, the output power increases four times.

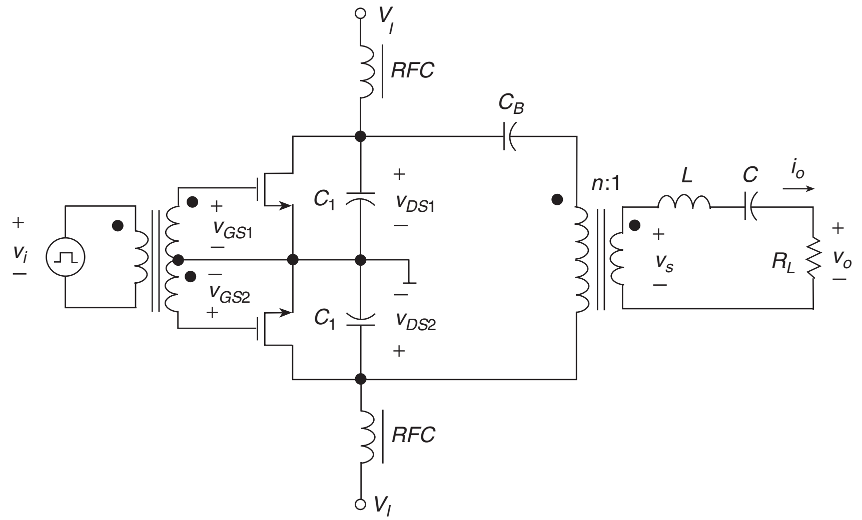

Figure 5.17 Push-pull Class E ZVS RF power amplifier with two RF chokes [5].

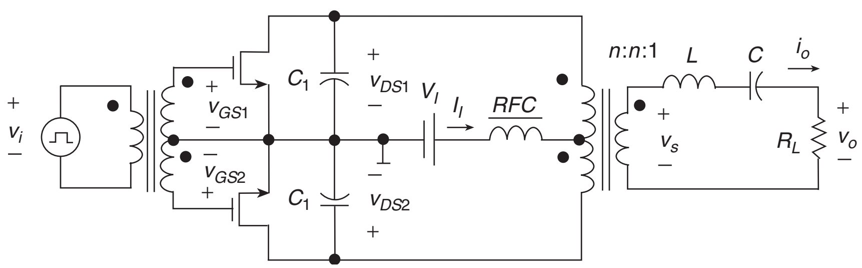

Figure 5.18 Push-pull Class E ZVS RF power amplifier.

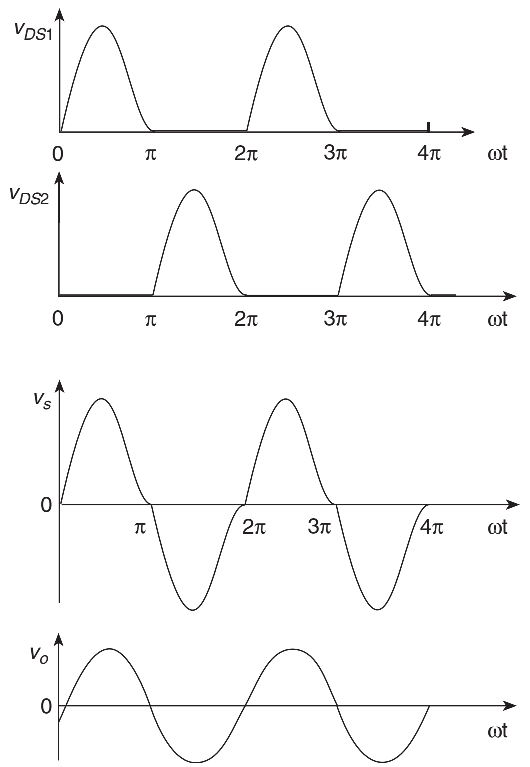

Figure 5.19 Waveforms in push-pull Class E ZVS RF power amplifier.

The load resistance seen by each transistor across each primary winding is given by

$$R = n^2 R_L \tag{5.256}$$

where n is the transformer turns ratio of the number of turns of one primary winding to the number of turns of the secondary winding. The relationships among component values are

$$\omega C_1 = \frac{8}{\pi(\pi^2+4)R} = \frac{8}{\pi(\pi^2+4)n^2 R_L} \tag{5.257}$$

$$\frac{\omega L_b}{R_L} = \frac{\pi(\pi^2-4)}{16} \quad \text{and} \tag{5.258}$$

$$\frac{1}{\omega C R_L} = \left(Q_L - \frac{\omega L_b}{R_L}\right) = \left[Q_L - \frac{\pi(\pi^2-4)}{16}\right]. \tag{5.259}$$

The output power is

$$P_O = \frac{32}{\pi^2+4}\frac{V_I^2}{n^2 R_L}. \tag{5.260}$$

The Class E ZVS symmetrical RF power amplifiers are shown in Figure 5.20 [50, 58]. The amplitudes of harmonics in the load are reduced in these amplifiers, like in push-pull amplifiers.

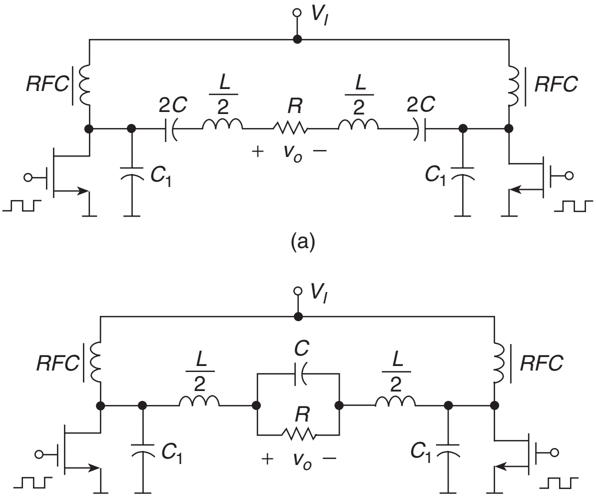

Figure 5.20 Class E ZVS symmetrical RF power amplifiers with reduced harmonics in the load. (a) Class E symmetrical amplifier with series-resonant circuit [50]. (b) Class E symmetrical amplifier with series-parallel circuit [58].

5.14 Class E ZVS RF Power Amplifier with Finite DC-feed Inductance

The Class E power amplifier can be operated with a finite dc-feed inductance L_f instead of an RF choke, as shown in Figure 5.21. The output network is formed by a parallel-series resonant circuit. The inductance L_f and the shunt capacitance C_1 form a parallel-resonant circuit and the capacitor C and the inductor L form a series resonant circuit. This circuit is easier to integrate because the dc-feed inductance is small. A small dc-feed inductance has a lower loss due to a smaller equivalent series resistance. In addition, if the amplifier is used as a radio transmitter with AM or any envelope modulation, it is easier to reduce distortion [17]. One such application of the Class E amplifier is in the envelope elimination and restoration (EER) system. The current and voltage waveforms of the Class E power amplifier with a dc-finite inductance are depicted in Figure 5.22. The peak-to-peak current of the finite dc-feed inductance L_f increases as L_f decreases and becomes larger than the dc input current I_I. Table 5.1 gives the parameters for the Class E amplifier with a

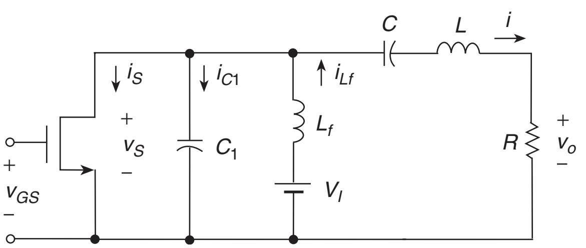

Figure 5.21 Class E power amplifier with finite dc-feed inductance L_f.

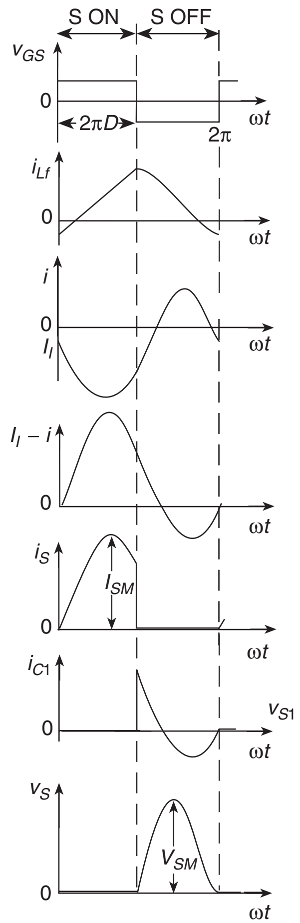

Figure 5.22 Waveforms in Class E power amplifier with finite dc-feed inductance L_f.

Table 5.1 Parameters of Class E amplifier with finite DC-feed inductance.

$\omega L_f/R_{DC}$	$\omega L_f/R$	$\omega C_1 R$	$\omega L_b/R$	RP_O/V_I^2
∞	∞	0.1836	1.152	0.5768
1000	574.40	0.1839	1.151	0.5774
500	289.05	0.1843	1.150	0.5781
200	116.02	0.1852	1.147	0.5801
100	58.340	0.1867	1.141	0.5834
50	29.505	0.1899	1.130	0.5901
20	12.212	0.1999	1.096	0.6106
15	9.3405	0.2056	1.077	0.6227
10	6.4700	0.2175	1.039	0.6470
5	3.6315	0.2573	0.9251	0.7263
3	2.5383	0.3201	0.7726	0.8461
2	2.0260	0.4142	0.5809	1.0130
1	1.3630	0.6839	0.0007	1.3630
0.9992	0.7320	0.6850	0.0000	1.3650

finite dc-feed inductance [41]. As $\omega L_f/R_{DC}$ decreases, ωCR and $P_O R/V_I^2$ increase, and $\omega L_a/R$ decreases. For the last set of parameters in Table 5.1, $L_b = 0$, $L_a = L$, and the resonant frequency of the components L and C is equal to the operating frequency so that $\omega = 1/\sqrt{LC}$. Assuming the value of the loaded quality factor Q_L, these components can be calculated from the following relationship [51]:

$$\frac{\omega L}{R} = \frac{1}{\omega CR} = Q_L. \tag{5.261}$$

Example 5.7

Design a Class E ZVS power amplifier to satisfy the following specifications: $V_I = 3.3\,\text{V}$, $P_{Omax} = 0.25\,\text{W}$, $f = 1\,\text{GHz}$, and the bandwidth $BW = 0.2\,\text{GHz}$. Assume $D = 0.5$.

Solution. Let us assume that $\omega L_f/R_{DC} = 1$. The load resistance is

$$R = 1.363\frac{V_I^2}{P_O} = 1.363 \times \frac{3.3^2}{0.25} = 59.372\,\Omega. \tag{5.262}$$

The shunt capacitance is

$$C_1 = \frac{0.6839}{\omega R} = \frac{0.6839}{2\pi \times 10^9 \times 59.372} = 1.833\,\text{pF}. \tag{5.263}$$

The dc feed inductance is

$$L_f = \frac{1.363R}{\omega} = \frac{1.363 \times 59.372}{2\pi \times 10^9} = 12.879\,\text{nF}. \tag{5.264}$$

The loaded quality factor is

$$Q_L = \frac{f_o}{BW} = \frac{10^9}{0.2 \times 10^9} = 5. \tag{5.265}$$

The inductance of the series-resonant circuit is

$$L = \frac{Q_L R}{\omega} = \frac{5 \times 59.372}{2\pi \times 10^9} = 47.246\,\text{nH}. \tag{5.266}$$

The capacitance of the series-resonant circuit is

$$C = \frac{1}{\omega R Q_L} = \frac{1}{2\pi \times 10^9 \times 59.372 \times 5} = 0.537\,\text{pF}. \tag{5.267}$$

The dc input resistance is

$$R_{DC} = \frac{V_I}{I_I} = \frac{\omega L_f}{1} = \frac{2\pi \times 10^9 \times 12.879 \times 10^{-9}}{1} = 80.921\,\Omega. \tag{5.268}$$

Assuming the efficiency $\eta_I = 0.8$, we obtain the dc input power

$$P_I = \frac{P_O}{\eta_I} = \frac{0.25}{0.8} = 0.3125\,\text{W} \tag{5.269}$$

and the dc input current

$$I_I = \frac{P_I}{V_I} = \frac{0.3125}{3.3} = 94.7\,\text{mA}. \tag{5.270}$$

5.15 Class E ZVS Amplifier with Parallel-series Resonant Circuit

A special case of a Class E ZVS amplifier with a finite dc inductance L_f is obtained, when the series components L and C are resonant at the operating frequency $f = \omega/(2\pi)$ [65]

$$\omega = \omega_s = \frac{1}{\sqrt{LC}}. \tag{5.271}$$

The resonant frequency of the parallel-resonant circuit is

$$\omega_p = \frac{1}{\sqrt{L_f C_1}}. \tag{5.272}$$

The ratio of the two resonant frequencies is

$$q = \frac{\omega_p}{\omega_s} = \frac{\omega_p}{\omega} = \frac{1}{\omega\sqrt{L_f C_1}} = \frac{\sqrt{LC}}{\sqrt{L_f C_1}}. \tag{5.273}$$

The output current is assumed to be sinusoidal

$$i = I_m \sin(\omega t + \phi). \tag{5.274}$$

From KCL,

$$i_{Lf} = i_S + i_{C1} + i. \tag{5.275}$$

When the switch is ON,

$$v_S = 0 \tag{5.276}$$

$$v_{Lf} = V_I \tag{5.277}$$

$$i_{C1} = \omega C_1 \frac{dv_S}{d(\omega t)} = 0 \tag{5.278}$$

$$i_{Lf} = \frac{1}{\omega L_f}\int_0^{\omega t} v_{Lf}\, d(\omega t) + i_{Lf}(0) = \frac{1}{\omega L_f}\int_0^{\omega t} V_I\, d(\omega t) + i_{Lf}(0) = \frac{V_I}{\omega L_f}\omega t + i_{Lf}(0) \tag{5.279}$$

and the switch current is given by

$$i_S = i_{Lf} - i = \frac{V_I}{\omega L_f}\omega t + i_{Lf}(0) - I_m \sin(\omega t + \phi) \quad \text{for} \quad 0 < \omega t \le \pi. \tag{5.280}$$

Since

$$i_S(0) = i_{Lf}(0) - I_m \sin\phi = 0 \tag{5.281}$$

$$i_{Lf}(0) = I_m \sin\phi. \tag{5.282}$$

Hence,

$$i_{Lf} = \frac{V_I}{\omega L_f}\omega t - I_m \sin\phi \tag{5.283}$$

and

$$i_S = \frac{V_I}{\omega L_f}\omega t + I_m[\sin\phi - \sin(\omega t + \phi)] \quad \text{for} \quad 0 < \omega t \le \pi. \tag{5.284}$$

When the switch is OFF, the switch current i_S is zero, $v_L = V_I - v_S$, and the current through the shunt capacitance C_1

$$i_{C1} = i_{Lf} - i = \omega C_1 \frac{dv_S}{d(\omega t)} = \frac{1}{\omega L_f} \int_{\pi}^{\omega t} v_L \, d(\omega t) + i_{Lf}(\pi) - I_m \sin(\omega t + \phi)$$

$$= \frac{1}{\omega L_f} \int_{\pi}^{\omega t} (V_I - v_S) \, d(\omega t) + i_{Lf}(\pi) - I_m \sin(\omega t + \phi) \qquad \text{for} \qquad \pi < \omega t \le 2\pi \quad (5.285)$$

where $v_S(\pi) = 0$ and

$$i_{Lf}(\pi) = \frac{\pi V_I}{\omega L_f} + I_m \sin\phi. \quad (5.286)$$

Equation (5.285) can be differentiated to obtain a linear nonhomogeneous second-order differential equation

$$\omega^2 L_f C_1 \frac{d^2 v_S}{d(\omega t)^2} + v_S - V_I - \omega L_f I_m \cos(\omega + \phi) = 0. \quad (5.287)$$

The general solution of the differential equation is

$$\frac{v_S}{V_I} = A_1 \cos(q\omega t) + A_2 \sin(q\omega t) + 1 + \frac{q^2 p}{q^2 - 1} \cos(\omega t + \phi) \quad (5.288)$$

where

$$p = \frac{\omega L_f I_m}{V_I} \quad (5.289)$$

$$q = \frac{\omega_r}{\omega} = \frac{1}{\omega\sqrt{L_f C_1}} \quad (5.290)$$

$$A_1 = \frac{qp}{q^2 - 1}[q \cos\phi \cos\pi q + (2q^2 - 1)\sin\phi \sin\pi q - \cos\pi q - \pi q \sin\pi q \quad (5.291)$$

and

$$A_2 = \frac{qp}{q^2 - 1}[q \cos\phi \sin\pi q - (2q^2 - 1)\sin\phi \cos\pi q + \pi q \cos\pi q - \sin\pi q. \quad (5.292)$$

The three unknowns in (5.288) are: p, q, and ϕ. Using the ZVS and ZDS conditions for v_S at $\omega t = 2\pi$ and the equation for the dc supply voltage

$$V_I = \frac{1}{2\pi} \int_{o}^{2\pi} v_S \, d(\omega t) \quad (5.293)$$

one obtains the numerical solution of (5.288)

$$q = \frac{\omega_r}{\omega} = \frac{1}{\omega\sqrt{L_f C_1}} \quad (5.294)$$

$$p = \frac{\omega L_f I_m}{V_I} = 1.21 \quad (5.295)$$

and

$$\phi = 195.155^\circ. \quad (5.296)$$

The dc supply current is given by

$$I_I = \frac{1}{2\pi} \int_0^{2\pi} i_S \, d(\omega t) = \frac{I_m}{2\pi} \left(\frac{\pi^2}{2p} + 2\cos\phi - \pi \sin\phi \right). \quad (5.297)$$

The component of the switch voltage at the fundamental frequency is dropped across the load R and therefore its reactive component is zero

$$V_{X1} = \frac{1}{\pi}\int_0^{2\pi} v_S \cos(\omega t + \phi)\, \mathrm{d}(\omega t) = 0. \tag{5.298}$$

The final results obtained from numerical solution are as follows:

$$\frac{\omega L_f}{R} = 0.732 \tag{5.299}$$

$$\omega C_1 R = 0.685 \tag{5.300}$$

$$\frac{\omega L}{R} = \frac{1}{\omega C R} = Q_L \tag{5.301}$$

$$\tan\ \psi = \frac{R}{\omega L_f} - \omega C_1 R \tag{5.302}$$

$$\psi = 145.856^{\circ} \tag{5.303}$$

$$V_m = 0.945 V_I \tag{5.304}$$

$$I_I = 0.826 I_m \tag{5.305}$$

$$I_m = 1.21 I_I \tag{5.306}$$

$$P_O = 1.365 \frac{V_I^2}{R} \tag{5.307}$$

$$I_{DM} = 2.647 I_I \tag{5.308}$$

$$V_{DSM} = 3.647 V_I \tag{5.309}$$

$$c_p = 0.1036 \tag{5.310}$$

and

$$f_{max} = 0.0798 \frac{P_O}{C_o V_I^2}. \tag{5.311}$$

5.16 Class E ZVS Amplifier with Nonsinusoidal Output Voltage

Figure 5.23 shows a circuit of the Class E amplifier with nonsinusoidal output voltage [18]. This circuit is obtained by replacing the resonant capacitor C in the Class E amplifier of Figure 5.1 with a blocking capacitor C_b. The loaded quality factor Q_L becomes low, and therefore the output voltage contains a lot of harmonics. The circuit of the Class E amplifier shown in Figure 5.23 is able to operate under ZVS and ZDS at any duty cycle D. Waveforms in the Class E amplifier with a nonsinusoidal output voltage are shown in Figure 5.24.

From KCL,

$$i_S = I_I - i_{C1} - i. \tag{5.312}$$

From KVL,

$$v_S = V_I + v_L + v_o. \tag{5.313}$$

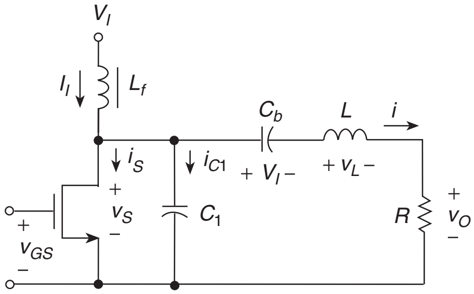

Figure 5.23 Class E amplifier with nonsinusoidal output voltage.

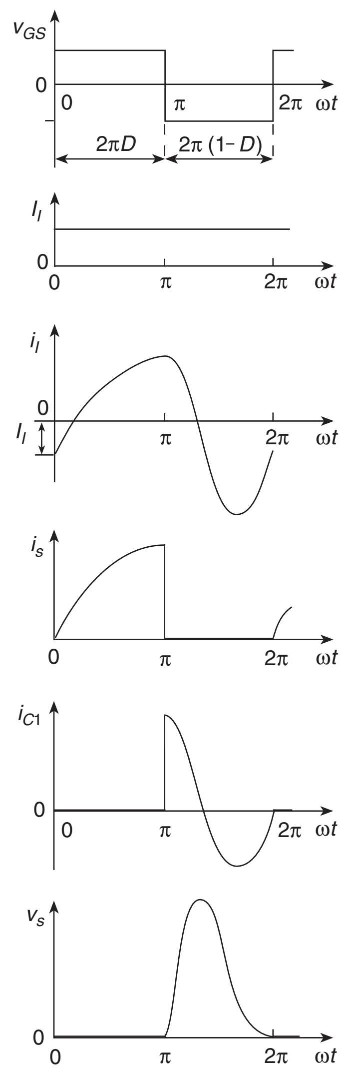

Figure 5.24 Waveforms of Class E amplifier with nonsinusoidal output voltage.

When the switch is ON for $0 < \omega t \le 2\pi D$ or $0 < t \le t_1$,

$$v_S = 0 \tag{5.314}$$

$$i_{C1} = C_1 \frac{dv_S}{d(\omega t)} = 0 \text{ and} \tag{5.315}$$

$$\frac{V_I}{s} = -sLI(s) + Li(0) - RI(s) \tag{5.316}$$

where $i(0)$ is the current through the inductor L at $t = 0$. Under the ZVS and ZDS conditions $v_S(0) = 0$ and $dv_S(0)/dt = 0$ at $t = 0$, $i_S(0) = 0$ and $i_{C1}(0) = 0$. Therefore,

$$i(0) = -I_I. \tag{5.317}$$

From (5.316),

$$i = \frac{V_I}{R} - \left(I_I + \frac{V_I}{R}\right) \exp\left(-\frac{R\omega t}{\omega L}\right) \tag{5.318}$$

resulting in

$$i_S = I_I - i = \left(I_I + \frac{V_I}{R}\right) \exp\left(-\frac{R\omega t}{\omega L}\right). \tag{5.319}$$

The dc supply current is obtained as

$$I_I = \frac{1}{2\pi} \int_0^{2\pi} i_S \, d(\omega t) = \frac{a}{1-a} \frac{V_I}{R} \tag{5.320}$$

where

$$a = D + \frac{Q}{2\pi A}\left[\exp\left(-\frac{2\pi AD}{Q}\right)\right] \tag{5.321}$$

$$A = \frac{f_0}{f} = \frac{1}{\omega\sqrt{LC_1}} \tag{5.322}$$

$$Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 C_1 R} \tag{5.323}$$

and

$$\omega_0 = \frac{1}{\sqrt{LC_1}}. \tag{5.324}$$

Substitution of (5.320) into (5.319) yields the normalized switch current

$$\frac{i_S}{I_I} = \frac{1}{a}\left[1 - \exp\left(-\frac{A\omega t}{Q}\right)\right] \quad \text{for} \quad 0 < \omega t \le 2\pi D \tag{5.325}$$

and

$$\frac{i_S}{I_I} = 0 \quad \text{for} \quad 2\pi D < \omega t \le 2\pi. \tag{5.326}$$

When the switch is OFF for $2\pi D < \omega t \le 2\pi$ or $t_1 < t \le T$,

$$i_S = 0 \tag{5.327}$$

$$V_S(s) = \frac{I_{C1}(s)}{sC_1} \tag{5.328}$$

$$I_{C1}(s) = I(s) + \frac{I_I}{sC_1} e^{-st_1} \tag{5.329}$$

and

$$V_S(s) = \frac{V_I}{s} e^{-st_1} - sLI(s) + Li(t_1)\, e^{-st_1} - RI(s) \tag{5.330}$$

where $i(t_1)$ is the initial condition of the inductor current at $t = t_1$. Thus,

$$V_S(s) = \left[\frac{1}{sC_1} \frac{\dfrac{V_I}{L} - \dfrac{I_I}{sLC_1} + si(t_1)}{s^2 + \dfrac{R}{L}s + \dfrac{1}{LC}} + \frac{I_I}{s^2 C} \right] e^{-st_1}. \tag{5.331}$$

For the oscillatory case ($Q > \frac{1}{2}$),

$$V_S(s) = \left[\frac{1}{sC_1} \frac{\dfrac{V_I}{L} - \dfrac{I_I}{sLC_1} + si(t_1)}{(s+\alpha)^2 + \omega_n^2} + \frac{I_I}{s^2 C} \right] e^{-st_1}. \tag{5.332}$$

where $\alpha = R/2L$ and $\omega_n = \omega_0\sqrt{1 - 1/4Q^2}$. Substituting (5.320) into (5.332) and taking the inverse Laplace transform, we obtain the normalized switch voltage

$$\frac{v_S}{V_I} = 0 \quad \text{for} \quad 0 < \omega t \le 2\pi D \tag{5.333}$$

and

$$\frac{v_S}{V_I} = \frac{1}{1-a} \left\{ 1 - \exp\left[-\frac{A(\omega t - 2\pi D)}{2Q} \right] \left[\cos\left(\frac{A\sqrt{4Q^2-1}(\omega t - 2\pi D)}{2Q} \right) - \frac{2Q^2\left(1 - \exp\left(-\dfrac{2\pi AD}{Q}\right)\right) - 1}{\sqrt{4Q^2-1}} \sin\left(\frac{A\sqrt{4Q^2-1}(\omega t - 2\pi D)}{2Q} \right) \right] \right\}$$
$$\text{for} \quad 2\pi D < \omega t \le 2\pi. \tag{5.334}$$

Imposing the ZVS and ZDS conditions on the switch voltage, we obtain the relationship among D, Q, and A in the form of a set of two equations:

$$\cos\left[\frac{\pi A(1-D)\sqrt{4Q^2-1}}{Q} \right] - \frac{2Q^2\left[1 - \exp\left(-\dfrac{2\pi AD}{Q}\right)\right] - 1}{\sqrt{4Q^2-1}} \sin\left[\frac{\pi A(1-D)\sqrt{4Q^2-1}}{Q} \right] = \exp\left[\frac{\pi A(1-D)}{Q} \right] \tag{5.335}$$

and

$$\tan\left[\frac{\pi A(1-D)\sqrt{4Q^2-1}}{Q} \right] = \sqrt{4Q^2-1}\, \frac{\exp\left(-\dfrac{2\pi AD}{Q}\right) - 1}{\exp\left(-\dfrac{2\pi AD}{Q} + 1\right)}. \tag{5.336}$$

These equations can be solved numerically. For $D = 0.5$, the solutions for which the ZVS and ZDS conditions are satisfied and the switch voltage is nonnegative are

$$A = 1.6029 \tag{5.337}$$

and

$$Q = 2.856. \tag{5.338}$$

The output voltage is given by

$$\frac{v_o}{V_I} = \frac{a}{a-1} + \frac{1}{1-a}\left[1 - \exp\left(-\frac{A\omega t}{2Q}\right)\right] \quad \text{for} \quad 0 < \omega t \le 2\pi D \tag{5.339}$$

and

$$\begin{aligned}\frac{v_o}{V_I} = \frac{a}{a-1} + \frac{1}{Q(1-a)}\sqrt{Q^2\left[1 - exp\left(-\frac{2\pi AD}{Q}\right)\right]^2 + exp\left(-\frac{2\pi AD}{Q}\right)} \\ \times \exp\left[-\frac{A(\omega t - 2\pi D)}{2Q}\right]\left\{\cos\left[\frac{A\sqrt{4Q^2-1}(\omega t - 2\pi D)}{2Q} - \psi\right]\right. \\ \left. - \frac{1}{\sqrt{4Q^2-1}}\sin\left[\frac{A\sqrt{4Q^2-1}(\omega t - 2\pi D)}{1Q} - \psi\right]\right\} \\ \text{for} \quad 2\pi D < \omega t \le 2\pi.\end{aligned} \tag{5.340}$$

where

$$\psi = \arctan\left\{\frac{\sqrt{4Q^2-1}}{2Q^2\left[1 - \exp\left(-\frac{2\pi AD}{Q}\right)\right] - 1}\right\}. \tag{5.341}$$

The major parameters of the amplifier at the duty cycle $D = 0.5$ are [18]:

$$P_O = 0.1788\frac{V_I^2}{R} \tag{5.342}$$

$$R_{DC} = \frac{V_I}{I_I} = 2.7801R \tag{5.343}$$

$$V_{SM} = 3.1014V_I \tag{5.344}$$

$$I_{SM} = 4.2704I_I \tag{5.345}$$

$$Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 C_1 R} = 2.856 \tag{5.346}$$

$$\omega C_1 R = 0.288 \tag{5.347}$$

$$\frac{\omega L}{R} = 2.4083 \tag{5.348}$$

$$c_p = \frac{P_O}{I_{SM} V_{SM}} = 0.0857 \tag{5.349}$$

and

$$f_{max} = 1.6108\frac{P_O}{C_o V_I^2} \tag{5.350}$$

where C_o is the transistor output capacitance. At $f = f_{max}$, $C_1 = C_o$.

Example 5.8

Design a Class E ZVS power amplifier with nonsinusoidal output voltage to satisfy the following specifications: $V_I = 100\,\text{V}$, $P_{Omax} = 80\,\text{W}$, and $f = 1.2\,\text{MHz}$.

Solution. Let us assume the duty cycle $D = 0.5$. The load resistance is

$$R = 0.1788\frac{V_I^2}{P_O} = 0.1788\frac{100^2}{80} = 22.35\,\Omega. \tag{5.351}$$

The dc input resistance seen by the power supply V_I is

$$R_{DC} = \frac{V_I}{I_I} = 2.7801R = 2.7801 \times 22.35 = 62.135\,\Omega. \tag{5.352}$$

Assuming the amplifier efficiency $\eta = 0.95$, the dc supply power is

$$P_I = \frac{P_O}{\eta} = \frac{80}{0.95} = 84.21\,\text{W}. \tag{5.353}$$

Hence, the dc supply current is

$$I_I = \frac{P_I}{V_I} = \frac{84.21}{100} = 0.8421\,\text{A}. \tag{5.354}$$

The voltage stress of the switch is

$$V_{SM} = 3.1014V_I = 3.1014 \times 100 = 310.14\,\text{V} \tag{5.355}$$

and the current stress of the switch is

$$I_{SM} = 4.2704I_I = 4.2704 \times 0.8421 = 3.59\,\text{A}. \tag{5.356}$$

The resonant frequency is

$$f_0 = Af = 1.6029 \times 1.2 \times 10^6 = 1.923\,\text{MHz}. \tag{5.357}$$

The inductance is

$$L = \frac{QR}{\omega_0} = \frac{2.856 \times 22.35}{2\pi \times 10^6 \times 1.923} = 5.283\,\mu\text{H}. \tag{5.358}$$

The shunt capacitance is

$$\frac{1}{Q\omega_0} = \frac{1}{2.856 \times 2\pi \times 1.923 \times 10^6} = 28.98\,\text{nF}. \tag{5.359}$$

5.17 Class E ZVS Power Amplifier with Parallel Resonant Circuit

Figure 5.25 shows a circuit of the Class E power amplifier with a parallel resonant circuit. This circuit is also called a Class E amplifier with only one capacitor and one inductor [11–15]. Another version of this amplifier is depicted in Figure 5.26 in which the dc supply voltage source V_I is connected in series with the inductor L and a blocking capacitor is connected in series with the load resistance R. The circuit of the amplifier is obtained from the conventional Class E ZVS power amplifier by replacing the RF choke with a dc-feed inductance and the series resonant circuit by a blocking capacitor C_B. Current and voltage waveforms are shown in Figure 5.27. The drain current waveform is an increasing ramp and the voltage waveform can satisfy both ZVS and ZDS conditions. The ZVS and ZDS conditions can be satisfied at any duty cycle D. From KCL and KVL,

$$i_S = i_L + i_C + i \tag{5.360}$$

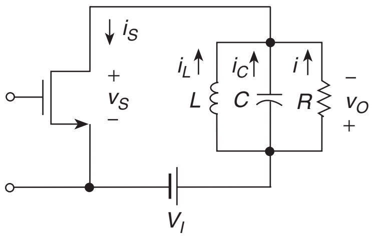

Figure 5.25 Class E amplifier with only one capacitor and one inductor.

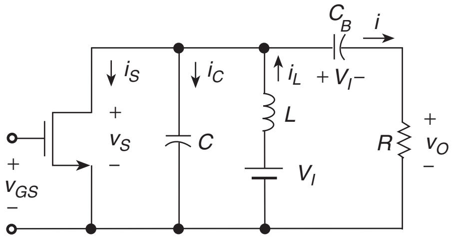

Figure 5.26 Another version of Class E amplifier with only one capacitor and one inductor.

and

$$v_S = V_I - v_o. \tag{5.361}$$

Let us consider the amplifier circuit of Figure 5.25. When the switch is ON for $0 < \omega t \leq 2\pi D$ or $0 < \omega t \leq t_1$,

$$v_S = 0 \tag{5.362}$$

$$v_o = V_I \tag{5.363}$$

$$i = \frac{V_I}{R} \tag{5.364}$$

$$i_C = \omega C \frac{\mathrm{d}v_o}{\mathrm{d}(\omega t)} = 0 \tag{5.365}$$

$$i_L = \frac{1}{L}\int_0^{\omega t} v_o \,\mathrm{d}(\omega t) + i_L(0) = \frac{1}{L}\int_0^{\omega t} V_I \,\mathrm{d}(\omega t) + i_L(0) = \frac{V_I \omega t}{\omega L} + i_L(0) \tag{5.366}$$

$$i_S = \frac{V_I}{R} + \frac{V_I \omega t}{\omega L} + i_L(0). \tag{5.367}$$

Since $i_S(0) = 0$, $i_L(0) = -V_I/R$. Hence,

$$i_L = \frac{V_I \omega t}{\omega L} - \frac{V_I}{R} \tag{5.368}$$

and

$$i_S = i_L + i = \frac{V_I \omega t}{\omega L}. \tag{5.369}$$

The dc supply voltage is

$$I_I = \frac{1}{2\pi}\int_0^{2\pi D} i_S \,\mathrm{d}(\omega t) = \frac{1}{2\pi}\int_0^{2\pi D} \frac{V_I \omega t}{\omega L}\mathrm{d}(\omega t) = \frac{\pi D^2 V_I}{\omega L} \tag{5.370}$$

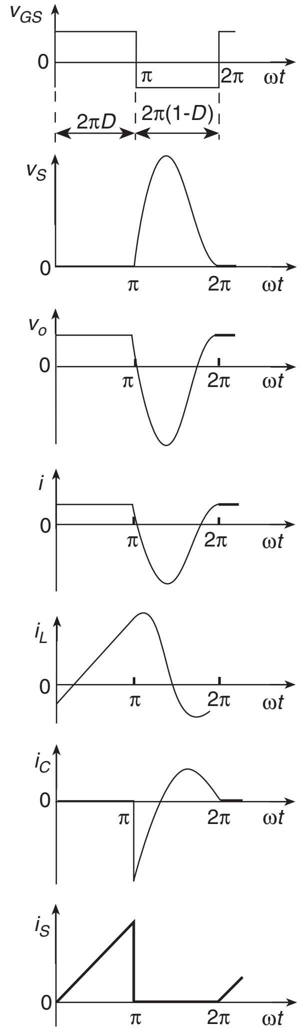

Figure 5.27 Waveforms of Class E amplifier with only one capacitor and one inductor.

where $2\pi D = \omega t_1$. Hence, the dc resistance seen by the dc supply voltage source is given by

$$R_{DC} = \frac{V_I}{I_I} = \frac{\omega L}{\pi D^2}. \tag{5.371}$$

The normalized switch current waveform is given by

$$\frac{i_S}{I_I} = \frac{\omega t}{\pi D^2} \quad \text{for} \quad 0 < \omega t \leq 2\pi D \tag{5.372}$$

and

$$\frac{i_S}{I_I} = 0 \quad \text{for} \quad 2\pi D < \omega t \leq 2\pi. \tag{5.373}$$

When the switch is OFF for $2\pi D < \omega t \leq 2\pi$ or $\omega t_1 < t \leq T$,

$$i_S = 0 \tag{5.374}$$

$$I_L(s) + I_C(s) + I(s) = 0 \tag{5.375}$$

$$I(s) = \frac{V_o(s)}{R} \tag{5.376}$$

$$I_C(s) = sCV_o(s) - Cv_o(t_1)\,\mathrm{e}^{-st_1} \tag{5.377}$$

$$I_L(s) = \frac{V_o(s)}{sL} + \frac{i_L(t_1)}{s}\mathrm{e}^{-st_1} \tag{5.378}$$

where $v_o(t_1) = V_I$ and $i_L(t_1) = V_I(t_1/L - 1/R)$. Hence,

$$V_o(s) = V_I \frac{s - \dfrac{t_1}{RC} + \dfrac{1}{RC}}{s^2 + \dfrac{s}{RC} + \dfrac{1}{LC}}. \tag{5.379}$$

For the oscillatory case ($Q > \frac{1}{2}$),

$$V_o(s) = V_I \frac{(s^2 - \omega_0^2 t_1 + 2\alpha)\,\mathrm{e}^{-st_1}}{(s+\alpha)^2 + \omega_n^2} \tag{5.380}$$

where

$$\alpha = \frac{1}{2RC} = \frac{\omega_0}{2Q} \tag{5.381}$$

and

$$\omega_n = \sqrt{\omega_0^2 - \alpha^2} = \omega_0\sqrt{1 - 1/4Q^2}. \tag{5.382}$$

Thus, the normalized switch voltage is

$$\frac{v_S}{V_I} = 0 \quad \text{for} \quad 0 < \omega t \le 2\pi D \tag{5.383}$$

and

$$\frac{v_S}{V_I} = 1 - \exp\left[-\frac{A}{2Q}(\omega t - 2\pi D)\right]\left\{\cos\left[\frac{A\sqrt{4Q^2-1}}{2Q}(\omega t - 2\pi D)\right] - \frac{4\pi AQD - 1}{\sqrt{4Q^2-1}}\sin\left[\frac{A\sqrt{4Q^2-1}}{2Q}(\omega t - 2\pi D)\right]\right\} \quad \text{for} \quad 2\pi D < \omega t \le 2\pi \tag{5.384}$$

where

$$\omega_0 = \frac{1}{\sqrt{LC}} \tag{5.385}$$

$$A = \frac{f_0}{f} = \frac{1}{\omega\sqrt{LC}} \tag{5.386}$$

and

$$Q = \omega_0 RC = \frac{R}{\omega_0 L}. \tag{5.387}$$

Using the ZVS and ZDS conditions for the switch voltage at $\omega t = 2\pi$, we obtain a set of two equations:

$$\cos\left[\frac{\pi A(1-D)\sqrt{4Q^2-1}}{Q}\right] - \frac{4\pi AQD - 1}{\sqrt{4Q^2-1}}\sin\left[\frac{\pi A(1-D)\sqrt{4Q^2-1}}{Q}\right] = \exp\left[\frac{\pi A(1-D)}{Q}\right] \tag{5.388}$$

and

$$\tan\left[\frac{\pi A(1-D)\sqrt{4Q^2-1}}{Q}\right] = \frac{\pi AD\sqrt{4Q^2-1}}{\pi AD - Q}. \tag{5.389}$$

These set of equations can be solved numerically. At $D = 0.5$, the solutions are

$$A = 1.5424 \tag{5.390}$$

and

$$Q = 1.5814. \tag{5.391}$$

The output voltage waveform is

$$\frac{v_o}{V_I} = 1 \quad \text{for} \quad 0 < \omega t \le 2\pi D \tag{5.392}$$

and

$$\frac{v_o}{V_I} = \exp\left[-\frac{A}{2Q}(\omega t - 2\pi D)\right]\left\{\cos\left[\frac{A\sqrt{4Q^2-1}}{2Q}(\omega t - 2\pi D)\right]\right.$$
$$\left.-\frac{4\pi AQD - 1}{\sqrt{4Q^2-1}}\sin\left[\frac{A\sqrt{4Q^2-1}}{2Q}(\omega t - 2\pi D)\right]\right\} \quad \text{for} \quad 2\pi D < \omega t \le 2\pi. \tag{5.393}$$

The most important parameters of the amplifier at the duty cycle $D = 0.5$ are given by [15]

$$R_{DC} = \frac{V_I}{I_I} = 0.522R \tag{5.394}$$

$$I_{SM} = 4I_I \tag{5.395}$$

$$V_{SM} = 3.849V_I \tag{5.396}$$

$$P_O = \pi ADQ\frac{V_I^2}{R} = 1.9158\frac{V_I^2}{R} \tag{5.397}$$

$$A = \frac{f_0}{f} = 1.5424 \tag{5.398}$$

$$Q = \omega_0 CR = \frac{R}{\omega_0 L} = 1.5814 \tag{5.399}$$

$$\omega CR = 1.0253 \tag{5.400}$$

$$\frac{\omega L}{R} = 0.41. \tag{5.401}$$

$$c_p = \frac{P_O}{I_{SM}V_{SM}} = 0.0649 \tag{5.402}$$

and

$$f_{max} = 0.5318\frac{P_O}{C_o V_I^2}. \tag{5.403}$$

Example 5.9

Design a Class E ZVS power amplifier with a parallel resonant circuit to satisfy the following specifications: $V_I = 3.3$ V, $P_{Omax} = 1$ W, and $f = 2.4$ GHz.

Solution. We will assume the duty cycle $D = 0.5$. The load resistance is

$$R = 1.9158\frac{V_I^2}{P_O} = 1.9158 \times \frac{3.3^2}{1} = 20.86\,\Omega. \tag{5.404}$$

The dc input resistance seen by the power supply V_I is

$$R_{DC} = \frac{V_I}{I_I} = 0.522R = 0.522 \times 20.86 = 10.89\,\Omega. \tag{5.405}$$

Assuming the amplifier efficiency $\eta = 0.8$, the dc supply power is

$$P_I = \frac{P_O}{\eta} = \frac{1}{0.8} = 1.25\,\text{W}. \tag{5.406}$$

Hence, the dc supply current is

$$I_I = \frac{P_I}{V_I} = \frac{1.25}{3.3} = 0.379\,\text{A}. \tag{5.407}$$

The voltage stress of the switch is

$$V_{SM} = 3.849V_I = 3.849 \times 3.3 = 12.7\,\text{V} \tag{5.408}$$

and the current stress of the switch is

$$I_{SM} = 4I_I = 4 \times 0.379 = 1.516\,\text{A}. \tag{5.409}$$

The resonant frequency is

$$f_0 = Af = 1.5424 \times 2.4 \times 10^9 = 3.7\,\text{GHz}. \tag{5.410}$$

The resonant inductance is

$$L = \frac{0.41R}{\omega} = \frac{0.41 \times 20.86}{2\pi \times 2.4 \times 10^9} = 0.567\,\text{nH}. \tag{5.411}$$

The resonant capacitance is

$$C = \frac{1.0253}{\omega R} = \frac{1.0253}{2\pi \times 2.4 \times 10^9 \times 22.86} = 2.974\,\text{pF}. \tag{5.412}$$

5.18 Amplitude Modulation of Class E ZVS RF Power Amplifier

The output voltage waveform in the Class E ZVS amplifier for continuous-wave (CW) operation, i.e., without modulation, is given by

$$v_o = V_c \cos \omega_c t. \tag{5.413}$$

where ω_c is the carrier angular frequency and V_c is the carrier amplitude. The amplitude of the output voltage of the Class E ZVS RF power amplifier V_m is directly proportional to the dc supply voltage V_I. For the duty cycle $D = 0.5$, the output voltage amplitude is

$$V_c = \frac{4}{\sqrt{\pi^2 + 4}} V_I. \tag{5.414}$$

This property can be used for obtaining amplitude modulation, as proposed in [17].

Figure 5.28 shows the circuit of a Class E ZVS RF power amplifier with drain amplitude modulation [17]. The source of the modulating voltage v_m is connected in series with the dc supply voltage source V_I, e.g., via a transformer. The voltage waveforms in the AM Class E amplifier are shown in Figure 5.29. A modulating voltage source v_m is connected in series with the dc voltage source V_I. The modulating voltage is given by

$$v_m = V_m \cos \omega_m t. \tag{5.415}$$

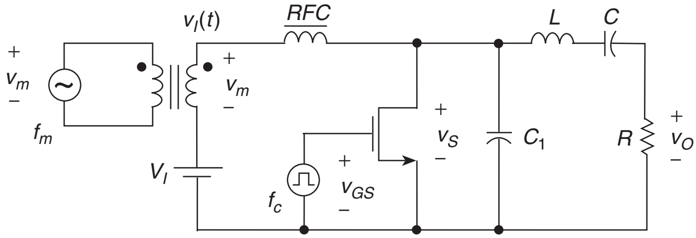

Figure 5.28 Class E ZVS RF power amplifier with drain amplitude modulation.

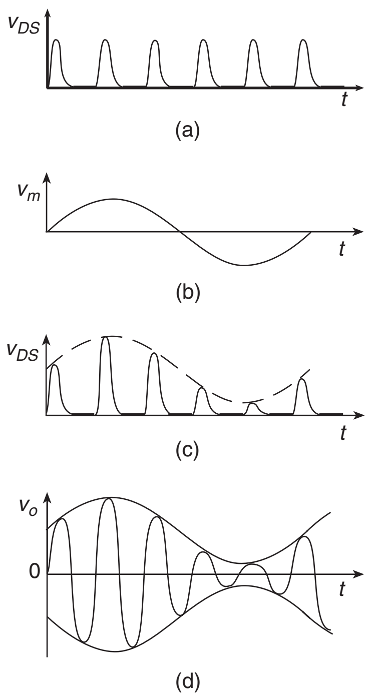

Figure 5.29 Waveforms in Class E ZVS RF power amplifier with drain amplitude modulation. (a) CW drain-to-source voltage v_{DS}. (b) Modulating voltage v_m. (c) AM drain-to-source voltage v_{DS}. (d) AM output voltage v_o.

Therefore, the supply voltage of the amplifier with amplitude modulation is

$$v_I(t) = V_I + v_m(t) = V_I + V_m \cos \omega_m t. \tag{5.416}$$

The amplitude of the output voltage is

$$V_m(t) = \frac{4}{\sqrt{\pi^2 + 4}}[V_I + v_m(t)] = \frac{4}{\sqrt{\pi^2 + 4}}(V_I + V_m \cos \omega_m t). \tag{5.417}$$

The switch voltage is $v_S = 0$ for $0 < \omega_c t \le \pi$ and

$$v_S = (V_I + V_m \cos \omega_m t)\pi \left(\omega_c t + \frac{3\pi}{2} - \frac{\pi}{2} \cos \omega_c t - \sin \omega_c t\right) \quad \text{for} \ \ \pi < \omega_c t \le 2\pi. \tag{5.418}$$

Assuming an ideal bandpass filter of the Class E amplifier, the output voltage waveform is

$$v_o = V_m(t)\cos\omega_c t = \frac{4}{\sqrt{\pi^2+4}}(V_I + V_m\cos\omega_m t)\cos\omega_c t$$
$$= \frac{4}{\sqrt{\pi^2+4}}V_I(1 + m\cos\omega_m t)\cos\omega_c t = V_c(1 + m\cos\omega_m t)\cos\omega_c t \quad (5.419)$$

where the modulation index is

$$m = \frac{V_m}{V_I}. \quad (5.420)$$

The output voltage waveform can be rearranged to the form

$$v_o = V_c\cos\omega_c t + \frac{mV_c}{2}\cos(\omega_c - \omega_m)t + \frac{mV_c}{2}\cos(\omega_c + \omega_m)t. \quad (5.421)$$

In reality, the voltage transfer function of the series-resonant circuit in the Class E amplifier under ZVS and ZDS conditions is not symmetrical with respect to the carrier frequency. The carrier frequency f_c is higher than the resonant frequency of the series-resonant circuit f_r (i.e., $f_c > f_r$). Therefore, the two sideband components are transmitted from the drain to the load resistance R with different magnitudes and different phase shifts (i.e., delays). The upper sideband is attenuated more than the lower sideband. In addition, the phase shift of the upper sideband is more negative than that of the lower sideband. The phase shift of the lower sideband may become even positive. These effects cause harmonic distortion of the envelope of the AM output voltage [17].

5.19 Summary

- The Class E ZVS RF power amplifier is defined as the circuit in which a single transistor is used and is operated as a switch, the transistor turns on at zero voltage, and the transistor may turn on at zero derivative.
- The transistor output capacitance, the choke parasitic capacitance, and the stray capacitance are absorbed into the shunt capacitance C_1 in the Class E ZVS power amplifier.
- The turn-on switching loss is zero.
- The operating frequency f is greater than the resonant frequency $f_0 = 1/(2\pi\sqrt{LC})$ of the series-resonant circuit. This results in an inductive load for the switch when it is ON.
- The antiparallel diode of the switch turns off at low di/dt and zero voltage, reducing reverse-recovery effects. Therefore, the MOSFET body diode can be used and there is no need for a fast diode.
- Zero-voltage-switching operation can be accomplished in the basic topology for load resistances ranging from zero to R_{opt}. Matching circuits can be used to match any impedance to the desired load resistance.
- The peak voltage across the transistor is about four times higher than the input dc voltage. Therefore, the circuit is suitable for low input voltage applications.
- The drive circuit is easy to build because the gate-to-source voltage of the transistor is referenced to ground.
- The circuit is very efficient and can be operated at high frequencies.

- The large choke inductance with a low current ripple can be replaced by a low inductance with a large current ripple. In this case, the equations describing the amplifier operation will change [24].
- The loaded quality factor of the resonant circuit can be small. In the extreme case, the resonant capacitor becomes a large dc-blocking capacitor. The mathematical description will change accordingly [18].
- The maximum operating frequency of Class E operation with ZVS and ZDS or with ZVS only is limited by the output capacitance of the switch and is given by (5.62).
- The maximum operating frequency of the Class E power amplifier increases as the duty cycle decreases.

5.20 References

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5.21 Review Questions

5.1 What is the ZVS technique?

5.2 Is the transistor output capacitance absorbed into the Class E ZVS inverter topology?

5.3 Is it possible to obtain the ZVS operation at any load using the basic topology of the Class E ZVS inverter?

5.4 Is the turn-on switching loss zero in the Class E ZVS inverter?

5.5 Is the turn-off switching loss zero in the Class E ZVS inverter?

5.6 Is it possible to achieve the ZVS condition at any operating frequency?

5.7 Is the basic Class E ZVS inverter short-circuit proof?

5.8 Is the basic Class E ZVS inverter open-circuit proof?

5.9 Is it possible to use a finite dc-feed inductance in series with the dc input voltage source V_I?

5.10 Is it required to use a high value of the loaded quality factor of the resonant circuit in the Class E ZVS inverter?

5.22 Problems

5.1 Design a Class E RF power amplifier for wireless communication applications to meet the following specifications: $V_I = 3.3\,\text{V}$, $P_O = 1\,\text{W}$, $f = 1\,\text{GHz}$, $C_o = 1\,\text{pF}$, $Q_L = 5$,

$r_{DS} = 0.01\,\Omega$, $r_{Lf} = 0.012\,\Omega$, $r_{C1} = 0.08\,\Omega$, $r_C = 0.05\,\Omega$, $r_L = 0.1\,\Omega$, and $t_f = 0$. Find the component values, reactances of reactive components, component stresses, and efficiency.

5.2 The Class E RF power amplifier given in Problem 5.1 had a load resistance $R_L = 50\,\Omega$. Design the impedance matching circuit.

5.3 Design an optimum Class E ZVS inverter to meet the following specifications: $P_O = 125$ W, $V_I = 48$ V, and $f = 2$ MHz. Assume $Q_L = 5$.

5.4 The rms value of the US utility voltage is from 92 to 132 V. This voltage is rectified by a bridge peak rectifier to supply a Class E ZVS inverter that is operated at a switch duty cycle of 0.5. What is the required value of the voltage rating of the switch?

5.5 Repeat Problem 5.4 for the European utility line, whose rms voltage is $220 \pm 15\,\%$.

5.6 Derive the design equations for the component values for the matching resonant circuit π2a shown in Figure 5.8(b).

5.7 Find the maximum operating frequency at which pure Class E operation is still achievable for $V_I = 200$ V, $P_O = 75$ W, and $C_{out} = 100$ pF.

5.8 Design a Class E power amplifier for wireless communications applications to meet the following specifications: $V_I = 12$ V, $P_O = 10$ W, $f = 2.4$ GHz, $Co = 1$ pF, $Q_L = 10$, $r_{DS} = 0.01$, $r_{Lf} = 0.01$, $r_{C1} = 0.08$, $r_C = 0.01$, $r_L = 0.1$, and $t_f = 0$. Find the component values, reactances of reactive components, component stresses, component losses, and efficiency.

5.9 The Class E amplifier given in Problem 5.8 has $R_L = 50$. Design the impedance matching circuit.

6

Class E Zero-current-switching RF Power Amplifier

6.1 Introduction

In this chapter, a Class E RF zero-current switching (ZCS) power amplifier [1–4] is presented and analyzed. In this amplifier, the switch is turned off at zero current, yielding zero turn-off switching loss. A shortcoming of the Class E ZCS amplifier is that the switch output capacitance is not included in the basic amplifier topology. The switch turns on at a nonzero voltage, and the energy stored in the switch output capacitance is dissipated in the switching device, reducing the efficiency. Therefore, the upper operating frequency of the Class E ZCS amplifier is lower than that of the Class E ZVS amplifier.

6.2 Circuit Description

A circuit of a Class E ZCS RF power amplifier is depicted in Figure 6.1(a). This circuit was introduced in [1]. It consists of a single transistor and a load network. The transistor operates cyclically as a switch at the desired operating frequency $f = \omega/(2\pi)$. The simplest type of load network consists of a resonant inductor L_1 connected in series with the dc source V_I, and an L-C-R series-resonant circuit. The resistance R is an ac load.

The equivalent circuit of the Class E ZCS amplifier is shown in Figure 6.1(b). The capacitance C is divided into two series capacitances, C_a and C_b, so that capacitance C_a is series resonant with L at the operating frequency $f = \omega/2\pi$

$$\omega = \frac{1}{\sqrt{LC_a}}. \tag{6.1}$$

RF Power Amplifiers Marian K. Kazimierczuk

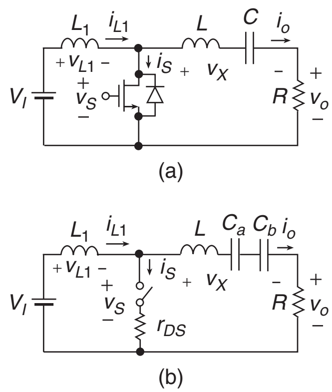

Figure 6.1 Class E RF zero-current-switching amplifier. (a) Circuit. (b) Equivalent circuit.

The additional capacitance C_b signifies the fact that the operating frequency f is lower than the resonant frequency of the series-resonant circuit when the switch is ON $f_{o1} = 1/(2\pi\sqrt{LC})$. The loaded quality factor Q_L is defined by the expression

$$Q_L = \frac{X_{Cr}}{R} = \frac{C_a + C_b}{\omega R C_a C_b}. \tag{6.2}$$

The choice of Q_L involves the usual tradeoff among (1) low harmonic content of the power delivered to R (high Q_L), (2) low change of amplifier performance with frequency (low Q_L), (3) high efficiency of the load network (low Q_L), and (4) high bandwidth (low Q_L).

6.3 Principle of Operation

The equivalent circuit of the amplifier is shown in Figure 6.1(b). It is based on the following assumptions:

(1) The elements of the load network are ideal.

(2) The loaded quality factor Q_L of the series-resonant circuit is high enough that the output current is essentially a sinusoid at the operating frequency.

(3) The switching action of the transistor is instantaneous and lossless; the transistor has zero output capacitance, zero saturation resistance, zero saturation voltage, and infinite 'off' resistance.

It is assumed for simplicity that the switch duty ratio is 50 %, that is, the switch is ON for half of the ac period and OFF for the remainder of the period. However, the duty ratio can be any arbitrarily chosen if the circuit component values are chosen to be appropriate for the chosen duty ratio. It will be explained in Section 6.4 that a duty ratio of 50 % is one of the conditions for optimum amplifier operation. The amplifier operation is determined by the switch when it is closed and by the transient response of the load network when the switch is open. The principle of operation of the amplifier is explained by the current and voltage waveforms, which are shown in Figure 6.2. Figure 6.2(a) depicts the waveforms for optimum operation. When the switch is open, its current i_S is zero. Hence, the inductor current i_{L1} is equal to a nearly sinusoidal output current i. The current i_{L1}

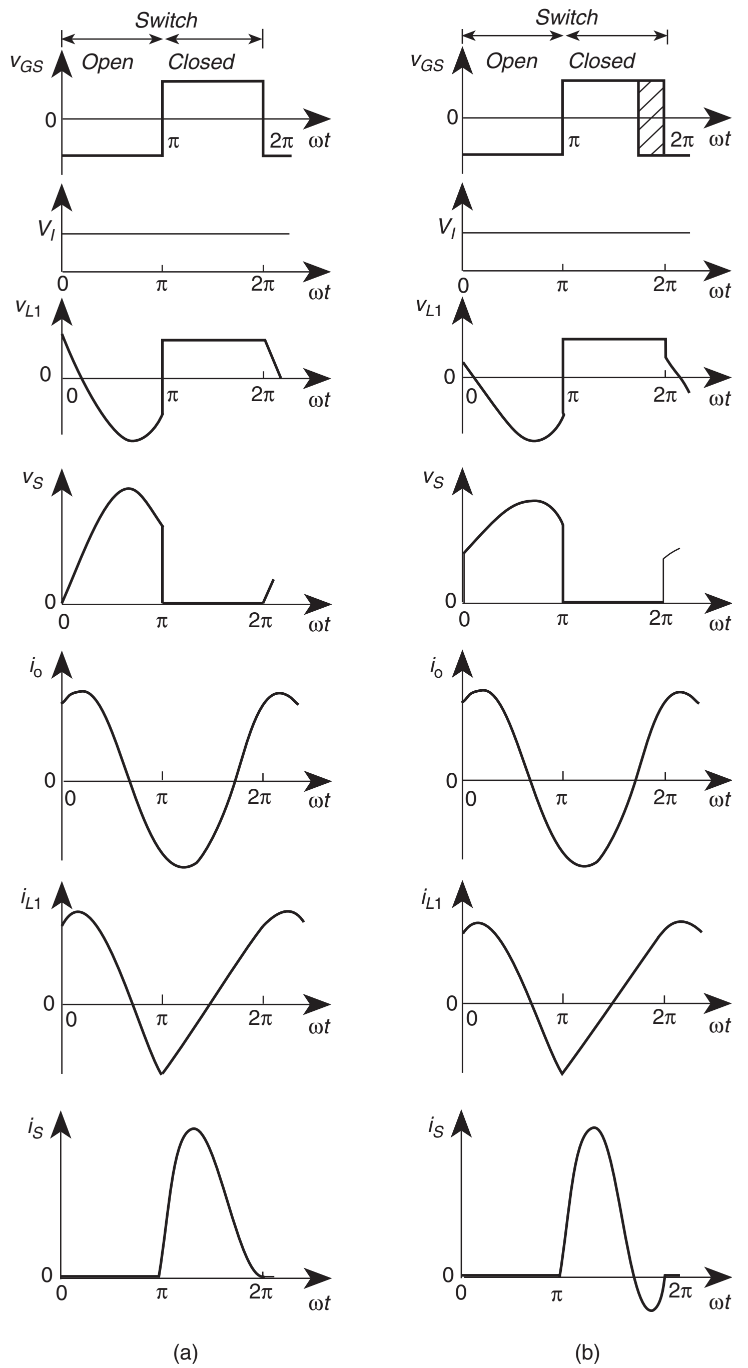

Figure 6.2 Current and voltage waveforms in the Class E ZCS amplifier. (a) For optimum operation. (b) For suboptimum operation.

produces the voltage drop v_{L1} across the inductor L_1. This voltage is approximately a section of a sine wave. The difference between the supply voltage V_I and the voltage v_{L1} is the voltage across the switch v_S. When the switch is closed, the voltage v_S is zero, and voltage v_{L1} equals the supply voltage V_I. This voltage produces the linearly increasing current i_{L1}. The difference between the current i_{L1} and current i flows through the switch.

In the Class E ZCS amplifier, it is possible to eliminate power losses due to on-to-off transition of the transistor, yielding high efficiency. Assuming that the transistor is turned off at $\omega t_{off} = 2\pi$, the ZCS condition at turn-off is

$$i_S(2\pi) = 0. \tag{6.3}$$

For optimum operation, the zero-derivative switching (ZDS) condition should also be satisfied:

$$\left.\frac{di_S}{d(\omega t)}\right|_{\omega t=2\pi} = 0. \tag{6.4}$$

If condition (6.3) is not satisfied, the transistor turns off at nonzero current. Consequently, there is a fall time of the drain (or collector) current during which the transistor acts as a current source. During the fall time, the drain current increases and the drain-source voltage increases. Since the transistor current and voltage overlap during the turn-off interval, there is a turn-off power loss. However, if the transistor current is already zero at turn-off, the transistor current fall time is also zero, there is no overlap of the transistor current and voltage, and the turn-off switching loss is zero.

Condition (6.3) eliminates dangerous voltage spikes at the output of the transistor. If this condition is not satisfied, the current i_S changes rapidly during turn-off of the transistor. Hence, the inductor current i_{L1} also changes rapidly during turn-off. Therefore, inductive voltage spikes appear at the output of the transistor and device failure may occur. The rapid change of i_{L1} during turn-off of the transistor causes a change of the energy stored in the inductor L_1. A part of this energy is dissipated in the transistor as heat, and the remainder is delivered to the series-resonant circuit L, C, and R. If condition (6.4) is satisfied, the switch current is always positive and the antiparallel diode never conducts. Furthermore, the voltage across the switch at the turn-off instant will be zero, that is, $v_S(2\pi) = 0$, and during the 'off' state the voltage v_S will start to increase from zero only gradually. This zero starting voltage v_S is desirable in the case of the real transistor because the energy stored in the parasitic capacitance across the transistor is zero at the instant the transistor switches off. The parasitic capacitance comprises the transistor capacitances, the winding capacitance of L_1, and stray winding capacitance. The optimum operating conditions can be accomplished by a proper choice of the load-network components. The load resistance at which the ZCS condition is satisfied is $R = R_{opt}$.

Figure 6.2(b) shows the waveforms for suboptimum operation. This operation occurs when only the ZCS condition is satisfied. If the slope of the switch current at the time the switch current reaches zero is positive, the switch current will be negative during a portion of the period. If the transistor is OFF, the antiparallel diode conducts the negative switch current. If the transistor is ON, either only the transistor conducts or both the transistor and the antiparallel diode conduct. The transistor should be turned off during the time interval the switch current is negative. When the switch current reaches zero, the antiparallel diode turns off.

The voltage across the inductor L_1 is described by the expression

$$v_{L1} = \omega L_1 \frac{di_{L1}}{d(\omega t)}. \tag{6.5}$$

At the switch turn-on, the derivative of the inductor current i_{L1} changes rapidly from a negative to a positive value. This causes a step change in the inductor voltage v_{L1} and consequently in the switch voltage v_S.

According to assumption (3), the conduction power loss and the turn-on switching power loss are neglected. The conduction loss dominates at low frequencies, and the turn-on switching loss dominates at high frequencies. The off-to-on switchingtime is especially important in high-frequency operation. The parasitic capacitance across the transistor is discharged from the voltage $2V_I$ to zero when the transistor switches on. This discharge requires a nonzero length of time. The switch current i_S is increasing during this time. Since the switch voltage v_S and the switch current i_S are simultaneously nonzero, the power is dissipated in the transistor. The off-to-on switching loss becomes comparable to saturation loss at high frequencies. Moreover, the transient response of the load network depends on the parasitic capacitance when the switch is open. This influence is neglected in this analysis. According to assumption (1), the power losses in the parasitic resistances of the load network are also neglected.

6.4 Analysis

6.4.1 Steady-state Current and Voltage Waveforms

The basic equations of the equivalent amplifier circuit shown in Figure 6.1(b) are

$$i_S = i_{L1} - i \tag{6.6}$$

$$v_S = V_I - v_{L1}. \tag{6.7}$$

The series-resonant circuit forces a sinusoidal output current

$$i = I_m \sin(\omega t + \varphi). \tag{6.8}$$

The switch is OFF for the interval $0 < \omega t \le \pi$. Therefore,

$$i_S = 0, \qquad \text{for} \quad 0 < \omega \le \pi. \tag{6.9}$$

From (6.6), (6.8), and (6.9),

$$i_{L1} = i = I_m \sin(\omega t + \varphi), \quad \text{for} \quad 0 < \omega t \le \pi. \tag{6.10}$$

The voltage across the inductor L_1 is

$$v_{L1} = \omega L_1 \frac{\mathrm{d}i_{L1}}{\mathrm{d}(\omega t)} = \omega L_1 I_m \cos(\omega t + \varphi), \quad \text{for} \quad 0 < \omega t \le \pi. \tag{6.11}$$

Hence, (6.7) becomes

$$v_S = V_I - v_{L1} = V_I - \omega L_1 I_m \cos(\omega t + \varphi), \quad \text{for} \quad 0 < \omega t \le \pi. \tag{6.12}$$

Using (6.10) and taking into account the fact that the inductor current i_{L1} is continuous,

$$i_{L1}(\pi+) = i_{L1}(\pi-) = I_m \sin(\pi + \varphi) = -I_m \sin\varphi. \tag{6.13}$$

The switch is ON for the interval $\pi < \omega t \le 2\pi$ during which

$$v_S = 0, \qquad \text{for} \quad \pi < \omega t \le 2\pi. \tag{6.14}$$

Substitution of this into (6.7) then produces

$$v_{L1} = V_I, \qquad \text{for} \quad \pi < \omega t \le 2\pi. \tag{6.15}$$

Thus, from (6.13) and (6.15), the current through the inductor L_1 is

$$i_{L1} = \frac{1}{\omega L_1}\int_{\pi}^{\omega t} v_{L1}(u)\,\mathrm{d}u + i_{L1}(\pi+) = \frac{1}{\omega L_1}\int_{\pi}^{\omega t} V_I(u)\,\mathrm{d}u + i_{L1}(\pi+)$$
$$= \frac{V_I}{\omega L_1}(\omega t - \pi) - I_m \sin\varphi, \qquad \text{for} \quad \pi < \omega t \le 2\pi. \tag{6.16}$$

From (6.6) and (6.8), the switch current is obtained as

$$i_S = i_{L1} - i = \frac{V_I}{\omega L_1}(\omega t - \pi) - I_m[\sin(\omega t + \varphi) + \sin\ \varphi], \qquad \text{for} \quad \pi < \omega t \le 2\pi. \tag{6.17}$$

Substituting the ZCS conditon $i_S(2\pi) = 0$ into (6.17),

$$I_m = V_I \frac{\pi}{2\omega L_1 \sin\varphi}. \tag{6.18}$$

Because $I_m > 0$,

$$0 < \varphi < \pi. \tag{6.19}$$

From (6.9), (6.17), and (6.18),

$$i_S = \begin{cases} 0, & 0 < \omega t \le \pi, \\ \dfrac{V_I}{\omega L_1}\left[\omega t - \dfrac{3\pi}{2} - \dfrac{\pi}{2\sin\varphi}\sin(\omega t + \varphi)\right], & \pi < \omega t \le 2\pi. \end{cases} \tag{6.20}$$

Substitution of the condition of optimum operation given by (6.4) into (6.20) yields

$$\tan\varphi = \frac{\pi}{2}. \tag{6.21}$$

From (6.19) and (6.21),

$$\varphi = \arctan\left(\frac{\pi}{2}\right) = 1.0039\,\text{rad} = 57.52^\circ. \tag{6.22}$$

Consideration of trigonometric relationships shows that

$$\sin\varphi = \frac{\pi}{\sqrt{\pi^2 + 4}} \tag{6.23}$$

$$\cos\varphi = \frac{2}{\sqrt{\pi^2 + 4}}. \tag{6.24}$$

From (6.20) and (6.21),

$$i_S = \begin{cases} 0, & 0 < \omega t \le \pi, \\ \dfrac{V_I}{\omega L_1}\left(\omega t - \dfrac{3\pi}{2} - \dfrac{\pi}{2}\cos\omega t - \sin\omega t\right), & \pi < \omega t \le 2\pi. \end{cases} \tag{6.25}$$

Using the Fourier formula, the supply dc current is

$$I_I = \frac{1}{2\pi}\int_{\pi}^{2\pi} i_S\ \mathrm{d}(\omega t) = \frac{V_I}{2\pi\omega L_1}\int_{\pi}^{2\pi}\left(\omega t - \frac{3\pi}{2} - \frac{\pi}{2}\cos\omega t - \sin\omega t\right)\mathrm{d}(\omega t) = \frac{V_I}{\pi\omega L_1}. \tag{6.26}$$

The amplitude of the output current can be found from (6.18), (6.23), and (6.26)

$$I_m = \frac{\sqrt{\pi^2 + 4}}{2}\frac{V_I}{\omega L_1} = \frac{\pi\sqrt{\pi^2 + 4}}{2} I_I = 5.8499 I_I. \tag{6.27}$$

Substitution of (6.27) into (6.25) yields the normalized steady-state switch current waveform

$$\frac{i_S}{I_I} = \begin{cases} 0, & 0 < \omega t \le \pi \\ \pi\left(\omega t - \dfrac{3\pi}{2} - \dfrac{\pi}{2}\cos\omega t - \sin\omega t\right), & \pi < \omega t \le 2\pi. \end{cases} \tag{6.28}$$

From (6.12), (6.18), and (6.21), the normalized switch voltage waveform is found as

$$\frac{v_S}{V_I} = \begin{cases} \frac{\pi}{2}\sin\omega t - \cos\omega t + 1, & 0 < \omega t \leq \pi \\ 0, & \pi < \omega t \leq 2\pi. \end{cases} \tag{6.29}$$

6.4.2 Peak Switch Current and Voltage

The peak switch current I_{SM} and voltage V_{SM} can be determined by differentiating waveforms (6.28) and (6.29), and setting the results equal to zero. Finally, we obtain

$$I_{SM} = \pi(\pi - 2\varphi)I_I = 3.562 I_I \tag{6.30}$$

and

$$V_{SM} = \left(\frac{\sqrt{\pi^2+4}}{2} + 1\right)V_I = 2.8621 V_I. \tag{6.31}$$

Neglecting power losses, the output power equals the dc input power $P_I = I_I V_I$. Thus, the power-output capability c_p can be computed from the expression

$$c_p = \frac{P_O}{I_{SM}V_{SM}} = \frac{I_I V_I}{I_{SM}V_{SM}} = 0.0981. \tag{6.32}$$

It has the same value as the Class E ZVS amplifier with a shunt capacitor. It can be proved that the maximum power output capability occurs at a duty ratio of 50 %.

6.4.3 Fundamental-frequency Components

The output voltage is sinusoidal and has the form

$$v_{R1} = V_m \sin(\omega t + \varphi) \tag{6.33}$$

where

$$V_m = RI_m. \tag{6.34}$$

The voltage v_X across the elements L, C_a, and C_b is not sinusoidal. The fundamental-frequency component v_{X1} of the voltage v_X appears only across the capacitor C_b because the inductance L and the capacitance C_a are resonant at the operating frequency f and their reactance $\omega L - 1/(\omega C_a) = 0$. This component is

$$v_{X1} = V_{X1}\cos(\omega t + \varphi) \tag{6.35}$$

where

$$V_{X1} = -\frac{I_m}{\omega C_b}. \tag{6.36}$$

The fundamental-frequency component of the switch voltage is

$$v_{S1} = v_{R1} + v_{X1} = V_m \sin(\omega t + \varphi) + V_{X1}\cos(\omega t + \varphi). \tag{6.37}$$

The phase shift between the voltages v_{R1} and v_{S1} is determined by the expression

$$\tan\psi = \frac{V_{X1}}{V_m} = -\frac{1}{\omega C_b R}. \tag{6.38}$$

Using (6.29) and the Fourier formulas, we can obtain

$$V_m = \frac{1}{\pi}\int_0^{2\pi} v_S \sin(\omega t + \varphi)\, \mathrm{d}(\omega t) = \frac{4}{\pi\sqrt{\pi^2+4}}V_I = 0.3419V_I \tag{6.39}$$

and

$$V_{X1} = \frac{1}{\pi}\int_0^{2\pi} v_S \cos(\omega t + \varphi)\, \mathrm{d}(\omega t) = -\frac{\pi^2+12}{4\sqrt{\pi^2+4}}V_I = -1.4681V_I. \tag{6.40}$$

Substituting (6.26) and (6.27) into (6.39) and (6.40),

$$V_m = \frac{8}{\pi(\pi^2+4)}\omega L_1 I_m \tag{6.41}$$

$$V_{X1} = -\frac{\pi^2+12}{2(\pi^2+4)}\omega L_1 I_m. \tag{6.42}$$

The fundamental-frequency components of the switch current

$$i_{s1} = I_{s1}\sin(\omega t + \gamma) \tag{6.43}$$

where

$$I_{s1} = I_I\sqrt{\left(\frac{\pi^2}{4} - 2\right)^2 + \frac{\pi^2}{2}} = 1.6389I_I \tag{6.44}$$

and

$$\gamma = 180^\circ + \arctan\left(\frac{\pi^2 - 8}{2\pi}\right) = 196.571^\circ. \tag{6.45}$$

The switch voltage is

$$v_{s1} = V_{s1}\sin(\omega t + \vartheta) \tag{6.46}$$

where

$$V_{s1} = \sqrt{V_m^2 + V_{X1}^2} = V_I\sqrt{\frac{16}{\pi^2(\pi^2+4)} + \frac{(\pi^2+12)^2}{16(\pi^2+4)}} = 1.5074V_I \tag{6.47}$$

and

$$\vartheta = \varphi + \psi = -19.372^\circ. \tag{6.48}$$

The phase ϕ of the input impedance of the load network at the operating frequency is

$$\phi = 180^\circ + \vartheta - \gamma = -35.945^\circ. \tag{6.49}$$

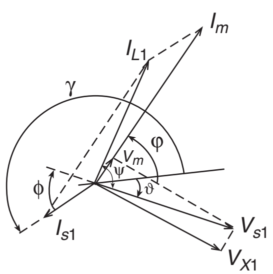

Figure 6.3 Phasor diagram of the fundamental-frequency components of the currents and voltages for optimum operation of Class E ZCS amplifier.

This indicates that the input impedance of the load network is capacitive. Figure 6.3 shows a phasor diagram for the fundemental-frequency currents and voltages for optimum operation of the Class E ZCS amplifier.

6.5 Power Relationships

The dc input power P_I is

$$P_I = I_I V_I. \tag{6.50}$$

From (6.39), the output power P_O is

$$P_O = \frac{V_m^2}{2R} = \frac{8}{\pi^2(\pi^2+4)}\frac{V_I^2}{R} = 0.05844\frac{V_I^2}{R}. \tag{6.51}$$

6.6 Element Values of Load Network

From (6.34), (6.38), (6.41), and (6.42),

$$\frac{\omega L_1}{R} = \frac{\pi(\pi^2+4)}{8} = 5.4466 \tag{6.52}$$

$$\omega C_b R = \frac{16}{\pi(\pi^2+12)} = 0.2329 \tag{6.53}$$

and

$$\psi = \arctan\left(\frac{V_{X1}}{V_m}\right) = -\arctan\left[\frac{\pi(\pi^2+12)}{16}\right] = -76.89^\circ. \tag{6.54}$$

Hence, according to Figure 6.1(b), the capacitor C_b should be connected in series with C_a, L, and R. The values of L and C_a can be found from formulas (6.1) and (6.2).
From (6.27) and (6.52),

$$I_m = \frac{4}{\pi\sqrt{\pi^2+4}}\frac{V_I}{R}. \tag{6.55}$$

From (6.26) and (6.52),

$$I_I = \frac{8}{\pi^2(\pi^2+4)}\frac{V_I}{R}. \tag{6.56}$$

The dc input resistance of the amplifier is obtained from (6.26) and (6.52)

$$R_{DC} \equiv \frac{V_I}{I_I} = \pi\omega L_1 = \frac{2\pi(\pi^2+4)}{(\pi^2+12)}\frac{1}{\omega C_b} = \frac{\pi^2(\pi^2+4)}{8}R = 17.11R. \tag{6.57}$$

The element values of the load network can be computed from the following expressions:

$$R = \frac{8}{\pi^2(\pi^2+4)}\frac{V_I^2}{P_O} = 0.05844\frac{V_I^2}{P_O} \tag{6.58}$$

$$L_1 = \frac{\pi(\pi^2+4)}{8}\frac{R}{\omega} = 5.4466\frac{R}{\omega} \tag{6.59}$$

$$C_b = \frac{16}{\pi(\pi^2+12)}\frac{1}{\omega R} = \frac{0.2329}{\omega R} \tag{6.60}$$

$$C = \frac{1}{\omega R Q_L} \tag{6.61}$$

and

$$L = \left[Q_L - \frac{\pi(\pi^2 + 12)}{16} \right] \frac{R}{\omega} = (Q_L - 4.2941)\frac{R}{\omega}. \tag{6.62}$$

It is apparent from (6.62) that the loaded-quality factor Q_L must be greater than 4.2941.

6.7 Design Example

Example 6.1

Design the Class E ZCS RF power amplifier of Figure 6.1(a) to meet the following specifications: $V_I = 5\,\text{V}$, $P_{Omax} = 1\,\text{W}$, and $f = 1\,\text{GHz}$.

Solution. It is sufficient to design the amplifier for the full power. From (6.58), the full-load resistance is

$$R = \frac{8}{\pi^2(\pi^2 + 4)} \frac{V_I^2}{P_O} = 0.05844 \times \frac{5^2}{1} = 1.146\,\Omega. \tag{6.63}$$

According to Section 6.6, the loaded-quality factor Q_L must be greater than 4.2941. Let $Q_L = 8$. Using (6.59), (6.61), and (6.62), the values of the elements of the load network are:

$$L_1 = \frac{\pi^2 + 4}{16} \frac{R}{f} = 0.8669 \times \frac{1.461}{10^9} = 1.2665\,\text{nH} \tag{6.64}$$

$$C = \frac{1}{\omega R Q_L} = \frac{1}{2 \times \pi \times 10^9 \times 1.461 \times 8} = 13.62\,\text{pF} \tag{6.65}$$

and

$$L = \left[Q_L - \frac{\pi(\pi^2 + 12)}{16} \right] \frac{R}{\omega} = \left[8 - \frac{\pi(\pi^2 + 12)}{16} \right] \times \frac{1.461}{2 \times \pi \times 10^9} = 0.862\,\text{nH}. \tag{6.66}$$

The maximum voltage across the switch can be obtained using (6.31) as

$$V_{SM} = \left(\frac{\sqrt{\pi^2 + 4}}{2} + 1 \right) V_I = 2.8621 \times 5 = 14.311\,\text{V}. \tag{6.67}$$

From (6.56), the dc input current is

$$I_I = \frac{8}{\pi^2(\pi^2 + 4)} \frac{V_I}{R} = 0.0584 \times \frac{5}{1.461} = 0.2\,\text{A}. \tag{6.68}$$

The maximum switch current is calculated using (6.30) as

$$I_{SM} = \pi(\pi - 2\varphi)I_I = 3.562 \times 0.2 = 0.7124\,\text{A} \tag{6.69}$$

and from (6.55) the maximum amplitude of the current through the resonant circuit is

$$I_m = \frac{4}{\pi\sqrt{\pi^2 + 4}} \frac{V_I}{R} = 0.3419 \times \frac{5}{1.461} = 1.17\,\text{A}. \tag{6.70}$$

The dc resistance seen by the dc power supply is

$$R_{DC} = \frac{\pi^2(\pi^2 + 4)}{8} R = 17.11 \times 1.461 = 25\,\Omega. \tag{6.71}$$

The resonant frequency of the L-C series-resonant circuit when the switch is ON is

$$f_{o1} = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.862 \times 10^{-9} \times 13.62 \times 10^{-12}}} = 1.469\,\text{GHz} \tag{6.72}$$

and the resonant frequency of the L_1-L-C series-resonant circuit when the switch is OFF is

$$f_{o2} = \frac{1}{2\pi\sqrt{C(L + L_1)}} = \frac{1}{2\pi\sqrt{13.62 \times 10^{-12}(1.2665 + 0.862) \times 10^{-9}}} = 0.9347\,\text{GHz}. \tag{6.73}$$

6.8 Summary

- In the Class E ZCS RF power amplifier, the transistor turns off at zero current, reducing turn-off switching loss to zero, even if the transistor switching time is an appreciable fraction of the cycle of the operating frequency.
- The transistor output capacitance is not absorbed into the topology of the Class E ZCS amplifier.
- The transistor turns on at nonzero voltage, causing turn-on power loss.
- The efficiency of the Class E ZCS amplifier is lower than that of the Class E ZVS amplifier, using the same transistor and the same operating frequency.
- The voltage stress in the Class E ZCS amplifier is lower than that of the Class E ZVS amplifier.
- The ZCS condition can be satisfied for load resistances ranging from a minimum value of R to infinity.
- The load network of the amplifier can be modified for impedance transformation and harmonic suppression.

6.9 References

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6.10 Review Questions

6.1 What is the ZCS technique?

6.2 What is the turn-off switching loss in the Class E ZCS amplifier?

6.3 What is the turn-on switching loss in the Class E ZCS amplifier?

6.4 Is the transistor output capacitance absorbed into the Class E ZCS amplifier topology?

6.5 Is the inductance connected in series with the dc input source V_I a large high-frequency choke in the Class E ZCS amplifier?

6.6 What are the switch voltage and current stresses for the Class D ZCS amplifier at $D = 0.5$?

6.7 Compare the voltage and current stresses for the Class E ZVS and ZCS amplifier at $D = 0.5$.

6.11 Problems

6.1 Design a Class E ZCS RF power amplifier to meet the following specifications: $V_I = 15\,\text{V}$, $P_O = 10\,\text{W}$, and $f = 900\,\text{MHz}$.

6.2 A Class E ZCS amplifier is powered from a 340-V power supply. What is the required voltage rating of the switch if the switch duty cycle is 0.5?

6.3 Design a Class E ZCS power amplifier to meet the following specifications: $V_I = 180\,\text{V}$, $P_O = 250\,\text{W}$, and $f = 200\,\text{kHz}$.

6.4 It has been found that a Class E ZCS amplifier has the following parameters: $D = 0.5$, $f = 400\,\text{kHz}$, $L_1 = 20\,\mu\text{H}$, and $P_O = 100\,\text{W}$. What is the maximum voltage across the switch in this amplifier?

6.5 Design the Class E ZCS power amplifier to meet the following specifications: $V_I = 100\,\text{V}$, $P_{Omax} = 50\,\text{W}$, and $f = 1\,\text{MHz}$.

7

Class DE RF Power Amplifier

7.1 Introduction

The Class DE RF switching-mode power amplifier [1–11] consists of two transistors, series-resonant circuit, and shunt capacitors connected in parallel with the transistors. It combines the properties of low voltage stress of the Class D power amplifier and zero-voltage switching of the Class E power amplifier. Switching losses are zero in the Class DE power amplifier, yielding high efficiency. In the Class DE power amplifier, the transistors are driven in such a way that there are time intervals (dead times) when both transistors are OFF. In this chapter, we will present the circuit of the Class DE amplifier, its principle of operation, analysis, and design procedure.

7.2 Analysis of Class DE RF Power Amplifier

A circuit of the Class DE RF power amplifier is depicted in Figure 7.1. It consists of two transistors, series-resonant circuit, and shunt capacitors connected in parallel with the transistors. The transistor output capacitances are absorbed into the shunt capacitances. At high frequencies, the shunt capacitances can be formed by the transistor output capacitances. We assume that the duty cycle of each transistor is fixed at $D = 0.25$. This means that the duty cycle of the dead time is also 0.25. In general, the duty cycle D can be in range from zero to 0.5. Figure 7.2 shows equivalent circuits for four time intervals during the cycle of the operating frequency f_s. Current and voltage waveforms are shown in Figure 7.3. During the dead-time intervals, the load current discharges one shunt capacitance and charges the other. When the transistor turns on, the Class E ZVS and ZDS conditions are satisfied. Therefore, the efficiency of the Class DE power amplifier is high.

The series-resonant circuit forces nearly a sinusoidal current

$$i_o = I_m \sin(\omega t + \phi) \tag{7.1}$$

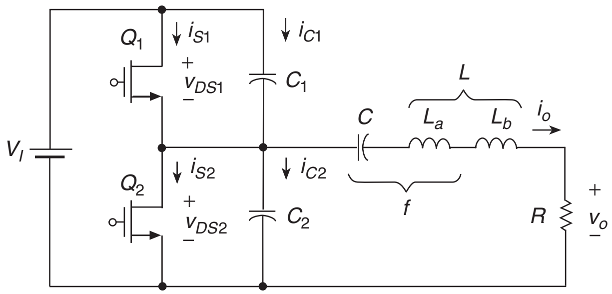

Figure 7.1 Class DE RF power amplifier.

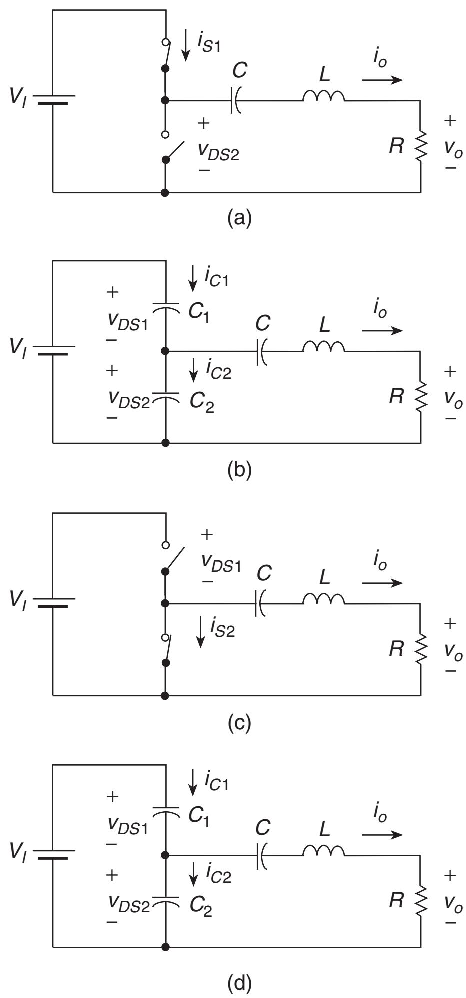

Figure 7.2 Equivalent circuits for Class DE RF power amplifier. (a) S_1 is ON and S_2 is OFF. (b) S_1 and S_2 are OFF. (c) S_1 is OFF and S_2 is ON. (d) S_1 and S_2 are OFF.

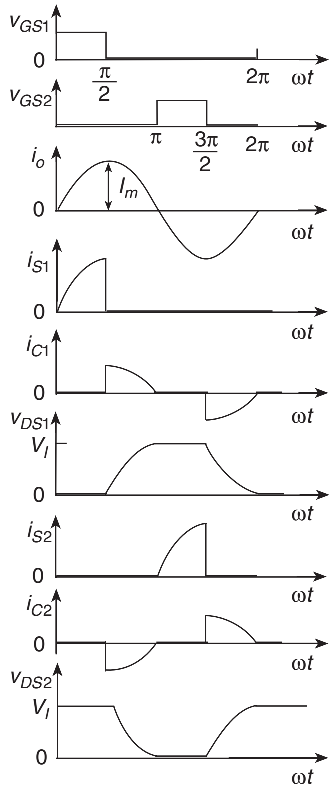

Figure 7.3 Voltage and current waveforms for Class DE RF power amplifier.

where I_m is the amplitude and ϕ is the phase of the output current. From KVL,

$$v_{DS1} + v_{DS2} = V_I. \tag{7.2}$$

(1) For the time interval $0 \le \omega t \le \pi/2$, switch S_1 is ON and switch S_2 is OFF. An equivalent circuit of the Class DE amplifier for this time interval is shown in Figure 7.2(a). The drain current of the bottom transistor is

$$i_{S2} = 0. \tag{7.3}$$

The drain-to-source voltage of the bottom transistor is

$$v_{DS2} = V_I \tag{7.4}$$

resulting in

$$i_{C2} = \omega C_2 \frac{\mathrm{d}v_{DS2}}{\mathrm{d}(\omega t)} = \omega C_2 \frac{\mathrm{d}V_I}{\mathrm{d}(\omega t)} = 0. \tag{7.5}$$

The voltage across the upper transistor is

$$v_{DS1} = 0 \tag{7.6}$$

yielding

$$i_{C1} = \omega C_1 \frac{\mathrm{d}v_{DS1}}{\mathrm{d}(\omega t)} = 0. \tag{7.7}$$

The output current i_o flows through switch S_1:

$$i_{S1} = i_o = I_m \sin(\omega t + \phi) \quad \text{for} \quad 0 \le \omega t \le \frac{\pi}{2}. \tag{7.8}$$

(2) For the time interval $\pi/2 \le \omega t \le \pi$, both switches S_1 and S_2 are OFF. An equivalent circuit of the Class DE amplifier for this time interval is shown in Figure 7.2(b). During this time interval, the capacitor C_1 is charged and capacitor C_2 is discharged. Therefore, the voltage v_{DS1} increases from zero to V_I and the voltage v_{DS2} decreases from V_I to zero. From KVL,

$$v_{DS1} = V_I - v_{DS2}. \tag{7.9}$$

Note that

$$\frac{\mathrm{d}v_{DS1}}{\mathrm{d}(\omega t)} = -\frac{\mathrm{d}v_{DS2}}{\mathrm{d}(\omega t)}. \tag{7.10}$$

From KCL,

$$-i_{C1} + i_{C2} = -i_o. \tag{7.11}$$

This leads to

$$-\omega C_1 \frac{\mathrm{d}v_{DS1}}{\mathrm{d}(\omega t)} + \omega C_2 \frac{\mathrm{d}v_{DS2}}{\mathrm{d}(\omega t)} = -I_m \sin(\omega t + \phi) \tag{7.12}$$

resulting in

$$-\omega C_1 \frac{\mathrm{d}(V_I - v_{DS2})}{\mathrm{d}(\omega t)} + \omega C_2 \frac{\mathrm{d}v_{DS2}}{\mathrm{d}(\omega t)} = -I_m \sin(\omega t + \phi). \tag{7.13}$$

Hence,

$$\omega C_1 \frac{\mathrm{d}v_{DS2}}{\mathrm{d}(\omega t)} + \omega C_2 \frac{\mathrm{d}v_{DS2}}{\mathrm{d}(\omega t)} = -I_m \sin(\omega t + \phi). \tag{7.14}$$

Rearrangement of this equation gives

$$\frac{\mathrm{d}v_{DS2}}{\mathrm{d}(\omega t)} = -\frac{I_m}{\omega(C_1 + C_2)} \sin(\omega t + \phi). \tag{7.15}$$

The condition for zero-derivative switching (ZDS) of voltage v_{DS2} at $\omega t = \pi$ is given by

$$\left.\frac{\mathrm{d}v_{DS2}(\omega t)}{\mathrm{d}(\omega t)}\right|_{\omega t = \pi} = 0. \tag{7.16}$$

Imposing this condition on (7.15), we obtain

$$\sin(\pi + \phi) = 0. \tag{7.17}$$

The solutions of this equation are

$$\phi = 0 \tag{7.18}$$

and

$$\phi = \pi. \tag{7.19}$$

Only $\phi = 0$ is the physical solution, which allows the charging of the capacitor C_1 and discharging of the capacitor C_2.

The voltage v_{DS2} is

$$v_{DS2} = -\frac{I_m}{\omega(C_1 + C_2)} \int_{\frac{\pi}{2}}^{\omega t} \sin \omega t \, \mathrm{d}(\omega t) + v_{DS2}\left(\frac{\pi}{2}\right) = \frac{I_m}{\omega(C_1 + C_2)} \cos \omega t + V_I. \tag{7.20}$$

The zero-voltage switching (ZVS) condition of voltage v_{DS2} at $\omega t = \pi$ is given by

$$v_{DS2}(\pi) = 0. \tag{7.21}$$

Imposing this condition on (7.20), we get

$$\frac{I_m}{\omega(C_1 + C_2)} = V_I. \tag{7.22}$$

Hence, the voltage across the bottom switch is

$$v_{DS2} = V_I(\cos \omega t + 1) \tag{7.23}$$

and the voltage across the upper switch is

$$v_{DS1} = V_I - v_{DS2} = -V_I \cos \omega t. \tag{7.24}$$

The current through the upper capacitor is

$$i_{C1} = \frac{i_o}{2} = \frac{1}{2} I_m \sin \omega t \tag{7.25}$$

and the current through the bottom capacitor is

$$i_{C2} = -\frac{i_o}{2} = -\frac{1}{2} I_m \sin \omega t. \tag{7.26}$$

(3) For the time interval $\pi \le \omega t \le 3\pi/2$, the switch S_1 is OFF and the switch S_2 is ON. An equivalent circuit of the Class DE amplifier for this time interval is shown in Figure 7.2(c). The drain-to-source voltage of the bottom transistor is

$$v_{DS2} = 0 \tag{7.27}$$

which gives

$$i_{C2} = \omega C_2 \frac{\mathrm{d}v_{DS2}}{\mathrm{d}(\omega t)} = 0. \tag{7.28}$$

The drain-to-source voltage of the upper transistor is

$$v_{DS1} = V_I \tag{7.29}$$

producing

$$i_{C1} = \omega C_1 \frac{\mathrm{d}v_{DS1}}{\mathrm{d}(\omega t)} = \omega C_1 \frac{\mathrm{d}V_I}{\mathrm{d}(\omega t)} = 0. \tag{7.30}$$

The load current is

$$i_o = I_m \sin \omega t \qquad \text{for} \qquad \pi \le \omega t \le \frac{3\pi}{2}. \tag{7.31}$$

Therefore, the current through the switch S_2 is

$$i_{S2} = -i_o = -I_m \sin \omega t \qquad \text{for} \qquad \pi \le \omega t \le \frac{3\pi}{2}. \tag{7.32}$$

(4) For the time interval $3\pi/2 \le \omega t \le 2\pi$, both switches are OFF. An equivalent circuit of the Class DE amplifier for this time interval is shown in Figure 7.2(d). From KCL,

$$i_{C1} - i_{C2} = i_o = I_m \sin \omega t. \tag{7.33}$$

Hence,

$$\omega C_1 \frac{dv_{DS1}}{d(\omega t)} - \omega C_2 \frac{dv_{DS2}}{d(\omega t)} = I_m \sin \omega t. \tag{7.34}$$

Since

$$v_{DS2} = V_I - v_{DS1} \tag{7.35}$$

we have

$$\omega C_1 \frac{dv_{DS1}}{d(\omega t)} - \omega C_2 \frac{d(V_I - v_{DS1})}{d(\omega t)} = I_m \sin \omega t \tag{7.36}$$

which gives

$$\omega C_1 \frac{dv_{DS1}}{d(\omega t)} + \omega C_2 \frac{dv_{DS1}}{d(\omega t)} = I_m \sin \omega t. \tag{7.37}$$

Thus,

$$\frac{dv_{DS1}}{d(\omega t)} = \frac{I_m}{\omega (C_1 + C_2)} \sin \omega t \tag{7.38}$$

resulting in

$$v_{DS1} = \frac{I_m}{\omega (C_1 + C_2)} \int_{\pi}^{\omega t} \sin \omega t \, d(\omega t) + v_{DS1}\left(\frac{3\pi}{2}\right) = -\frac{I_m}{\omega (C_1 + C_2)} \cos \omega t + V_I. \tag{7.39}$$

Applying the ZVS condition to the above equation at $\omega t = 2\pi$,

$$v_{DS1}(2\pi) = 0 \tag{7.40}$$

we obtain

$$\frac{I_m}{\omega (C_1 + C_2)} = V_I \tag{7.41}$$

Hence, the voltage across the switch S_1 is

$$v_{DS1} = V_I (1 - \cos \omega t) \tag{7.42}$$

and the voltage across the switch S_2 is

$$v_{DS2} = V_I - v_{DS1} = V_I \cos \omega t. \tag{7.43}$$

The output current and voltage are

$$i_o = I_m \sin \omega t \tag{7.44}$$

and

$$v_o = V_m \sin \omega t \tag{7.45}$$

where $V_m = RI_m$.

The dc component of the current through the shunt capacitance C_1 is zero for steady-state operation. Therefore, the dc input current is

$$I_I = \frac{1}{2\pi} \int_0^{2\pi} i_{S1} d(\omega t) = \frac{1}{2\pi} \int_0^{\frac{\pi}{2}} I_m \sin \omega t \, d(\omega t) = \frac{I_m}{2\pi} = \frac{\omega (C_1 + C_2)}{2\pi} V_I. \tag{7.46}$$

The dc input resistance of the amplifier is

$$R_{I(DC)} = \frac{V_I}{I_I} = \frac{2\pi}{\omega (C_1 + C_2)}. \tag{7.47}$$

7.3 Components

The output voltage is

$$v_o = V_m \sin \omega t \tag{7.48}$$

where $V_m = RI_m$. The fundamental component of the voltage across inductance L is

$$v_{L1} = V_{Lm} \cos \omega t \tag{7.49}$$

where $V_{Lm} = \omega L I_m$. From Fourier analysis, the amplitude of the output voltage is

$$\begin{aligned} V_m &= \frac{1}{\pi}\int_0^{2\pi} v_{DS2} \sin \omega t \, \mathrm{d}(\omega t) \\ &= \frac{1}{\pi}\left[\int_0^{\frac{\pi}{2}} V_I \sin \omega t \, \mathrm{d}(\omega t) + \int_{\frac{\pi}{2}}^{\pi} V_I (\cos \omega t + 1) \sin \omega t \, \mathrm{d}(\omega t) + \int_{\frac{3\pi}{2}}^{2\pi} V_I \cos \omega t \sin \omega t \, \mathrm{d}(\omega t)\right] \\ &= \frac{V_I}{\pi} \end{aligned} \tag{7.50}$$

and

$$\begin{aligned} V_{Lm} &= \frac{1}{\pi}\int_0^{2\pi} v_{DS2} \cos \omega t \, \mathrm{d}(\omega t) \\ &= \frac{1}{\pi}\left[\int_0^{\frac{\pi}{2}} V_I \cos^2 \omega t \, \mathrm{d}(\omega t) + \int_{\frac{\pi}{2}}^{\pi} V_I (\cos \omega t + 1) \cos \omega t \, \mathrm{d}(\omega t) + \int_{\frac{3\pi}{2}}^{2\pi} V_I \cos^2 \omega t (\mathrm{d}\omega t)\right] \\ &= \frac{V_I}{2}. \end{aligned} \tag{7.51}$$

Hence,

$$\frac{V_{Lm}}{V_m} = \frac{\omega L_b}{R} = \frac{\pi}{2}. \tag{7.52}$$

The fundamental component of the voltage v_{DS2} is

$$v_{s1} = v_o + v_{L1} = V_m \sin \omega t + V_{Lm} \cos \omega t = V_I \left(\frac{1}{\pi} \sin \omega t + \frac{1}{2} \cos \omega t\right). \tag{7.53}$$

The output power is

$$P_O = \frac{V_m^2}{2R} = \frac{V_I^2}{2\pi^2 R}. \tag{7.54}$$

Assuming that the efficiency is 100 %, the output power can also be expressed as

$$P_O = P_I = I_I V_I = \frac{\omega(C_1 + C_2)V_I^2}{2\pi}. \tag{7.55}$$

Therefore,

$$\omega(C_1 + C_2)R = \frac{1}{\pi}. \tag{7.56}$$

Assuming that $C_1 = C_2$,

$$\omega C_1 R = \omega C_2 R = \frac{1}{2\pi}. \tag{7.57}$$

The loaded-quality factor is defined as

$$Q_L = \frac{\omega L}{R}. \tag{7.58}$$

Hence,

$$L_a = L - L_b = \left(Q_L - \frac{\pi}{2}\right)\frac{R}{\omega}. \tag{7.59}$$

Since $\omega^2 = 1/(L_a C)$,

$$C = \frac{1}{\omega^2 L_a} = \frac{1}{\omega R(Q_L - \frac{\pi}{2})}. \tag{7.60}$$

7.4 Device Stresses

The maximum drain current is given by

$$I_{DMmax} = I_{m(max)} = \frac{V_m}{R} = \frac{V_I}{\pi R}. \tag{7.61}$$

The maximum drain-to-source voltage is

$$V_{DSmax} = V_I. \tag{7.62}$$

The maximum voltage across the series capacitor C is

$$V_{Cm(max)} = \frac{I_{m(max)}}{\omega C}. \tag{7.63}$$

The maximum voltage across the series inductor L is

$$V_{Lm(max)} = \omega L I_{m(max)}. \tag{7.64}$$

7.5 Design Equations

The design equations for the component values are:

$$R = \frac{V_I^2}{2\pi^2 P_O} \tag{7.65}$$

$$C_1 = \frac{1}{2\pi\omega R} = \frac{\pi P_O}{\omega V_I^2} \tag{7.66}$$

$$L = \frac{Q_L R}{\omega} \tag{7.67}$$

$$C = \frac{1}{\omega R(Q_L - \frac{\pi}{2})}. \tag{7.68}$$

7.6 Maximum Operating Frequency

There is a maximum operating frequency f_{max} at which both the ZVS and ZDS conditions are satisfied. This frequency is limited by the transistor output capacitance C_o and is determined by

$$C_o = C_1 = C_2 = \frac{1}{4\pi^2 R f_{max}} \tag{7.69}$$

resulting in

$$f_{max} = \frac{1}{4\pi^2 R C_o} = \frac{P_O}{2C_o V_I^2} \tag{7.70}$$

where C_o is the transistor output capacitance.

Example 7.1

Design a Class DE RF power amplifier to meet the following specifications: $P_O = 0.25$ W, $V_I = 3.3$ V, and $f = 1$ GHz. Assume that the transistor output capacitance C_o is linear and is equal to 1.5 pF.

Solution. Assuming $Q_L = 10$, we can calculate the component values as follows:

$$R = \frac{V_I^2}{2\pi^2 P_O} = \frac{3.3^2}{2\pi^2 \times 0.25} = 2.2\Omega \tag{7.71}$$

$$C_1 = \frac{1}{2\pi\omega R} = \frac{\pi P_O}{\omega V_I^2} = \frac{\pi \times 0.25}{2\pi \times 10^9 \times 3.3^2} = 11.5\,\text{pF} \tag{7.72}$$

$$C_{1(ext)} = C_1 - C_o = 11.5 - 1.5 = 10\,\text{pF} \tag{7.73}$$

$$L = \frac{Q_L R}{\omega} = \frac{10 \times 2.2}{2\pi \times 10^9} = 3.5\,\text{nH} \tag{7.74}$$

and

$$C = \frac{1}{\omega R(Q_L - \frac{\pi}{2})} = \frac{1}{2\pi \times 10^9 \times 2.2 \times (10 - \frac{\pi}{2})} = 8.58\,\text{pF}. \tag{7.75}$$

These components have low values and can be integrated. The output network may need a matching circuit.

The resonant frequency of the series-resonant circuit is

$$f_o = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{3.5 \times 10^{-9} \times 8.58 \times 10^{-12}}} = 0.9184\,\text{GHz}. \tag{7.76}$$

The ratio of the operating frequency to the resonant frequency is

$$\frac{f}{f_o} = \frac{1}{0.9184} = 1.0931. \tag{7.77}$$

Thus, the operation is well above the resonance.

Assuming the amplifier efficiency $\eta = 0.94$, the dc supply power is

$$P_I = \frac{P_O}{\eta} = \frac{0.25}{0.94} = 0.266\,\text{W} \tag{7.78}$$

and the dc supply current is

$$I_I = \frac{P_I}{V_I} = \frac{0.266}{3.3} = 0.08\,\text{A}. \tag{7.79}$$

The amplitude of the output voltage is

$$V_m = \sqrt{2P_O R} = \sqrt{2 \times 0.25 \times 2.2} = 1.0488\,\text{V} \tag{7.80}$$

The amplitude of the output current I_m and the MOSFET current stress I_{SMmax} are

$$I_{SMmax} = I_{m(max)} = \frac{V_m}{R} = \frac{1.0488}{2.2} = 0.4767\,\mathrm{A}. \tag{7.81}$$

The voltage stress of the MOSFETs is

$$V_{SM} = V_I = 3.3\,\mathrm{V}. \tag{7.82}$$

The MOSFET should have $V_{DSS} = 5\,\mathrm{V}$ and $I_{DSmax} = 1\,\mathrm{A}$. The maximum voltage across the series capacitor C is

$$V_{Cmax} = \frac{I_{m(max)}}{\omega C} = \frac{0.4767}{2\pi \times 10^9 \times 8.58 \times 10^{-12}} = 8.84\,\mathrm{V}. \tag{7.83}$$

7.7 Class DE Amplifier with Only One Shunt Capacitor

Figure 7.4 shows the two circuits of Class DE power amplifier with only one shunt capacitor. A single shunt capacitance C_s can be connected in parallel with the bottom transistor as in Figure 7.4(a) or in parallel with the upper transistor as shown in Figure 7.4(b). Figure 7.5 shows equivalent circuits for the Class DE power amplifier with only one capacitor connected in parallel with the bottom transistor. Voltage and current waveforms in the Class

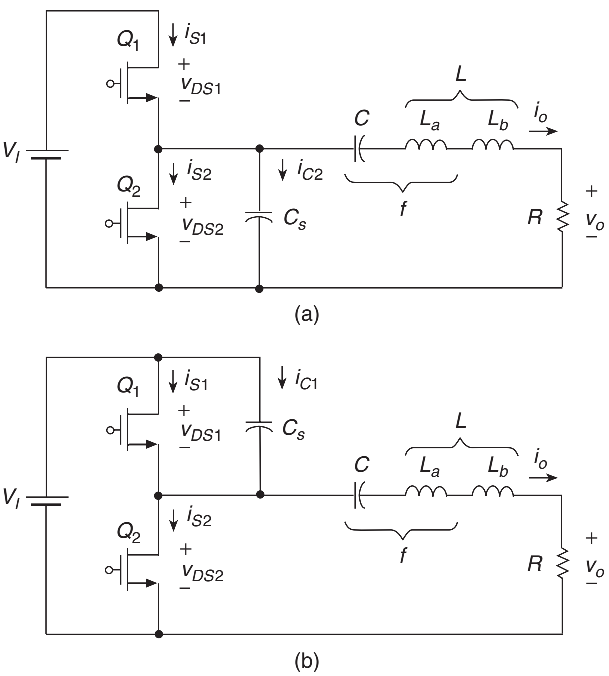

Figure 7.4 Class DE RF power amplifiers with only one shunt capacitance. (a) Shunt capacitance across the bottom transistor. (b) Shunt capacitance across the upper transistor.

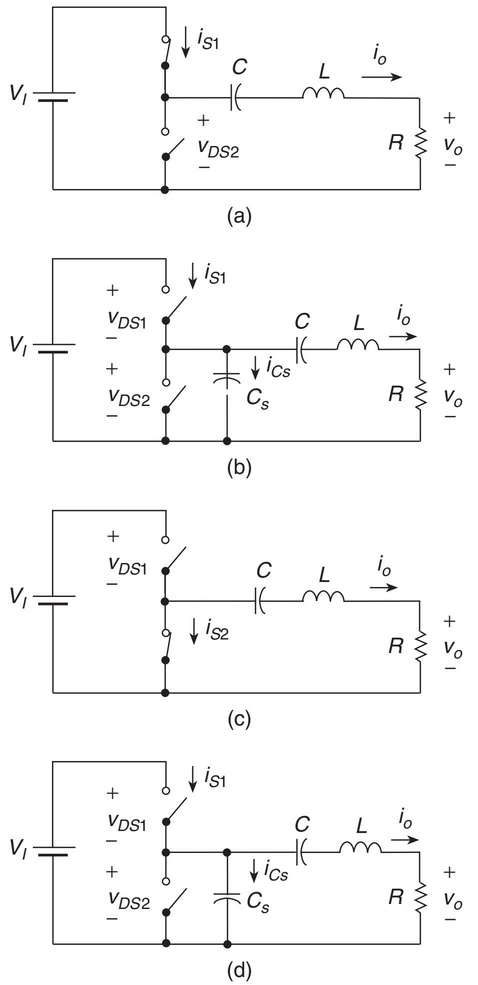

Figure 7.5 Equivalent circuits for Class DE RF power amplifiers with only one shunt capacitance connected in parallel with the bottom transistor. (a) S_1 is ON and S_2 is OFF. (b) Both transistors are OFF. (c) S_1 is OFF and S_2 is ON. (d) Both transistors are OFF.

DE power amplifier with only one capacitor connected in parallel with the bottom transistor are depicted in Figure 7.6.

(1) For $0 < \omega t \leq \pi/2$, the switch S_1 is ON and the switch S_2 is OFF. The equivalent circuit for this time interval is depicted in Figure 7.5(a). The analysis for this time interval is the same as that already presented for the case with two shunt capacitances C_1 and C_2.

(2) For $\pi/2 < \omega t \leq \pi$, both switches S_1 and S_2 are OFF. The equivalent circuit for this time interval is depicted in Figure 7.5(b). The current through the shunt capacitor is

$$i_{Cs} = \omega C_s \frac{dv_{DS2}}{d(\omega t)} = -i_o = -I_m \sin(\omega t + \phi). \tag{7.84}$$

Using the ZDS condition, $\phi = 0$. The output current waveform is

$$i_o = I_m \sin \omega t. \tag{7.85}$$

The current through the shunt capacitor is

$$i_{Cs} = -i_o = -I_m \sin \omega t. \tag{7.86}$$

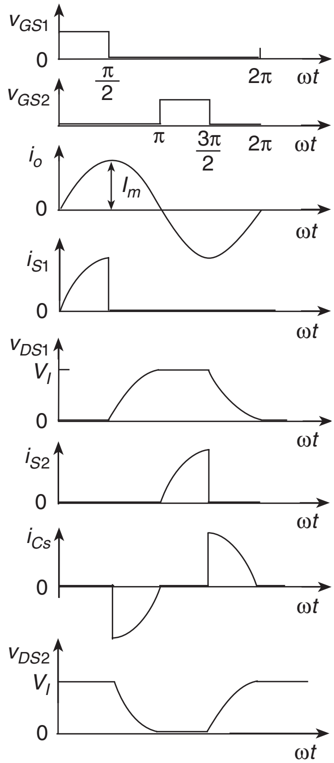

Figure 7.6 Voltage and current waveforms in Class DE RF power amplifiers with only one shunt capacitance connected in parallel with the bottom transistor.

The drain-to-source voltage v_{DS2} is given by

$$v_{DS2} = \frac{1}{\omega C_s} \int_{\frac{\pi}{2}}^{\omega t} i_{Cs} \mathrm{d}(\omega t) + V_I = -\frac{I_m}{\omega C_s} \int_{\frac{\pi}{2}}^{\omega t} \sin \omega t \, \mathrm{d}(\omega t) + V_I = \frac{I_m}{\omega C_s} \cos \omega t + V_I. \tag{7.87}$$

Imposing the ZVS condition, we obtain

$$\frac{I_m}{\omega C_s} = V_I. \tag{7.88}$$

Hence,

$$v_{DS2} = V_I(\cos \omega t + 1) \tag{7.89}$$

and

$$v_{DS1} = V_I - v_{DS2} = -V_I \cos \omega t. \tag{7.90}$$

(3) For $\pi < \omega t \leq 3\pi/2$, the switch S_1 is OFF and the switch S_2 is ON. The equivalent circuit for this time interval is depicted in Figure 7.5(c). The analysis for this time interval is the same as that already presented for the case with two shunt capacitances C_1 and C_2.

(4) For $3\pi/2 < \omega t \leq 2\pi$, switches S_1 and S_2 are OFF. The equivalent circuit for this time interval is depicted in Figure 7.5(d). The current through the shunt capacitor is

$$i_{Cs} = \omega C_s \frac{\mathrm{d}v_{DS2}}{\mathrm{d}(\omega t)} = -i_o = -I_m \sin(\omega t + \phi). \tag{7.91}$$

The drain-to-source voltage v_{DS2} is given by

$$v_{DS2} = \frac{1}{\omega C_s} \int_{\frac{\pi}{2}}^{\omega t} i_{Cs} \mathrm{d}(\omega t) = -\frac{I_m}{\omega C_s} \int_{\frac{\pi}{2}}^{\omega t} \sin \omega t \, \mathrm{d}(\omega t) = \frac{I_m}{\omega C_s} \cos \omega t + V_I. \tag{7.92}$$

Imposing the ZVS condition, we obtain

$$\frac{I_m}{\omega C_s} = V_I. \tag{7.93}$$

Hence,

$$v_{DS2} = V_I(\cos \omega t + 1) \tag{7.94}$$

and

$$v_{DS1} = V_I - v_{DS2} = -V_I \cos \omega t. \tag{7.95}$$

The dc input current is

$$I_I = \frac{1}{2\pi} \int_0^{2\pi} i_{S1} \mathrm{d}(\omega t) = \frac{1}{2\pi} \int_0^{\frac{\pi}{2}} I_m \sin \omega t \, \mathrm{d}(\omega t) = \frac{I_m}{2\pi} = \frac{\omega C_s}{2\pi} V_I. \tag{7.96}$$

The dc input resistance of the amplifier is

$$R_{I(DC)} = \frac{V_I}{I_I} = \frac{2\pi}{\omega C_s}. \tag{7.97}$$

7.8 Components

The output power is

$$P_O = \frac{V_m^2}{2R} = \frac{V_I^2}{2\pi^2 R} \tag{7.98}$$

Neglecting power losses, the efficiency is 100 %. The output power can be expressed as

$$P_O = P_I = I_I V_I = \frac{\omega C_s V_I^2}{\pi}. \tag{7.99}$$

Hence,

$$\omega C_s R = \frac{1}{\pi}. \tag{7.100}$$

Thus,

$$C_s = 2C_1 = \frac{1}{\pi \omega R}. \tag{7.101}$$

The single-shunt capacitance external to the transistors is given by

$$C_{s(ext)} = C_s - 2C_o = 2C_{1(ext)} \tag{7.102}$$

where C_o is the transistor output capacitance and $C_{s(ext)}$ is the external shunt capacitance. In the Example 7.1, the two external shunt capacitances $C_{1(ext)} = 10\,\text{pF}$ can be replaced by a single-shunt capacitance $C_{s(ext)} = 2C_{1(ext)} = 20\,\text{pF}$.

7.9 Cancellation of Nonlinearities of Transistor Output Capacitances

The transistor output capacitance C_o is nonlinear. The output capacitance C_o is large when the drain-to-source voltage v_{DS} is low, and it is low when the drain-to-source voltage v_{DS} is high. The total voltage across both transistor output capacitances is constant

$$v_{DS1} + v_{DS2} = V_I. \tag{7.103}$$

In the Class DE amplifier, the transistor capacitances are connected in parallel for the ac component [1]. When the voltage v_{DS2} across the lower output capacitance C_{o2} is low, the capacitance C_{o2} is high. At the same time, the voltage v_{DS1} is high and C_{o1} is low. Therefore, there is a partial cancellation of the nonlinearities of the total transistor output capacitance $C_{ot} = C_{o1} + C_{o2}$. The parallel combination of the shunt capacitance is approximately constant. For this reason, the expressions for load network components and the output power of the Class DE RF power amplifier are not affected by the nonlinearity of the transistor output capacitances [10, 11]. The shunt capacitances can be composed of only nonlinear transistor output capacitances or can be the combinations of nonlinear transistor output capacitances and linear external capacitances.

7.10 Summary

- The Class DE RF amplifier consists of two transistors, a series-resonant circuit, and shunt capacitors.
- The transistors in the Class DE amplifier are operated as switches.
- There is a dead time in the gate-to-source voltages during which the drain-to-source voltage of one transistor goes from high to low, and the drain-to-source voltage of the other transistor goes from low to high, and vice versa.
- The transistor output capacitances are absorbed into the shunt capacitances.
- Switching losses in the Class DE RF power amplifier are zero due to ZVS operation, like in the Class E amplifier.
- The voltage stress of power MOSFETs is low and equal to the supply voltage V_I, like in the Class D amplifier.
- The nonlinearity of the transistor output capacitances does affect the values of the load network components and the output power.
- The Class DE power amplifier can be implemented with only one shunt capacitor and the output capacitances of the transistors can be included in the circuit topology.

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7.12 Review Questions

7.1 How large are the switching losses in the Class DE RF power amplifier?

7.2 How large are the current and voltage stresses in the Class DE power amplifier?

7.3 How many transistors are used in the Class DE amplifier?

7.4 How are the transistors driven in the Class DE amplifier?

7.5 Can the Class DE power amplifier operate under ZVS condition?

7.6 Can the Class DE power amplifier operate under ZDS condition?

7.7 Is there any limitation for the operating frequency of the Class DE power amplifier?

7.8 How does the nonlinearity of the transistor output capacitances affect the values of the load network components and the output power of the Class DE power amplifier?

7.13 Problems

7.1 Design a Class DE amplifier to meet the following specifications: $V_I = 5\,\text{V}$, $P_O = 1\,\text{W}$, and $f = 4\,\text{GHz}$. Assume that the output capacitances of the transistors are linear and equal to 1 pF.

7.2 Design a Class DE amplifier with a single shunt capacitance to meet the following specifications: $V_I = 5\,\text{V}$, $P_O = 1\,\text{W}$, and $f = 4\,\text{GHz}$. Assume that the output capacitances of the transistors are linear and equal to 1 pF.

8

Class F RF Power Amplifier

8.1 Introduction

Class F RF power amplifiers [1–24] utilize multiple-harmonic resonators in the output network to shape the drain-to-source voltage such that the transistor loss is reduced and the efficiency is increased. These circuits are also called polyharmonic or multi-resonant power amplifiers. The drain current flows when the drain-to-source voltage is flat and low, and the drain-to-source voltage is high when the drain current is zero. Therefore, the product of the drain current and the drain-to-source voltage is low, reducing the power dissipation in the transistor. This method of improving the efficiency is the oldest technique and was invented by Tyler in 1919 [1]. Class F power amplifiers with lumped-element resonant circuits tuned to the third harmonic or to the third and fifth harmonics have been widely used in high-power amplitude-modulated (AM) broadcast radio transmitters in the LF range (30–300 kHz), the MF range (0.3–3 MHz), and the HF range (3–30 MHz). Class F power amplifiers with a quarter-wavelength transmission line controls all the odd harmonics and are used in VHF (30–300 MHz) frequency-modulated (FM) broadcast radio transmitters [1]. They are also used in UHF (300 MHz to 3 GHz) FM broadcast radio transmitters [2]. Dielectric resonators can be used in place of lumped-element resonant circuits. Output power of 40 W has been achieved at 11 GHz with an efficiency of 77 %. In this chapter, we will present Class F power amplifier circuits, principle of operation, analysis, and design example.

There are two groups of Class F RF power amplifiers:

- odd harmonic Class F power amplifiers;
- even harmonic Class F power amplifiers.

In Class F amplifiers with odd harmonics, the drain-to-source voltage contains only odd harmonics and the drain current contains only even harmonics. Therefore, the input impedance of the load network represents an open circuit at odd harmonics and a short circuit at even harmonics. The drain-to-source voltage v_{DS} of Class F amplifiers with odd harmonics is

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symmetrical for the lower and upper half of the cycle. In general, the drain-to-source voltage v_{DS} of Class F amplifiers with odd harmonics is given by

$$v_{DS} = V_I - V_m \cos \omega_o t + \sum_{n=3,5,7,\ldots}^{\infty} V_{mn} \cos n\omega_o t \tag{8.1}$$

and the drain current is

$$i_D = I_I + I_m \cos \omega_o t + \sum_{n=2,4,6,\ldots}^{\infty} I_{mn} \cos n\omega_o t. \tag{8.2}$$

In Class F amplifiers with even harmonics, the drain-to-source voltage contains only even harmonics and the drain current contains only odd harmonics. Hence, the load network represents an open circuit at even harmonics and a short circuit at odd harmonics. The input impedance of the load network at each harmonic frequency is either zero or infinity. The drain-to-source voltage v_{DS} of Class F amplifiers is not symmetrical for the lower and upper half of the cycle.

The drain-to-source voltage v_{DS} of Class F amplifiers with even harmonics is given by

$$v_{DS} = V_I - V_m \cos \omega_o t + \sum_{n=2,4,6,\ldots}^{\infty} V_{mn} \cos n\omega_o t \tag{8.3}$$

and the drain current is

$$i_D = I_I + I_m \cos \omega_o t + \sum_{n=3,5,7,\ldots}^{\infty} I_{mn} \cos n\omega_o t. \tag{8.4}$$

No real power is generated at harmonics because there is either no current or no voltage present at each harmonic frequency. There are finite and infinite order Class F power amplifiers.

Two particular categories of high-efficiency Class F power amplifiers can be distinguished:

- Class F power amplifiers with maximally flat drain-to-source voltage;
- Class F power amplifiers with maximum drain efficiency.

For Class F amplifiers with symmetrical drain-to-source voltage v_{DS}, the maximally flat voltage occurs at two values of $\omega_o t$, at the bottom part of the waveform and the top part of the waveform. For Class F amplifiers with unsymmetrical drain-to-source voltage v_{DS}, the maximally flat voltage occurs only at one value of $\omega_o t$ at the bottom part of the waveform. All the derivatives of voltage v_{DS} are zero at $\omega_0 t$ at which the voltage v_{DS} is maximally flat.

8.2 Class F RF Power Amplifier with Third Harmonic

The circuit of the Class F RF power amplifier with a third harmonic resonator [1] is shown in Figure 8.1. This circuit is called the Class F_3 power amplifier. It consists of a transistor, load network, and RF choke (RFC). The load network consists of a parallel-resonant LC circuit tuned to the operating frequency f_o and a parallel-resonant circuit tuned to the third harmonic $3f_o$. The two resonant circuits are connected in series. The ac power is delivered to the load resistance R.

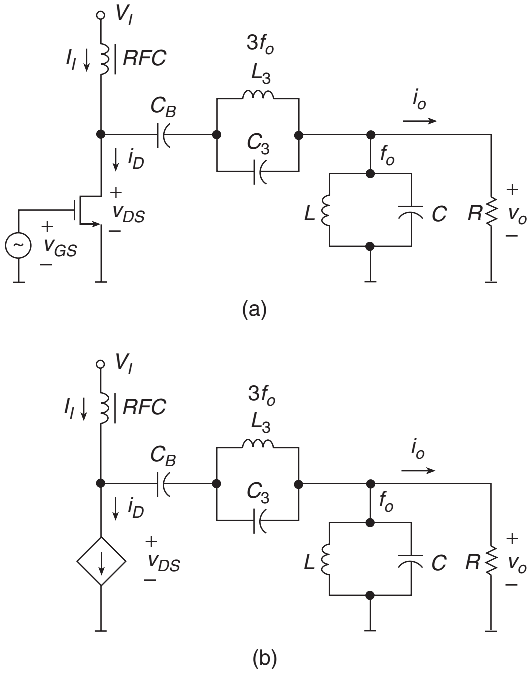


Figure 8.1 Class F_3 power amplifier with a third harmonic resonator. (a) Circuit. (b) Equivalent circuit.

Figure 8.2 shows the voltage and current waveforms of the Class F power amplifier with a third harmonic. The drain current waveform may have any conduction angle $2\theta < 2\pi$, which contains higher harmonics. In most applications, the conduction angle is $\theta = \pi$, like in the Class B amplifier. In this case, the drain current is a half-sine wave given by

$$i_D = \begin{cases} I_{DM}\cos\omega_o t & \text{for} \quad -\dfrac{\pi}{2} < \omega_o t \le \dfrac{\pi}{2} \\ 0 & \text{for} \quad \dfrac{\pi}{2} < \omega_o t \le \dfrac{3\pi}{2}. \end{cases} \tag{8.5}$$

where I_{DM} is the peak value of the drain current, which occurs at $\omega t = 0$. Other values of the conduction angle θ are also possible.

The output resonant circuit tuned to the fundamental frequency f_o acts like a band-pass filter and filters out all the harmonics of the drain current. Therefore, the output voltage waveform is sinusoidal

$$v_o = -V_m\cos\omega_o t. \tag{8.6}$$

The fundamental component of the drain-to-source voltage is

$$v_{ds1} = v_o = -V_m\cos\omega_o t. \tag{8.7}$$

The voltage across the parallel-resonant circuit tuned to the third harmonic $3f_o$ is

$$v_{ds3} = V_{m3}\cos 3\omega_o t. \tag{8.8}$$

The third harmonic voltage waveform is 180° out of phase with respect to the fundamental frequency voltage v_{ds1}. The voltages at all other harmonic frequencies are zero. The drain-to-source voltage waveform is given by

$$v_{DS} = V_I + v_{ds1} + v_{ds3} = V_I - V_m\cos\omega_o t + V_{m3}\cos 3\omega_o t. \tag{8.9}$$

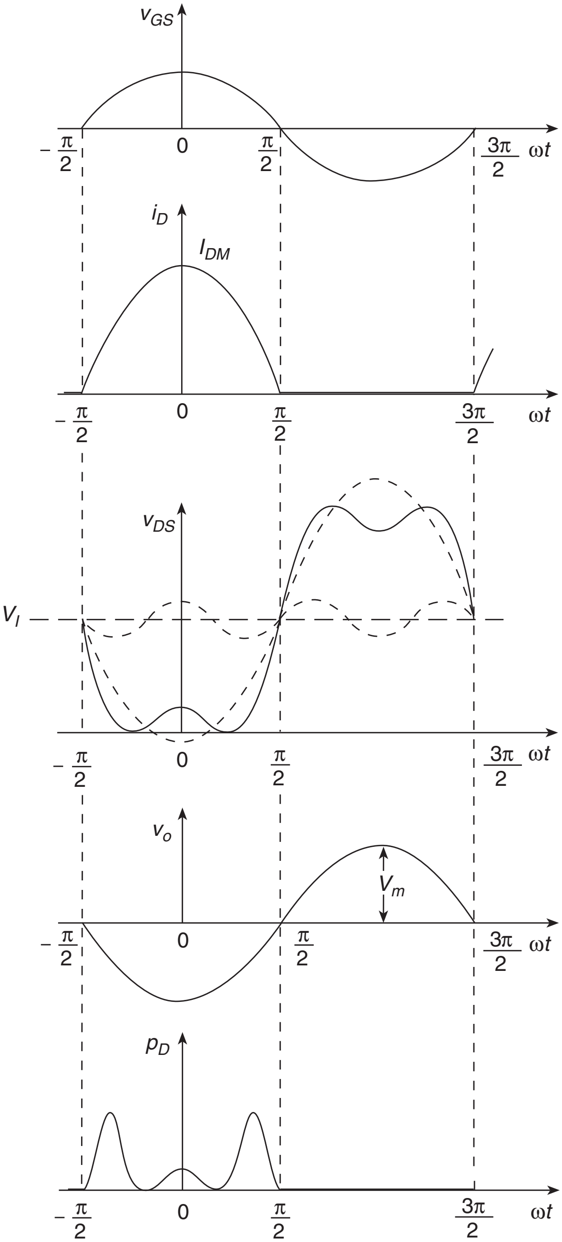


Figure 8.2 Waveforms in Class F RF power amplifier with a third harmonic peaking.

The third harmonic flattens the drain-to-source voltage v_{DS} so that the drain current i_D flows when the drain-to-source voltage v_{DS} is low, as shown in Figure 8.2. When the drain-to-source voltage v_{DS} is high, the drain current is zero. Therefore, the average value of the drain voltage-current product is reduced. This results in both high drain efficiency η_D and high output-power capability c_p.

When the third harmonic is added to the fundamental component, the peak-to-peak swing $2V_{pk}$ of the ac component of the composed drain-to-source voltage waveform is reduced, and the peak value of the resulting ac component of the drain-to-source voltage V_{pk} is lower than the amplitude of the fundamental component V_m. The waveform of v_{DS} changes as the ratio V_{m3}/V_m increases.

(1) For $0 < V_{m3}/V_m < 1/9$, the resulting waveform of v_{DS} has a single valley at $\omega_o t = 0$ and a single peak at $\omega t = \pi$. In other words, the minimum value of v_{DS} occurs at $\omega_o t = 0$ and the maximum value of v_{DS} occurs at $\omega_o t = \pi$. The peak value of the ac component of v_{DS} is

$$V_{pk} = V_I - V_{DSmin} = V_I - v_{DS}(0) = V_I - V_I + V_m - V_{m3} = V_m - V_{m3}. \quad (8.10)$$

For an ideal transistor, $V_{DSmin} = 0$ and $V_{pk} = V_I$. Hence,

$$\frac{V_m}{V_{pk}} = \frac{V_m}{V_I} = \frac{V_m}{V_m - V_{m3}} = \frac{1}{1 - \dfrac{V_{m3}}{V_m}}. \quad (8.11)$$

As V_{m3}/V_m increases, V_m/V_I also increases.

(2) At $V_{m3}/V_m = 1/9$, the waveform of v_{DS} is maximally flat. The minimum value of v_{DS} occurs at $\omega_o t = 0$ and the maximum value of v_{DS} occurs at $\omega_o t = \pi$.

(3) For $V_{m3}/V_m > 1/9$, the waveform of v_{DS} exhibits a ripple with double peaks, as shown in Figure 8.2. There is a local maximum value of v_{DS} at $\omega_o t = 0$ and two local adjacent minimum values. In addition, there is a local minimum value of v_{DS} at $\omega_o t = \pi$ and two local adjacent maximum values.

(4) As V_{m3}/V_m increases from 1/9, the ratio V_m/V_I reaches its maximum value at $V_{m3}/V_m = 1/6$ and is given by

$$\frac{V_m}{V_{pk}} = \frac{V_m}{V_I} = \frac{2}{\sqrt{3}}. \quad (8.12)$$

(5) For $V_{m3}/V_m > 1/6$, the waveform of v_{DS} has double valleys and a peak at low voltages and double peaks and one valley at high voltages. The ratio V_m/V_{pk} decreases as V_{m3}/V_m increases.

It is possible to increase the ratio V_m/V_I by adding a small amount of third harmonic, while maintaining $v_{DS} \geq 0$. When V_I and I_I are held constant, P_I is constant. As V_m increases, $P_O = V_m I_m/2$ increases, and therefore $\eta_D = P_O/P_I$ also increases by the same factor by which V_m is increased.

8.2.1 Maximally Flat Class F$_3$ Amplifier

In order to find the maximally flat waveform of v_{DS}, we determine its derivative and set the result to be equal to zero

$$\frac{dv_{DS}}{d(\omega_o t)} = V_m \sin\omega_o t - 3V_{m3}\sin 3\omega_o t = V_m \sin\omega_o t - 3V_{m3}(3\sin\omega_o t - 4\sin^3\omega_o t)$$
$$= \sin\omega_o t(V_m - 9V_{m3} + 12V_{m3}\sin^2\omega_o t) = 0. \quad (8.13)$$

One solution of this equation at non-zero values of V_m and V_{m3} is

$$\sin\omega_o t_m = 0 \quad (8.14)$$

which gives the location of one extremum (a minimum or a maximum) of v_{DS} at

$$\omega_o t_m = 0 \tag{8.15}$$

and the other extremum of v_{DS} at

$$\omega_o t_m = \pi. \tag{8.16}$$

The other solutions are

$$\sin \omega_o t_m = \pm\sqrt{\frac{9V_{m3} - V_m}{12V_{m3}}} \quad \text{for} \quad V_{m3} \geq \frac{V_m}{9}. \tag{8.17}$$

The real solutions of this equation do not exist for $V_{m3} < V_m/9$. For $\omega_o t > V_m/9$, there are two minimum values of v_{DS} at

$$\omega_o t_m = \pm \arcsin\sqrt{\frac{9V_{m3} - V_m}{12V_{m3}}} \tag{8.18}$$

and two maximum values of v_{DS} at

$$\omega_o t_m = \pi \pm \arcsin\sqrt{\frac{9V_{m3} - V_m}{12V_{m3}}}. \tag{8.19}$$

For

$$\frac{V_{m3}}{V_m} = \frac{1}{9} \tag{8.20}$$

the location of three extrema of v_{DS} converges to 0 and the location of the other three extrema converges to π. In this case, the waveform of v_{DS} is maximally flat at $\omega_o t = 0$ and at $\omega_o t = \pi$.

For the symmetrical and maximally flat drain-to-source voltage v_{DS}, the minimum value of v_{DS} occurs at $\omega_o t = 0$ and the maximum value occurs at $\omega_o t = \pi$. The Fourier coefficients for the maximally flat waveform of v_{DS} can be derived by setting all the derivatives of v_{DS} to zero. The first and the second-order derivatives of voltage v_{DS} are

$$\frac{\mathrm{d}v_{DS}}{\mathrm{d}(\omega_o t)} = V_m \sin \omega_o t - 3V_{m3} \sin 3\omega_o t \tag{8.21}$$

and

$$\frac{\mathrm{d}^2 v_{DS}}{\mathrm{d}(\omega_o t)^2} = V_m \cos \omega_o t - 9V_{m3} \cos 3\omega_o t. \tag{8.22}$$

The first-order derivative is zero at $\omega_o t = 0$ and $\omega_o t = \pi$, but it does not generate an equation because $\sin 0 = \sin \pi = 0$. From the second-order derivative of v_{DS} at $\omega_o t = 0$,

$$\left.\frac{\mathrm{d}^2 v_{DS}}{\mathrm{d}(\omega_o t)^2}\right|_{\omega_o t = 0} = V_m - 9V_{m3} = 0. \tag{8.23}$$

Thus, a maximally flat waveform of the drain-to-source voltage occurs for

$$\frac{V_{m3}}{V_m} = \frac{1}{9}. \tag{8.24}$$

Idealy, the minimum drain-to-source voltage is

$$V_{DSmin} = v_{DS}(0) = V_I - V_m + V_{m3} = V_I - V_m + \frac{V_m}{9} = V_I - \frac{8}{9}V_m = 0 \tag{8.25}$$

producing the maximum amplitude of the output voltage

$$V_m = \frac{9}{8}V_I. \tag{8.26}$$

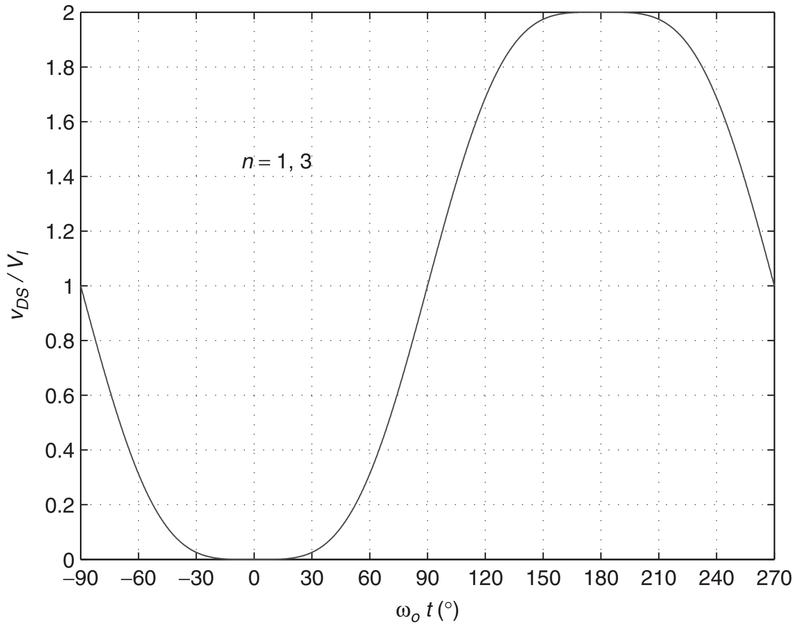


Figure 8.3 Maximally flat waveform of the normalized drain-to-source voltage v_{DS}/V_I for Class F RF power amplifier with third harmonic peaking.

Similarly,

$$V_{DSmin} = v_{DS}(0) = V_I - V_m + V_{m3} = V_I - 9V_{m3} + V_{m3} = V_I - 8V_{m3} = 0 \tag{8.27}$$

yielding the amplitude of the third harmonic voltage

$$V_{m3} = \frac{V_I}{8}. \tag{8.28}$$

The maximum drain-to-source voltage is

$$V_{DSM} = v_{DS}(\pi) = V_I + V_m - V_{m3} = V_I + \frac{9}{8}V_I - \frac{1}{8}V_I = 2V_I. \tag{8.29}$$

The normalized maximally flat waveform of v_{DS} is

$$\frac{v_{DS}}{V_I} = 1 - \frac{V_m}{V_I}\cos\omega_o t + \frac{V_{m3}}{V_I}\cos 3\omega_o t = 1 - \frac{9}{8}\cos\omega_o t + \frac{1}{8}\cos 3\omega_o t. \tag{8.30}$$

Figure 8.3 shows the maximally flat waveform of the normalized drain-to-source voltage v_{DS}/V_I with the third harmonic peaking.

The fundamental component of the load current is equal to the drain current

$$i_o = i_{d1} = I_m \cos\omega_o t \tag{8.31}$$

where

$$I_m = \frac{1}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} i_D \cos\omega_o t \, \mathrm{d}(\omega_o t) = \frac{1}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} I_{DM}\cos^2\omega_o t \, \mathrm{d}(\omega_o t) = \frac{I_{DM}}{2} \tag{8.32}$$

or

$$I_m = \frac{V_m}{R} = \frac{9(V_I - V_{DSmin})}{8R}. \tag{8.33}$$

The dc input current is equal to the component of the drain current

$$I_I = \frac{1}{2\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} i_D \, d(\omega_o t) = \frac{1}{2\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} I_{DM}\cos\omega_o t \, d(\omega_o t) = \frac{I_{DM}}{\pi} = \frac{2}{\pi}I_m = \frac{9(V_I - V_{DSmin})}{4\pi R}. \tag{8.34}$$

The dc input power is given by

$$P_I = V_I I_I = \frac{9V_I(V_I - V_{DSmin})}{4\pi R} = \frac{9V_I(V_I - V_{DSmin})}{4\pi R}. \tag{8.35}$$

The drain current i_D contains the dc component, fundamental component, and even harmonics. The drain-to-source voltage v_{DS} contains the dc component, fundamental component, and third harmonic. Therefore, the output power contains only the fundamental component

$$P_O = \frac{1}{2}I_m V_m = \frac{1}{2} \times \frac{I_{DM}}{2} \times \frac{9}{8}(V_I - V_{DSmin}) = \frac{9}{32}I_{DM}(V_I - V_{DSmin}) \tag{8.36}$$

or

$$P_O = \frac{V_m^2}{2R} = \frac{81(V_I - V_{DSmin})^2}{128R} = 0.6328\frac{(V_I - V_{DSmin})^2}{R}. \tag{8.37}$$

Hence, the drain efficiency of the amplifier is

$$\eta_D = \frac{P_O}{P_I} = \frac{1}{2}\left(\frac{I_m}{I_I}\right)\left(\frac{V_m}{V_I}\right) = \frac{1}{2} \times \frac{\pi}{2} \times \left(\frac{V_m}{V_I}\right) = \frac{\pi}{4}\frac{V_m}{V_I} = \frac{\pi}{4} \times \frac{9}{8}\left(1 - \frac{V_{DSmin}}{V_I}\right)$$
$$= \frac{9\pi}{32}\left(1 - \frac{V_{DSmin}}{V_I}\right) = 0.8836\left(1 - \frac{V_{DSmin}}{V_I}\right). \tag{8.38}$$

The ratio of the drain efficiency $\eta_{D(F3)}$ for the Class F amplifier with the third harmonic to the efficiency of Class B amplifier at $V_{DSmin} = 0$ is

$$\frac{\eta_{D(F3)}}{\eta_{D(B)}} = \frac{9}{8} = 1.125. \tag{8.39}$$

Thus, the improvement in the efficiency of the Class F amplifier over that of the Class B amplifier is by a factor of 1.125 from 78.54 % to 88.36 %. Note that the amplitude of the fundamental component $V_m = V_I$ is increased in the Class F amplifier with the third harmonic to $V_m = 9V_I/8 = 1.125V_I$. Thus, the drain efficiency η_D and the ratio V_m/V_I are increased by the same factor, equal to 1.125.

The output-power capability is

$$c_p = \frac{P_O}{V_{DSM} I_{DM}} = \frac{P_O}{2V_I I_{DM}} = \frac{9}{64}\left(1 - \frac{V_{DSmin}}{V_I}\right) = 0.1406\left(1 - \frac{V_{DSmin}}{V_I}\right) \tag{8.40}$$

where the maximum value of the drain-to-source voltage is $V_{DSM} = 2V_I$. The dc input resistance is

$$R_{DC} = \frac{V_I}{I_I} = \frac{4\pi}{9}R \approx 1.396R. \tag{8.41}$$

Example 8.1

Design a Class F power amplifier employing third harmonic peaking with a maximally flat drain-to-source voltage to deliver a power of 10 W at $f_c = 800$ MHz. The required bandwidth is $BW = 100$ MHz. The dc power supply voltage is 12 V and $V_{DSmin} = 1$ V.

Solution. The maximum amplitude of the fundamental component of the drain-to-source voltage is

$$V_m = \frac{9}{8}(V_I - V_{DSmin}) = \frac{9}{8}(12 - 1) = 12.375\,\text{V}. \tag{8.42}$$

The amplitude of the third harmonic is

$$V_{m3} = \frac{V_m}{9} = \frac{12.375}{9} = 1.375\,\text{V}. \tag{8.43}$$

The load resistance is

$$R = \frac{V_m^2}{2P_O} = \frac{12.375^2}{2 \times 10} = 7.657\,\Omega. \tag{8.44}$$

The maximum drain-to-source voltage is

$$V_{DSmax} = 2V_I = 2 \times 12 = 24\,\text{V}. \tag{8.45}$$

The amplitude of the fundamental component of the drain current is

$$I_m = \frac{V_m}{R} = \frac{12.375}{7.657} = 1.616\,\text{A}. \tag{8.46}$$

The maximum drain current is

$$I_{DM} = 2I_m = 2 \times 1.616 = 3.232\,\text{A}. \tag{8.47}$$

The dc supply current is

$$I_I = \frac{I_{DM}}{\pi} = \frac{3.232}{\pi} = 1.029\,\text{A}. \tag{8.48}$$

The dc supply power is

$$P_I = I_I V_I = 1.029 \times 12 = 12.348\,\text{W}. \tag{8.49}$$

The drain power loss of the transistor is

$$P_D = P_I - P_O = 12.348 - 10 = 2.348\,\text{W}. \tag{8.50}$$

The drain efficiency is

$$\eta_D = \frac{P_O}{P_I} = \frac{10}{12.348} = 80.98\,\%. \tag{8.51}$$

The dc resistance presented by the amplifier to the dc source is

$$R_{DC} = \frac{4\pi}{9}R = 1.396 \times 7.657 = 10.689\,\Omega. \tag{8.52}$$

The loaded quality factor is

$$Q_L = \frac{f_c}{BW} = \frac{800}{100} = 8. \tag{8.53}$$

The definition of the loaded quality factor for a parallel-resonant circuit is

$$Q_L = \frac{R}{\omega_c L} = \omega_c CR. \tag{8.54}$$

The inductance of the resonant circuit tunned the fundamental is

$$L = \frac{R}{\omega_c Q_L} = \frac{7.657}{2\pi \times 0.8 \times 10^9 \times 8} = 0.19\,\text{nH}. \tag{8.55}$$

The capacitance‘ of the resonant circuit tunned the fundamental is

$$C = \frac{Q_L}{\omega_c R} = \frac{8}{2\pi \times 0.8 \times 10^9 \times 7.657} = 207.855\,\text{pF}. \tag{8.56}$$

8.2.2 Maximum Drain Efficiency Class F_3 Amplifier

The maximum drain efficiency η_D and the maximum output-power capability c_p do not occur for the maximally flat drain-to-source voltage v_{DS}, but when the waveform v_{DS} exhibits slight ripple. From (8.17),

$$\cos\omega_o t_m = \sqrt{1-\sin^2\omega_o t_m} = \sqrt{1-\frac{9V_{m3}-V_m}{12V_{m3}}} = \sqrt{\frac{1}{4}+\frac{V_m}{12V_{m3}}}. \tag{8.57}$$

The minimum value of v_{DS} occurs at $\omega_o t_m$ and is equal to zero

$$v_{DS}(\omega_o t_m) = V_I - V_m\cos\omega_o t_m + V_{m3}\cos 3\omega_o t_m = 0 \tag{8.58}$$

which gives

$$\begin{aligned} V_I &= V_m\cos\omega_o t_m - V_{m3}\cos 3\omega_o t_m = V_m\cos\omega_o t_m - V_{m3}(4\cos^3\omega_o t_m - 3\cos\omega_o t_m) \\ &= \cos\omega_o t_m(V_m - 4V_{m3}\cos^2\omega_o t_m + 3V_{m3}) = \sqrt{\frac{1}{4}+\frac{V_m}{12V_{m3}}}\left(\frac{2V_m}{3}+2V_{m3}\right) \\ &= V_m\left[\left(\frac{2}{3}+\frac{2V_{m3}}{V_m}\right)\sqrt{\frac{1}{4}+\frac{V_m}{12V_{m3}}}\right] \end{aligned} \tag{8.59}$$

Hence,

$$\frac{V_m}{V_I} = \frac{1}{\sqrt{\frac{1}{4}+\frac{V_m}{12V_{m3}}}\left(\frac{2}{3}+\frac{2V_{m3}}{V_m}\right)}. \tag{8.60}$$

In order to maximize the drain efficiency

$$\eta_D = \frac{P_O}{P_I} = \frac{1}{2}\left(\frac{I_m}{I_I}\right)\left(\frac{V_m}{V_I}\right) \tag{8.61}$$

at a given waveform of drain current i_D, i.e., at a given ratio I_m/I_I, we will maximize the ratio V_m/V_I. As V_{m3} increases when V_m is held constant, V_I first decreases, reaches its minimum value, and then slowly increases. To maximize V_m/V_I, we take the derivative and set it to zero

$$\frac{d\left(\frac{V_m}{V_I}\right)}{d\left(\frac{V_{m3}}{V_m}\right)} = \frac{9}{\left(9+\frac{3V_m}{V_{m3}}\right)^{\frac{2}{3}}}\frac{1}{\frac{2}{3}+\frac{2V_{m3}}{V_m}}\left(\frac{V_m}{V_{m3}}\right)^2 - \frac{12}{\sqrt{9+\frac{3V_m}{V_{m3}}}}\frac{1}{\left(\frac{2}{3}+\frac{2V_{m3}}{V_m}\right)^2} = 0 \tag{8.62}$$

producing

$$18\left(\frac{V_{m3}}{V_m}\right)^2 + 3\left(\frac{V_{m3}}{V_m}\right) - 1 = 0. \tag{8.63}$$

Solution of this equation yields

$$\frac{V_{m3}}{V_m} = \frac{1}{6} \approx 0.1667. \tag{8.64}$$

Using this relationship, we obtain

$$\begin{aligned} V_I &= V_m\cos\omega_o t_m - V_{m3}\cos 3\omega_o t_m = V_m\cos\omega_o t_m - V_{m3}(4\cos^3\omega_o t_m - 3\cos\omega_o t_m) \\ &= V_m\sqrt{\frac{1}{4}+\frac{1}{2}}\left[1-\frac{2}{3}\left(\frac{1+2}{4}\right)+\frac{1}{2}\right] = V_m\frac{\sqrt{3}}{2} \end{aligned} \tag{8.65}$$

resulting in

$$\frac{V_m}{V_I} = \frac{2}{\sqrt{3}} \approx 1.1547. \tag{8.66}$$

Finally,

$$V_{m3} = \frac{V_m}{6} = \frac{V_I}{3\sqrt{3}} \tag{8.67}$$

yielding

$$\frac{V_{m3}}{V_I} = \frac{1}{3\sqrt{3}} \approx 0.19245. \tag{8.68}$$

The minimum values of v_{DS} occur at

$$\omega_o t_m = \pm \arcsin\sqrt{\frac{9V_{m3} - V_m}{12V_{m3}}} = \pm \arcsin\left(\frac{1}{2}\right) = \pm 30^\circ. \tag{8.69}$$

The normalized waveform v_{DS}/V_I for achieving the maximum drain efficiency is given by

$$\frac{v_{DS}}{V_I} = 1 - \frac{V_m}{V_I}\cos\omega_o t + \frac{V_{m3}}{V_I}\cos 3\omega_o t = 1 - \frac{2}{\sqrt{3}}\cos\omega_o t + \frac{1}{3\sqrt{3}}\cos 3\omega_o t. \tag{8.70}$$

The normalized drain-to-source voltage waveform v_{DS}/V_I for the Class F amplifier with the third harmonic peaking for the maximum drain efficiency is shown in Figure 8.4. The drain efficiency is

$$\eta_D = \frac{P_O}{P_I} = \frac{1}{2}\frac{I_m}{I_I}\frac{V_m}{V_I} = \frac{1}{2} \times \frac{\pi}{2} \times \frac{2}{\sqrt{3}} = \frac{\pi}{2\sqrt{3}} \approx 90.67\,\%. \tag{8.71}$$

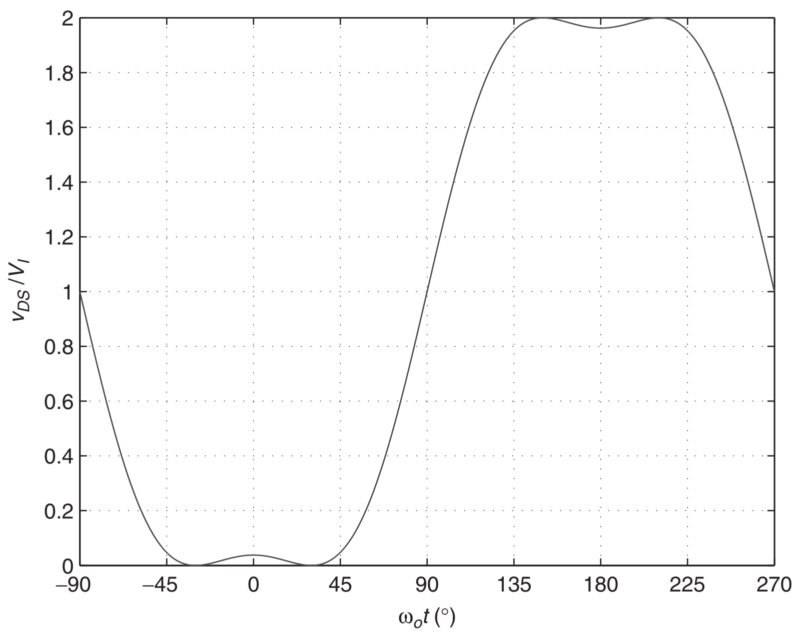


Figure 8.4 Waveform of the normalized drain-to-source voltage v_{DS}/V_I for Class F RF power amplifier with third harmonic peaking, yielding the maximum drain efficiency and output-power capability.

The ratio of the maximum drain efficiency $\eta_{Dmax(F3)}$ of the Class F amplifier with the third harmonic to the drain efficiency of the Class B amplifier at $V_{DSmin} = 0$ is

$$\frac{\eta_{Dmax(F3)}}{\eta_{D(B)}} = \frac{2}{\sqrt{3}} \approx 1.1547. \tag{8.72}$$

Thus, the maximum drain efficiency is increased from 78.54 % to 90.67 % by the same factor as the ratio V_m/V_I is increased, i.e., by a factor of $2/\sqrt{3} \approx 1.1547$.

In general, the drain efficiency of the Class F amplifier is given by

$$\eta_D = \eta_B \left(\frac{V_m}{V_I}\right) = \frac{\pi}{4}\left(\frac{V_m}{V_I}\right). \tag{8.73}$$

Hence,

$$\eta_D = \frac{\pi}{4}\left(\frac{1}{1 - \dfrac{V_{m3}}{V_m}}\right) \quad \text{for} \quad 0 \le \frac{V_{m3}}{V_m} \le \frac{1}{9}. \tag{8.74}$$

and

$$\eta_D = \frac{\pi}{4\sqrt{\dfrac{1}{4} + \dfrac{V_m}{12V_{m3}}\left(\dfrac{2}{3} + \dfrac{2V_{m3}}{V_m}\right)}} \quad \text{for} \quad \frac{V_{m3}}{V_m} \ge \frac{1}{9}. \tag{8.75}$$

Figure 8.5 shows V_m/V_I as a function of V_{m3}/V_m. The drain efficiency η_D as a function of V_{m3}/V_m is shown in Figure 8.6.

The ratio of the maximum drain efficiency $\eta_{Dmax(F3)}$ of the Class F amplifier with the third harmonic to the drain efficiency of the Class B amplifier with maximally flat waveform

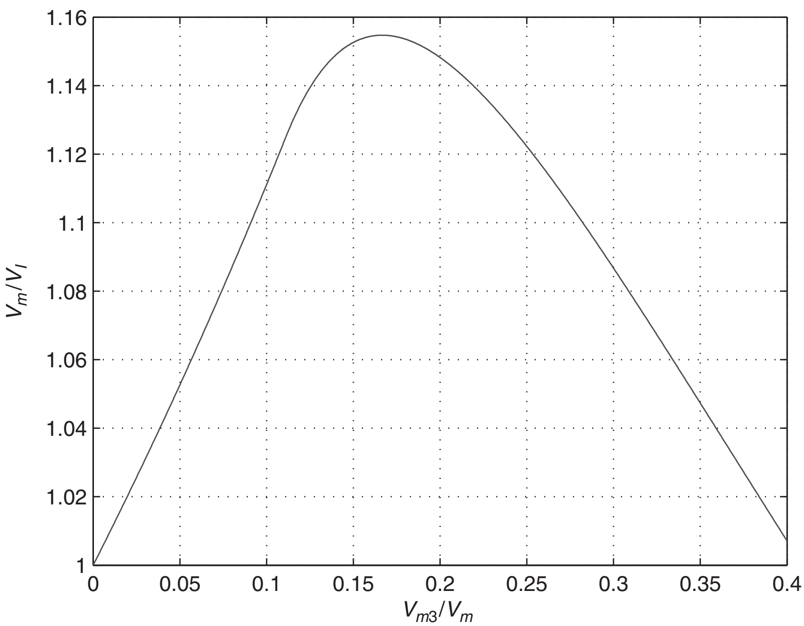


Figure 8.5 Ratio V_m/V_I as a function of V_{m3}/V_m for Class F RF power amplifier with third harmonic peaking.

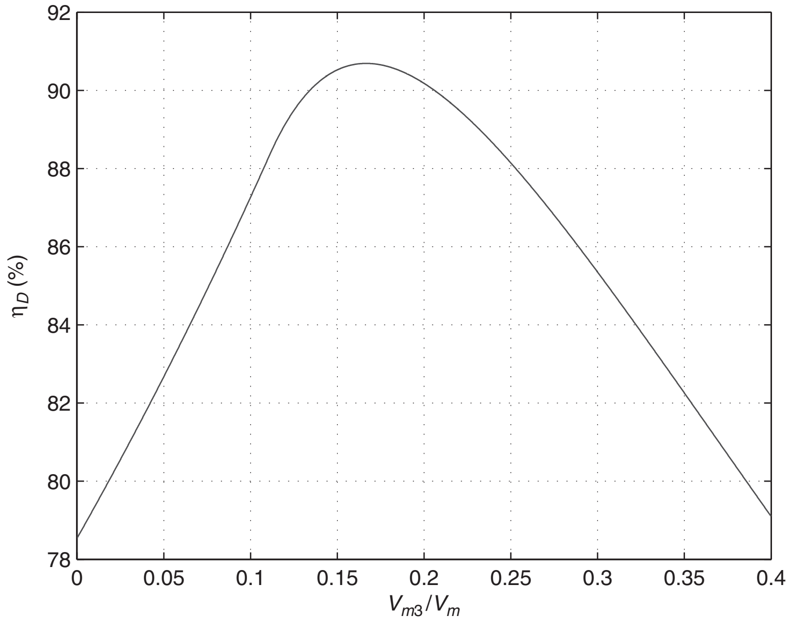


Figure 8.6 Drain efficiency η_D as a function of V_{m3}/V_m for Class F RF power amplifier with third harmonic peaking.

v_{DS} at $V_{DSmin} = 0$ is

$$\frac{\eta_{Dmax(F3)}}{\eta_{D(F3)}} = \frac{16}{9\sqrt{3}} \approx 1.0264. \tag{8.76}$$

Thus, the improvement in the efficiency is by 2.64 %. The maximum drain-to-source voltage is

$$V_{DSM} = 2V_I. \tag{8.77}$$

The maximum output-power capability is

$$c_p = \frac{P_O}{I_{DM}V_{DSM}} = \frac{\eta_D P_I}{I_{DM}V_{DSM}} = \eta_D \frac{I_I V_I}{I_{DM}V_{DSM}} = \left(\frac{\pi}{2\sqrt{3}}\right)\left(\frac{1}{\pi}\right)\left(\frac{1}{2}\right) = \frac{1}{4\sqrt{3}} \approx 0.1443. \tag{8.78}$$

The maximum output-power capability at any value of V_{m3}/V_m is given by

$$c_p = \frac{1}{2}\left(\frac{I_m}{I_{DM}}\right)\left(\frac{V_m}{V_{DSM}}\right) = \frac{1}{2}\left(\frac{I_m}{I_{DM}}\right)\left(\frac{V_m}{2V_I}\right) = \frac{1}{8}\left(\frac{V_m}{V_I}\right) = c_{p(B)}\left(\frac{V_m}{V_I}\right). \tag{8.79}$$

Thus,

$$c_p = \frac{1}{8}\left(\frac{1}{1-\frac{V_{m3}}{V_m}}\right) \quad \text{for} \quad 0 \le \frac{V_{m3}}{V_m} \le \frac{1}{9}. \tag{8.80}$$

and

$$c_p = \frac{1}{8\sqrt{\frac{1}{4}+\frac{V_m}{12V_{m3}}\left(\frac{2}{3}+\frac{2V_{m3}}{V_m}\right)}} \quad \text{for} \quad \frac{V_{m3}}{V_m} \ge \frac{1}{9}. \tag{8.81}$$

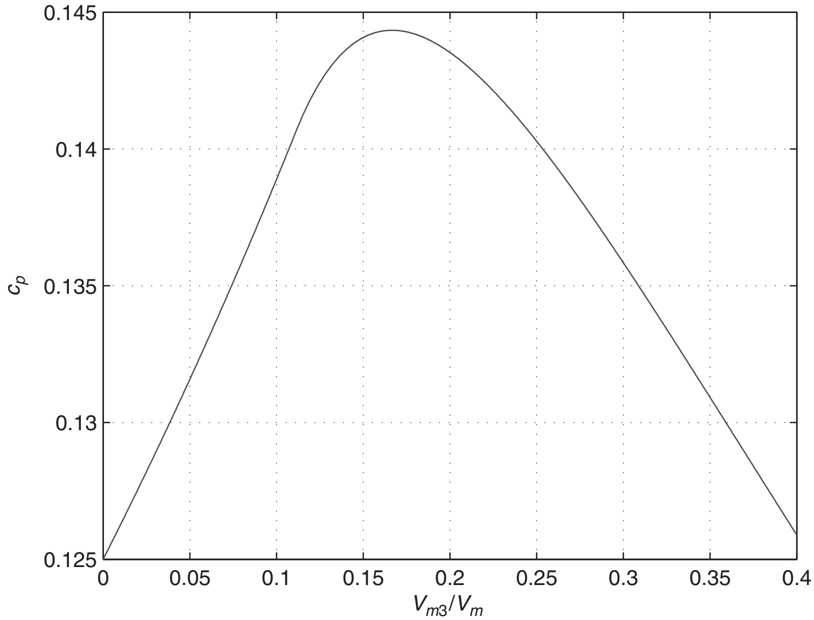


Figure 8.7 Output power capability c_p as a function of V_{m3}/V_m for Class F RF power amplifier with third harmonic peaking.

Figure 8.7 shows c_p as a function of V_{m3}/V_m. For odd harmonic Class F amplifiers, the maximum drain efficiency and the maximum output power capability occur at the same voltage amplitude ratios.

The dc resistance of the amplifier presented to the dc power supply is

$$R_{DC} = \frac{V_I}{I_I} = \frac{\pi\sqrt{3}}{2}R \approx 1.36\,\mathrm{R}. \tag{8.82}$$

Example 8.2

Design a Class F power amplifier employing third harmonic peaking and having a maximum drain efficiency to deliver a power of 10 W at $f_c = 800$ MHz with $BW = 100$ MHz. The dc power supply voltage is 12 V and $V_{DSmin} = 1$ V.

Solution. The maximum amplitude of the fundamental component of the drain-to-source voltage is

$$V_m = \frac{2}{\sqrt{3}}(V_I - V_{DSmin}) = 1.1547(12 - 1) = 12.7\,\mathrm{V}. \tag{8.83}$$

The amplitude of the third harmonic is

$$V_{m3} = \frac{V_m}{6} = \frac{12.7}{6} = 2.117\,\mathrm{V}. \tag{8.84}$$

The load resistance is

$$R = \frac{V_m^2}{2P_O} = \frac{12.7^2}{2 \times 10} = 8.0645\,\Omega. \tag{8.85}$$

The maximum drain-to-source voltage is

$$V_{DSmax} = 2V_I = 2 \times 12 = 24\,\text{V}. \tag{8.86}$$

The amplitude of the fundamental component of the drain current is

$$I_m = \frac{V_m}{R} = \frac{12.7}{8.0645} = 1.575\,\text{A}. \tag{8.87}$$

The maximum drain current is

$$I_{DM} = 2I_m = 2 \times 1.575 = 3.15\,\text{A}. \tag{8.88}$$

The dc supply current is

$$I_I = \frac{I_{DM}}{\pi} = \frac{3.15}{\pi} = 1.00267\,\text{A}. \tag{8.89}$$

The dc supply power is

$$P_I = I_I V_I = 1.00267 \times 12 = 12.03204\,\text{W}. \tag{8.90}$$

The drain power loss of the transistor is

$$P_D = P_I - P_O = 12.03204 - 10 = 2.03204\,\text{W}. \tag{8.91}$$

The drain efficiency is

$$\eta_D = \frac{P_O}{P_I} = \frac{10}{12.03204} = 83.11\,\%. \tag{8.92}$$

The dc resistance presented by the amplifier to the dc power supply is

$$R_{DC} = \frac{\pi\sqrt{3}}{4}R \approx 1.36 \times 8.0645 = 10.97\,\Omega. \tag{8.93}$$

From Example 8.1, $L = 0.19$ nH and $C = 207.855$ pF.

The loaded quality factor of the resonant circuit tunned to the fundamental of the carrier frequency is

$$Q_L = \frac{f_c}{BW} = \frac{800}{100} = 8. \tag{8.94}$$

The resonant circuit components of the output circuit are

$$L = \frac{R}{\omega_c Q_L} = \frac{8.0645}{2\pi \times 0.8 \times 10^9 \times 8} = 0.2\,\text{nH} \tag{8.95}$$

and

$$C = \frac{Q_L}{\omega_c R} = \frac{8}{2\pi \times 0.8 \times 10^9 \times 8.0645} = 197.35\,\text{pF}. \tag{8.96}$$

8.3 Class F RF Power Amplifier with Third and Fifth Harmonics

8.3.1 Maximally Flat Class F_{35} Amplifier

The circuit of a Class F RF power amplifier with both the third and fifth harmonics is depicted in Figure 8.8. This circuit is called the Class F_{35} amplifier. The voltage and

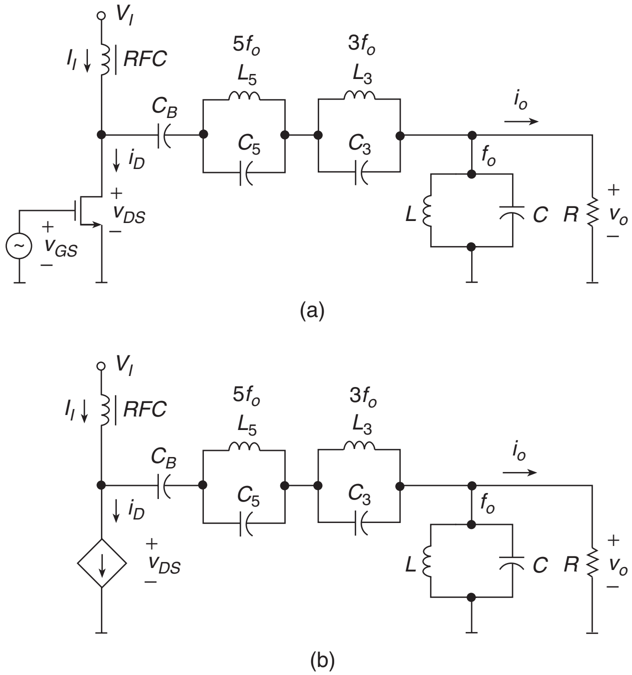


Figure 8.8 Class F power amplifier with third and fifth harmonic resonators. (a) Circuit. (b) Equivalent circuit.

current waveforms are shown in Figure 8.9. The drain-to-source voltage waveform is

$$v_{DS} = V_I - V_m \cos \omega_o t + V_{m3} \cos 3\omega_o t - V_{m5} \cos 5\omega_o t. \tag{8.97}$$

The derivatives of this voltage are

$$\frac{\mathrm{d}v_{DS}}{\mathrm{d}(\omega_o t)} = V_m \sin \omega_o t - 3V_{m3} \sin 3\omega_o t + 5V_{m5} \sin 5\omega_o t \tag{8.98}$$

$$\frac{\mathrm{d}^2 v_{DS}}{\mathrm{d}(\omega_o t)^2} = V_m \cos \omega_o t - 9V_{m3} \cos 3\omega_o t + 25V_{m5} \cos 5\omega_o t \tag{8.99}$$

$$\frac{\mathrm{d}^3 v_{DS}}{\mathrm{d}(\omega_o t)^3} = -V_m \sin \omega_o t + 27V_{m3} \sin 3\omega_o t - 125V_{m5} \sin 5\omega_o t \tag{8.100}$$

and

$$\frac{\mathrm{d}^4 v_{DS}}{\mathrm{d}(\omega_o t)^4} = -V_m \cos \omega_o t + 81V_{m3} \cos 3\omega_o t - 625V_{m5} \cos 5\omega_o t. \tag{8.101}$$

All the terms of the first and third-order derivatives are zero at $\omega_o t = 0$ and do not give any equations. The second and the fourth-order derivatives are zero at $\omega_o t = 0$, yielding a set of two simultaneous equations

$$\left.\frac{\mathrm{d}^2 v_{DS}}{\mathrm{d}(\omega_o t)^2}\right|_{\omega_o t = 0} = V_m - 9V_{m3} + 25V_{m5} = 0 \tag{8.102}$$

and

$$\left.\frac{\mathrm{d}^4 v_{DS}}{\mathrm{d}(\omega_o t)^4}\right|_{\omega_o t = 0} = -V_m + 81V_{m3} - 625V_{m5} = 0. \tag{8.103}$$

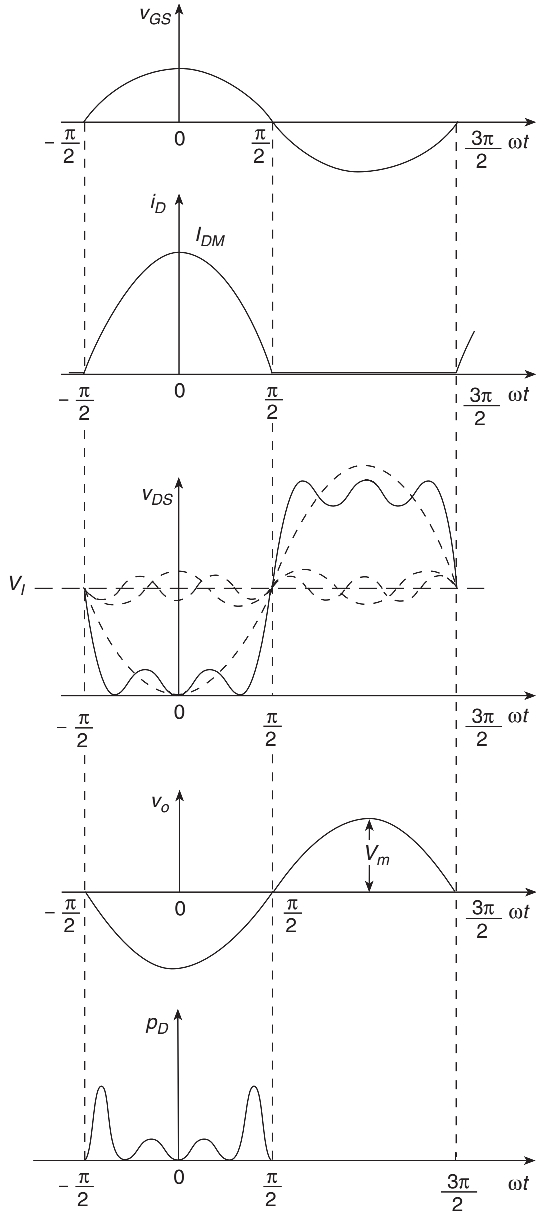


Figure 8.9 Waveforms in Class F RF power amplifier with third and fifth harmonic peaking.

Solution of this set of equations gives

$$V_{m3} = \frac{V_m}{6} \tag{8.104}$$

$$V_{m5} = \frac{V_m}{50} \tag{8.105}$$

and

$$V_{m5} = \frac{3}{25} V_{m3}. \tag{8.106}$$

Now

$$v_{DS}(0) = V_I - V_m + V_{m3} - V_{m5} = V_I - V_m + \frac{V_m}{6} - \frac{V_m}{50} = V_I - \frac{64}{75}V_m = 0 \quad (8.107)$$

producing

$$V_m = \frac{75}{64}V_I \approx 1.1719V_I. \quad (8.108)$$

Similarly,

$$v_{DS}(0) = V_I - V_m + V_{m3} - V_{m5} = V_I - 6V_{m3} + V_{m3} - \frac{3V_{m3}}{25} = V_I - \frac{128}{25}V_{m3} = 0 \quad (8.109)$$

yielding

$$V_{m3} = \frac{25}{128}V_I \approx 0.1953V_I. \quad (8.110)$$

Finally,

$$v_{DS}(0) = V_I - V_m + V_{m3} - V_{m5} = V_I - 50V_{m5} + \frac{25}{3}V_{m5} - V_{m5} = V_I - \frac{128}{3}V_{m5} = 0 \quad (8.111)$$

leading to

$$V_{m5} = \frac{3}{128}V_I \approx 0.02344V_I. \quad (8.112)$$

The maximum drain-to-source voltage is

$$V_{DSM} = v_{DS}(\pi) = V_I + V_m - V_{m3} + V_{m5} = V_I + \frac{75}{64}V_I - \frac{25}{128}V_I + \frac{3}{128}V_I = 2V_I. \quad (8.113)$$

The normalized maximally flat voltage is given by

$$\begin{aligned}\frac{v_{DS}}{V_I} &= 1 - \frac{V_m}{V_I}\cos\omega_o t + \frac{V_{m3}}{V_m}\cos 3\omega_o t - \frac{V_{m5}}{V_m}\cos 5\omega_o t \\ &= 1 - \frac{75}{64}\cos\omega_o t + \frac{25}{128}\cos 3\omega_o t - \frac{3}{128}\cos 5\omega_o t. \end{aligned} \quad (8.114)$$

Figure 8.10 shows the maximally flat waveform of the normalized drain-to-source voltage v_{DS}/V_I with the third and fifth harmonics.

The amplitude of the fundamental component of the current is

$$I_m = \frac{V_m}{R} = \frac{75}{64}\frac{V_I}{R}. \quad (8.115)$$

The dc input current is

$$I_I = \frac{2}{\pi}I_m = \frac{75}{32\pi}\frac{V_I}{R}. \quad (8.116)$$

The dc input power is

$$P_I = I_I V_I = \frac{75}{32\pi}\frac{V_I^2}{R}. \quad (8.117)$$

The output power is

$$P_O = \frac{V_m^2}{2R} = \frac{5625}{8192}\frac{V_I^2}{R} \approx 0.6866\frac{V_I^2}{R}. \quad (8.118)$$

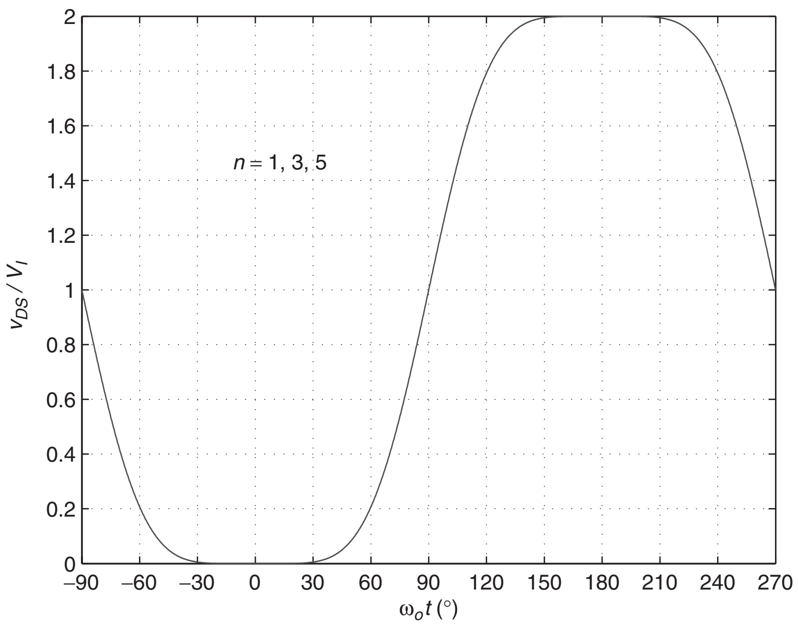


Figure 8.10 Maximally flat waveform of the normalized drain-to-source voltage v_{DS}/V_I for Class F RF power amplifier with third and fifth harmonics.

The drain efficiency is

$$\eta_D = \frac{P_O}{P_I} = \frac{75\pi}{256} \approx 92.04\,\% \tag{8.119}$$

The maximum drain-to-source voltage is

$$V_{DSM} = 2V_I. \tag{8.120}$$

The output-power capability is

$$c_p = \frac{P_O}{V_{DSM} I_{DM}} = \frac{1}{2}\left(\frac{V_m I_m}{2V_I I_{DM}}\right) = \frac{1}{4} \times \frac{75}{64} \times \frac{1}{2} = \frac{75}{512} = 0.1465. \tag{8.121}$$

The dc input resistance is

$$R_{DC} = \frac{V_I}{I_I} = \frac{32\pi}{75} R \approx 1.34R. \tag{8.122}$$

Example 8.3

Design a Class F power amplifier employing third and fifth harmonics peaking with a maximally flat drain-to-source voltage to deliver a power of 16 W at $f_c = 1$ GHz. The required bandwidth is $BW = 100$ MHz. The dc power supply voltage is 24 V and $V_{DSmin} =$ 1 V.

Solution. The maximum amplitude of the fundamental component of the drain-to-source voltage is

$$V_m = \frac{75}{64}(V_I - V_{DSmin}) = \frac{75}{64}(24 - 1) = 26.953\text{ V}. \tag{8.123}$$

The amplitude of the third harmonic is

$$V_{m3} = \frac{V_m}{6} = \frac{26.953}{6} = 4.492\,\text{V}. \tag{8.124}$$

$$V_{m5} = \frac{V_m}{50} = \frac{26.953}{50} = 0.5391\,\text{V}. \tag{8.125}$$

The load resistance is

$$R = \frac{V_m^2}{2P_O} = \frac{26.953^2}{2 \times 16} = 22.7\,\Omega. \tag{8.126}$$

The maximum drain-to-source voltage is

$$V_{DSmax} = 2V_I = 2 \times 24 = 48\,\text{V}. \tag{8.127}$$

The amplitude of the fundamental component of the drain current is

$$I_m = \frac{V_m}{R} = \frac{26.953}{22.7} = 1.187\,\text{A}. \tag{8.128}$$

The maximum drain current is

$$I_{DM} = 2I_m = 2 \times 1.187 = 2.374\,\text{A}. \tag{8.129}$$

The dc supply current is

$$I_I = \frac{I_{DM}}{\pi} = \frac{2.374}{\pi} = 0.7556\,\text{A}. \tag{8.130}$$

The dc supply power is

$$P_I = I_I V_I = 0.7556 \times 24 = 18.1344\,\text{W}. \tag{8.131}$$

The drain power loss of the transistor is

$$P_D = P_I - P_O = 18.1344 - 16 = 2.1344\,\text{W}. \tag{8.132}$$

The drain efficiency is

$$\eta_D = \frac{P_O}{P_I} = \frac{16}{18.1344} = 88.23\,\%. \tag{8.133}$$

The dc resistance presented by the amplifier to the dc source is

$$R_{DC} = \frac{32\pi}{75}R = 1.34 \times 22.7 = 30.418\,\Omega. \tag{8.134}$$

The loaded quality factor is

$$Q_L = \frac{f_c}{BW} = \frac{1000}{100} = 10. \tag{8.135}$$

The inductance of the resonant circuit tunned the fundamental frequency is

$$L = \frac{R}{\omega_c Q_L} = \frac{22.7}{2\pi \times 10^9 \times 10} = 0.36128\,\text{nH}. \tag{8.136}$$

The capacitance of the resonant circuit tunned the fundamental frequency is

$$C = \frac{Q_L}{\omega_c R} = \frac{10}{2\pi \times 10^9 \times 22.7} = 70.11\,\text{pF}. \tag{8.137}$$

8.3.2 Maximum Drain Efficiency Class F_{35} Amplifier

The maximum drain efficiency and the output-power capability of the Class F amplifier with the third and fifth harmonics is obtained for [20]

$$\frac{V_m}{V_I} = 1.2071 \tag{8.138}$$

$$\frac{V_{m3}}{V_I} = 0.2804 \tag{8.139}$$

$$\frac{V_{m5}}{V_I} = 0.07326 \tag{8.140}$$

$$\frac{V_{m3}}{V_m} = 0.2323 \tag{8.141}$$

$$\frac{V_{m5}}{V_m} = 0.0607 \tag{8.142}$$

and

$$\frac{V_{m5}}{V_{m3}} = 0.2613. \tag{8.143}$$

The normalized waveform v_{DS}/V_I for the maximum drain efficiency is given by

$$\begin{aligned}\frac{v_{DS}}{V_I} &= 1 - \frac{V_m}{V_I}\cos\omega_o t + \frac{V_{m3}}{V_m}\cos\omega_o t + \frac{V_{m5}}{V_I}\cos 5\omega_o t \\ &= 1 - 1.2071\cos\omega_o t + 0.2804V_m\cos\omega_o t + 0.07326\cos 5\omega_o t.\end{aligned} \tag{8.144}$$

The waveform of normalized drain-to-source voltage v_{DS}/V_I in the Class F amplifier with the third and fifth harmonics, yielding the maximum drain efficiency and the maximum output-power capability, is shown in Figure 8.11.

The amplitude of the fundamental component of the drain current is

$$I_m = \frac{V_m}{R} = \frac{1.207V_I}{R}. \tag{8.145}$$

The dc supply current is

$$I_I = \frac{2}{\pi}I_m = \frac{2\times 1.207}{\pi}\frac{V_I}{R} = 0.7684\frac{V_I}{R}. \tag{8.146}$$

The dc supply current is

$$P_I = I_I V_I = 0.7684\frac{V_I^2}{R}. \tag{8.147}$$

The output power is

$$P_O = \frac{V_m^2}{2R} = \frac{1}{2}\frac{(1.207V_I)^2}{R} \approx 0.7284\frac{V_I^2}{R}. \tag{8.148}$$

The maximum drain efficiency is

$$\eta_D = \frac{P_O}{P_I} = \frac{1}{2}\left(\frac{I_m}{I_I}\right)\left(\frac{V_m}{V_I}\right) = \frac{1}{2}\left(\frac{\pi}{2}\right)(1.207) = \frac{\pi}{4}\times 1.207 \approx 94.8\,\%. \tag{8.149}$$

Hence,

$$\frac{\eta_{Dmax}}{\eta_{D(F35)}} = \frac{94.8}{92.4} = 1.026. \tag{8.150}$$

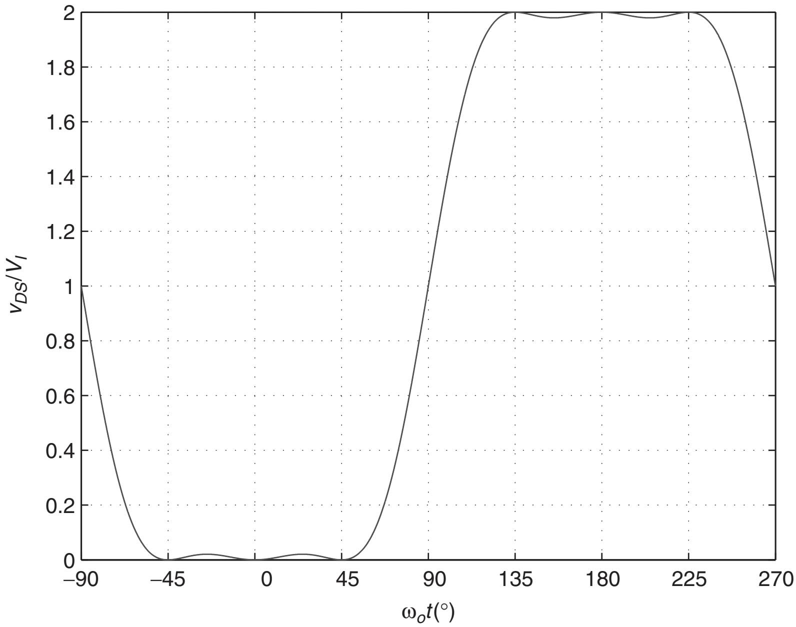


Figure 8.11 Waveform of the normalized drain-to-source voltage v_{DS}/V_I for Class F RF power amplifier with third and fifth harmonics, yielding the maximum drain efficiency and output-power capability.

The maximum value of v_{DS} is

$$V_{DSM} = 2V_I = \frac{2}{1.207}V_m = 1.657V_m. \tag{8.151}$$

The output-power capability is

$$c_p = \frac{P_O}{I_{DM}V_{DSM}} = \frac{1}{2}\left(\frac{I_m}{I_{DM}}\right)\left(\frac{V_m}{V_{DSM}}\right) = \frac{1}{2} \times \left(\frac{1}{2}\right) \times \left(\frac{1}{1.657}\right) = 0.1509. \tag{8.152}$$

The dc input resistance is

$$R_{DC} = \frac{V_I}{I_I} \approx 1.3R. \tag{8.153}$$

The normalized voltage amplitudes in odd harmonics Class F power amplifiers that give the maximally flat waveform of v_{DS} are given in Table 8.1. The normalized voltage amplitudes of these amplifiers that ensure the maximum drain efficiency are given in in Table 8.2.

Table 8.1 Voltage amplitudes in odd harmonic maximally flat Class F power amplifiers.

Class	$\frac{V_m}{V_I}$	$\frac{V_{m3}}{V_I}$	$\frac{V_{m5}}{V_I}$	$\frac{V_{m3}}{V_m}$	$\frac{V_{m5}}{V_m}$
F_3	$\frac{9}{8} = 1.125$	$\frac{1}{8} = 0.125$	0	$\frac{1}{9} = 0.1111$	–
F_{35}	$\frac{75}{64} = 1.1719$	$\frac{25}{128} = 0.1953$	$\frac{3}{128} = 0.02344$	$\frac{1}{6} = 1.667$	$\frac{1}{50} = 0.02$

Table 8.2 Voltage amplitudes in odd harmonic maximum drain efficiency Class F power amplifiers.

Class	$\frac{V_m}{V_I}$	$\frac{V_{m3}}{V_I}$	$\frac{V_{m5}}{V_I}$	$\frac{V_{m3}}{V_m}$	$\frac{V_{m5}}{V_m}$
F_3	$\frac{2}{\sqrt{3}} = 1.1547$	$\frac{1}{3\sqrt{3}} = 0.1925$	0	$\frac{1}{6} = 0.1667$	–
F_{35}	1.207	0.2804	0.07326	0.2323	0.0607

8.4 Class F RF Power Amplifier with Third, Fifth, and Seventh Harmonics

For the maximally flat Class F_{357} amplifier,

$$V_{m3} = \frac{V_{m1}}{5} \tag{8.154}$$

$$V_{m5} = \frac{V_{m1}}{25} \tag{8.155}$$

$$V_{m7} = \frac{V_{m1}}{245} \tag{8.156}$$

$$V_{m1} = \frac{1225}{1024} V_I \tag{8.157}$$

$$V_{m3} = \frac{245}{2024} V_I \tag{8.158}$$

$$V_{m5} = \frac{49}{1024} V_I \tag{8.159}$$

and

$$V_{m7} = \frac{5}{1024} V_I. \tag{8.160}$$

8.5 Class F RF Power Amplifier with Parallel-resonant Circuit and Quarter-wavelength Transmission Line

Figure 8.12(a) shows the circuit of a Class F power amplifier with a quarter-wavelength transmission line and a parallel-resonant circuit. The quarter-wavelength transmission line is equivalent to an infinite number of parallel-resonant circuits, which behave like an open circuit for odd harmonics and like a short circuit for even harmonics, as shown in Figure 8.12(b). This circuit is referred to as Class F_∞ amplifier. It is widely used in VHF and UHF frequency-modulated (FM) radio transmitters [2]. Current and voltage waveforms are shown in Figure 8.13. The drain current waveform is a half sine wave. The conduction angle of the drain current is $\theta = \pi$. The drain-to-source voltage is a square wave. These current and voltage waveforms constitute the waveforms of an ideal Class F amplifier. The

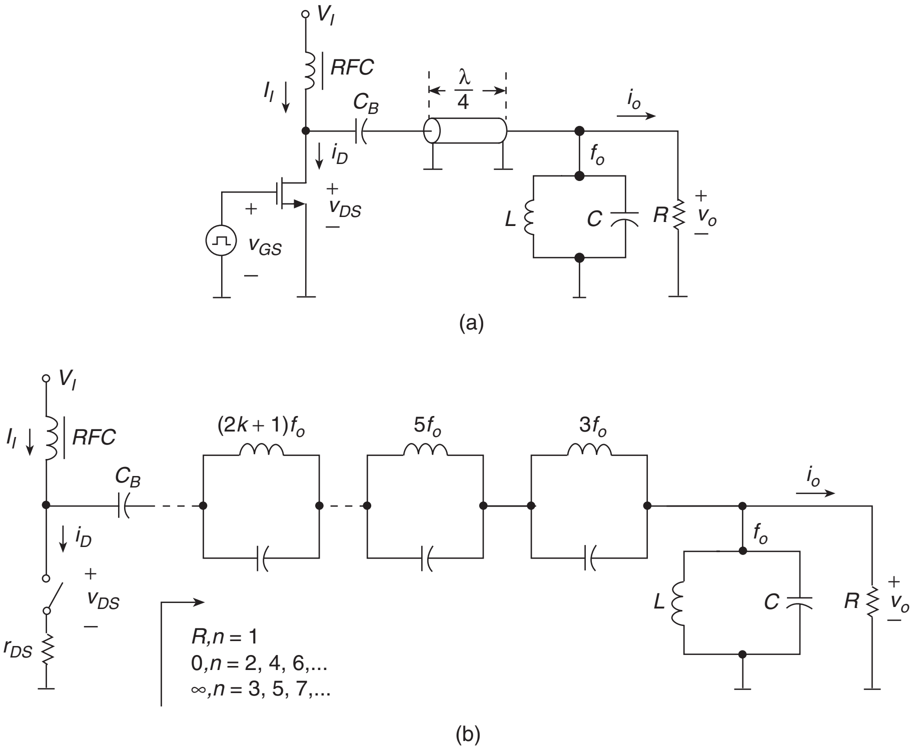


Figure 8.12 Class F power amplifier employing a parallel-resonant circuit and a quarter-wavelength transmission line, where $n = (2k + 1)$ with $k = 0, 1, 2, 3, \ldots$. (a) Circuit. (b) Equivalent circuit at $Z_o = R$.

transmission line may be difficult to integrate, thus preventing full integration of the Class F power amplifier on a single chip. At $f = 2.4\,\text{GHz}$, $\lambda/4 \approx 2\,\text{cm}$.

The drain current waveform i_D can be expanded into a Fourier trigonometric series

$$i_D = I_{DM}\left[\frac{1}{\pi} + \frac{1}{2}\cos\omega_o t + \frac{2}{\pi}\sum_{n=2}^{\infty}\frac{\cos\left(\frac{n\pi}{2}\right)}{1-n^2}\cos n\omega_o t\right]$$
$$= I_{DM}\left[\frac{1}{\pi} + \frac{1}{2}\cos\omega_o t + \frac{2}{3\pi}\cos 2\omega_o t - \frac{2}{15\pi}\cos 4\omega_o t + \frac{2}{35\pi}\cos 6\omega_o t + \cdots\right]. \tag{8.161}$$

Thus, the drain current waveform i_D contains only the dc component, fundamental component, and even harmonic components.

The drain-to-source voltage v_{DS} is a square-wave and it can be expanded into a Fourier series

$$v_{DS} = V_I\left\{1 + \frac{4}{\pi}\left[\cos\omega_o t - \frac{1}{3}\cos 3\omega_o t + \frac{1}{5}\cos 5\omega_o t - \frac{1}{7}\cos 7\omega_o t + \cdots\right]\right\}. \tag{8.162}$$

The drain-to-source voltage v_{DS} contains only the dc component, fundamental component, and odd harmonic components.

The peak value of the drain-to-source voltage is

$$V_{DSM} = 2V_I\, V_{DSM} = 2V_I \tag{8.163}$$

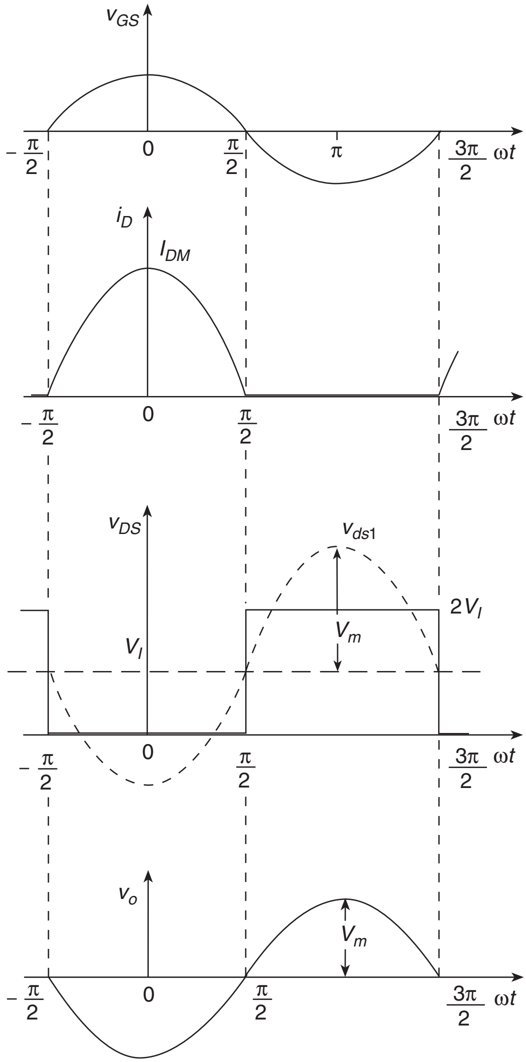


Figure 8.13 Voltage and current waveforms for Class F RF power amplifier with a parallel-resonant circuit and a quarter-wavelength transmission line.

The amplitude of the drain-to-source voltage is

$$V_m = \frac{4}{\pi}(V_I - V_{DSmax}) \tag{8.164}$$

and the amplitude of the drain current is

$$I_m = \frac{I_{DM}}{2}. \tag{8.165}$$

The input impedance of the load network seen by the drain and source terminals at the fundamental frequency f_o is

$$Z(f_o) = \frac{V_m}{I_m} = \frac{\frac{4}{\pi}(V_I - V_{DSmin})}{\frac{1}{2}I_{DM}} = \frac{8}{\pi}\frac{(V_I - V_{DSmax})}{I_{DM}} = R_i = \frac{Z_o^2}{R}; \tag{8.166}$$

the impedance at even harmonics is

$$Z(nf_o) = Z(2kf_o) = \frac{V_{m(n)}}{I_{m(n)}} = \frac{0}{\text{finite value}} = 0 \quad \text{for} \quad n = 2, 4, 6, \ldots \tag{8.167}$$

and the impedance at odd harmonics is

$$Z(nf_o) = Z[(2k+1)f_o] = \frac{V_{m(n)}}{I_{m(n)}} = \frac{\text{finite value}}{0} = \infty \quad \text{for} \quad n = 3, 5, 7, \ldots \tag{8.168}$$

where $k = 1, 2, 3, \ldots$.

The input impedance of the transmission line is given by

$$Z_i(l) = Z_o \frac{Z_L + jZ_o \tan\left(\frac{2\pi}{\lambda} l\right)}{Z_o + jZ_L \tan\left(\frac{2\pi}{\lambda} l\right)} \tag{8.169}$$

where l is the length of the transmission line, the wavelength in the transmission line is

$$\lambda = \frac{v_p}{f} = \frac{c}{f\sqrt{\epsilon_r}} \tag{8.170}$$

where $v_p = c/\sqrt{\epsilon_r}$ is the phase velocity, c is the speed of light, ϵ_r is the relative dielectric constant, and λ_o is the wavelength in free space.

The input impedance of the transmission line for $l = \lambda/4$ is

$$Z_i\left(\frac{\lambda}{4}\right) = Z_o \frac{Z_L + jZ_o \tan\left(\frac{2\pi}{\lambda}\frac{\lambda}{4}\right)}{Z_o + jZ_L \tan\left(\frac{2\pi}{\lambda}\frac{\lambda}{4}\right)} = \frac{Z_o^2}{Z_L}. \tag{8.171}$$

In this case, the transmission line acts like an *impedance inverter*. The input impedance of the transmission line for $l = \lambda/2$ is

$$Z_i\left(\frac{\lambda}{2}\right) = Z_o \frac{Z_L + jZ_o \tan\left(\frac{2\pi}{\lambda}\frac{\lambda}{2}\right)}{Z_o + jZ_L \tan\left(\frac{2\pi}{\lambda}\frac{\lambda}{2}\right)} = Z_L. \tag{8.172}$$

The transmission line under this condition acts like an *impedance repeater*. The capacitor in the parallel-resonant circuit represents a short circuit for all harmonics. The quarter-wavelength transmission line converts short circuits at its output to open circuits at its input at odd harmonics, and it converts short circuits to short circuits at even harmonics. The high impedances at odd harmonics shape a square wave of the drain-to-source voltage v_{DS} shown in Figure 8.13 because this voltage waveform contains only odd harmonics. The low impedances at even harmonics shape a half sine wave of the drain current i_D since this current waveform contains only even harmonics. Ideally, the voltage and current waveforms in the Class F and Class D amplifiers are identical.

The fundamental component of the drain-to-source voltage is given by

$$V_m = \frac{4}{\pi}(V_I - V_{DSmin}). \tag{8.173}$$

The fundamental component of the drain current is

$$i_{d1} = I_m \cos \omega_o t \tag{8.174}$$

where

$$I_m = \frac{1}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} i_D \cos\omega_o t \, d(\omega_o t) = \frac{1}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} I_{DM}\cos^2\omega_o t \, d(\omega_o t) = \frac{I_{DM}}{2} \tag{8.175}$$

or

$$I_m = \frac{V_m}{R_i} = \frac{4(V_I - V_{DSmin})}{\pi R_i}. \tag{8.176}$$

The dc input current is equal to the dc component of the drain current

$$I_I = \frac{1}{2\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} i_D \, d(\omega_o t) = \frac{1}{2\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} I_{DM}\cos\omega_o t \, d(\omega_o t) = \frac{I_{DM}}{\pi} = \frac{2}{\pi}I_m = \frac{8(V_I - V_{DSmin})}{\pi^2 R_i}. \tag{8.177}$$

The dc input power is given by

$$P_I = I_I V_I = \frac{I_{DM} V_I}{\pi}. \tag{8.178}$$

The output power is

$$P_O = \frac{1}{2}I_m V_m = \frac{1}{2} \times \frac{I_{DM}}{2} \times \frac{4}{\pi}(V_I - V_{DSmin}) = \frac{1}{\pi}I_{DM}(V_I - V_{DSmin}) \tag{8.179}$$

or

$$P_O = \frac{V_m^2}{2R_i} = \frac{8(V_I - V_{DSmin})^2}{\pi^2 R_i}. \tag{8.180}$$

The drain efficiency of the amplifier is

$$\eta_D = \frac{P_O}{P_I} = 1 - \frac{V_{DSmin}}{V_I}. \tag{8.181}$$

The output-power capability is

$$c_p = \frac{P_O}{I_{DM} V_{DSM}} = \frac{1}{2\pi}\left(1 - \frac{V_{DSmin}}{V_I}\right) = 0.159\left(1 - \frac{V_{DSmin}}{V_I}\right). \tag{8.182}$$

The dc resistance is

$$R_{DC} = \frac{V_I}{I_I} = \frac{\pi^2}{8}R_i. \tag{8.183}$$

The rms value of the drain current is

$$I_{DSrms} = \sqrt{\frac{1}{2\pi}\int_{\pi/2}^{\pi/2} I_m^2 \cos^2\omega_o t d(\omega_o t)} = \frac{I_m}{2}. \tag{8.184}$$

If the transistor is operated as a switch, the conduction loss in the MOSFET on-resistance r_{DS} is given by

$$P_{rDS} = r_{DS} I_{DSrms}^2 = \frac{r_{DS} I_m^2}{4}. \tag{8.185}$$

The output power is

$$P_O = \frac{R_i I_m^2}{2}. \tag{8.186}$$

Hence, the drain efficiency is

$$\eta_D = \frac{P_O}{P_I} = \frac{P_O}{P_O + P_{rDS}} = \frac{1}{1 + \frac{P_{rDS}}{P_O}} = \frac{1}{1 + \frac{r_{DS}}{2R_i}} = \frac{R_i}{R_i + \frac{r_{DS}}{2}} = 1 - \frac{R_i}{R_i + \frac{r_{DS}}{2}}. \tag{8.187}$$

Example 8.4

Design a Class F power amplifier with a quarter-wave transmission line to deliver a power of 16 W to a load of 50 Ω at $f_c = 5$ GHz. The dc power supply voltage is 28 V and $V_{DSmin} =$ 1 V. The MOSFET on-resistance is $r_{DS} = 0.2\,\Omega$.

Solution. The maximum amplitude of the fundamental component of the drain-to-source voltage is

$$V_m = \frac{4}{\pi}(V_I - V_{DSmin}) = \frac{4}{\pi}(28 - 1) = 34.377\,\text{V}. \tag{8.188}$$

The input resistance of the load circuit is

$$R_i = \frac{V_m^2}{2P_O} = \frac{34.377^2}{2 \times 16} = 36.93\,\Omega. \tag{8.189}$$

The maximum drain-to-source voltage is

$$V_{DSmax} = 2V_I = 2 \times 28 = 56\,\text{V}. \tag{8.190}$$

The amplitude of the fundamental component of the drain current is

$$I_m = \frac{V_m}{R} = \frac{34.377}{36.93} = 0.9308\,\text{A}. \tag{8.191}$$

The maximum drain current is

$$I_{DM} = 2I_m = 2 \times 0.9308 = 1.8616\,\text{A}. \tag{8.192}$$

The dc supply current is

$$I_I = \frac{I_{DM}}{\pi} = \frac{1.8616}{\pi} = 0.5926\,\text{A}. \tag{8.193}$$

The dc supply power is

$$P_I = I_I V_I = 0.5926 \times 28 = 16.5928\,\text{W}. \tag{8.194}$$

The drain power loss of the transistor is

$$P_D = P_I - P_O = 16.5928 - 16 = 0.5928\,\text{W}. \tag{8.195}$$

The drain efficiency is

$$\eta_D = \frac{P_O}{P_I} = \frac{16}{16.5928} = 96.42\,\%. \tag{8.196}$$

The dc resistance presented by the amplifier to the dc source is

$$R_{DC} = \frac{\pi^2}{8}R_i = 1.2337 \times 36.93 = 45.56\,\Omega. \tag{8.197}$$

The characteristic impedance of the transmission line is

$$Z_o = \sqrt{R_i R} = \sqrt{36.93 \times 50} = 42.97\,\Omega. \tag{8.198}$$

Assume the dielectric constant used for construction of the transmission line to be $\epsilon_r = 2.1$. The wavelength in the transmission line is

$$\lambda = \frac{c}{\sqrt{\epsilon_r} f_c} = \frac{3 \times 10^8}{\sqrt{2.1} \times 5 \times 10^9} = 4.14\,\text{cm}. \tag{8.199}$$

Hence, the length of the transmission line is

$$l_{TL} = \frac{\lambda}{4} = \frac{4.14}{4} = 1.035\,\text{cm}. \tag{8.200}$$

Assume that the loaded quality factor $Q_L = 7$. The loaded quality factor for a parallel-resonant circuit is defined as

$$Q_L = \frac{R}{\omega L} = \omega C R_L. \tag{8.201}$$

Hence, the inductance of the resonant circuit tunned the fundamental is

$$L = \frac{R}{\omega_c Q_L} = \frac{50}{2\pi \times 5 \times 10^9 \times 7} = 0.227\,\text{nH}. \tag{8.202}$$

The capacitance of the resonant circuit tuned to the fundamental is

$$C = \frac{Q_L}{\omega_c R} = \frac{7}{2\pi \times 5 \times 10^9 \times 50} = 4.456\,\text{pF}. \tag{8.203}$$

Neglecting switching losses, the drain efficiency is

$$\eta_D = \frac{R_i}{R_i + \frac{r_{DS}}{2}} = \frac{36.93}{36.93 + \frac{0.2}{2}} = 99.73\,\%. \tag{8.204}$$

8.6 Class F RF Power Amplifier with Second Harmonic

8.6.1 Maximally Flat Class F_2 Amplifier

A circuit of the Class F RF power amplifier with a second harmonic peaking is shown in Figure 8.14. This circuit is called the Class F_2 amplifier. Figure 8.15 shows the voltage and current waveforms. The drain current waveform is a square wave. The drain-to-source voltage waveform v_{DS} is given by

$$v_{DS} = V_I - V_m \cos \omega_o t + V_{m2} \cos 2\omega_o t. \tag{8.205}$$

The derivative of this voltage with respect to $\omega_o t$ is

$$\begin{aligned}\frac{\mathrm{d}v_{DS}}{\mathrm{d}(\omega_o t)} &= V_m \sin \omega_o t - 2V_{m2} \sin 2\omega_o t \\ &= V_m \sin \omega_o t - 4V_{m2} \sin \omega_o t \cos \omega_o t \\ &= \sin \omega_o t (V_m - 4V_{m2} \cos \omega_o t) = 0.\end{aligned} \tag{8.206}$$

One solution of this equation is

$$\sin \omega_o t_m = 0 \tag{8.207}$$

yielding the extremum at

$$\omega_o t_m = 0 \tag{8.208}$$

and the maximum at

$$\omega_o t_m = \pi. \tag{8.209}$$

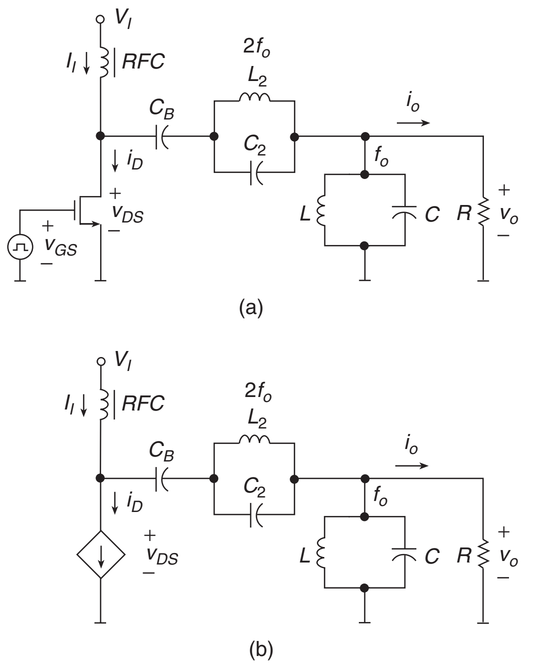


Figure 8.14 Class F power amplifier with second harmonic. (a) Circuit. (b) Equivalent circuit.

The other solution is

$$\cos \omega_o t_m = \frac{V_m}{4V_{m2}} \le 1. \tag{8.210}$$

For $0 \le V_{m2}/V_m \le 1/4$, the waveform of v_{DS} has only one minimum value at $\omega_o t = 0$ and only one maximum value at $\omega_o t = \pi$. For $V_{DSmin} = 0$,

$$\frac{V_m}{V_{pk}} = \frac{V_m}{V_I} = \frac{V_m}{V_m - V_{m2}} = \frac{1}{1 - \dfrac{V_{m2}}{V_m}}. \tag{8.211}$$

The waveform of v_{DS} is maximally flat for

$$\frac{V_{m2}}{V_m} = \frac{1}{4}. \tag{8.212}$$

The solutions do not exist for $V_{m2} > V_m/4$.

Using the trigonometric identity $\sin^2 \omega_o t + \cos^2 \omega_o t = 1$, we get

$$\sin^2 \omega_o t = 1 - \cos^2 \omega_o t = 1 - \frac{V_m^2}{16V_{m2}^2}. \tag{8.213}$$

Hence,

$$\sin \omega_o t_m = \pm\sqrt{1 - \frac{V_m^2}{16V_{m2}^2}} = \pm\sqrt{\frac{16v_{m2}^2 - V_m^2}{16V_{m2}^2}} \quad \text{for} \quad V_{m2} \ge \frac{V_m}{4}. \tag{8.214}$$

For $V_{m2} < V_m/4$, real solutions of this equation do not exist. For $V_{m2} > V_m/4$, the waveform of v_{DS} has two maxima at

$$\omega_o t_m = \pm \arcsin\sqrt{1 - \frac{V_m^2}{16V_{m2}^2}}. \tag{8.215}$$

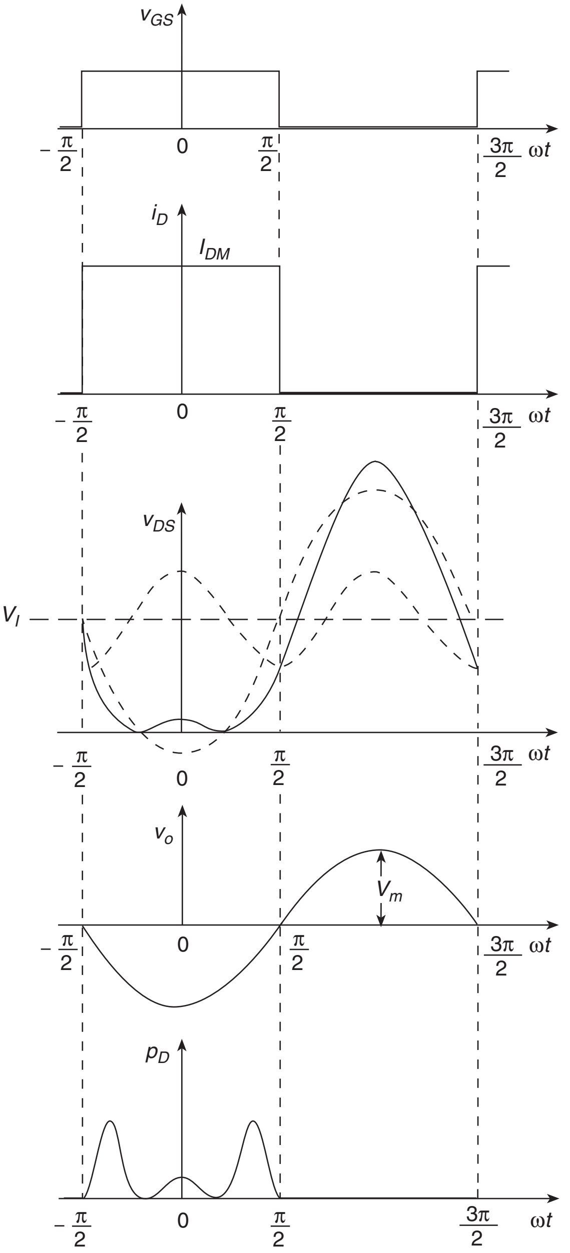


Figure 8.15 Waveforms in Class F RF power amplifier with a second harmonic.

The derivatives of the voltage v_{DS} are

$$\frac{\mathrm{d}v_{DS}}{\mathrm{d}(\omega_o t)} = V_m \sin \omega_o t - 2V_{m2} \sin 2\omega_o t \tag{8.216}$$

and

$$\frac{\mathrm{d}^2 v_{DS}}{\mathrm{d}(\omega_o t)^2} = V_m \cos \omega_o t - 4V_{m2} \cos 2\omega_o t. \tag{8.217}$$

Both terms of the first-order derivative at $\omega_o t = 0$ are zero. Using the second derivative, we get

$$\left.\frac{d^2 v_{DS}}{d(\omega_o t)^2}\right|_{\omega_o t=0} = V_m - 4V_{m2} = 0 \tag{8.218}$$

yielding

$$V_{m2} = \frac{V_m}{4}. \tag{8.219}$$

Hence,

$$v_{DS}(0) = V_{m2} = V_I - V_m + V_{m2} = V_I - V_m + \frac{V_m}{4} = V_I - \frac{3}{4}V_m = 0 \tag{8.220}$$

producing

$$V_m = \frac{4}{3}V_I. \tag{8.221}$$

Similarly,

$$v_{DS}(0) = V_{m2} = V_I - V_m + V_{m2} = V_I - 4V_{m2} + V_{m2} = V_I - 3V_{m2} = 0 \tag{8.222}$$

which gives

$$V_{m2} = \frac{V_I}{3}. \tag{8.223}$$

The maximum value of the drain-to-source voltage is

$$V_{DSM} = v_{DS}(\pi) = V_I + V_m + V_{m2} = V_I + \frac{4}{3}V_I + \frac{1}{3}V_I = \frac{8}{3}V_I. \tag{8.224}$$

The normalized maximally flat waveform v_{DS}/V_I is given by

$$\frac{v_{DS}}{V_I} = 1 - \frac{V_m}{V_I}\cos\omega_o t + \frac{V_{m2}}{V_m}\cos 2\omega_o t = 1 - \frac{4}{3}\cos\omega_o t + \frac{1}{3}\cos 2\omega_o t. \tag{8.225}$$

Figure 8.16 depicts the maximally flat waveform of the normalized drain-to-source voltage v_{DS}/V_I with a second harmonic peaking.

The amplitude of the drain current is

$$I_m = \frac{I_{DM}}{2} \tag{8.226}$$

or

$$I_m = \frac{V_m}{R} = \frac{4}{3}\frac{V_I}{R}. \tag{8.227}$$

Hence, the maximum drain current is

$$I_{DM} = 2I_m = \frac{8}{3}\frac{V_m}{R} = \frac{32}{9}\frac{V_I}{R}. \tag{8.228}$$

The dc input current is

$$I_I = \frac{I_{DM}}{2} = \frac{\pi}{4}I_m = \frac{\pi}{3}\frac{V_I}{R}. \tag{8.229}$$

The dc input power is

$$P_I = I_I V_I = \frac{\pi}{3}\frac{V_I^2}{R}. \tag{8.230}$$

The output power is

$$P_O = \frac{V_m^2}{2R} = \frac{8}{9}\frac{V_I^2}{R}. \tag{8.231}$$

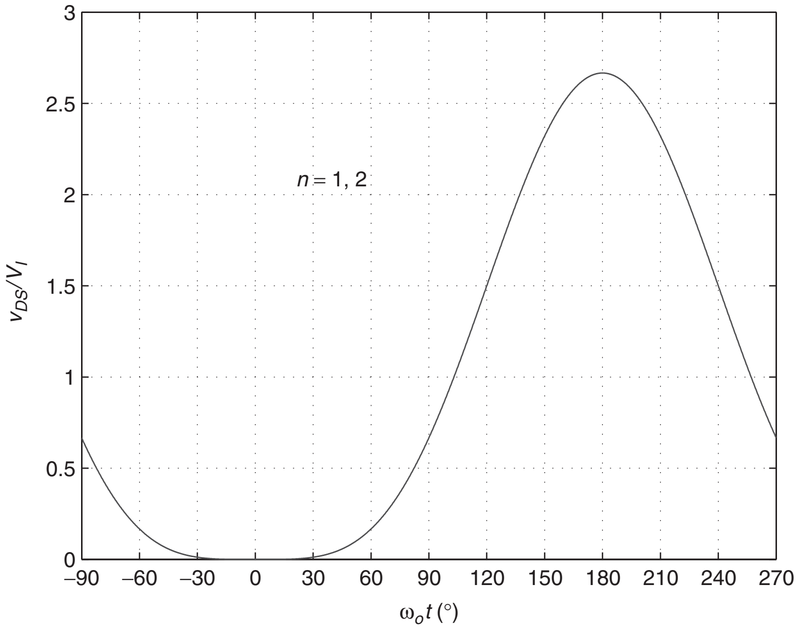


Figure 8.16 Maximally flat waveform of the drain-to-source voltage v_{DS}/V_I for Class F RF power amplifier with the second harmonic peaking.

The drain efficiency is

$$\eta_D = \frac{P_O}{P_I} = \frac{1}{2}\left(\frac{I_m}{I_I}\right)\left(\frac{V_m}{V_I}\right) = \frac{1}{2} \times \frac{4}{\pi} \times \frac{4}{3} = \frac{8}{3\pi} \approx 84.88\,\%. \tag{8.232}$$

The ratio of the drain efficiency of the Class F amplifier with the second harmonic and maximally flat voltage to the drain efficiency of the Class B amplifier at $V_{DSmin} = 0$ is given by

$$\frac{\eta_{D(F2)}}{\eta_{D(B)}} = \frac{32}{3\pi^2} \approx 1.0808. \tag{8.233}$$

The output-power capability is

$$c_p = \frac{P_O}{I_{DM}V_{DSM}} = \frac{\eta_D P_I}{I_{DM}V_{DSM}} = \frac{\eta_D I_I V_I}{I_{DM}V_{DSM}} = \left(\frac{8}{3\pi}\right)\left(\frac{1}{2}\right)\left(\frac{3}{8}\right) = \frac{1}{2\pi} \approx 0.1592. \tag{8.234}$$

The dc input resistance is

$$R_{DC} = \frac{V_I}{I_I} = \frac{3}{\pi}R \approx 0.9549R. \tag{8.235}$$

Example 8.5

Design a Class F power amplifier employing second harmonic peaking with a maximally flat drain-to-source voltage to deliver a power of 25 W at $f_c = 900$ MHz with $BW = 100$ MHz. The dc power supply voltage is 32 V and $V_{DSmin} = 1$ V.

Solution. The maximum amplitude of the fundamental component of the drain-to-source voltage is

$$V_m = \frac{4}{3}(V_I - V_{DSmin}) = \frac{4}{3}(32 - 1) = 41.333\,\text{V}. \tag{8.236}$$

The amplitude of the third harmonic is

$$V_{m2} = \frac{V_m}{4} = \frac{41.333}{4} = 10.332\,\text{V}. \tag{8.237}$$

The load resistance is

$$R = \frac{V_m^2}{2P_O} = \frac{41.333^2}{2 \times 25} = 34.168\,\Omega. \tag{8.238}$$

The maximum drain-to-source voltage is

$$V_{DSmax} = \frac{8V_I}{3} = \frac{8 \times 32}{3} = 85.333\,\text{V}. \tag{8.239}$$

The amplitude of the fundamental component of the drain current is

$$I_m = \frac{V_m}{R} = \frac{41.333}{34.168} = 1.2097\,\text{A}. \tag{8.240}$$

The maximum drain current is

$$I_{DM} = \frac{\pi}{2} I_m = \frac{\pi}{2} \times 1.2097 = 1.9\,\text{A}. \tag{8.241}$$

The dc supply current is

$$I_I = \frac{I_{DM}}{2} = \frac{1.9}{2} = 0.95\,\text{A}. \tag{8.242}$$

The dc supply power is

$$P_I = I_I V_I = 0.95 \times 32 = 30.4\,\text{W}. \tag{8.243}$$

The drain power loss of the transistor is

$$P_D = P_I - P_O = 30.4 - 25 = 5.4\,\text{W}. \tag{8.244}$$

The drain efficiency is

$$\eta_D = \frac{P_O}{P_I} = \frac{25}{30.4} = 82.23\,\%. \tag{8.245}$$

The dc resistance presented by the amplifier to the dc source is

$$R_{DC} = \frac{3}{\pi} R = 0.9549 \times 34.168 = 32.627\,\Omega. \tag{8.246}$$

The loaded quality factor of the resonant circuit tuned to the fundamental is

$$Q_L = \frac{f_c}{BW} = \frac{900}{100} = 9. \tag{8.247}$$

Hence, the components of this circuit are

$$L = \frac{R}{\omega_c Q_L} = \frac{34.168}{2\pi \times 0.9 \times 10^9 \times 9} = 0.671\,\text{nH} \tag{8.248}$$

and

$$C = \frac{Q_L}{\omega_c R} = \frac{9}{2\pi \times 0.9 \times 10^9 \times 34.168} = 46.58\,\text{pF}. \tag{8.249}$$

8.6.2 Maximum Drain Efficiency Class F_2 Amplifier

The optimum amplitudes of the fundamental component and the second harmonic for achieving the maximum drain efficiency for the Class F amplifier with the second harmonic are derived below. The minimum value of the drain-to-source voltage with ripple is given by

$$v_{DS}(\omega_o t_m) = V_I - V_m \cos \omega_o t_m + V_{m2} \cos 2\omega_o t_m = 0. \tag{8.250}$$

Using (8.210),

$$\begin{aligned} V_I &= V_m \cos \omega_o t_m - V_{m2} \cos 2\omega_o t_m = V_m \cos \omega_o t_m - V_{m2}(2\cos^2 \omega_o t_m - 1) \\ &= V_m \times \frac{V_m}{4V_{m2}} - V_{m2}\left(2 \times \frac{V_m^2}{16V_{m2}^2} - 1\right) = \frac{V_m^2}{8V_{m2}} + V_{m2} \end{aligned} \tag{8.251}$$

producing

$$\frac{V_m}{V_I} = \frac{1}{\dfrac{V_{m2}}{V_m} + \dfrac{V_m}{8V_{m2}}}. \tag{8.252}$$

To maximize the ratio V_m/V_I, we take the derivative and set it equal to zero

$$\frac{d\left(\dfrac{V_m}{V_I}\right)}{d\left(\dfrac{V_{m2}}{V_m}\right)} = \frac{-1 + \dfrac{1}{8}\left(\dfrac{V_m}{V_{m2}}\right)^2}{\left(\dfrac{V_{m2}}{V_m} + \dfrac{V_m}{8V_{m2}}\right)^2} = 0 \tag{8.253}$$

yielding the optimum ratio of the amplitudes of the second harmonic and the fundamental component

$$\frac{V_{m2}}{V_m} = \frac{1}{2\sqrt{2}} \approx 0.3536. \tag{8.254}$$

The dc supply voltage V_I can be expressed in terms of V_m to give

$$V_I = V_m \cos \omega_o t_m - V_{m2}(2\cos^2 \omega_o t_m - 1) = \frac{V_m}{\sqrt{2}} - 2\sqrt{2}\left(2 \times \frac{1}{2} - 1\right) = \frac{V_m}{\sqrt{2}} \tag{8.255}$$

producing

$$\frac{V_m}{V_I} = \sqrt{2}. \tag{8.256}$$

Finally,

$$V_{m2} = \frac{V_m}{2\sqrt{2}} = \frac{V_I}{2} \tag{8.257}$$

yielding

$$\frac{V_{m2}}{V_I} = \frac{1}{2}. \tag{8.258}$$

When V_I reaches its minimum value, the ratio V_m/V_I reaches the maximum value, and therefore the drain efficiency also reaches the maximum value. The maximum drain-to-source voltage is

$$V_{DSM} = v_{DS}(\pi) = V_I + V_m + V_{m2} = V_I + \sqrt{2}V_I + 0.5V_I = (1.5 + \sqrt{2})V_I \approx 2.914V_I. \tag{8.259}$$

The two minimum values of v_{DS} for the maximum efficiency are located at

$$\omega_o t_m = \pm \arcsin\sqrt{1 - \frac{1}{16}\left(\frac{V_m}{V_{m3}}\right)^2} = \pm \arcsin\sqrt{1 - \frac{1}{16}\left(2\sqrt{2}\right)^2} = \pm \arcsin\frac{1}{\sqrt{2}} = \pm 45^\circ. \tag{8.260}$$

The normalized waveform v_{DS}/V_I at the maximum drain efficiency is given by

$$\frac{v_{DS}}{V_I} = 1 - \frac{V_m}{V_I}\cos\omega_o t + \frac{V_{m2}}{V_I}\cos 2\omega_o t = 1 - \sqrt{2}\cos\omega_o t + \frac{1}{2}\cos 2\ \omega_o t \tag{8.261}$$

and is illustrated in Figure 8.17.

The amplitude of the fundamental component of the drain current is

$$I_m = \frac{V_m}{R} = \frac{\sqrt{2}V_I}{R}. \tag{8.262}$$

The dc supply current is

$$I_I = \frac{\pi}{4} I_m = \frac{\pi\sqrt{2}}{4}\frac{V_I}{R}. \tag{8.263}$$

The dc supply power is

$$P_I = I_I V_I = \frac{\pi\sqrt{2}}{4}\frac{V_I^2}{R}. \tag{8.264}$$

The output power is

$$P_O = \frac{V_m^2}{2R} = \frac{(\sqrt{2}V_I)^2}{2R} = \frac{V_I^2}{R}. \tag{8.265}$$

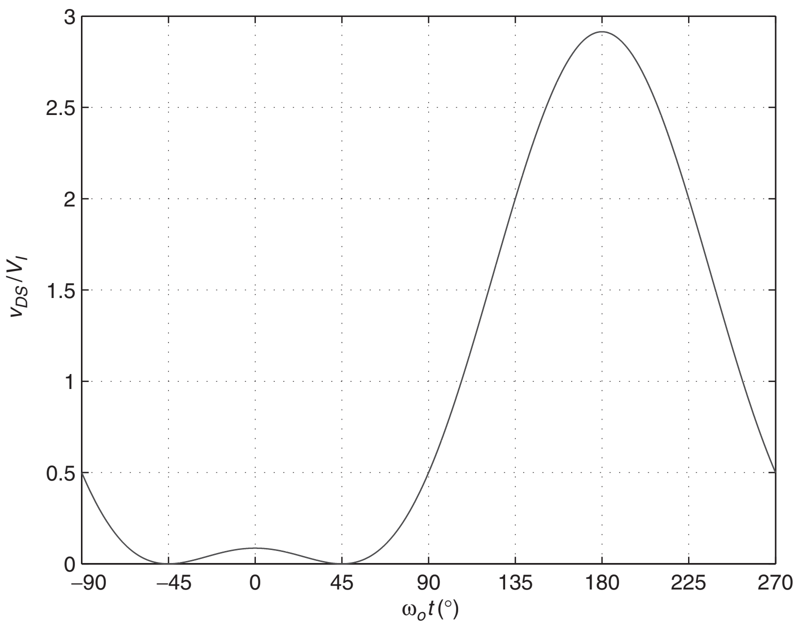


Figure 8.17 Waveform of the drain-to-source voltage v_{DS}/V_I for Class F RF power amplifier with the second harmonic peaking at the maximum drain efficiency.

The drain efficiency is

$$\eta_D = \frac{P_O}{P_I} = \frac{1}{2}\frac{I_m}{I_I}\frac{V_m}{V_I} = \frac{1}{2} \times \frac{4}{\pi} \times \sqrt{2} = \frac{2\sqrt{2}}{\pi} \approx 90.03\,\%. \tag{8.266}$$

The ratio of the maximum drain efficiency $\eta_{Dmax(F2)}$ to the drain efficiency with maximally flat voltage is

$$\frac{\eta_{Dmax(F2)}}{\eta_{D(F2)}} = \frac{3\sqrt{2}}{4} \approx 1.0607. \tag{8.267}$$

The ratio of the maximum drain efficiency $\eta_{Dmax(F2)}$ to the drain efficiency of the Class B amplifier is

$$\frac{\eta_{Dmax(F2)}}{\eta_{D(B)}} = \frac{32}{3\pi^2} \approx 1.0808. \tag{8.268}$$

The drain efficiency of the Class B amplifier with a square-wave drain current and a sinusoidal drain-to-source voltage is given by

$$\eta_B = \frac{P_O}{P_I} = \frac{2}{\pi}. \tag{8.269}$$

The drain efficiency of the Class F amplifier with the second harmonic is expressed as

$$\eta_D = \eta_B \left(\frac{V_m}{V_I}\right) = \frac{2}{\pi}\left(\frac{V_m}{V_I}\right). \tag{8.270}$$

Thus,

$$\eta_D = \frac{2}{\pi}\left(\frac{1}{1 - \frac{V_{m2}}{V_m}}\right) \qquad \text{for} \quad 0 \le \frac{V_{m2}}{V_m} \le \frac{1}{4} \tag{8.271}$$

and

$$\eta_D = \frac{2}{\pi} \times \frac{1}{\frac{V_{m2}}{V_m} + \frac{V_m}{8V_{m2}}} \qquad \text{for} \quad \frac{V_{m2}}{V_m} \ge \frac{1}{4}. \tag{8.272}$$

The ratio V_m/V_I as a function of V_{m2}/V_m is shown in Figure 8.18. Figure 8.19 shows the drain efficiency as function of V_{m2}/V_m.

The output-power capability is

$$c_p = \frac{P_{O(max)}}{I_{DM}V_{DSM}} = \frac{\eta_D P_I}{I_{DM}V_{DSM}} = \eta_D \frac{I_I V_I}{I_{DM}V_{DSM}} = \frac{2\sqrt{2}}{\pi} \times \frac{1}{2} \times \frac{1}{\sqrt{2}+1.5}$$
$$= \frac{\sqrt{2}}{\pi(\sqrt{2}+1.5)} \approx 0.1545. \tag{8.273}$$

The maximum output power capability at any value of V_{m3}/V_m is given by

$$c_p = \frac{P_{O(max)}}{I_{DM}V_{DSM}} = \frac{1}{2}\left(\frac{I_m}{I_{DM}}\right)\left(\frac{V_m}{V_{DSM}}\right) = \frac{1}{2}\left(\frac{2}{\pi}\right)\left(\frac{V_m}{V_{DSM}}\right) = \frac{1}{\pi}\left(\frac{V_m}{V_I}\right)\left(\frac{V_I}{V_{DSM}}\right). \tag{8.274}$$

Thus,

$$c_p = \frac{1}{2\pi} \qquad \text{for} \quad 0 \le \frac{V_{m2}}{V_m} \le \frac{1}{4} \tag{8.275}$$

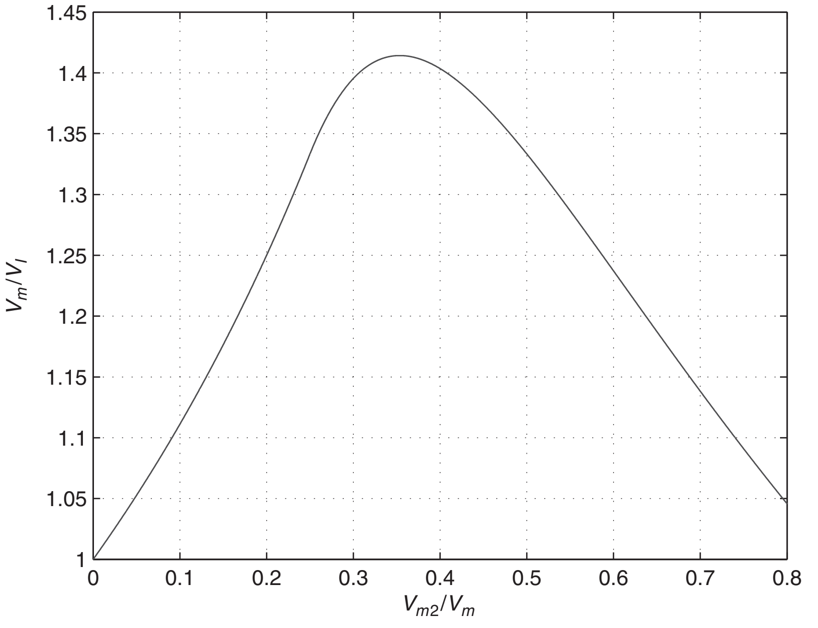


Figure 8.18 Ratio V_m/V_I as a function of V_{m2}/V_m for Class F power amplifier with second harmonic.

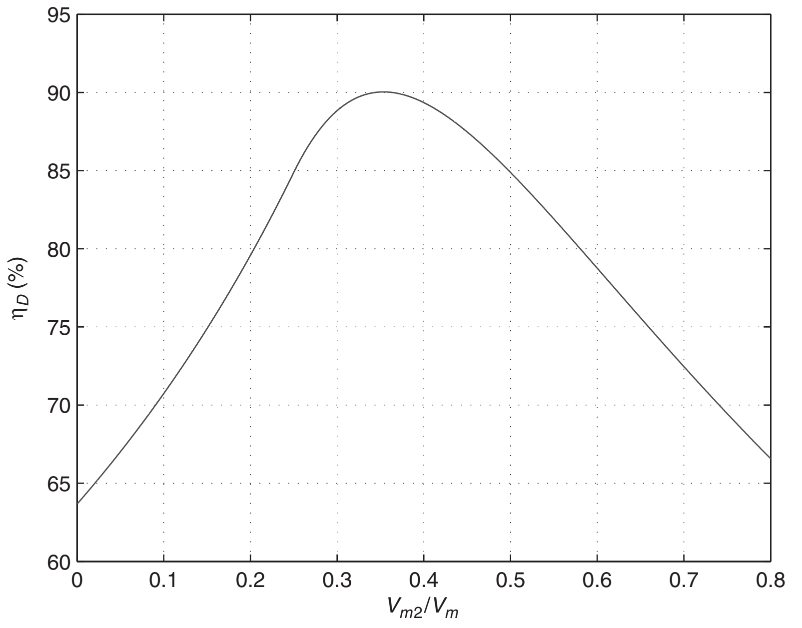


Figure 8.19 Drain efficiency η_D as a function of V_{m2}/V_m for Class F power amplifier with second harmonic.

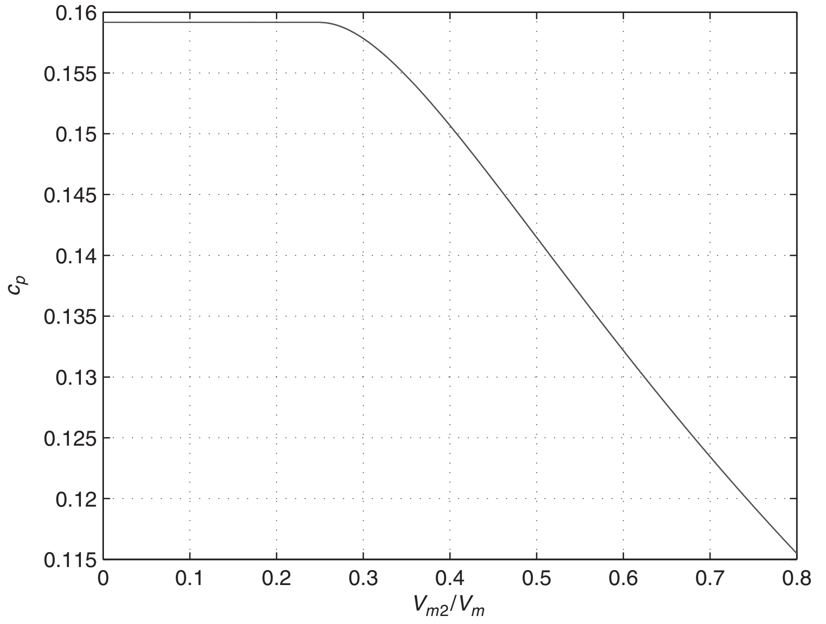


Figure 8.20 Output power capability c_p as a function of V_{m2}/V_m for Class F power amplifier with second harmonic.

and

$$c_p = \frac{1}{\pi} \frac{1}{1 + \dfrac{2V_{m2}}{V_m} + \dfrac{V_m}{8V_{m2}}} \qquad \text{for} \qquad \frac{V_{m2}}{V_m} > \frac{1}{4}. \tag{8.276}$$

Fig 8.20 shows c_p as a function of V_{m2}/V_m.

The dc input resistance is

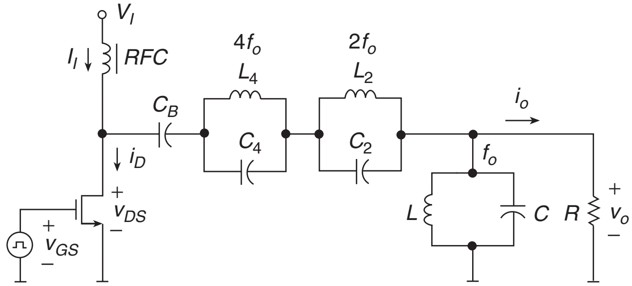
$$R_{DC} = \frac{V_I}{I_I} = \frac{4}{\pi\sqrt{2}} R \approx 0.9R. \tag{8.277}$$

8.7 Class F RF Power Amplifier with Second and Fourth Harmonics

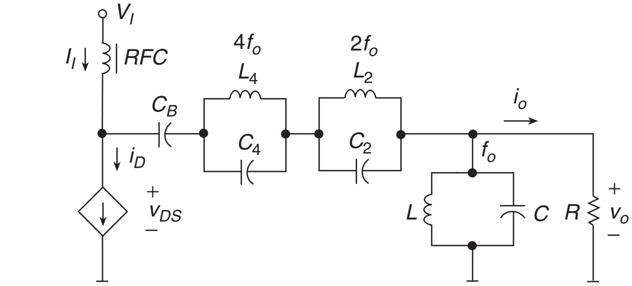
8.7.1 Maximally Flat Class F_{24} Amplifier

Figure 8.21 shows the circuit of a Class F RF power amplifier with the second and fourth harmonics. This circuit is called the Class F_{24} amplifier. The voltage and current waveforms are depicted in Figure 8.22. The drain-to-source voltage is

$$v_{DS} = V_I - V_m \cos \omega_o t + V_{m2} \cos 2\omega_o t - V_{m4} \cos 4\omega_o t. \tag{8.278}$$



(a)



(b)

Figure 8.21 Class F power amplifier with second and fourth harmonics. (a) Circuit. (b) Equivalent circuit.

The derivatives of this voltage are

$$\frac{\mathrm{d}v_{DS}}{\mathrm{d}(\omega_o t)} = V_m \sin \omega_o t - 2V_{m2} \sin 2\omega_o t + 4V_{m4} \sin 4\omega_o t \tag{8.279}$$

$$\frac{\mathrm{d}^2 v_{DS}}{\mathrm{d}(\omega_o t)^2} = V_m \cos \omega_o t - 4V_{m2} \cos 2\omega_o t + 16V_{m4} \cos 4\omega_o t \tag{8.280}$$

$$\frac{\mathrm{d}^3 v_{DS}}{\mathrm{d}(\omega_o t)^3} = -V_m \sin \omega_o t + 8V_{m2} \sin 2\omega_o t - 64V_{m4} \sin 4\omega_o t \tag{8.281}$$

and

$$\frac{\mathrm{d}^4 v_{DS}}{\mathrm{d}(\omega_o t)^4} = -V_m \cos \omega_o t + 16V_{m2} \cos 2\omega_o t - 256V_{m4} \cos 4\omega_o t. \tag{8.282}$$

The first and third-order derivatives are zero at $\omega_o t = 0$ and do not generate any equations. Using the second and forth-order derivatives, we obtain

$$\left.\frac{\mathrm{d}^2 v_{DS}}{\mathrm{d}(\omega_o t)^2}\right|_{\omega_o t=0} = V_m - 4V_{m2} + 16V_{m4} = 0 \tag{8.283}$$

and

$$\left.\frac{\mathrm{d}^4 v_{DS}}{\mathrm{d}(\omega_o t)^4}\right|_{\omega_o t=0} = V_m - 16V_{m2} + 256V_{m4} = 0. \tag{8.284}$$

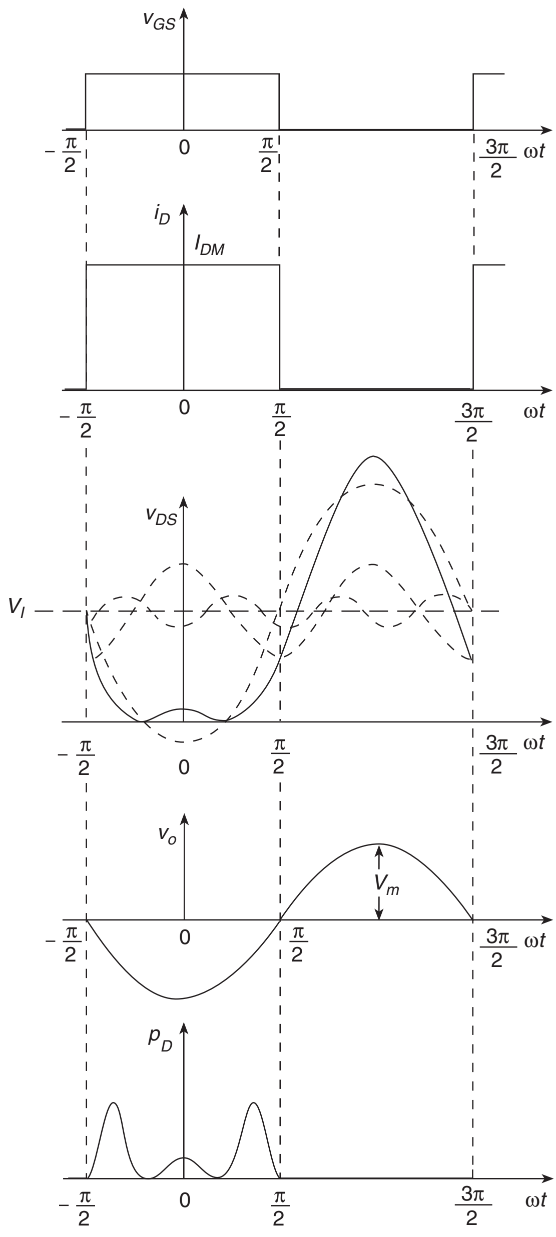


Figure 8.22 Waveforms in Class F RF power amplifier with second and fourth harmonics peaking.

Solution for this set of equations yields

$$V_{m2} = \frac{5}{16} V_m \tag{8.285}$$

$$V_{m4} = \frac{1}{64} V_m \tag{8.286}$$

and

$$V_{m2} = 20V_{m4}. \tag{8.287}$$

Hence,

$$v_{DS}(0) = V_I - V_m + V_{m2} - V_{m4} = V_I - V_m + \frac{5}{16}V_m - \frac{1}{64}V_m = V_I - \frac{45}{64}V_m = 0 \tag{8.288}$$

producing

$$V_m = \frac{64}{45}V_I. \tag{8.289}$$

Now

$$v_{DS}(0) = V_I - V_m + V_{m2} - V_{m4} = V_I - \frac{16}{5}V_{m2} + V_{m2} - \frac{1}{20}V_{m2} = V_I - \frac{9}{4}V_{m2} = 0 \tag{8.290}$$

yielding

$$V_{m2} = \frac{4}{9}V_I. \tag{8.291}$$

Finally,

$$v_{DS}(0) = V_I - V_m + V_{m2} - V_{m4} = V_I - 64V_{m4} + 20V_{m4} - V_{m4} = 0 \tag{8.292}$$

giving

$$V_{m4} = \frac{1}{45}V_I. \tag{8.293}$$

The maximum drain-to-source voltage is

$$\begin{aligned} V_{DSM} &= v_{DS}(\pi) = V_I + V_m + V_{m2} - V_{m4} = V_I + \frac{64}{45}V_I + \frac{4}{9}V_I - \frac{1}{45}V_I \\ &= \frac{128}{45}V_I \approx 2.844V_I. \end{aligned} \tag{8.294}$$

The normalized maximally flat waveform v_{DS}/V_I is given by

$$\begin{aligned} \frac{v_{DS}}{V_I} &= 1 - \frac{V_m}{V_I}\cos\omega_o t + \frac{V_{m2}}{V_I}\cos 2\omega_o t - \frac{V_{m4}}{V_I}\cos 4\omega_o t \\ &= 1 - \frac{64}{45}\cos\omega_o t + \frac{4}{9}\cos 2\omega_o t - \frac{1}{45}\cos 4\omega_o t. \end{aligned} \tag{8.295}$$

and is shown in Figure 8.23.

The amplitude of the fundamental component of the drain current is

$$I_m = \frac{2}{\pi}I_{DM} \tag{8.296}$$

and

$$I_m = \frac{V_m}{R} = \frac{64}{45}\frac{V_I}{R}. \tag{8.297}$$

Thus,

$$I_{DM} = \frac{\pi}{2}I_m = \frac{32\pi}{45}\frac{V_I}{R}. \tag{8.298}$$

The dc input current is

$$I_I = \frac{I_{DM}}{2} = \frac{\pi I_m}{4} = \frac{16\pi}{45}\frac{V_I}{R}. \tag{8.299}$$

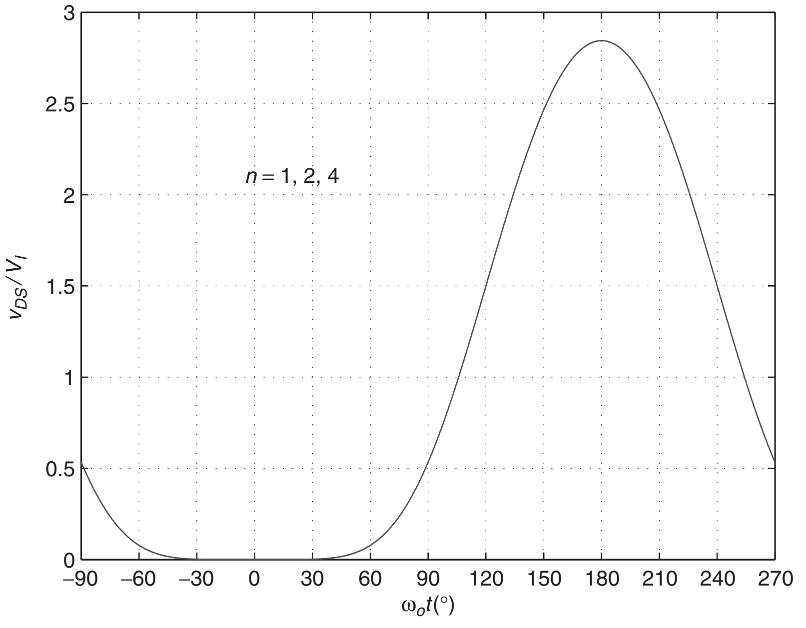


Figure 8.23 Maximally flat waveform of the drain-to-source voltage v_{DS}/V_I for Class F RF power amplifier with the second and fourth harmonic peaking.

The dc input power is

$$P_I = I_I V_I = \frac{16\pi}{45} \frac{V_I^2}{R}. \tag{8.300}$$

The output power is

$$P_O = \frac{V_m^2}{2R} = \frac{2048}{2025} \frac{V_I^2}{R}. \tag{8.301}$$

The drain efficiency of the amplifier is

$$\eta_D = \frac{P_O}{P_I} = \frac{128}{45\pi} \approx 90.54\,\%. \tag{8.302}$$

The maximum drain-to-source voltage is

$$V_{DSM} = v_{DS}(\pi) V_I + V_m + V_{m2} - V_{m4} = V_I \left(1 + \frac{64}{45} + \frac{4}{9} - \frac{1}{45}\right) = \frac{128}{45} V_I \approx 2.844 V_I. \tag{8.303}$$

The output-power capability is

$$c_p = \frac{P_O}{V_{DSM} I_{DM}} = \frac{\eta_D P_I}{V_{DSM} I_{DM}} = \eta_D \left(\frac{I_I}{I_{DM}}\right)\left(\frac{V_I}{V_{DSM}}\right)$$
$$= \frac{128}{45\pi} \times \frac{1}{2} \times \frac{45}{128} = \frac{1}{2\pi} \approx 0.1592. \tag{8.304}$$

The dc input resistance is

$$R_{DC} = \frac{V_I}{I_I} = \frac{45}{16\pi} R \approx 0.8952R. \tag{8.305}$$

8.7.2 Maximum Drain Efficiency Class F_{24} Amplifier

To find the extrema of the drain-to-source voltage v_{DS}, we take the derivative of this voltage and set the result equal to zero

$$\begin{aligned}\frac{dv_{DS}}{d(\omega t)} &= V_m \sin\omega_o t - 2V_{m2}\sin 2\omega_o t + 4V_{m4}\sin 4\omega_o t \\ &= V_m \sin\omega_o t - 4V_{m2}\sin\omega_o t\cos\omega_o t + 8V_{m4}\sin 2\omega_o t\cos 2\omega_o t \\ &= V_m \sin\omega_o t - 4V_{m2}\sin\omega_o t\cos\omega_o t + 16V_{m4}\sin\omega_o t\cos\omega_o t\cos 2\omega_o t \\ &= \sin\omega_o t(V_m - 4V_{m2}\cos\omega_o t + 16V_{m4}\cos\omega_o t\cos 2\omega_o t) \\ &= \sin\omega_o t[V_m - 4V_{m2}\cos\omega_o t + 16V_{m4}\cos\omega_o t(2\cos^2\omega_o t - 1)] = 0. \end{aligned} \quad (8.306)$$

Hence, one solution is

$$\sin\omega_o t_m = 0 \quad (8.307)$$

which gives

$$\omega_o t_m = 0 \quad (8.308)$$

and

$$\omega_o t_m = \pi. \quad (8.309)$$

The second equation is

$$32V_{m4}\cos^3\omega_o t - (4V_{m2} + 16V_{m4})\cos\omega_o t + V_m = 0 \quad (8.310)$$

which can be rearranged to the form

$$32\frac{V_{m4}}{V_m}\cos^3\omega_o t - \left(4\frac{V_{m2}}{V_m} + 16\frac{V_{m4}}{V_m}\right)\cos\omega_o t + 1 = 0. \quad (8.311)$$

This equation can only be solved numerically. The Fourier coefficients of the drain-to-source voltage waveform v_{DS} for achieving the maximum drain efficiency are [20]

$$\frac{V_m}{V_I} = 1.5 \quad (8.312)$$

$$\frac{V_{m2}}{V_I} = 0.389 \quad (8.313)$$

and

$$\frac{V_{m4}}{V_I} = 0.0556. \quad (8.314)$$

The normalized waveform v_{DS}/V_I is given by

$$\begin{aligned}\frac{v_{DS}}{V_I} &= 1 - \frac{V_m}{V_I}\cos\omega_o t + \frac{V_{m2}}{V_I}\cos 2\omega_o t - \frac{V_{m4}}{V_I}\cos 4\omega_o t \\ &= 1 - 1.5\cos\omega_o t + 0.5835\cos 2\omega_o t - 0.0834\cos 4\omega_o t. \end{aligned} \quad (8.315)$$

The waveform of v_{DS}/V_I for the maximum efficiency is shown in Figure 8.24.

The amplitude of the fundamental component of the drain current is

$$I_m = \frac{V_m}{R} = \frac{1.5V_I}{R}. \quad (8.316)$$

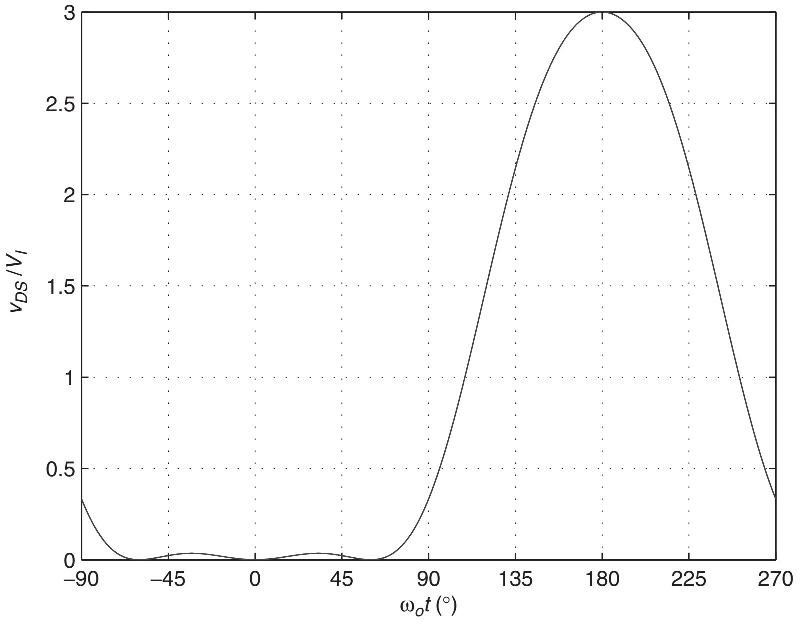


Figure 8.24 Waveform of the normalized drain-to-source voltage v_{DS}/V_I for Class F RF power amplifier with the second and fourth harmonic peaking for the maximum drain efficiency.

The maximum drain current is

$$I_{DM} = \frac{\pi}{2} I_m = \frac{\pi}{2} \times 1.5 \times \frac{V_I}{R} \approx 2.356 \frac{V_I}{R}. \tag{8.317}$$

The dc supply current is

$$I_I = \frac{\pi}{4} I_m = \frac{\pi}{4} \times 1.5 \times \frac{V_I}{R} \approx 1.178 \frac{V_I}{R}. \tag{8.318}$$

The dc input current is

$$P_I = I_I V_I = 1.178 \frac{V_I^2}{R}. \tag{8.319}$$

The output power is

$$P_O = \frac{V_m^2}{2R} = \frac{(1.5V_I)^2}{2R} = 1.125 \frac{V_I^2}{R}. \tag{8.320}$$

The drain efficiency is

$$\eta_D = \frac{P_O}{P_I} = 0.955. \tag{8.321}$$

The maximum drain-to-source voltage is

$$V_{DSM} = v_{DS}(\pi) = V_I(1 + 1.5 + 0.389 - 0.0556) = 2.833 V_I. \tag{8.322}$$

The output-power capability is

$$c_p = \frac{P_O}{P_I} = \frac{1}{2} \left(\frac{I_m}{I_{DM}} \right) \left(\frac{V_m}{V_{DSM}} \right) = \frac{1}{2} \times \frac{2}{\pi} \times \frac{1}{2.833} = 0.1123. \tag{8.323}$$

Table 8.3 Voltage amplitudes in even harmonic maximally flat Class F power amplifiers.

Class	$\frac{V_m}{V_I}$	$\frac{V_{m2}}{V_I}$	$\frac{V_{m4}}{V_I}$	$\frac{V_{m2}}{V_m}$	$\frac{V_{m4}}{V_m}$
F_2	$\frac{4}{3} = 1.333$	$\frac{1}{3} = 0.333$	0	$\frac{1}{4} = 0.25$	0
F_{24}	$\frac{64}{45} = 1.422$	$\frac{4}{9} = 0.4444$	$\frac{1}{45} = 0.02222$	$\frac{5}{16} = 0.3125$	$\frac{1}{64} = 0.01563$

Table 8.4 Voltage amplitudes in even harmonic maximum drain efficiency Class F power amplifiers.

Class	$\frac{V_m}{V_I}$	$\frac{V_{m2}}{V_I}$	$\frac{V_{m4}}{V_I}$	$\frac{V_{m2}}{V_m}$	$\frac{V_{m4}}{V_m}$
F_2	$\sqrt{2} = 1.4142$	$\frac{1}{2} = 0.5$	0	$\frac{1}{4} = 0.25$	0
F_{24}	1.5	0.5835	0.0834	0.389	0.0556

The dc input resistance is

$$R_{DC} = \frac{V_I}{I_I} = 0.8489R. \tag{8.324}$$

The normalized voltage amplitudes in the even harmonic Class F amplifiers that ensure the maximally flat waveform of v_{DS} are given in Table 8.3. The ratios of voltage amplitudes that give the maximum drain efficiency are given in Table 8.4.

8.8 Class F RF Power Amplifier with Second, Fourth, and Sixth Harmonics

For the maximally flat Class F_{246} amplifier,

$$V_{m2} = \frac{175}{512} V_m \tag{8.325}$$

$$V_{m4} = \frac{7}{256} V_m \tag{8.326}$$

$$V_{m6} = \frac{1}{512} V_m \tag{8.327}$$

$$V_{m1} = \frac{64}{45} V_I \tag{8.328}$$

$$V_{m2} = \frac{4}{9} V_I \tag{8.329}$$

and

$$V_{m4} = \frac{1}{45} V_I. \tag{8.330}$$

8.9 Class F RF Power Amplifier with Series-resonant Circuit and Quarter-wavelength Transmission Line

A Class F RF power amplifier with a series-resonant circuit and a quarter-wavelength transmission line was introduced in [7] and is shown in Figure 8.25(a). The quarter-wavelength transmission line is equivalent to an infinite number of series-resonant circuits, which behaves like short circuits for odd harmonics and like open circuits for even harmonics. The equivalent circuit of the amplifier is depicted in Figure 8.25(b). Voltage and current waveforms of the amplifier are shown in Figure 8.26. The drain current is a square wave and the drain-to-source voltage is a half-sine wave. The circuit and its waveforms are dual to those of the Class F amplifier with a parallel-resonant circuit. The drain current and the drain-to-source voltage waveforms are interchanged. This circuit will be referred to as the Class F_s amplifier or Class F inverse amplifier.

The drain current is a square wave and it can be expanded into a Fourier series

$$i_D = I_{DM}\left\{\frac{1}{2}+\frac{2}{\pi}\left[\cos\omega_o t-\frac{1}{3}\cos 3\omega_o t+\frac{1}{5}\cos 5\omega_o t-\frac{1}{7}\cos 7\omega_o t+\cdots\right]\right\}. \quad (8.331)$$

The drain current i_D contains only the dc component, fundamental component, and odd harmonic components.

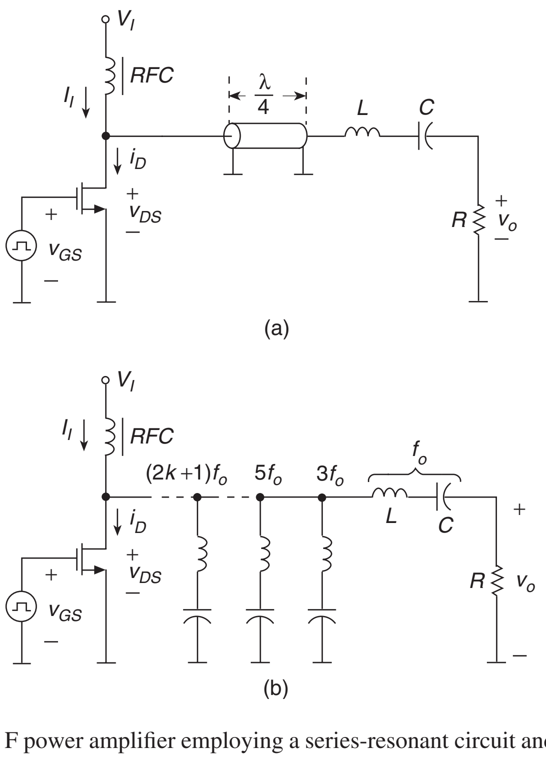


Figure 8.25 Class F power amplifier employing a series-resonant circuit and a quarter-wavelength transmission line, where $n = 2k + 1$ with $k = 0, 1, 2, \ldots$. (a) Circuit. (b) Equivalent circuit at $Z_o = R$.

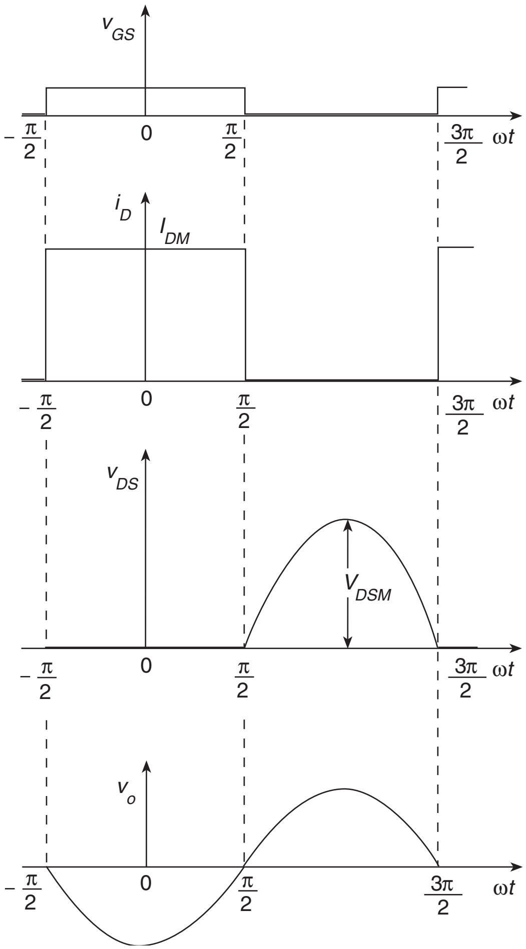


Figure 8.26 Voltage and current waveforms in Class F RF power amplifier with a series-resonant circuit and a quarter-wavelength transmission line [7].

The drain-to-source voltage waveform v_{DS} can be expanded into a Fourier trigonometric series

$$\begin{aligned} v_{DS} &= V_{DSM}\left[\frac{1}{\pi} + \frac{1}{2}\cos\omega_o t + \frac{2}{\pi}\sum_{n=2}^{\infty}\frac{\cos\left(\frac{n\pi}{2}\right)}{1-n^2}\cos n\omega_o t\right] \\ &= V_{DSM}\left[\frac{1}{\pi} + \frac{1}{2}\cos\omega_o t + \frac{2}{3\pi}\cos 2\omega_o t - \frac{2}{15\pi}\cos 4\omega_o t + \frac{2}{35\pi}\cos 6\omega_o t + \cdots\right]. \end{aligned} \tag{8.332}$$

Thus, the drain current waveform i_D contains only the dc component, fundamental component, and the even harmonic components.

The output voltage is equal to the fundamental component of the drain-to-source voltage

$$v_o = -V_m \cos\omega_o t \tag{8.333}$$

where

$$V_m = \frac{V_{DSM}}{2} = \frac{\pi}{2}V_I. \tag{8.334}$$

The output current is equal to the fundamental component of the drain current

$$i_o = i_{d1} = I_m \cos \omega_o t \tag{8.335}$$

where

$$I_m = \frac{2}{\pi} I_{DM} = \frac{4}{\pi} I_I \tag{8.336}$$

or

$$I_m = \frac{V_m}{R} = \frac{\pi V_I}{2R}. \tag{8.337}$$

The dc supply current is equal to the dc component of the drain current

$$I_I = \frac{I_{DM}}{2} = \frac{\pi}{4} I_m = \frac{\pi^2 V_I}{8R}. \tag{8.338}$$

and dc supply voltage is

$$V_I = \frac{V_{DSM}}{\pi}. \tag{8.339}$$

Hence, the dc supply power is

$$P_I = I_I V_I = \frac{I_{DM} V_I}{2} = \frac{I_{DM} V_{DSM}}{2\pi}. \tag{8.340}$$

The output power is

$$P_O = \frac{1}{2} I_m V_m = \frac{1}{2} \times \frac{2}{\pi} \times I_{DM} \frac{V_{DSM}}{2} = \frac{1}{2\pi} I_{DM} V_{DSM} \tag{8.341}$$

or

$$P_O = \frac{V_m^2}{2R} = \frac{\pi^2 V_I^2}{8R}. \tag{8.342}$$

The efficiency of the amplifier is

$$\eta = \frac{P_O}{P_I} = 1. \tag{8.343}$$

The output-power capability is

$$c_p = \frac{P_O}{I_{DM} V_{DSM}} = \frac{1}{2\pi} = 0.159. \tag{8.344}$$

The dc resistance is

$$R_{DC} = \frac{V_I}{I_I} = \frac{8}{\pi^2} R \approx 0.811R. \tag{8.345}$$

The input impedance of the load network seen by the drain and source terminals at the fundamental frequency f_o is

$$Z(f_o) = \frac{V_m}{I_m} = R_i = \frac{Z_o^2}{R}; \tag{8.346}$$

the impedance at even harmonics is

$$Z(nf_o) = Z(2kf_o) = \frac{V_{m(n)}}{I_{m(n)}} = \frac{\text{finite value}}{0} = \infty \quad \text{for} \quad n = 2, 4, 6, \ldots \tag{8.347}$$

and the impedance at odd harmonics is

$$Z(nf_o) = Z[(2k+1)f_o] = \frac{V_{m(n)}}{I_{m(n)}} = \frac{0}{\text{finite value}} = 0 \quad \text{for} \quad n = 3, 5, 7, \ldots \tag{8.348}$$

where $k = 1, 2, 3, \ldots$.

The rms value of the drain current is

$$I_{DSrms} = \sqrt{\frac{1}{2\pi}\int_{\pi/2}^{\pi/2} I_{DM}^2 \, d(\omega_o t)} = \frac{I_{DM}}{\sqrt{2}}. \tag{8.349}$$

If the transistor is operated as a switch, the conduction loss in the MOSFET on-resistance r_{DS} is given by

$$P_{rDS} = r_{DS} I_{DSrms}^2 = \frac{r_{DS} I_m^2}{2} = \frac{\pi^2}{8} r_{DS} I_m^2. \tag{8.350}$$

The output power is

$$P_O = \frac{r_{DS} I_m^2}{2}. \tag{8.351}$$

Thus, the drain efficiency is

$$\eta_D = \frac{P_O}{P_I} = \frac{P_O}{P_O + P_{rDS}} = \frac{1}{1 + \frac{P_{rDS}}{P_O}} = \frac{1}{1 + \frac{\pi^2}{4}\frac{r_{DS}}{R_i}}. \tag{8.352}$$

Example 8.6

Design a Class F power amplifier with a quarter-wave transmission line to deliver a power of 10 W at $f_c = 5$ GHz. The required bandwidth is $BW = 500$ MHz. The dc power supply voltage is 28 V and $V_{DSmin} = 1$ V. The load resistance is $R = 50\,\Omega$. The MOSFET on-resistance is $r_{DS} = 0.2\,\Omega$.

Solution. The maximum amplitude of the fundamental component of the drain-to-source voltage is

$$V_m = \frac{\pi}{2}(V_I - V_{DSmin}) = \frac{\pi}{2}(28 - 1) = 42.41 \text{ V}. \tag{8.353}$$

The input resistance of the load circuit is

$$R_i = \frac{V_m^2}{2P_O} = \frac{42.41^2}{2 \times 10} = 89.93\,\Omega. \tag{8.354}$$

The maximum drain-to-source voltage is

$$V_{DSmax} = \pi V_I = \pi \times 28 = 87.965 \text{ V}. \tag{8.355}$$

The amplitude of the fundamental component of the drain current is

$$I_m = \frac{V_m}{R_i} = \frac{42.41}{89.93} = 0.472 \text{ A}. \tag{8.356}$$

The maximum drain current is

$$I_{DM} = \frac{\pi}{2} I_m = \frac{\pi}{2} \times 0.472 = 0.7414 \text{ A}. \tag{8.357}$$

The dc supply current is

$$I_I = \frac{I_{DM}}{2} = \frac{0.7414}{2} = 0.37 \text{ A}. \tag{8.358}$$

The dc supply power is

$$P_I = I_I V_I = 0.37 \times 28 = 10.36 \text{ W}. \tag{8.359}$$

The drain power loss of the transistor is

$$P_D = P_I - P_O = 10.36 - 10 = 0.36\,\text{W}. \tag{8.360}$$

The drain efficiency is

$$\eta_D = \frac{P_O}{P_I} = \frac{10}{10.36} = 96.525\,\%. \tag{8.361}$$

The dc resistance presented by the amplifier to the dc source is

$$R_{DC} = \frac{8}{\pi^2}R = 0.81 \times 89.93 = 72.84\,\Omega. \tag{8.362}$$

The characteristic impedance of the transmission line is

$$Z_o = \sqrt{R_i R} = \sqrt{89.93 \times 50} = 67\,\Omega. \tag{8.363}$$

Assume the dielectric constant used for construction of the transmission line to be $\epsilon_r = 2.1$. The wavelength in the transmission line is

$$\lambda = \frac{c}{\sqrt{\epsilon_r} f_c} = \frac{3 \times 10^8}{\sqrt{2.1} \times 5 \times 10^9} = 4.14\,\text{cm}. \tag{8.364}$$

Hence, the length of the transmission line is

$$l_{TL} = \frac{\lambda}{4} = \frac{4.14}{4} = 1.035\,\text{cm}. \tag{8.365}$$

The loaded quality factor is

$$Q_L = \frac{f_c}{BW} = \frac{5 \times 10^9}{500 \times 10^6} = 10. \tag{8.366}$$

The loaded quality factor for a series-resonant circuit is defined as

$$Q_L = \frac{\omega_c L}{R} = \omega_c CR. \tag{8.367}$$

Hence, the inductance of the resonant circuit tuned to the fundamental is

$$L = \frac{Q_L R}{\omega_c} = \frac{10 \times 50}{2\pi \times 5 \times 10^9} = 15.915\,\text{nH}. \tag{8.368}$$

The capacitance of the resonant circuit is tunned to the fundamental frequency is

$$C = \frac{1}{\omega_c Q_L R} = \frac{1}{2\pi \times 5 \times 10^9 \times 10 \times 50} = 0.06366\,\text{pF}. \tag{8.369}$$

The drain efficiency is

$$\eta_D = \frac{1}{1 + \dfrac{\pi^2}{4}\dfrac{r_{DS}}{R_i}} = \frac{1}{1 + \dfrac{\pi^2}{4}\dfrac{0.2}{89.93}} = 99.45\,\%. \tag{8.370}$$

8.10 Summary

- The drain efficiency and the output-power capability for Class F RF power amplifiers are given in Table 8.5, where $\eta_{D(MF)}$ and $c_{p(MF)}$ are the drain efficiency and the output-power capability for the maximally flat waveform of v_{DS}, respectively, and B_s is the Class B amplifier with rectangular drain current and sinusoidal drain-to-source voltage.
- The load network in Class F RF power amplifiers has resonant circuits tuned to the fundamental frequency and to one or more harmonic frequencies.

Table 8.5 Drain efficiencies and output power capabilities of Class F power amplifiers.

Class	$\eta_{D(MF)}$	η_{Dmax}	$c_{p(MF)}$	c_{pmax}
B	–	$\frac{\pi}{4} = 78.54\,\%$	–	$\frac{1}{8} = 0.125$
F_3	$\frac{9\pi}{32} = 88.36\,\%$	$\frac{\pi}{2\sqrt{3}} = 90.67\,\%$	$\frac{9}{64} = 0.1406$	$\frac{1}{4\sqrt{3}} = 0.1443$
F_{35}	$\frac{75\pi}{256} = 90.23\,\%$	$92.04\,\%$	$\frac{225}{1536} = 0.1465$	0.2071
F_2	$\frac{8}{3\pi} = 84.88\,\%$	$\frac{2\sqrt{2}}{\pi} = 90.03\,\%$	$\frac{1}{2\pi} = 0.1592$	$\frac{\sqrt{2}}{\pi(\sqrt{2}+1.5)} = 0.1545$
F_{24}	$\frac{128}{45\pi} = 90.54\,\%$	$95.5\,\%$	$\frac{1}{2\pi} = 0.1592$	0.1061
F_∞	$100\,\%$	$100\,\%$	$\frac{1}{2\pi} = 0.1592$	$\frac{1}{2\pi} = 0.1592$
B_s	–	$\frac{2}{\pi} = 63.67\,\%$	–	$\frac{1}{2\pi} = 0.1592$

- The addition of harmonics of the correct amplitudes and phases into the drain-to-source voltage flattens the v_{DS} voltage, reducing the transistor loss and improving the efficiency.
- The circuit diagram of the Class F power amplifier is complex.
- The Class F RF power amplifier consists of a transistor and a parallel or series resonant circuit tuned to the fundamental frequency and resonant circuits tuned to the harmonics.
- The transistor in the Class F amplifier with third harmonic or with third and fifth harmonic is operated as a dependent current source.
- The conduction angle of the drain or collector current in the Class F power amplifier is usually 180°.
- The drain efficiency of the idealized Class F power amplifier at $V_{DSmin} = 0$ for the maximally flat waveform of v_{DS} (1) with the third harmonic is 88.36 %, (2) with the third and fifth harmonics is 90.23 %, (3) with the second harmonic is 84.88 %, (4) with the second and fourth harmonics is 90.54 %, and (5) and with the transmission line is 100 %.
- The maximum drain efficiency of the idealized Class F power amplifier (1) with the third harmonic is 90.67 %, (2) with the third and fifth harmonics is 92.04 %, and (3) with the second harmonic is 90.03 %.
- The Class F power amplifier with a parallel-resonant circuit and a quarter-wavelength transmission line ideally has a square wave drain-to-source voltage v_{DS} and a half-sinusoidal drain current i_D.
- The Class F power amplifier with a series resonant-circuit and a quarter-wavelength transmission line forms a square-wave drain current i_D and a half-sinusoidal drain-to-source voltage v_{DS}.
- Transmission lines can be used to create higher-order harmonics in Class F power amplifiers.
- There is a trade-off between the efficiency and circuit complexity of Class F amplifiers.

- The transistor output capacitance can dominate the total impedance of the output network and hamper the synthesis of the desired output network with high impedances at harmonics in the Class F power amplifiers.

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8.12 Review Questions

8.1 What is the principle of operation of Class F RF power amplifiers?

8.2 What is the topology of the Class F power amplifier?

8.3 Are the topologies of Class F power amplifiers simple or complex?

8.4 What is the mode of operation of the transistor in the Class F power amplifier?

8.5 How high is the efficiency of Class F power amplifiers?

8.6 What should be the amplitude of the third harmonic voltage in the Class F amplifier with the third harmonic peaking?

8.7 How can the transmission lines be used to control the impedances at harmonics in Class F power amplifiers?

8.8 What are the applications of the Class F RF power amplifiers?

8.9 Does the transistor output capacitance affect the operation of Class F RF power amplifiers?

8.10 List the applications of Class F RF power amplifiers.

8.13 Problems

8.1 Design a Class F power amplifier employing third-harmonic peaking with a maximally flat drain-to-source voltage to deliver a power of 100 W at $f = 2.4$ MHz. The dc power supply voltage is 120 V and $V_{DSmin} = 1$ V.

8.2 Design a Class F power amplifier employing third-harmonic peaking with maximally drain efficiency to deliver a power of 100 W at $f = 2.4$ MHz. The dc power supply voltage is 120 V and $V_{DSmin} = 1$ V.

8.3 Design a Class F power amplifier employing second-harmonic peaking with a maximally flat drain-to-source voltage to deliver a power of 1 W at $f = 2.4$ MHz. The dc power supply voltage is 5 V and $V_{DSmin} = 0.3$ V.

8.4 Design a Class F_3 RF power amplifier to meet the following specifications: $V_I = 48$ V, $P_O = 100$ W, $V_{DSmin} = 2$ V, $\theta = 60^\circ$, $R_L = 50\,\Omega$, $f_c - 88$ MHz, and BW $= 10$ MHz.

9

Linearization and Efficiency Improvement of RF Power Amplifiers

9.1 Introduction

High efficiency and high linearity of power amplifiers are of primary importance in wireless communications systems [1–41]. High efficiency is required for low energy consumption, a longer battery lifetime, and thermal management. Linearity is required for achieving low distortion of the amplified signals. The RF power amplifier specifications call for IM levels of −60 dBc. Linearity and efficiency enhancement techniques of RF power amplifiers, which are used in transmitters, are studied in this chapter. Signals used in modern communications systems have a time-varying envelope and time-varying angle. A nonconstant envelope signal requires linear power amplifiers in the transmitters. The output power in CDMA2000 and WCDMA transmitters may vary in a wide dynamic range of 80 dB. The average output power is usually lower than the peak power by 15–25 dB. The transmitters must be designed for the maximum output power. The maximum efficiency of linear power amplifiers such as Class A, AB, and B amplifiers occurs at the maximum output power. In this case, the maximum amplitude of the drain-to-source voltage V_m is nearly equal to the supply voltage V_I. This corresponds to the AM modulation index equal to 1. However, the average AM modulation index is in the range of 0.2–0.3, when the amplitude of the drain-to-source voltage V_m is much lower than the supply voltage V_I. Therefore, the statistical average efficiency of linear power amplifiers with AM modulation is much lower than their maximum efficiency. In wireless communications, there are two types of transmitters: base-station transmitters and handset transmitters.

Ideal characteristics of a power amplifier are depicted in Figure 9.1. The efficiency η should be very high over a wide range of the output power, as shown in Figure 9.1(a). The output voltage v_o should be a linear function of the input voltage v_i, as shown in

RF Power Amplifiers Marian K. Kazimierczuk

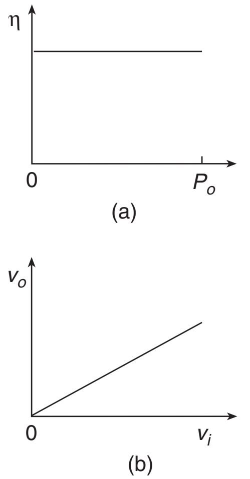

Figure 9.1 Ideal characteristics of a power amplifier. (a) Efficiency η as a function of the output power. (b) Output voltage v_o as a function of the input voltage v_i.

Figure 9.1(b). In this case, the voltage gain of the amplifier $A_v = v_o/v_i$ is constant over a wide range of the input voltage.

Power control of RF transmitters is required in modern digital wireless communications. In CDMA, power control is used in both base-station transmitters and handset transmitters. In base-station transmitters, a higher power is required to transmit the signal to the edges of a cell. In handset transmitters, the output power should be transmitted at variable levels so that the power levels of the signals received at the base station are similar for all users.

9.2 Predistortion

Nonlinear distortion in the output signal of a power amplifier is caused by changes in the slope of the transfer function $v_o = f(v_i)$. These changes are caused by the nonlinear properties of transistors (MOSFETs or BJTs) used in the power amplifier. The changes in the slope of the transfer function cause changes in the gain at different values of the input voltage or power. There are several techniques for reduction of nonlinear distortion.

Predistortion is a technique that seeks to linearize a power amplifier by making suitable modifications to the amplitude and phase of the power amplifier input signal. A block diagram of a power amplifier with predistortion is depicted in Figure 9.2. A nonlinear block is connected in the signal path to compensate for the nonlinearity of the power amplifier. This block is called a *predistorter* or a *predistortion linearizer*. Predistortion can be performed either at RF or baseband frequencies. Predistortion at baseband frequencies is favorable because of digital signal processing (DSP) capabilities. This is called 'digital predistortion.'

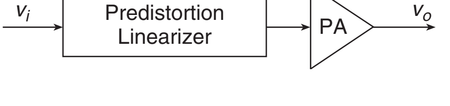

Figure 9.2 Block diagram of power amplifier with a predistortion system.

DSP ICs can be used to perform digital predistortion. It is an open-loop system, which is inherently stable. The disadvantage of the system is the difficulty in generating the transfer function of the predistorter such that the transfer function of the overall system is linear.

Example 9.1

A power amplifier voltage transfer function has the gain $A_1 = 100$ for small signals and the gain $A_2 = 50$ for large signals, as depicted in Figure 9.3(a). Find the transfer function of the predistorter to achieve a linear transfer function of the overall system.

Solution. Let us assume the gain of the predistorter for small signals to be $A_3 = 1$. The ratio of the two gains of the power amplifiers is

$$\frac{A_1}{A_2} = \frac{100}{50} = 2. \tag{9.1}$$

Hence, the gain of the predistorter for large signals is

$$A_4 = 2. \tag{9.2}$$

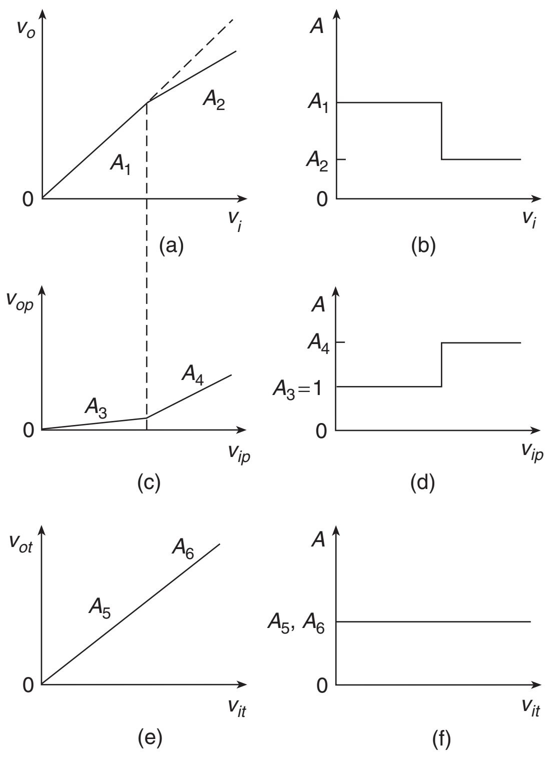

Figure 9.3 Transfer functions of a power amplifier using predistortion. (a) Nonlinear transfer function of a power amplifier $v_o = f(v_i)$. (b) Transfer function of the predistorter A. (c) Transfer function of the predistorter $v_{op} = f(v_{ip})$. (d) Transfer function of a predistortion amplifier A_p. (e) Transfer function of the total amplifier A. (f) Linearized transfer function of the overall system $v_{ot} = f(v_{it})$.

The gain of the overall system for low signals is

$$A_5 = A_1 A_3 = 100 \times 1 = 100 \tag{9.3}$$

and for large signals is

$$A_6 = A_2 A_4 = 50 \times 2 = 100. \tag{9.4}$$

Thus, $A_6 = A_5$, resulting in a linear transfer function over a wide range of the input signal. Figure 9.3 illustrates the linearization of a power amplifier using the predistortion technique.

9.3 Feedforward Linearization Technique

A block diagram of a basic power amplifier system with feedforward linearization technique [1–6] is shown in Figure 9.4(a). The feedforward linearization technique relies on the cancellation of the distortion signal. This is done by generating a proper error voltage and

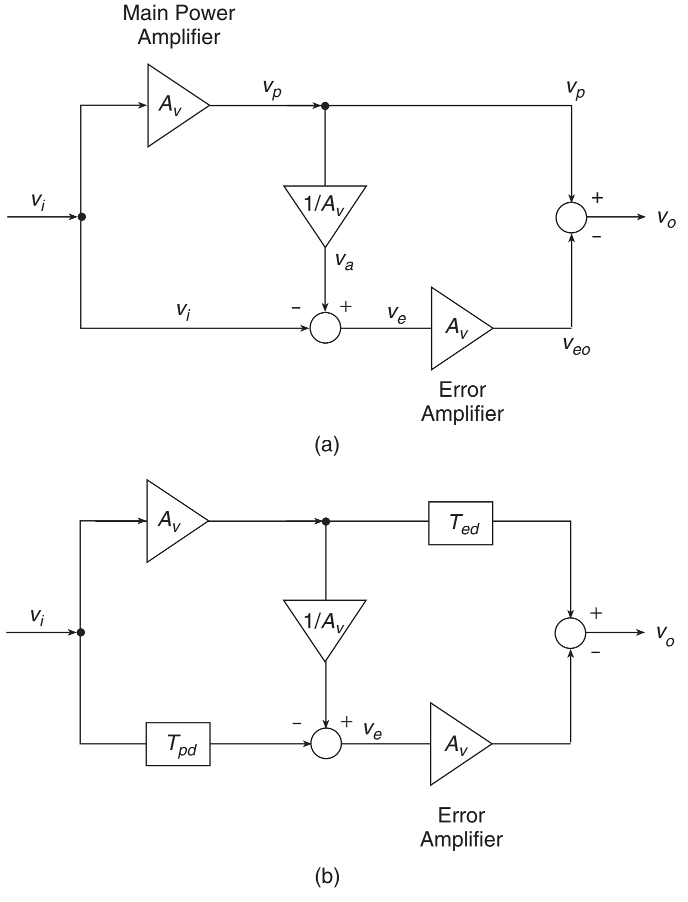

Figure 9.4 Block diagram of a power amplifier system with feedforward linearization. (a) Basic feedforward system. (b) Feedforward system with delay blocks.

subtracting it from the distorted output voltage of a nonlinear power amplifier. The input voltage v_i is applied to two channels. The voltage in one channel is applied to the input of the main power amplifier with the voltage gain A_v and it is amplified. The other channel uses the original input voltage v_i as a reference signal for future comparison. The output voltage of the main power amplifier containsthe undistorted and amplified input voltage $A_v v_i$ and the distortion voltage v_d produced by the amplifier nonlinearity

$$v_p = A_v v_i + v_d. \tag{9.5}$$

The total output voltage of the main power amplifier is attenuated by a circuit with the transfer function equal to $1/A_v$

$$v_a = \frac{v_p}{A_v} = v_i + \frac{v_d}{A_v}. \tag{9.6}$$

The attenuated voltage is compared with the original voltage v_i in the subtractor to produce an error voltage v_e:

$$v_e = v_a - v_i = v_i + \frac{v_d}{A_v} - v_i = \frac{v_d}{A_v}. \tag{9.7}$$

The error voltage v_e is amplified by the error amplifier with the voltage gain A_v, yielding

$$v_{eo} = A_v v_e = A_v \frac{v_d}{A_v} = v_d. \tag{9.8}$$

Next, the output voltage of the main power amplifier v_p is compared with the error amplifier output voltage v_{eo} by a subtractor, producing the output voltage of the entire system

$$v_o = v_p - v_{eo} = A_v v_i + v_d - v_d = A_v v_i. \tag{9.9}$$

It can be seen that the distortion signal v_d is cancelled out for perfect amplitude and phase matching at each subtractor. In addition to the cancellation of the nonlinear component, the system suppresses the noise added to the signal in the main power amplifier. The first loop is called the signal cancellation loop and the second loop is called the error cancellation loop.

The feedforward linearization system offers a wide bandwidth, reduced noise level, and is stable in spite of large phase shifts at RF because the output signal is not applied to the input. Wideband operation is useful in multicarrier wireless communications such as wireless base stations.

The main disadvantage of the feedforward linearization system is the gain mismatch and the phase (delay) mismatch in the signal channels. The amount of linearization depends on the amplitude and phase matching at each subtractor. If the firstloop from the input voltage v_i to the first subtractor has a relative gain mismatch $\Delta A/A$ and the phase mismatch $\Delta\phi$, the attenuation of the magnitude of the intermodulation terms in the output voltage is [2, 4]

$$A_{IM} = \sqrt{1 - 2\left(1 + \frac{\Delta A}{A}\right)\cos\Delta\phi + \left(1 + \frac{\Delta A}{A}\right)^2}. \tag{9.10}$$

To improve the signal quality, delay blocks can be added as shown in Figure 9.4(b). A delay block T_{pd} is added in the bottom path to compensate for the delay introduced by the main power amplifier and the attenuator. The other delay block T_{ed} is added in the upper path to compensate for the delay introduced by the error amplifier. The delay blocks can be implemented using passive lumped-element networks or transmission lines. However, these blocks dissipate power and reduce the amplifier efficiency. In addition, the design of wideband delay blocks is difficult.

9.4 Negative Feedback Linearization Technique

A block diagram of a power amplifier with a negative feedback is depicted in Figure 9.5. The transfer function $v_o = f(v_s)$ of a power amplifier can be considerably linearized through the application of negative feedback, reducing nonlinear distortion. Large changes in the open-loop gain cause much smaller changes in the closed-loop gain.

Let us assume that the output voltage of a power amplifier without negative feedback contains a distortion component (an intermodulation term of a harmonic) given by

$$v_d = V_d \sin \omega_d t. \tag{9.11}$$

The distortion component of the output voltage of the power amplifier with negative feedback is

$$v_{df} = V_{df} \sin \omega_d t. \tag{9.12}$$

We wish to find the relationship between v_d and v_{df}. The feedback voltage is

$$v_f = \beta v_{df}. \tag{9.13}$$

The reference voltage for the distortion component is zero. Hence, the input voltage of the power amplifier is

$$v_e = -v_f = -\beta v_{df}. \tag{9.14}$$

The distortion component at the power amplifier output is

$$v_{od} = Av_e = -Av_f = -\beta A v_{df}. \tag{9.15}$$

Thus, the output voltage contains two terms: the original distortion component generated by the power amplifier v_d andthe component v_{od}, which represents the effect of negative feedback. Hence, the overall output voltage is

$$v_{df} = v_d - v_{od} = v_d - \beta A v_{df} \tag{9.16}$$

yielding

$$v_{df} = \frac{v_d}{1 + \beta A} = \frac{v_d}{1 + T}. \tag{9.17}$$

Since A is generally a function of frequency, the amplitude of the distortion component must be evaluated at the frequency of the distortion component $f_d = \omega_d/(2\pi)$.

It follows from (9.17) that the nonlinear distortion reduction is large when the loop gain $T = \beta A$ is high. However, the voltage gain of RF power amplifiers is low, and therefore the loop gain T is also low at RF. In addition, stability of the loop is of great concern due to a large number of poles introduced by various parasitic components. A high-order power amplifier may have an excessive phase shift, causing oscillations. For a large loop gain $T = \beta A$,

$$A_f \approx \frac{1}{\beta}. \tag{9.18}$$

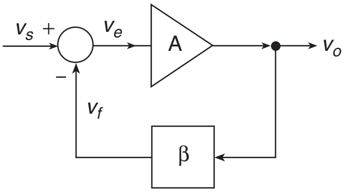

Figure 9.5 Block diagram of a power amplifier system with negative feedback.

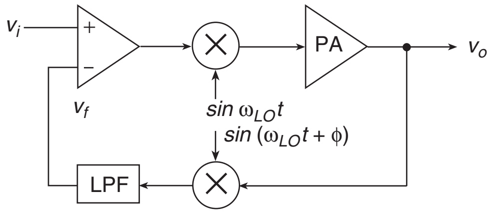

Figure 9.6 Block diagram of a power amplifier system with negative feedback linearization.

This equation indicates that the gain depends only on the linear feedback network β. The output voltage of a power amplifier is given by

$$v_o = A_f v_s \approx \frac{v_i}{\beta}. \tag{9.19}$$

Thus, the transfer function is nearly linear. However, the feedback amplifier requires a larger input voltage in order to produce the same output as the amplifier without feedback.

Figure 9.6 shows a block diagram of a power amplifier with negative feedback and frequency translation. The forward path consists of a low-frequency high-gain error amplifier, an up-conversion mixer, and a power amplifier. The mixer converts the frequency of the input signal f_i to an RF frequency $f_{RF} = f_i + f_{LO}$, where f_{LO} is the frequency of the local oscillator. The feedback path consists of a down-conversion mixer and a low-pass filter (LPF). The mixer converts the frequency of the RF signal f_{RF} to the input frequency $f_i = f_{RF} - f_{LO}$. In this system, ahigh loop gain T is obtained at low frequencies. Therefore, the negative feedback is very effective in reducing the nonlinear distortion. The total phase shift in the upper path by the error amplifier, the mixer, and the power amplifier usually exceeds 180°. Therefore, a phase shift is added to the local oscillator to ensure loop stability.

Example 9.2

An open-loop power amplifier has a gain $A_1 = 100$ for small signals and a gain $A_2 = 50$. The transfer function of the feedback network is $\beta = 0.1$. Find the closed-loop gains of the power amplifier with negative feedback.

Solution. The ratio of the open-loop gains is

$$\frac{A_1}{A_2} = \frac{100}{50} = 2. \tag{9.20}$$

The relative change in the gain of the open-loop power amplifier is

$$\frac{A_1 - A_2}{A_1} = \frac{100 - 50}{100} = 50\,\%. \tag{9.21}$$

The loop gain for small signals is

$$T_1 = \beta A_1 = 0.1 \times 100 = 10 \tag{9.22}$$

and the closed-loop gain for small signals is

$$A_{f1} = \frac{A_1}{1 + T_1} = \frac{100}{1 + 10} = 9.091. \tag{9.23}$$

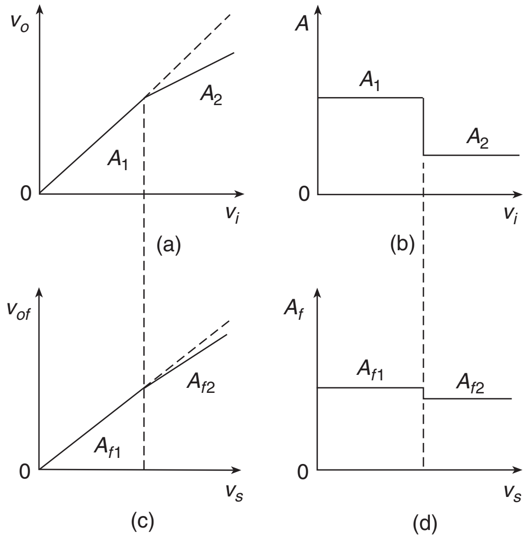

Figure 9.7 Transfer functions of a power amplifier without and with negative feedback. (a) Transfer function $v_o = f(v_i)$ without negative feedback. (b) Transfer function A without negative feedback. (c) Transfer function $V_{of} = v_s$ with negative feedback. (d) Transfer function A_f with negative feedback.

The loop gain for large signals is

$$T_2 = \beta A_2 = 0.1 \times 50 = 5 \tag{9.24}$$

and the closed-loop gain for large signals is

$$A_{f2} = \frac{A_2}{1 + T_2} = \frac{50}{1 + 5} = 8.333. \tag{9.25}$$

The ratio of the closed-loop gain is

$$\frac{A_{f1}}{A_{f2}} = \frac{9.091}{8.333} = 1.091. \tag{9.26}$$

The relative change in the gain of the closed-loop power amplifier is

$$\frac{A_{f1} - A_{f2}}{A_{f1}} = \frac{9.091 - 8.333}{9.091} = 8.333\,\%. \tag{9.27}$$

Thus, the open-loop transfer function changes by a factor of 2, whereas the closed-loop transfer function changes by a factor of 1.091. Figure 9.7 illustrates the linearization of a power amplifier using the negative feedback technique.

Example 9.3

The relationship between the input voltage v_e and the output voltage v_o of a power stage of an RF amplifier is given by

$$v_o = A v_e^2. \tag{9.28}$$

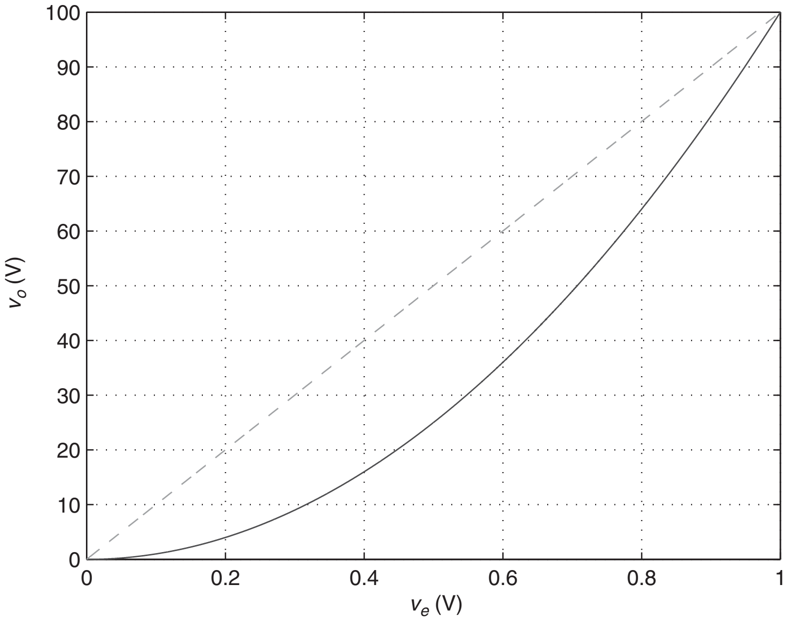

Figure 9.8 Output voltage as a function of input voltage for a power amplifier without negative feedback.

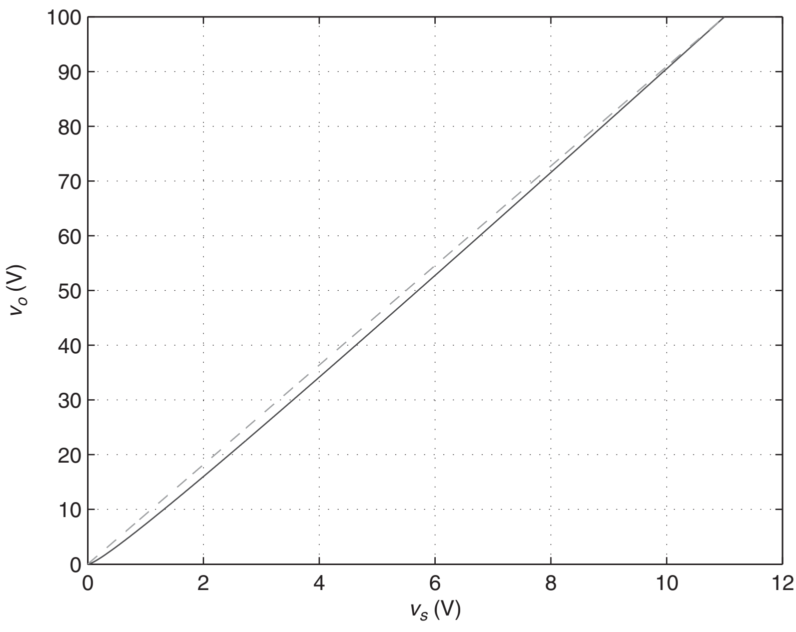

Figure 9.9 Output voltage as function of input voltage for a power amplifier with negative feedback.

Find the relationship between the input voltage v_s and the output voltage v_o for the RF amplifier with negative feedback. Draw the plots of $v_o = f(v_e)$ and $v_o = f(v_s)$ for $A = 100$ and $\beta = 1/10$.

Solution. The error voltage is

$$v_e = v_s - \beta v_o. \tag{9.29}$$

Hence, the output voltage with negative feedback is

$$v_o = A(v_s - \beta v_o)^2. \tag{9.30}$$

Solving for v_s, we get

$$v_s = \beta v_o + \sqrt{\frac{v_o}{A}} \tag{9.31}$$

or

$$v_o = \frac{v_s}{\beta} - \frac{1}{\beta}\sqrt{\frac{v_o}{A}}. \tag{9.32}$$

Figures 9.8 and 9.9 illustrates the linearization of a power amplifier using the negative feedback technique. It can be seen that the plot of the amplifier with negative feedback is much more linear.

9.5 Envelope Elimination and Restoration

A block diagram of a power amplifier with an envelope elimination and restoration system (EER) is shown in Figure 9.10. The concept of the EER system was first presented by Kahn in 1952 [8]. It has been used to increase linearity and efficiency of radio transmitters. One path of the EER system consists of an RF limiter and an RF nonlinear high-efficiency switching-mode power amplifier. The other path consists of an envelope detector and an AM modulator, which is a low-frequency power amplifier.

An amplitude modulated (AM) and phase modulated (PM) voltage applied to the amplifier input can be represented by

$$v_i = V(t)\cos[\omega_c t + \theta(t)]. \tag{9.33}$$

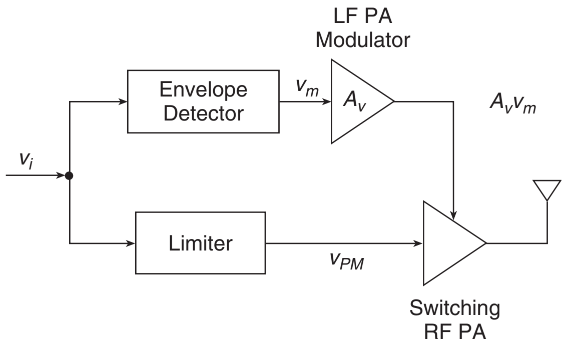

Figure 9.10 Block diagram of power amplifier with envelope elimination and restoration (EER).

This voltage is applied to both an RF limiter and an envelope detector. The output voltage of an RF limiter is a constant-envelope phase-modulated (CE PM) voltage

$$v_{PM} = V_c \cos[\omega_c t + \theta(t)]. \tag{9.34}$$

The RF limiter should be designed to minimize amplitude-to-phase conversion. The output voltage of an envelope detector is

$$v_m(t) = V(t). \tag{9.35}$$

The envelope of the input voltage is extracted using an envelope detector, usually a diode AM detector to obtain the AM modulating voltage $v_m(t)$. Next, it is amplified by a low-frequency power amplifier with the voltage gain A_v, producing a modulating voltage $A_v v_m(t)$. The envelope is eliminated from the AM-PM voltage using a limiter. The PM voltage v_{PM} is simultaneously amplified and amplitude modulated by an RF power amplifier to restore the original amplitude and phase information contained in the input signal v_i. The PM voltage v_{PM} is applied to the gate of the transistor in the RF power amplifier and the modulating voltage v_m is used to modulate the supply voltage of the RF power amplifier, producing $v_{DD} = v_m(t) A_v V_{DD}$. In switching-mode RF power amplifiers, the amplitude of the output voltage is directly proportional to the supply voltage V_{DD}. A very important advantage of the EER system is that a linear RF power amplifier is not required. Linear RF power amplifiers exhibit very low efficiency, which decreases when the AM modulation index decreases. The average AM modulation index is in the range from 0.2 to 0.3. A highly efficient nonlinear RF power amplifier can be used in the EER system. Switching-mode RF power amplifiers, such as Class D, E, DE, and F amplifiers, exhibit a very high efficiency, greater than 90 %. Their efficiency is approximately independent of the AM modulation index. An amplitude-modulated high-efficiency Class E power amplifier was studied in [15].

The EER system suffers from a couple of shortcomings. First, there is mismatch between the total phase shift (delay) and gain in the two paths. It is a hard task because the phase shift in the low-frequency path is much larger than that in the high-frequency path. The intermodulation term due to the phase mismatch is given by [10]

$$IDM \approx 2\pi BW_{RF}^2 \Delta\tau^2 \tag{9.36}$$

where BW_{RF} is the bandwidth of the RF signal and $\Delta\tau$ is the delay mismatch.

Second, phase distortion is introduced by the limiter. Limiters are built using nonlinear devices and exhibit a considerable AM-to-PM conversion at high frequencies, corrupting the phase $\theta(t)$ of the PM signal v_{PM}. Third, the phase distortion is introduced by the nonlinear transistor capacitance due to the AM modulation of the supply voltage of the RF power amplifier.

9.6 Envelope Tracking

The main goal of the envelope tracking technique is to maintain a high efficiency over a wide range of the input power. A block diagram of a power amplifier with envelope tracking (ET) is shown in Figure 9.11 [17–21]. This system is also called a power amplifier with adaptive biasing or a power amplifier with dynamic control of supply voltage. The system consists of two paths. One path contains a linear power amplifier and a delay block T_d. The other path consists of an envelope detector and a dc-dc switching-mode power converter. The ET system is similar to the EER system, but it uses a linear power amplifier and does not have an RF limiter as the EER system does. Instead of modulating the RF power

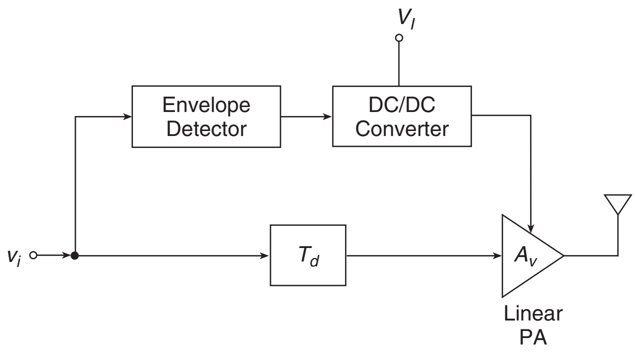

Figure 9.11 Block diagram of power amplifier with envelope tracking (ET).

amplifier, it adjusts the level of the supply voltage so that V_m/V_I is always close to 1 to allow efficient linear amplification.

The drain efficiency of current-mode power amplifiers decreases as the amplitude of the drain-to-source voltage V_m decreases. In the current-mode power amplifiers, such as Class A, AB, B, and C amplifiers, the transistor is operated as a dependent current source. In these amplifiers, the amplitude of the drain-to-source voltage V_m is proportional to the amplitude of the input voltage V_{im}. The drain efficiency of the power amplifiers at the resonant frequency is given by

$$\eta_D = \frac{P_{DS}}{P_I} = \frac{1}{2}\left(\frac{I_m}{I_I}\right)\left(\frac{V_m}{V_I}\right). \tag{9.37}$$

It can be seen from this equation that the drain efficiency η_D is proportional to the ratio V_m/V_I. As V_m decreases due to an increase in V_{im} at fixed value of the supply voltage V_I, the ratio V_m/V_I also decreases, reducing the drain efficiency. If the supply voltage V_I is increased proportionally to the amplitude of the input voltage V_{im}, the ratio V_{im}/V_I, then the ratio V_m/V_I can be maintained at a fixed value close to 1 (e.g., 0.9), resulting in high efficiency at any amplitude V_m. When V_{im} is varied in current-mode power amplifiers, the conduction angle of the drain current remains approximately constant and the ratio I_m/I_I is also constant. For instance, for the Class B amplifier, $I_m/I_I = \pi/2$. The efficiency is improved, while low distortion of the RF linear power amplifier is preserved.

9.7 The Doherty Amplifier

The Doherty power amplifier was first proposed in 1936 [23]. The main purpose of this system is to maintain high efficiency over a wide range of the input voltage of a linear power amplifier with AM signals. The average efficiency of linear power amplifiers with AM modulation is very low because the average AM modulation index is very low, usually from 0.2 to 0.3. The Doherty power amplifier architecture delivers high efficiency with input signals that have high peak-to-average power ratio (PAR) in the range of 6–10 dB. This system has the potential to deliver high efficiency in base station transmitters. Current and emerging wireless systems produce high PAR signals, including WCDMA, CDMA2000, and OFDM (orthogonal frequency division multiplexing). For example, at 3-dB output power backoff (i.e., at 50 % of full power), the efficiency of the Class B power amplifier

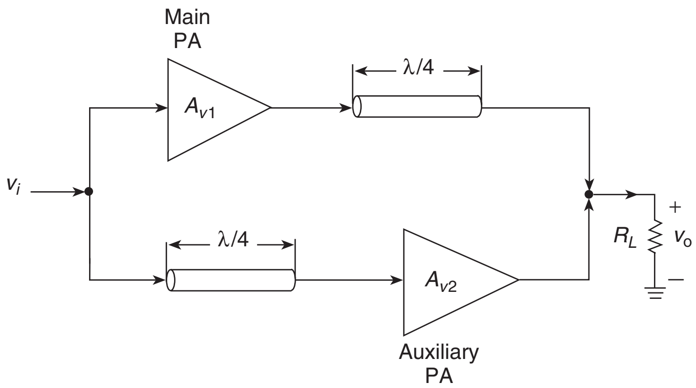

Figure 9.12 Block diagram of Doherty power amplifier.

decreases to $\pi/4 = 39\,\%$. The transmitter accounts for a high percentage of the overall power consumption. Increased efficiency can lower the electricity cost and minimize cooling requirements.

A block diagram of a Doherty power amplifier is shown in Figure 9.12. It consists of a *main power amplifier* and an *auxiliary power amplifier*. The main power amplifier is called a *carrier amplifier* and the auxiliary power amplifier is called a *peak amplifier*. The main power amplifier is usually a Class B (or a Class AB) amplifier and the auxiliary amplifier is usually a Class C amplifier. A quarter-wave transformer is connected at the output of the main power amplifier. Also, a quarter-wave transformer is connected at the input of the auxiliary amplifier to compensate for a 90° phase shift introduced by the transformer in the main amplifier. The transistors in both amplifiers are operated as dependent current sources. The main amplifier saturates at high input power and therefore its voltage gain decreases. The auxiliary amplifier turns on at high input power when the main power amplifier reaches saturation. As a result, the linearity of the overall system is improved at high input power levels. The system maintains good efficiency under power backoff conditions. It is common for power amplifiers in mobile transmitters to operate at output power levels of 10–40 dB backoff from peak power.

9.7.1 Condition for High Efficiency Over Wide Power Range

The efficiency of power amplifiers at the resonant frequency is given by

$$\eta_D = \frac{P_{DS}}{P_I} = \frac{1}{2}\left(\frac{I_m}{I_I}\right)\left(\frac{V_m}{V_I}\right) = \frac{1}{2}\left(\frac{I_m}{I_I}\right)\left(\frac{RI_m}{V_I}\right). \tag{9.38}$$

When V_{im} is varied in current-mode power amplifiers (like the Class B and C amplifiers), the peak drain current is proportional to V_{im}, the conduction angle of the drain current remains approximately constant, and therefore the ratio I_m/I_I also remains constant. The ratio V_m/V_I should be held close to 1 (e.g., equal to 0.9) to achieve high efficiency at any value of the output voltage amplitude. This can be accomplished if $V_m/V_I = RI_m/V_I$ is maintained at a fixed value close to 1. At a fixed supply voltage V_I, the following condition should be satisfied:

$$V_m = RI_m = \text{Const} \tag{9.39}$$

where R is the load resistance. If I_m is decreased, R should be increased so that the product RI_m remains constant. This idea is partially realized in the Doherty amplifier because the carrier power amplifier is presented with a *modulated load impedance*, *dynamic load variation*, *drive-dependent load resistance* or *load-pulling effect*, which enables high efficiency and a constant voltage gain over a wide input power range. The main shortcoming of the Doherty power amplifier is its poor linearity. However, recent advancements in digital and analog predistortion and feedforward techniques can dramatically reduce nonlinear distortion.

9.7.2 Impedance Modulation Concept

The load impedance of a current source can be modified by applying a current from another current source. Consider the circuit depicted in Figure 9.13. The current source I_1 represents the transistor of one power amplifier operated as a current source. Similarly, the current source I_2 represents the transistor of another power amplifier operated as a current source. From KCL,

$$I_L = I_1 + I_2 \tag{9.40}$$

and from Ohm's law,

$$R_L = \frac{V_L}{I_L}. \tag{9.41}$$

The load impedance seen by the current sources I_1 is

$$R_1 = \frac{V}{I_1} = \frac{V}{I_L - I_2} = \frac{V}{I_L\left(1 - \frac{I_2}{I_L}\right)} = \frac{V_L}{I_L\left(1 - \frac{I_2}{I_1 + I_2}\right)} = R_L\left(1 + \frac{I_2}{I_1}\right). \tag{9.42}$$

Thus, the load resistance seen by the current source I_1 is dependent on the current I_2. If the currents I_1 and I_2 are in phase, the load voltage V is increased, and therefore the current source I_1 sees a higher load resistance. For $I_2 = I_1$,

$$R_1 = 2R_L. \tag{9.43}$$

For $I_2 = 0$,

$$R_1 = R_L. \tag{9.44}$$

The changes of the load resistance R_1 in the circuit shown in Figure 9.13 are not compatible with power amplifier requirements because R_1 decreases as I_2/I_1 decreases.

Similarly, the load resistance of the current source I_2 is expressed as

$$R_2 = \frac{V}{I_2} = R_L\left(1 + \frac{I_1}{I_2}\right). \tag{9.45}$$

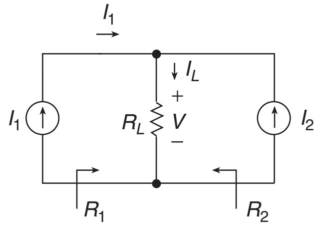

Figure 9.13 Circuit with load impedance modulation.

9.7.3 Equivalent Circuit of the Doherty Amplifier

Both the main and auxiliary power amplifiers should deliver the maximum powers at the maximum overall output power. As the overall power is decreased, the power of each amplifier should be reduced.

The peak drain current is proportional to the peak gate-to-source voltage in Class A, AB, B, and C amplifiers. The maximum output power occurs at the maximum value of the fundamental component of the drain current I_{1max}. The amplitude of the fundamental component of the drain-to-source voltage also has the maximum value $V_{m(max)}$ at the maximum output power $P_{O(max)}$. The load resistance seen by the drain and source terminal at the operating frequency for $P_{o(max)}$ is

$$R_1 = R_{1(opt)} = \frac{V_{m(max)}}{I_{1max}}. \tag{9.46}$$

As the drive power is decreased, the output power decreases, and the fundamental component of the drain current decreases. In order to maintain a constant value of the fundamental component of the drain-to-source voltage V_m when I_{d1m} is decreased, the load resistance R_1 should be increased. Therefore, an *impedance inverter* is needed between the load R_L and the current source I_1 as shown in Figure 9.14. A quarter-wave transmission line transformer acts as an impedance inverter

$$R_1 = \frac{Z_0^2}{R_3} \tag{9.47}$$

where Z_0 is the characteristic impedance of the transmission line. As R_3 increases, R_1 decreases. Lumped-component impedance inverters are depicted in Figure 9.15. The components of these inverters are given by

$$\omega L = \frac{1}{\omega C} = Z_0. \tag{9.48}$$

The phase shift of the impedance inverter shown in Figure 9.15(a) is -90° and the phase shift of impedance inverter shown in Figure 9.15(b) is 90°.

An equivalent circuit of the most basic form of the Doherty amplifier is shown in Figure 9.14. The load resistance of the quarter-wave transformer is given by

$$R_3 = \frac{V}{I_3} = \frac{V}{I_L - I_2} = \frac{V}{I_L\left(1 - \frac{I_2}{I_L}\right)} = R_L \frac{1}{\frac{I_3}{I_2 + I_3}} = R_L\left(1 + \frac{I_2}{I_3}\right). \tag{9.49}$$

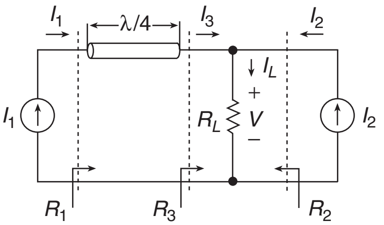

Figure 9.14 Equivalent circuit of the Doherty power amplifier.

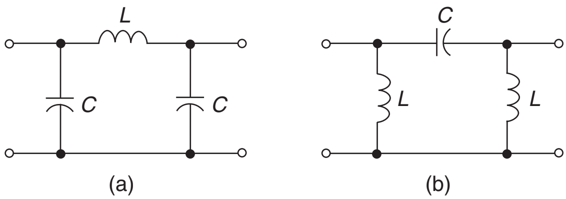

Figure 9.15 Lumped-component impedance inverters. (a) Two-capacitor single-inductor π impedance inverter. (b) Two-inductor single-capacitor π impedance inverter.

The load resistance of the current source I_1 is

$$R_1 = \frac{Z_0^2}{R_3} = \frac{Z_0^2}{R_L\left(1 + \frac{I_2}{I_3}\right)}. \tag{9.50}$$

As I_2 decreases, R_1 increases. Therefore, as I_1 decreases, V_m remains constant. For $I_2 = I_3$,

$$R_1 = \frac{Z_0^2}{2R_L} \tag{9.51}$$

and for $I_2 = 0$,

$$R_1 = \frac{Z_0^2}{R_L}. \tag{9.52}$$

As I_2 decreases from its maximum value to zero, the load resistance of the current source I_1 increases from $Z_0^2/(2R_L)$ to Z_0^2/R_L.

The load resistance of the current source I_2 is given by

$$R_2 = R_L\left(1 + \frac{I_3}{I_2}\right). \tag{9.53}$$

9.7.4 Power and Efficiency of Doherty Amplifier

The output power averaged over a cycle of the modulating frequency of the main power amplifier with AM single-frequency signal is

$$P_M = \left(1 - \frac{m}{\pi} + \frac{m^2}{4}\right)P_C \tag{9.54}$$

where P_C is the power of the carrier and m is the modulation index. The average output power of the auxiliary amplifier is

$$P_A = \left(\frac{m}{\pi} + \frac{m^2}{4}\right)P_C. \tag{9.55}$$

For $m = 1$,

$$P_M = \left(1 - \frac{1}{\pi} + \frac{1}{4}\right) = 0.9317P_C \tag{9.56}$$

$$P_A = \left(\frac{1}{\pi} + \frac{1}{4}\right) = 0.5683P_C \tag{9.57}$$

and the total average output power is

$$P_T = P_M + P_A = 1.5P_C. \tag{9.58}$$

For $m = 0$,

$$P_T = P_M = P_C \tag{9.59}$$

and

$$P_A = 0. \tag{9.60}$$

The drain efficiency averaged over a cycle of the modulating frequency of the main power amplifier with AM single-frequency signal is

$$\eta_M = \left(1 - \frac{m}{\pi} + \frac{m^2}{4}\right)\eta_C \tag{9.61}$$

where η_C is the drain efficiency for the operation at carrier only ($m = 0$). The average drain efficiency of the auxiliary amplifier is

$$\eta_{AV} = 1.15\left(\frac{1}{2} + \frac{\pi}{8}m\right)\eta_C. \tag{9.62}$$

The average drain efficiency of the overall amplifier is

$$\eta_{AV} = \frac{1 + \dfrac{m^2}{2}}{1 + 1.15\dfrac{2}{\pi}m}\eta_C. \tag{9.63}$$

The total average efficiency can be expressed in terms of the amplitude of the input voltage V_i normalized with respect to its maximum value V_{imax}

$$\eta = \frac{\pi}{2}\left(\frac{V_i}{V_{imax}}\right) \qquad \text{for} \qquad 0 \le V_i \le \frac{V_{imax}}{2} \tag{9.64}$$

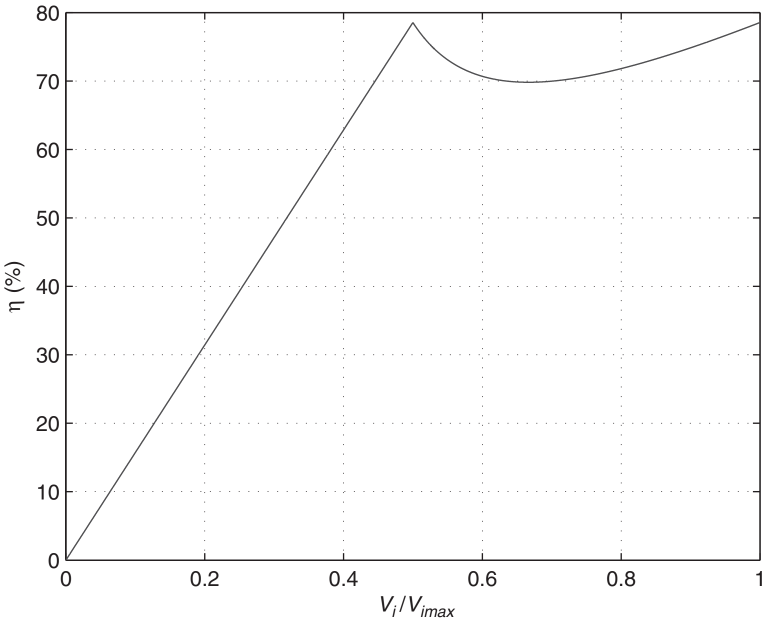

Figure 9.16 Efficiency of the Doherty power amplifier as a function of the normalized input voltage V_i/V_{imax}.

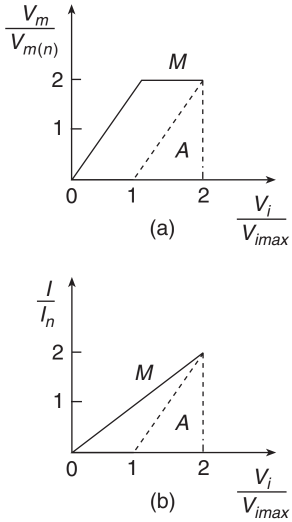

Figure 9.17 Voltage and current amplitudes of the main power amplifier (M) and the auxiliary power amplifier (A) as functions of V_i/V_{imax} in the Doherty system. (a) $V_m/V_{m(n)}$ versus V_i/V_{imax}. (b) I/I_n versus V_i/V_{imax}.

and

$$\eta = \frac{\pi}{2}\frac{\left(\frac{V_i}{V_{im}}\right)^2}{3\left(\frac{V_i}{V_{imax}}\right) - 1} \quad \text{for} \quad \frac{V_{imax}}{2} \le V_i \le V_{imax}. \tag{9.65}$$

Figure 9.16 depicts the overall efficiency of the Doherty power amplifier as a function of the normalized input voltage amplitude V_i/V_{imax}. Figure 9.17 shows the normalized voltage and current amplitudes in the main power amplifier (M) and the auxiliary power amplifier as functions of the normalized input voltage amplitude V_i/V_{imax}.

9.8 Outphasing Power Amplifier

An outphasing power amplifier system was invented by Chireix in 1935 [31] to improve the efficiency of AM signal power amplifiers. The system is depicted in Figure 9.18. It consists of a signal component separator (SCS) and phase modulator, two identical nonlinear high-efficiency power amplifiers, and RF power adder (combiner). Power amplifiers can be Class D, E, or DE switching-mode constant-envelope amplifiers. The signal separator and phase modulator is a complex stage. The outphasing power amplifier was studied in [33–40]. The subsequent analysis shows the possibility of a linear amplification with nonlinear components (LINC) [32].

A bandpass signal with amplitude and phase modulation can be described by

$$v_i = V(t)\cos[\omega_c t + \phi(t)] = \cos[\omega_c t + \phi(t)]\cos[\arccos V(t)]. \tag{9.66}$$

Using the trigonometric relationship,

$$\cos\alpha\cos\beta = \frac{1}{2}\cos(\alpha+\beta) + \frac{1}{2}\cos(\alpha-\beta) \tag{9.67}$$

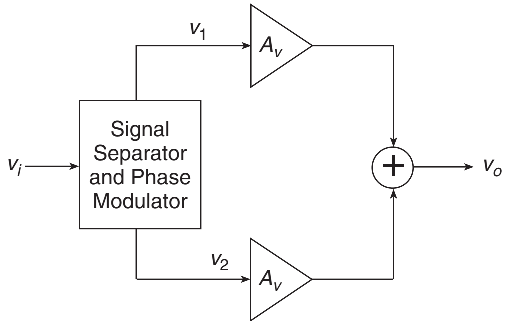

Figure 9.18 Block diagram of outphasing power amplifier system.

the AM signal can be represented as the sum of two constant-envelope phase-modulated signals

$$v_i = v_1 + v_2 \tag{9.68}$$

where

$$v_1 = \frac{1}{2}\cos[\omega_c t + \phi(t) + \arccos V(t)] \tag{9.69}$$

and

$$v_2 = \frac{1}{2}\cos[\omega_c t + \phi(t) - \arccos V(t)]. \tag{9.70}$$

Both the amplitude and phase information of the original input signal v_i is included in the phase-modulated component signals v_1 and v_2. The constant-envelope signals v_1 and v_2 can be amplified by nonlinear high-efficiency switching-mode power amplifiers, such as Class D, E, and DE circuits.

Using the trigonometric formula,

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} \tag{9.71}$$

the two amplified signals can be added up to form the output signal

$$v_o = A_v(v_1 + v_2) = A_v\cos[\omega_c t + \phi(t)]\cos[\arccos V(t)] = A_v V(t)\cos[\omega_c t + \phi(t)]. \tag{9.72}$$

The output signal v_o contains the same AM and PM information as v_i does. The practical implementation of the outphasing power amplifier is very difficult. The signal component separator is not an easy circuit to design and build. The phase of the voltages v_1 and v_2 must be modulated by a highly nonlinear functions $\pm\arccos V(t)$.

An alternative method of splitting the input signal is given below. The input signal is

$$v_i = V(t)\cos[\omega_c t + \phi(t)] = \cos[\omega_c t + \phi(t)]\sin[\arcsin V(t)]. \tag{9.73}$$

From the trigonometric relationship,

$$\cos\alpha\sin\beta = \frac{1}{2}\sin(\alpha+\beta) - \frac{1}{2}\sin(\alpha-\beta) \tag{9.74}$$

we obtain two constant-amplitude phase-modulated voltages

$$v_1 = \frac{1}{2}\sin[\omega_c t + \phi(t) + \arcsin V(t)] \tag{9.75}$$

and

$$v_2 = -\frac{1}{2}\sin[\omega_c t + \phi(t) - \arcsin V(t)]. \tag{9.76}$$

From the relationship

$$\sin\alpha - \sin\beta = 2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2} \tag{9.77}$$

we obtain

$$v_o = A_v(v_1 + v_2) = \cos[\omega_c t + \phi(t)]\sin[\arcsin V(t)] = A_v V(t)\cos[\omega_c t + \phi(t)]. \tag{9.78}$$

There are two more combinations of splitting and adding the signals.

9.9 Summary

- The predistortion technique linearizes power amplifiers by making suitable modifications to the amplitude and phase of its input signal so that the overall transfer function of the system is linear.
- The principle of operation of the feedforward linearization system of power amplifiers is based on cancellation of the distortion signal produced by the nonlinearity of the power amplifier.
- The feedforward linearization system of power amplifiers is wideband, low noise, and stable.
- The feedforward linearization system of power amplifiers suffers from gain and phase (delay) mismatches of signals in two channels.
- The negative feedback reduces nonlinear distortion by a factor $1 + T$.
- The EER system improves the linearity and efficiency of RF power transmitters with amplitude and phase modulation.
- In the EER system, the AM-PM signal is first decomposed into a constant-envelope PM signal by a limiter and an envelope signal by an envelope detector. Next, the two signals are recombined by an AM modulation of an RF power amplifier to restore the amplitude and phase of the original signal.
- Highly nonlinear RF power amplifiers can be used in the EER system, such as Class D, E, DE, and F amplifiers.
- A phase shift and gain mismatch in two signal paths is a drawback of the EER system.
- The phase distortion, introduced by the limiter and the nonlinearity of the transistor output capacitance, is a shortcoming of the EER system.
- The envelope tracking technique adjusts the supply voltage of a power amplifier so that $V_m/V_I \approx 1$, yielding high efficiency of a linear power amplifier.
- The load impedance of power amplifiers can be modulated by driving the load with another current-source power amplifier.
- A Doherty power amplifier requires an impedance inverter to increase its load resistance when its output power decreases. Therefore, the amplitude of the drain-to-source voltage $V_m = R_1 I_1$ remains constant when I_1 decreases.

- The outphasing power amplifier separates the AM input signal into two constant-envelope signals, amplifies them by high-efficiency nonlinear amplifiers, and combines the amplified signals into one output signal.

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9.11 Review Questions

9.1 Explain the principle of reduction of nonlinear distortion using the predistortion technique.

9.2 Explain the principle of operation of the feedforward linearization system of power amplifiers.

9.3 List advantages of the feedforward linearization system of power amplifiers.

9.4 List disadvantages of the feedforward linearization system of power amplifiers.

9.5 How large is the reduction of nonlinear distortion using the negative feedback technique?

9.6 Explain the principle of operation of the EER system.

9.7 What are advantages of the EER system?

9.8 What are disadvantages of the EER system?

9.9 What is the principle of operation of envelope tracking?

9.10 What is the difference between the EER and envelope tracking techniques?

9.11 What is the main goal of using the Doherty power amplifier?

9.12 Why is an impedance inverter used in a Doherty power amplifier?

9.13 Is the linearity of the Doherty amplifier good?

9.14 Explain the principle of operation of the outphasing power amplifier.

9.12 Problems

9.1 An RF power amplifier has the voltage gain $A_1 = 180$ for $v_s = 0$ to 1 V and the voltage gain $A_2 = 60$ for $v_s = 1$ to 2 V. Find the voltage transfer function of predistorter to obtain a linear function of the entire amplifier.

9.2 The voltage gains of the power stage of nn RF power amplifier are $A_1 = 900$ for $v_i = 0$–1 V, $A_2 = 600$ for $v_i = 1$–2 V, and $A_3 = 300$ for $v_i = 2$–3 V. Negative feedback with $\beta = 0.1$ is used to linearize the amplifier. Find the gains of the amplifier with negative feedback.

9.3 The relationship between the input voltage v_e and the output voltage v_o of an power stage of an RF amplifier is given by

$$v_o = A v_e^2.$$

Find the relationship between the input voltage v_s and v_o of the amplifier with negative feedback. Draw the plots $v_o = f(v_e)$ and $v_o = f(v_s)$ at $A = 50$ and $\beta = 1/5$. Compare the nonlinearity of both plots.

9.4 The relationship between the input voltage v_e and the output voltage v_o of an power stage of an RF amplifier is given by

$$v_o = 100\left(v_e + \frac{v_e^2}{2}\right).$$

Find the relationship between the input voltage v_s and v_o of the amplifier with negative feedback. Draw the plots $v_o = f(v_e)$ and $v_o = f(v_s)$ at $\beta = 1/20$. Compare the nonlinearity of both plots.

10

Integrated Inductors

10.1 Introduction

Capacitors can easily be integrated in CMOS technology using polysilicon layers, resulting in high capacitances per unit area. One of the largest challenges in making completely monolithic intergated circuits is to make integrated inductors [1–45]. Lack of a good integrated inductor is the most important disadvantage of standard IC process for many applications. Front-end radio-frequency (RF) integrated circuits (ICs) require monolithic integrated inductors to construct resonant circuits, impedance matching networks, band-pass filters, low-pass filters, and high-impedance chokes. RF IC inductors are essential elements of communication circuit blocks, such as low-noise amplifiers (LNAs), mixers, intermediate frequency filters (IFFs), and voltage-controlled oscillators (VCOs) that drive cellular-phone transmitters. Monolithic inductors are also used to bias RF amplifiers, RF oscillators, tuning varactors, PIN diodes, transistors, and monolithic circuits. The structure of planar spiral inductors is based on the standard CMOS and BiCMOS technologies as the cost is one of the major factors. Applications of RF IC inductors include cell phone, wireless local network (WLN), TV tuners, and radars. On-chip and off-chip inductors are used in RF circuits. There are four major types of integrated RF inductors: planar spiral inductors, meander inductors, bondwire inductors, and MEMS inductors. In this chapter, integrated inductors will be discussed.

10.2 Skin Effect

At low frequencies, the current density J in a conductor is uniform. In this case, the ac resistance of the conductor R_{ac} is equal to the dc resistance R_{ac}, and the conduction power loss is identical for ac and dc current if $I_{rms} = I_{dc}$. In contrast, the current density J at high frequencies is not uniform due to eddy currents. Eddy currents are present in any conducting material, which is subjected to a time-varying magnetic field. The skin effect is the tendency of high-frequency current to flow mostly near the surface of a conductor. This

RF Power Amplifiers Marian K. Kazimierczuk

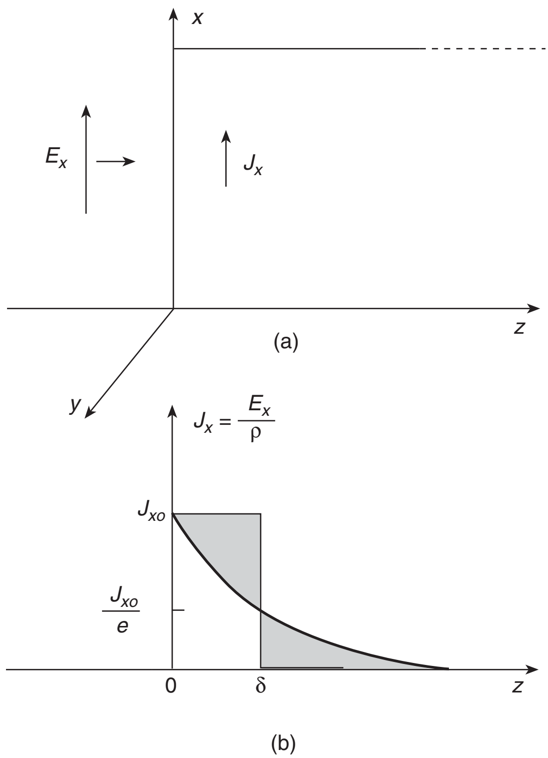

Figure 10.1 Semiinfinite slab. (a) Conductor. (b) Current density distribution. (c) Equivalent circuit.

causes an increase in the effective ac resistance of a conductor R_{ac} above its dc resistance R_{dc}, resulting in an increase in conduction power loss and heat.

Consider a semiinfinite slab of a conductor with conductivity $\sigma = 1/\rho$ that occupies $z \leq 0$, where ρ is the resistivity of the conductor. The semiinfinite conductor is shown in Figure 10.1(a). The harmonic electric field incident on the conductor is given by

$$E_x(z) = E_{xo}\, \mathrm{e}^{-\frac{z}{\delta}} \tag{10.1}$$

where E_{xo} is the amplitude of the electric field just on the surface of the conductor and the skin depth is

$$\delta = \frac{1}{\sqrt{\pi \mu \sigma f}} = \sqrt{\frac{\rho}{\pi \mu f}}. \tag{10.2}$$

The current density in the conductor is

$$J_x(z) = \sigma E_x(z) = \frac{E_{xo}}{\rho} = J_{xo}\, \mathrm{e}^{-\frac{z}{\delta}} \tag{10.3}$$

where $J_{xo} = \sigma E_{xo} = E_{xo}/\rho$ is the amplitude of the current density at the surface. The current density distribution in the conductor is shown in Figure 10.1(b). The current density J_x exponentially decays as a function of depth z into the conductor.

The current flows in the x-direction through the surface extending into the z-direction from $z = 0$ to $z = \infty$. The surface has a width w in the y-direction. The total current is obtain as

$$I = \int_{z=0}^{z=\infty} \int_{y=0}^{w} J_{xo}\, e^{-\frac{z}{\delta}}\, dy\, dz = J_{xo} w \delta. \tag{10.4}$$

Thus, the equivalent current is obtained if we assume that the current density J_x is constant, equal to J_{xo}, from the surface down to skin depth δ and is zero for $z > \delta$. The area of a rectangle of sides J_{xo} and δ is equivalent to the area the exponential curve. It can be shown that 95 % of the total current flows in the thickness of 3δ and 99.3 % of the total current flows in the thickness of 5δ.

The voltage across the distance l in the x-direction is

$$V = E_{xo} l = \rho J_{xo} l. \tag{10.5}$$

Hence, the ac resistance of the semiinfinite conductor of width w and length l that extends from $z = 0$ to $z = \infty$ is

$$R_{ac} = \frac{V}{I} = \frac{\rho}{\delta}\left(\frac{l}{w}\right) = \frac{1}{\sigma\delta}\left(\frac{l}{w}\right)\sqrt{\pi\rho\mu f}. \tag{10.6}$$

As the skin depth δ decreases with increasing frequency, the ac resistance R_{ac} increases.

Example 10.1

Calculate the skin depth at $f = 10\,\text{GHz}$ for (a) copper, (b) aluminum, (c) silver, and (d) gold.

Solution. (a) The resistivity of copper at $T = 20\,^\circ\text{C}$ is $\rho_{Cu} = 1.724 \times 10^{-8}\,\Omega\text{m}$. The skin depth of the copper at $f = 10\,\text{GHz}$ is

$$\delta_{Cu} = \sqrt{\frac{\rho_{Cu}}{\pi\mu_0 f}} = \sqrt{\frac{1.724 \times 10^{-8}}{\pi \times 4\pi \times 10^{-7} \times 10 \times 10^9}} = 0.6608\,\mu\text{m}. \tag{10.7}$$

(b) The resistivity of aluminum at $T = 20\,^\circ\text{C}$ is $\rho_{Al} = 2.65 \times 10^{-8}\,\Omega\text{m}$. The skin depth of the aluminum at $f = 10\,\text{GHz}$ is

$$\delta_{Al} = \sqrt{\frac{\rho_{Al}}{\pi\mu_0 f}} = \sqrt{\frac{2.65 \times 10^{-8}}{\pi \times 4\pi \times 10^{-7} \times 10 \times 10^9}} = 0.819\,\mu\text{m}. \tag{10.8}$$

(c) The resistivity of silver at $T = 20\,^\circ\text{C}$ is $\rho_{Ag} = 1.59 \times 10^{-8}\,\Omega\text{m}$. The skin depth of the silver at $f = 10\,\text{GHz}$ is

$$\delta_{Ag} = \sqrt{\frac{\rho_{Ag}}{\pi\mu_0 f}} = \sqrt{\frac{1.59 \times 10^{-8}}{\pi \times 4\pi \times 10^{-7} \times 10 \times 10^9}} = 0.6346\,\mu\text{m}. \tag{10.9}$$

(d) The resistivity of gold at $T = 20\,^\circ\text{C}$ is $\rho_{Au} = 2.44 \times 10^{-8}\,\Omega\text{m}$. The skin depth of the gold at $f = 10\,\text{GHz}$ is

$$\delta_{Au} = \sqrt{\frac{\rho Au}{\pi\mu_0 f}} = \sqrt{\frac{2.44 \times 10^{-8}}{\pi \times 4\pi \times 10^{-7} \times 10 \times 10^9}} = 0.786\,\mu\text{m}. \tag{10.10}$$

10.3 Resistance of Rectangular Trace

Consider a trace (a slab) of a conductor of thickness h, length l, and width w, as shown in Figure 10.2(a). The dc resistance of the trace is

$$R_{ac} = \rho \frac{l}{wh}. \tag{10.11}$$

The harmonic electric field incident on the conductor is given by

$$E_x(z) = E_{xo}\, \mathrm{e}^{-\frac{z}{\delta}}. \tag{10.12}$$

The current density at high frequencies is given by

$$J_x = J_{xo} e^{-\frac{z}{\delta}} \quad \text{for} \quad 0 \le z \le h \tag{10.13}$$

and

$$J_x = 0 \quad \text{for} \quad z > h. \tag{10.14}$$

The current density distribution in the trace conductor is shown in Figure 10.2(b). Hence, the current the total current flowing in the conductor is

$$I = \int_{z=0}^{h} \int_{y=0}^{w} J_{xo}\, \mathrm{e}^{-\frac{z}{\delta}}\, \mathrm{d}y\, \mathrm{d}z = w\delta J_{xo}\left(1 - \mathrm{e}^{-\frac{h}{\delta}}\right). \tag{10.15}$$

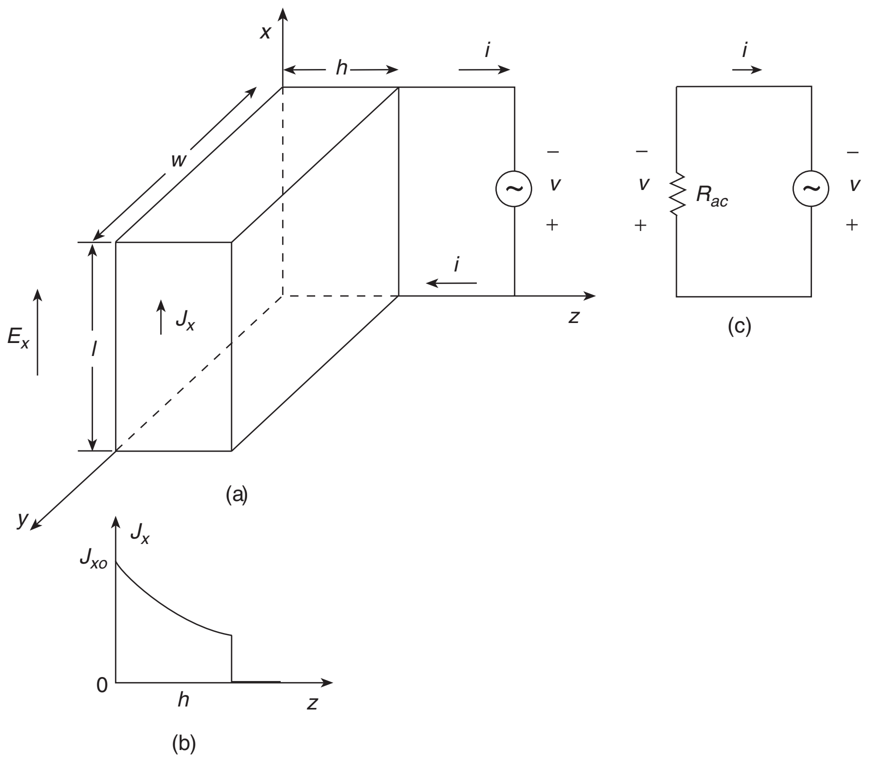

Figure 10.2 Slab conductor. (a) Slab. (b) Current density distribution. (c) Equivalent circuit.

Since $E_{xo} = \rho J_{xo}$, the voltage drop across the conductor in the x-direction of the current flow at $z = 0$ is

$$V = E_{xo} l = \frac{J_{xo}}{\sigma} l = \rho J_{xo} l. \tag{10.16}$$

The ac trace resistance is given by

$$R_{ac} = \frac{V}{I} = \frac{\rho l}{w\delta\left(1 - e^{-h/\delta}\right)} = \frac{l}{w\left(1 - e^{-h/\delta}\right)}\sqrt{\pi\rho\mu f} \quad \text{for} \quad \delta < h. \tag{10.17}$$

At high frequencies, the ac resistance of the metal trace R_{ac} is increased by the skin effect. The current density in the metal trace increases on the side of the substrate. The equivalent circuit of the trace conductor for high frequencies is shown in Figure 10.2(c).

The ratio of the ac-to-dc resistances is

$$F_R = \frac{R_{ac}}{R_{dc}} = \frac{h}{\delta(1 - e^{-\frac{h}{\delta}})}. \tag{10.18}$$

The ratio R_{ac}/R_{dc} as a function of h/δ is shown in Figure 10.3. As h/δ increases with increasing frequency, R_{ac}/R_{dc} increases.

For very high frequencies, this expression can be approximated by

$$R_{ac} \approx \frac{h}{\delta} = h\sqrt{\frac{\pi\mu f}{\rho}} \quad \text{for} \quad \delta < \frac{h}{5}. \tag{10.19}$$

The power loss in the metal trace resistance due to the flow of a sinusoidal current of amplitude I_m is given by

$$P_{Rac} = \frac{1}{2} R_{ac} I_m^2. \tag{10.20}$$

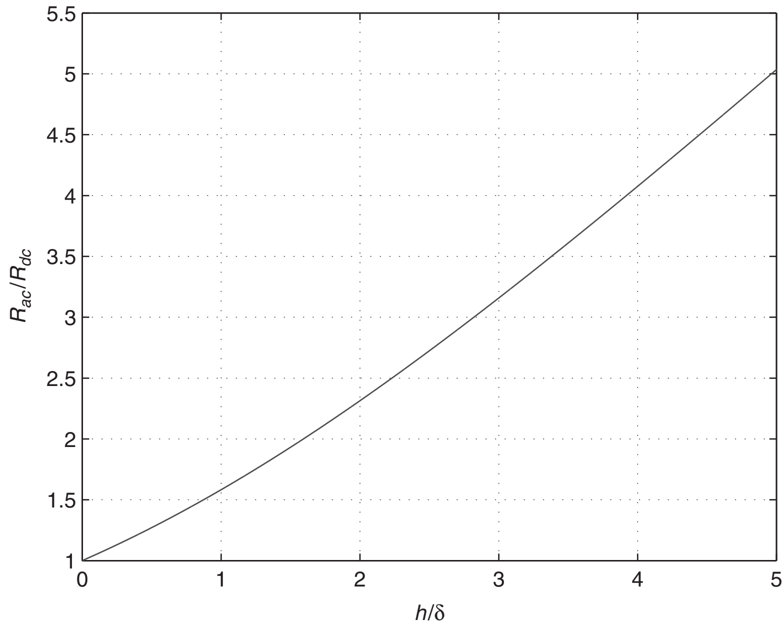

Figure 10.3 R_{ac}/R_{dc} as a function of h/δ.

Example 10.2

Calculate the dc resistance of a copper trace with $l = 50\,\mu\text{m}$, $w = 1\,\mu\text{m}$, and $h = 1\,\mu\text{m}$. Find the ac resistance of the trace at 10 GHz. Calculate F_R.

Solution. The resistivity of copper at $T = 20\,^\circ\text{C}$ is $\rho_{Cu} = 1.724 \times 10^{-8}\ \Omega\text{m}$. The dc resistance of the trace is

$$R_{dc} = \rho_{Cu}\frac{l}{wh} = \frac{1.724 \times 10^{-8} \times 50 \times 10^{-6}}{1 \times 10^{-6} \times 1 \times 10^{-6}} = 0.862\,\Omega. \tag{10.21}$$

The skin depth of the copper at $f = 10\,\text{GHz}$ is

$$\delta_{Cu} = \sqrt{\frac{\rho_{Cu}}{\pi\mu_0 f}} = \sqrt{\frac{1.724 \times 10^{-8}}{\pi \times 4\pi \times 10^{-7} \times 10 \times 10^9}} = 0.6608\,\mu\text{m}. \tag{10.22}$$

The ac resistance of the trace is

$$R_{ac} = \frac{\rho l}{w\delta\left(1 - e^{-h/\delta}\right)} = \frac{1.724 \times 10^{-8} \times 50 \times 10^{-6}}{1 \times 10^{-6} \times 0.6608 \times 10^{-6}\left(1 - e^{-\frac{1}{0.6608}}\right)} = 1.672\,\Omega. \tag{10.23}$$

The ratio of the ac-to-dc resistances is

$$F_R = \frac{R_{ac}}{R_{dc}} = \frac{1.672}{0.862} = 1.93. \tag{10.24}$$

10.4 Inductance of Straight Rectangular Trace

A straight rectangular trace inductor is shown in Figure 10.4. The self-inductance of a solitary rectangular straight conductor at low frequencies, where the current density is uniform and skin effect can be neglected, is given by Grover's formula [7]

$$L = \frac{\mu_0 l}{2\pi}\left[\ln\left(\frac{2l}{w+h}\right) + \frac{w+h}{3l} + 0.50049\right]\ (\text{H}) \quad \text{for} \quad w \le 2l \quad \text{and} \quad h \le 2l \tag{10.25}$$

where l, w, and h are the conductor length, width, and thickness in m, respectively. Since $\mu_0 = 2\pi \times 10^{-7}$, the trace inductances is

$$L = 2 \times 10^{-7} l\left[\ln\left(\frac{2l}{w+h}\right) + \frac{w+h}{3l} + 0.50049\right]\ (\text{H}) \quad \text{for} \quad w \le 2l \quad \text{and} \quad h \le 2l \tag{10.26}$$

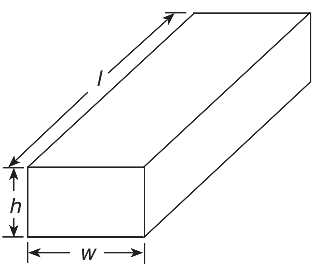

Figure 10.4 Straight rectangular trace inductor inductor.

where all dimentions are in m. Grover's formula can be also expressed as

$$L = 0.0002l\left[\ln\left(\frac{2l}{w+h}\right) + \frac{w+h}{3l} + 0.50049\right](\mathrm{nH}) \quad \text{for} \quad w \le 2l \quad \text{and} \quad h \le 2l \tag{10.27}$$

where l, w, and h are the conductor length, width, and thickness in μm, respectively. The inductance L of a trace increases as l increases and w decreases. Straight traces are segments of inductors of more complex structures of integrated planar inductors, such as square integrated planar inductors. They are also used as interconnections in integrated circuits.

The quality factor of the straight rectangular trace at frequency $f = \omega/(2\pi)$ is defined as

$$Q = 2\pi\frac{\text{Peak magnetic energy stored in } L}{\text{Energy dissipated per cycle}} = \frac{\omega L}{R_{ac}}. \tag{10.28}$$

Another definition of the quality factor is given by

$$Q = 2\pi\frac{\text{Peak magnetic energy} - \text{Peak electric energy}}{\text{Energy dissipated per cycle}}. \tag{10.29}$$

Example 10.3

Calculate the inductance of a straight rectangular trace with $l = 50\,\mu\mathrm{m}$, $w = 1\,\mu\mathrm{m}$, $h = 1\,\mu\mathrm{m}$, and $\mu_r = 1$. Find its quality factor Q at a frequency of 10 GHz.

Solution. The inductance of the straight trace is

$$\begin{aligned} L &= \frac{\mu_0 l}{2\pi}\left[\ln\left(\frac{2l}{w+h}\right) + \frac{w+h}{3l} + 0.50049\right] \\ &= \frac{4\pi\times10^{-7}\times50\times10^{-6}}{2\pi}\left[\ln\left(\frac{2\times50}{1+1}\right) + \frac{1+1}{3\times50} + 0.50049\right] \\ &= 0.04426\,\mathrm{nH}. \end{aligned} \tag{10.30}$$

From Example 10.2, $R_{ac} = 1.672\,\Omega$. The quality factor of the inductor is

$$Q = \frac{\omega L}{R_{ac}} = \frac{2\pi\times10\times10^{9}\times0.4426\times10^{-9}}{1.672} = 1.7092. \tag{10.31}$$

10.5 Meander Inductors

Meander inductors have a simple layout as shown in Figure 10.5. It is sufficient to use only one metal layer because both metal contacts are on the same metal level. This simplifies the technological process by avoiding of two levels of photolithography. They usually have a low inductance per unit area and a large ac resistance because the length of the trace is large, causing a large dc resistance. This results in a low quality factor. The meander inductor can be divided into straight segments. The self-inductance of each straight segment is given by (10.27). The total self-inductance is equal to the sum of the self-inductances of all straight segments and is given by [44]

$$L_{self} = 2L_a + 2L_b + NL_c + (N+1)L_s \tag{10.32}$$

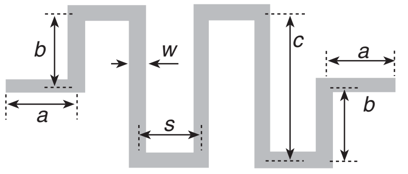

Figure 10.5 Meander inductor.

where

$$L_a = \frac{\mu_0 a}{2\pi}\left[\ln\left(\frac{2a}{w+h}\right) + \frac{w+h}{3a} + 0.50049\right] \quad \text{(H)} \tag{10.33}$$

$$L_b = \frac{\mu_0 b}{2\pi}\left[\ln\left(\frac{2b}{w+h}\right) + \frac{w+h}{3b} + 0.50049\right] \quad \text{(H)} \tag{10.34}$$

$$L_c = \frac{\mu_0 c}{2\pi}\left[\ln\left(\frac{2c}{w+h}\right) + \frac{w+h}{3c} + 0.50049\right] \quad \text{(H)} \tag{10.35}$$

$$L_s = \frac{\mu_0 s}{2\pi}\left[\ln\left(\frac{2s}{w+h}\right) + \frac{w+h}{3s} + 0.50049\right] \quad \text{(H)} \tag{10.36}$$

N is the number of segments of length c and L_a, L_b, L_c, and L_s are self-inductances of the segments. All dimensions are in m.

The mutual inductance between two equal parallel conductors (segments) shown in Figure 10.6 is given by [7]

$$M = \pm\frac{\mu_0 l}{2\pi}\left\{\ln\left[\frac{l}{s} + \sqrt{1+\left(\frac{l}{s}\right)^2}\right] - \sqrt{1+\left(\frac{s}{l}\right)^2} + \frac{s}{l}\right\} \quad \text{(H)} \tag{10.37}$$

where l is the length of the parallel conductors and s is the center-to-center separation of the conductors. All dimensions are in m. If the currents flow in both parallel segments in the same direction, the mutual inductance is positive. If the currents flow in the parallel segments in the opposite directions, the mutual inductance is negative. The mutual inductance of perpendicular conductors is zero.

It can be shown that the input inductance of a transformer with the inductance of each winding equal to L and both currents flowing in the same direction is

$$L_i = L + M. \tag{10.38}$$

The input inductance of a transformer with the inductance of each winding equal to L and both currents flowing in the opposite directions is

$$L_i = L - M. \tag{10.39}$$

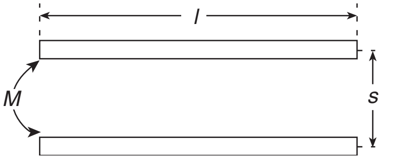

Figure 10.6 Two equal parallel straight rectangular conductors.

The total inductance of the meander inductor is the sum of the self-inductances of all segments and the positive and negative mutual inductances.

The monomial equation for the total inductance of the meander inductor is given by [44]

$$L = 0.00266a^{0.0603}h^{0.4429}N^{0.954}s^{0.606}w^{-0.173} \text{ (nH).} \tag{10.40}$$

where $a = c/2$. All dimensions are in μm. The accuracy of this equation is better than 12 %.

Example 10.4

Calculate the inductance of a meander inductor with $N = 5$, $w = 40\,\mu\text{m}$, $s = 40\,\mu\text{m}$, $a = 40\,\mu\text{m}$, $h = 100\,\mu\text{m}$, and $\mu_r = 1$.

Solution. The inductance of the trace is

$$\begin{aligned} L &= 0.00266a^{0.0603}h^{0.4429}N^{0.954}s^{0.606}w^{-0.173} \\ &= 0.00266 \times 40^{0.0603}100^{0.4429}5^{0.954}40^{0.606}40^{-0.173} = 0.585\,\text{nH}. \end{aligned} \tag{10.41}$$

10.6 Inductance of Straight Round Conductor

The inductance of a round straight inductor of length l and conductor radius a, when the effects of nearby conductors (including the return current) and the skin effect can be neglected, is given by [7]

$$L = \frac{\mu_0 l}{2\pi}\left[\ln\left(\frac{2l}{a}\right) + \frac{a}{l} - \frac{3}{4}\right] \text{ (H)} \tag{10.42}$$

where the dimensions are in m. This equation can be simplified to the form

$$L = 0.0002l\left[\ln\left(\frac{2l}{a}\right) + \frac{a}{l} - 0.75\right] \text{ (nH)} \tag{10.43}$$

where the dimensions are in μm.

Another expression for a round straight conductor is

$$L = \frac{\mu l}{8\pi} \quad \text{for} \quad \delta > a. \tag{10.44}$$

The inductance at high frequencies decreases with frequency. The high-frequency inductance of a round straight inductor when the skin effect cannot be neglected is given by

$$L_{HF} = \frac{l}{4\pi a}\sqrt{\frac{\mu\rho}{\pi f}} \quad \text{for} \quad \delta < a. \tag{10.45}$$

Example 10.5

Calculate the inductance of a round straight inductor with $l = 2\,\text{mm}$, $a = 0.1\,\text{mm}$, and $\mu_r = 1$ for $\delta > a$. Also, calculate the inductance of a copper round straight inductor with $a = 0.1\,\text{mm}$ at $f = 1\,\text{GHz}$.

Solution. The inductance of the round straight inductor for $\delta < a$ (low-frequency operation) is given by

$$L = \frac{\mu_0 l}{2\pi}\left[\ln\left(\frac{2l}{a}\right) + \frac{a}{l} - 0.75\right] \quad \text{(H)}$$

$$= \frac{4\pi \times 10^{-7} \times 2 \times 10^{-3}}{2\pi}\left[\ln\left(\frac{2 \times 2}{0.1}\right) + \frac{0.1}{2} - 0.75\right] = .1.1956\,\text{nH}. \quad (10.46)$$

The skin depth in copper at $f = 1\,\text{GHz}$ is

$$\delta = \sqrt{\frac{\rho}{\pi \mu_o f}} = \sqrt{\frac{1.724 \times 10^{-8}}{\pi \times 4\pi \times 10^{-7} \times 1 \times 10^{9}}} = 2.089\,\mu\text{m}. \quad (10.47)$$

Since $a = 0.1\,\text{mm} = 100\,\mu\text{m}$, $\delta \ll a$. Therefore, the operation at $f = 1\,\text{GHz}$ is the high-frequency operation. The inductance of the round straight inductor at $f = 1\,\text{GHz}$ is

$$L_{HF} = \frac{l}{4\pi a}\sqrt{\frac{\mu_0 \rho}{\pi f}} = \frac{2 \times 10^{-3}}{4\pi \times 0.1 \times 10^{-3}}\sqrt{\frac{4\pi \times 10^{-7} \times 1.724 \times 10^{-8}}{\pi \times 1 \times 10^{9}}} = 4.179\,\text{pH}. \quad (10.48)$$

Hence,

$$F_R = \frac{L_{HF}}{L_{LF}} = \frac{4.179 \times 10^{-12}}{1.1956 \times 10^{-9}} = 3.493 \times 10^{-3} = \frac{1}{286}. \quad (10.49)$$

10.7 Inductance of Circular Round Wire Loop

The inductance of a circular loop of round wire with loop radius r and wire radius a is [7]

$$L = \mu_0 a\left[\ln\left(\frac{8r}{a}\right) - 1.75\right] \quad \text{(H)}. \quad (10.50)$$

All dimensions are in m. The dependence of the inductance on wire radius is weak.

The inductance of a round wire square loop is

$$L = \frac{\mu_0 l}{2\pi}\left[\ln\left(\frac{l}{4a}\right) - 0.52401\right] \quad \text{(H)} \quad (10.51)$$

where l is the loop length and a is the wire radius. All dimensions are in m.

10.8 Inductance of Two-parallel Wire Loop

Consider a loop formed by two parallel conductors whose length is l radius is r, and the distance between the conductors is s, where $l \gg s$. The conductors carry equal currents in opposite directions. The inductance of the two-parallel wire loop is [7]

$$L = \frac{\mu_0 l}{\pi}\left[\ln\left(\frac{s}{r}\right) - \frac{s}{l} + 0.25\right] \quad \text{(H)}. \quad (10.52)$$

10.9 Inductance of Rectangle of Round Wire

The inductance of a rectangle of round wir eof radius r with rectangle side lengths x and y is given by [7]

$$L = \frac{\mu_0}{\pi}\left[x \ln\left(\frac{2x}{a}\right) + y \ln\left(\frac{2y}{a}\right) + 2\sqrt{x^2+y^2} - x \operatorname{arcsinh}\left(\frac{x}{y}\right) - y \operatorname{arcsinh}\left(\frac{y}{x}\right) - 1.75(x+y)\right] \tag{10.53}$$

10.10 Inductance of Polygon Round Wire Loop

The inductance of a single-loop polygon can be approximated by [7]

$$L \approx \frac{\mu p}{2\pi}\left[\ln\left(\frac{2p}{r}\right) - \ln\left(\frac{p^2}{A}\right) + 0.25\right] \text{ (H)} \tag{10.54}$$

where p is the perimeter of the coil, A is the area enclosed by the coil, and a is the wire radius. All dimensions are in m. The inductance is strongly dependent on the perimeter and weakly dependent on the loop area and wire radius. Inductors enclosing the same perimeter with similar shape have approximately the same inductance.

The inductance of a triangle is [7]

$$L = \frac{\mu_0 l}{2\pi}\left[\ln\left(\frac{l}{3a}\right) - 1.15546\right] \text{ (H)} \tag{10.55}$$

where a is the wire radius. The inductance of a square is

$$L = \frac{\mu_0 l}{2\pi}\left[\ln\left(\frac{l}{4a}\right) - 0.52401\right] \text{ (H).} \tag{10.56}$$

The inductance of a pentagon is

$$L = \frac{\mu_0 l}{2\pi}\left[\ln\left(\frac{l}{5a}\right) - 0.15914\right] \text{ (H).} \tag{10.57}$$

The inductance of a hexagon is

$$L = \frac{\mu_0 l}{2\pi}\left[\ln\left(\frac{l}{6a}\right) + 0.09848\right] \text{ (H).} \tag{10.58}$$

The inductance of an octagon is

$$L = \frac{\mu_0 l}{2\pi}\left[\ln\left(\frac{l}{8a}\right) + 0.46198\right] \text{ (H).} \tag{10.59}$$

All dimensions in m.

10.11 Bondwire Inductors

Bondwire inductors are frequently used in RF ICs. Figure 10.7 shows a bondwire inductor. It can be regarded as a fraction of a round turn. Bondwires are often used for chip connections.

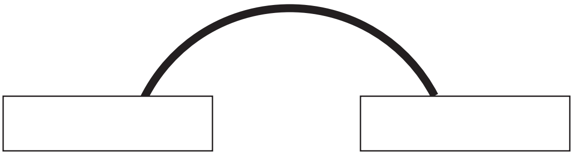

Figure 10.7 Bondwire inductor.

The main advantage of bondwire inductors is a very small series resistance. Standard bondwires have a relatively large diameter of about 25 µm and can handle substantial currents with low loss. They may be placed well above any conductive planes to reduce parasitic capacitances and hence increase the self-resonant frequency f_r. Typical inductances range from 2 to 5 nH. Since bondwires have much larger cross-sectional area than the traces of planar spiral inductors, these inductors have lower resistances and hence a higher quality factor Q_{Lo}, typically $Q_{Lo} = 20–50$ at 1 GHz. These inductors may be placed well above any conductive planes to reduce parasitic capacitances, yielding a higher self-resonant frequency f_r. The inductance of bondwire inductors is given by

$$L \approx \frac{\mu_0 l}{2\pi}\left[\ln\left(\frac{2l}{a}\right) - 0.75\right] \text{ (H)} \tag{10.60}$$

where l is the length of the bondwire and a is the radius of bondwire. A standard bondwire inductance with $l = 1$ mm gives $L = 1$ nH, or 1 nH/mm. The resistance is

$$R = \frac{l}{2\pi a\delta\sigma} \tag{10.61}$$

where σ is the conductivity and δ is the skin depth. For aluminum, $\sigma = 4 \times 10^7$ S/m. The skin depth for aluminum is $\delta_{Al} = 2.5$ µm at 1 GHz. The resistance per unit length is $R_{Al}/l \approx 0.2$ Ω/mm at 2 GHz. The major disadvantage is low predictability. Bondwire inductance depends on bonding geometry and the existence of neighboring bondwires, making accurate prediction of the inductance value difficult. However, once the configuration for a particular inductance is known, it is rather repeatable in subsequent bondings. Common bondwire metals include gold and aluminum. Gold is generally preferred because of its higher conductivity and flexibility. This allows higher quality factor Q_{Lo} with shorter physical lengths to be bonded for a given die height.

Example 10.6

Calculate the inductance of a round bondwire inductor with $l = 2$ mm, $a = 0.2$ mm, and $\mu_r = 1$ for $\delta > a$.

Solution. The inductance of the round straight inductor for $\delta < a$ is

$$L \approx \frac{\mu_0 l}{2\pi}\left[\ln\left(\frac{2l}{a}\right) - 0.75\right] = \frac{4\pi \times 10^{-7} \times 2 \times 10^{-3}}{2\pi}\left[\ln\left(\frac{2 \times 2}{0.2}\right) - 0.75\right] = 0.8982 \text{ nH}. \tag{10.62}$$

10.12 Single-turn Planar Inductors

The self-inductance is a special case of mutual inductance. Therefore, an expression for the self-inductance of an inductor can be derived using the concept of the mutual inductance. This is demonstrated by deriving an expression for the inductance of a single-turn planar inductor.

Figure 10.8 shows a single-turn strip inductor of width w, inner radius b, and outer radius a. The concept of mutual inductance can be used to determine a self-inductance. Consider two circuits: a line circuit along the inner surface of the strip and a line current along the axis (middle) of the strip. The vector magnetic potential caused by circuit 2 is

$$\mathbf{A}_2 = I_2 \oint_{C_2} \frac{\mu \mathrm{d}\mathbf{l_2}}{4\pi h}. \tag{10.63}$$

The integration of the vector magnetic potential $\mathbf{A}_2$ along circuit 1 resulting from the current I_2 in circuit 2 gives the mutual inductance between the two circuits

$$M = \frac{1}{I_2} \oint_{C_1} \mathbf{A}_2 \cdot \mathrm{d}\mathbf{l_1}. \tag{10.64}$$

Hence, the mutual inductance of two conductors is given by

$$M = \frac{\mu}{4\pi} \oint_{C_1} \oint_{C_2} \frac{\mathrm{d}\mathbf{l_1} \cdot \mathrm{d}\mathbf{l_2}}{h}. \tag{10.65}$$

From geometrical consideration and the following substitution

$$\theta = \pi - 2\phi \tag{10.66}$$

we have

$$\mathrm{d}l_1 = |\mathrm{d}\mathbf{l_1}| = a\mathrm{d}\theta = -2a\mathrm{d}\phi \tag{10.67}$$

yielding

$$\mathrm{d}\mathbf{l_1} \cdot \mathrm{d}\mathbf{l_2} = \mathrm{d}l_1\, \mathrm{d}l_2 \cos\theta = 2a \cos 2\phi \mathrm{d}l_2 \mathrm{d}\phi \tag{10.68}$$

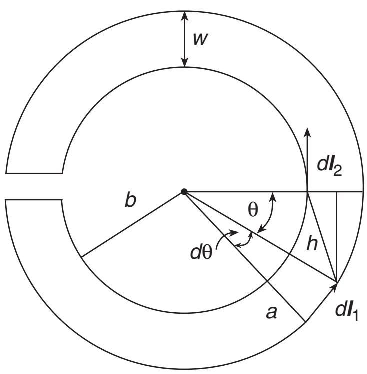

Figure 10.8 Single-turn planar inductor.

and

$$h = \sqrt{(a\cos\theta - b)^2 + (a\sin\theta)^2} = \sqrt{a^2 + b^2 - 2ab\cos\theta} = \sqrt{a^2 + b^2 + 2ab\cos 2\phi}$$
$$= \sqrt{a^2 + b^2 + 2ab(1 - 2\sin^2\phi)} = \sqrt{(a+b)^2 - 4ab\sin^2\phi} = (a+b)\sqrt{1 - k^2\sin^2\phi} \tag{10.69}$$

where

$$k^2 = \frac{4ab}{(a+b)^2} = 1 - \frac{(a-b)^2}{(a+b)^2}. \tag{10.70}$$

Also,

$$\oint \mathrm{d}l_2 = 2\pi b. \tag{10.71}$$

The mutual inductance is

$$M = \frac{\mu}{4\pi}\oint \mathrm{d}l_2 \int_0^{2\pi} \frac{a\cos\theta\,\mathrm{d}\theta}{\sqrt{a^2 + b^2 - 2ab\cos\theta}} = \frac{2\mu ab}{a+b}\int_0^{\pi/2} \frac{(2\sin^2\phi - 1)\,\mathrm{d}\phi}{\sqrt{1 - k^2\sin^2\phi}}$$
$$= \mu\sqrt{ab}k\int_0^{\pi/2} \frac{(2\sin^2\phi - 1)\,\mathrm{d}\phi}{\sqrt{1 - k^2\sin^2\phi}} = \mu\sqrt{ab}\left[\left(\frac{2}{k} - k\right)K(k) - \frac{2}{k}E(k)\right] \tag{10.72}$$

where the complete elliptic integrals are

$$E(k) = \int_0^{\pi/2} \sqrt{1 - k^2\sin^2\phi}\,\mathrm{d}\phi \tag{10.73}$$

and

$$K(k) = \int_0^{\pi/2} \frac{\mathrm{d}\phi}{\sqrt{1 - k^2\sin^2\phi}}. \tag{10.74}$$

The width of the strip is

$$w = a - b \tag{10.75}$$

resulting in

$$a + b = 2a - w \tag{10.76}$$

$$ab = a(a - w) = a^2\left(1 - \frac{w}{a}\right) \tag{10.77}$$

and

$$k^2 = 1 - \frac{w^2}{(2a - w)^2}. \tag{10.78}$$

If $w/a \ll 1$, $k \approx 1$, $E(k) \approx 1$,

$$k^2 = 1 - \frac{w^2}{4a^2\left(1 - \dfrac{w}{2a}\right)^2} \approx 1 - \frac{w^2}{4a^2} \tag{10.79}$$

and

$$K(k) \approx \ln\frac{4}{\sqrt{1 - k^2}} \approx \ln\left[\frac{4}{\sqrt{1 - \left(1 - \dfrac{w^2}{4a^2}\right)}}\right] = \ln\left(\frac{8a}{w}\right). \tag{10.80}$$

Hence, the self-inductance of a single-turn inductor is given by

$$L \approx \mu a \left[\ln\left(\frac{8a}{w}\right) - 2\right] = \frac{\mu l}{2\pi}\left[\ln\left(\frac{8a}{w}\right) - 2\right] \quad \text{for} \quad a \gg w \tag{10.81}$$

where $l = 2\pi a$.

Example 10.7

Calculate the inductance of a single-turn round planar inductor with $a = 100\,\mu\text{m}$, $w = 1\,\mu\text{m}$, and $\mu_r = 1$ for $\delta > a$.

Solution. The inductance of the single-turn round planar inductor for $\delta < a$ is

$$L \approx \mu a \left[\ln\left(\frac{8a}{w}\right) - 2\right] = 4\pi \times 10^{-7} \times 100 \times 10^{-6}\left[\ln\left(\frac{8 \times 100}{1}\right) - 2\right] = 0.58868\,\text{nH}. \tag{10.82}$$

10.13 Inductance of Planar Square Loop

The self-inductance of a square coil made of rectangular wire of length $l \gg w$ and width w is

$$L \approx \frac{\mu_0 l}{\pi}\left[\operatorname{arcsinh}\left(\frac{l}{2w}\right) - 1\right] \ [\text{H}]. \tag{10.83}$$

Both l and w are in m.

10.14 Planar Spiral Inductors

10.14.1 Geometries of Planar Spiral Inductors

Planar spiral inductors are the most widely used RF IC inductors. Figure 10.9 shows square, octagonal, hexagonal, and circular planar spiral inductors. A planar spiral inductor consists of a low-resistivity metal trace (aluminum, copper, gold, or silver), silicon dioxide SiO_2 of thickness t_{ox}, and a silicon substrate, as shown in Figure 10.10. A metal layer embedded in silicon dioxide is used to form the metal spiral. The topmost metal layer is usually the thickest and thus most conductive. In addition, a larger distance of the topmost metal layer to the substrate reduces the parasitic capacitance $C = \epsilon A_m / t_{ox}$, increasing the self-resonant frequency $f_r = 1/(2\pi\sqrt{LC})$ where A_m is the metal trace area. The substrate is a thick silicon (Si), gallium arsenide (GaAs), or silicon germanium (SiGe) layer of thickness of the order of 500–700 μm. The thin silicon dioxide SiO_2 of thickness 0.4–3 μm is used to isolate the metal strips of the inductor from the silicon substrate. The outer end of the spiral is connected directly to a port. The connection of the innermost turn is made through an *underpass* by another metal layer or via an *air bridge*. The whole structure is connected to pads and surrounded by a ground plane. Commonly used shapes of spiral inductors are square, rectangular, hexagonal, octagonal, and circular. Hexagonal and octagonal spiral

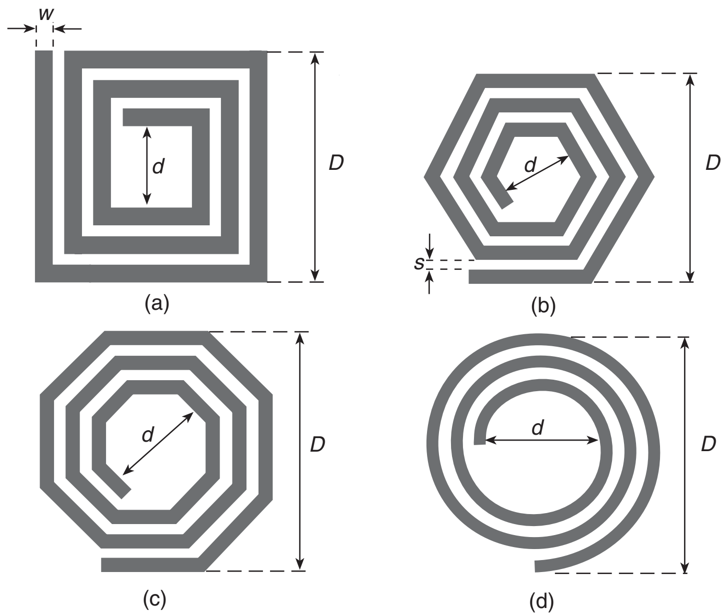

Figure 10.9 Planar spiral integrated inductors. (a) Square inductor. (b) Hexagonal inductor. (c) Octagonal inductor. (d) Circular inductor.

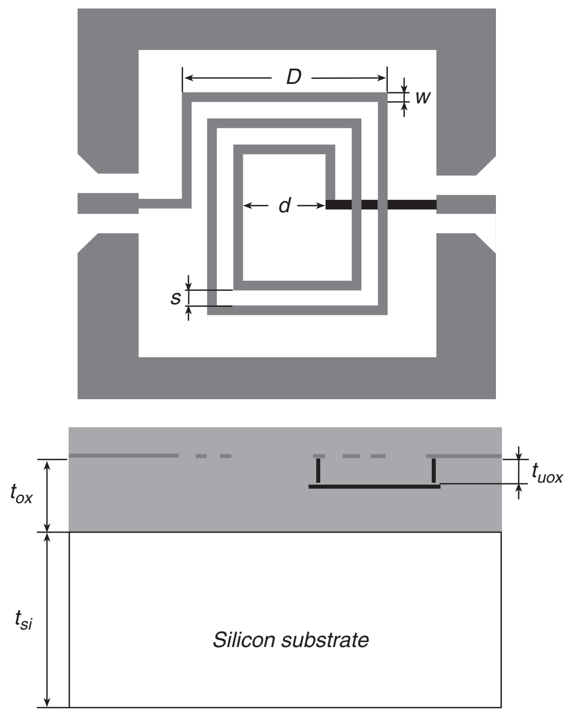

Figure 10.10 Cross-section of a planar RF IC inductor.

inductors generally have less inductance and less series resistance per turn as compared to square spirals. Since the hexagonal and octagonal structures occupy larger chip aream they are rarely used. A planar spiral integrated inductor can only be fabricated with technologies having two or more metal layers because the inner connection requires a metal layer different than that used for the spiral to connect the inside turn of the coil to outside. Square inductor shapes are the most compatible with IC layout tools. They are easily designed with Manhattan style physical layout tools, such as MAGIC. Hexagonal, octagonal, and higher-order polygon spiral inductors have higher quality factor Q_{Lo} than square spirals. The parameters of interest for integrated inductors are the inductance L_s, the quality factor Q_{Lo}, and the self-resonant frequency (SRF) f_r. The typical inductances are the range from 1 to 30 nH, the typical quality factors are from 5 to 20, and the typical self-resonant frequencies are from 2 to 20 GHz. The geometrical parameters of an inductor are the number of turns N, the metal strip width w, the metal strip height h, the turn-to-turn space s, the inner diameter d, the outer diameter D, the thickness of the silicon substrate t_{Si}, the thickness of the oxide layer t_{ox}, and the thickness of oxide between the metal strip and the underpass t_{uox}. The typical metal strip width w is 30 μm, the typical turn-to-turn spacing s is 20 μm, and the typical metal strip sheet resistance is $R_{sheet} = \rho/h = 0.03$ to 0.1 Ω per square. IC inductors require a lot of chip area. Therefore, practical IC inductors have small inductances, typically $L \leq 10$ nH, but they can be as high as 30 nH. The typical inductor size is from 130 × 130 μm to 1000 × 1000 μm. The inductor area is $A = D^2$. The topmost metal layer is usually used for construction of IC inductors because it is the thickest and thus has the lowest resistance. In addition, the distance from the topmost layer to the substrate is the largest, reducing the parasitic capacitances. The preferred metalization for integrated inductors is a low-resistivity, inert metal, such as gold. Other low-resistivity metals such as silver and copper do not offer the same level of resistance to atmospheric sulfur and moisture. Platinum, another noble metal, is two times more expensive than gold and has higher resistivity. The thickness of the metal h should be higher than 2δ one skin depth δ on the top of the trace and one skin depth on the bottom. The direction of the magnetic flux is perpendicular to the substrate. The magnetic field penetrates the substrate, inducing eddy currents. A high substrate conductivity tends to lower the inductor quality factor Q_{Lo}. The optimization of the spiral geometry and the metal trace width leads to minimization of the trace ohmic resistance and the substrate capacitance. To reduce power losses in the substrate, the substrate can be made of high resistive silicon oxide such as silicon-on-insultaor (SOI), thick dielectric layers, or thick and multilayer conductor lines.

10.14.2 Inductance of Square Planar Inductors

Several expressions have been developed to estimate RF spiral inductances. In general, the inductance of planar inductors L increases when the number of turns N and the area of an inductor A increase.

The inductance of a single loop of area A and any shape (e.g., circular, square, rectangular, hexagonal, or octagonal) is given by

$$L \approx \mu_0 \sqrt{\pi A}. \tag{10.84}$$

Hence, the inductance of a single-turn round inductor of radius r is

$$L \approx \pi \mu_0 r = 4\pi^2 \times 10^{-7} r = 4 \times 10^{-6} r \text{ (H)}. \tag{10.85}$$

For example, $L = 4$ nH for $r = 1$ mm.

The inductance of an arbitrary planar spiral inductor with N turns, often used in ICs, is given by

$$L \approx \pi \mu_0 r N^2 = 4\pi^2 \times 10^{-7} r N^2 = 4 \times 10^{-6} r N^2 \text{ (H)} \tag{10.86}$$

where N is the number of turns and r is the spiral radius.

Bryan's Formula

The inductance of the square planar spiral inductor is given by an empirical Bryan's equation [7]

$$L = 6.025 \times 10^{-7}(D + d)N^{\frac{5}{3}} \ln\left[4\left(\frac{D + d}{D - d}\right)\right] \text{ (H)} \tag{10.87}$$

where the outermost diameter is

$$D = d + 2Nw + 2(N - 1)s. \tag{10.88}$$

N is the number of turns, and D and d are the outermost and innermost diameters of the inductor in m, respectively.

Example 10.8

Calculate the inductance of a square planar spiral inductor with $N = 5$, $d = 60\,\mu\text{m}$, $w = 30\,\mu\text{m}$, $s = 20\,\mu\text{m}$, and $\mu_r = 1$ for $\delta > h$. Use Bryan's formula.

Solution. The outermost diameter is

$$D = d + 2Nw + 2(N - 1)s = 60 + 2 \times 5 \times 30 + 2 \times (5 - 1) \times 20 = 520\,\mu\text{m}. \tag{10.89}$$

The inductance of the round planar inductor for $\delta < h$ is

$$\begin{aligned} L &= 6.025 \times 10^{-7}(D + d)N^{\frac{5}{3}} \ln\left[4\left(\frac{D + d}{D - d}\right)\right] \text{ (H)} \\ &= 6.025 \times 10^{-7}(520 + 60) \times 10^{-6} \times 5^{\frac{5}{3}} \ln\left[4\left(\frac{520 + 60}{520 - 60}\right)\right] = 8.266\,\text{nH}. \end{aligned} \tag{10.90}$$

Wheeler's Formula

The inductance of a square spiral inductor is given by modified Wheeler's formula [3, 32]

$$L = 1.17\mu_0 N^2 \frac{D + d}{1 + 2.75\dfrac{D - d}{D + d}} \text{ (H)} \tag{10.91}$$

where

$$D = d + 2Nw + 2(N - 1)s. \tag{10.92}$$

All dimensions are in m.

Example 10.9

Calculate the inductance of a square planar spiral inductor with $N = 5$, $d = 60\,\mu\text{m}$, $w = 30\,\mu\text{m}$, $s = 20\,\mu\text{m}$, and $\mu_r = 1$ for $\delta > h$. Use Wheeler's formula.

Solution. The outermost diameter is

$$D = d + 2Nw + 2(N - 1)s = 60 + 2 \times 5 \times 30 + 2 \times (5 - 1) \times 20 = 520\,\mu\text{m}. \quad (10.93)$$

The inductance of the square planar inductor for $\delta < h$ is

$$L = 1.17\mu_0 N^2 \frac{D + d}{1 + 2.75\dfrac{D - d}{D + d}} \text{ (H)}$$
$$= 1.17 \times 4\pi \times 10^{-7} \times 5^2 \times \frac{(520 + 60) \times 10^{-6}}{1 + 2.75 \times \dfrac{520 - 60}{520 + 60}} = 6.7\,\text{nH}. \quad (10.94)$$

Greenhouse Formula

The inductance of spiral inductors is given by Greenhouse's equation [13]

$$L = 10^{-9} DN^2 \left\{ 180 \left[\log\left(\frac{D}{4.75(w + h)} \right) + 5 \right] + \frac{1.76(w + h)}{D} + 7.312 \right\} \text{ (nH)}. \quad (10.95)$$

All dimensions are in m.

Example 10.10

Calculate the inductance of a square planar spiral inductor with $N = 5$, $d = 60\,\mu$m, $w = 30\,\mu$m, $s = 20\,\mu$m, $h = 20\,\mu$m, and $\mu_r = 1$ for $\delta > h$. Use Greenhouse's formula.

Solution. The outermost diameter is

$$D = d + 2Nw + 2(N - 1)s = 60 + 2 \times 5 \times 30 + 2 \times (5 - 1) \times 20 = 520\,\mu\text{m}. \quad (10.96)$$

The inductance of the single-turn round planar inductor for $\delta < h$ is

$$L = 10^{-9} DN^2 \left\{ 180 \left[\log\left(\frac{D}{4.75(w + h)} \right) + 5 \right] + \frac{1.76(w + h)}{D} + 7.312 \right\} \text{ (nH)}$$
$$= 10^{-9} \times 520 \times 5^2 \left\{ 180 \left[\log\left(\frac{520}{4.75(30 + 20)} \right) + 5 \right] + \frac{1.76(30 + 20)}{520} + 7.312 \right\}$$
$$= 12.59\,\text{nH}. \quad (10.97)$$

Grover's Formula

The inductance of a square or rectangular spiral planar inductor with a rectangular cross-section of the trace is expressed as [7, 30]

$$L = \frac{\mu_0 l}{2\pi} \left[\ln\left(\frac{2l}{w + h} \right) + \frac{w + h}{3l} + 0.50049 \right] \text{ (H)} \quad (10.98)$$

where the length of the metal trace is

$$l = 2(D + w) + 2N(2N - 1)(w + s) \quad (10.99)$$

the outer diameter is

$$D = d + 2Nw + 2(N - 1)s \tag{10.100}$$

and h is the metal trace height. All dimensions l, w, s, and h are in m. This equation is identical to that for a straight trace inductor. It accounts for the self-inductance only and neglects mutual inductances between the traces.

Example 10.11

Calculate the inductance of a square planar spiral inductor with $N = 5$, $d = 60\,\mu\text{m}$, $w = 30\,\mu\text{m}$, $s = 20\,\mu\text{m}$, $h = 20\,\mu\text{m}$, and $\mu_r = 1$ for $\delta > h$. Use Grover's formula.

Solution. The outer diameter is

$$D = d + 2Nw + 2(N - 1)s = 60 + 2 \times 5 \times 30 + 2 \times (5 - 1) \times 20 = 520\,\mu\text{m}. \tag{10.101}$$

The length of the metal trace is

$$\begin{aligned} l &= 2(D + w) + 2N(2N - 1)(w + s) = 2(520 + 30) + 2 \times 5(2 \times 5 - 1)(30 + 20) \\ &= 5600\,\mu\text{m}. \end{aligned} \tag{10.102}$$

The inductance of the square spiral planar inductor for $\delta < h$ is

$$\begin{aligned} L &= \frac{\mu_0 l}{2\pi}\left[\ln\left(\frac{2l}{w + h}\right) + \frac{w + h}{3l} + 0.50049\right] \text{ (H)} \\ &= \frac{4\pi \times 10^{-7} \times 5600 \times 10^{-6}}{2\pi}\left[\ln\left(\frac{2 \times 5600}{30 + 20}\right) + \frac{(30 + 20)}{3 \times 5600} + 0.50049\right] \\ &= 6.608\,\text{nH}. \end{aligned} \tag{10.103}$$

Rosa's Formula

The equation for the inductance of square planar spiral inductors is expressed as [1, 8, 32]

$$L = 0.3175\mu N^2 (D + d)\left\{\ln\left[\frac{2.07(D + d)}{D - d}\right] + \frac{0.18(D - d)}{D + d} + 0.13\left(\frac{D - d}{D + d}\right)^2\right\} \text{ (H)}. \tag{10.104}$$

All dimensions are in m.

Example 10.12

Calculate the inductance of a square planar spiral inductor with $N = 5$, $d = 60\,\mu\text{m}$, $w = 30\,\mu\text{m}$, $s = 20\,\mu\text{m}$, $h = 20\,\mu\text{m}$, and $\mu_r = 1$ for $\delta > h$. Use Rosa's formula.

Solution. The outer diameter is

$$D = d + 2Nw + 2(N - 1)s = 60 + 2 \times 5 \times 30 + 2 \times (5 - 1) \times 20 = 520\,\mu\text{m}. \tag{10.105}$$

The inductance of the square spiral planar inductor for $\delta < h$ is

$$L = 0.3175\mu_0 N^2 (D + d) \left\{ \ln\left[\frac{2.07(D + d)}{D - d}\right] + \frac{0.18(D - d)}{D + d} + 0.13\left(\frac{D - d}{D + d}\right)^2 \right\}$$
$$= 0.3175 \times 4\pi \times 10^{-7} \times 5^2 (520 + 60) \times 10^{-6}$$
$$\times \left\{ \ln\left[\frac{2.07(520 + 60)}{520 - 60}\right] + \frac{0.18(520 - 60)}{520 + 60} + 0.13\left(\frac{520 - 60}{520 + 60}\right)^2 \right\}$$
$$= 6.849\,\text{nH}. \quad (10.106)$$

Cranin's Formula

An empirical expression for the inductance of spiral planar inductors that has less than 10 % error for inductors in the range from 5 to 50 nH is given by [21]

$$L \approx 1.3 \times 10^{-7} \frac{A_m^{5/3}}{A_{tot}^{1/6} w^{1.75} (w + s)^{0.25}} \text{ (nH)} \quad (10.107)$$

where A_m is the metal area, A_{tot} is the total inductor area, w is the metal trace width, and s is the spacing between metal traces. All dimensions are in m.

Example 10.13

Calculate the inductance of a square planar spiral inductor with $N = 5$, $d = 60\,\mu\text{m}$, $w = 30\,\mu\text{m}$, $s = 20\,\mu\text{m}$, $h = 20\,\mu\text{m}$, and $\mu_r = 1$ for $\delta > h$. Use Cranin's formula.

Solution. The outer diameter is

$$D = d + 2Nw + 2(N - 1)s = 60 + 2 \times 5 \times 30 + 2 \times (5 - 1) \times 20 = 520\,\mu\text{m}. \quad (10.108)$$

The length of the metal trace is

$$l = 2(D + w) + 2N(2N - 1)(w + s) = 2(520 + 30) + 2 \times 5(2 \times 5 - 1)(30 + 20)$$
$$= 5600\,\mu\text{m}. \quad (10.109)$$

Hence, the total area of the inductor is

$$A_{tot} = D^2 = (520 \times 10^{-6})^2 = 0.2704 \times 10^{-6}\,\text{m}^2. \quad (10.110)$$

The metal area is

$$A_m = wl = 30 \times 10^{-6} \times 5600 \times 10^{-6} = 0.168 \times 10^{-6}\,\text{m}^2 \quad (10.111)$$

The inductance of the square spiral planar inductor for $\delta < h$ is

$$L \approx 1.3 \times 10^{-7} \frac{A_m^{5/3}}{A_{tot}^{1/6} w^{1.75} (w + s)^{0.25}} \text{ (H)}$$
$$= 1.3 \times 10^{-7} \frac{(0.168 \times 10^{-6})^{5/3}}{(0.2704 \times 10^{-6})^{1/6} [30 \times 10^{-6}]^{1.75} [(30 + 20) \times 10^{-6}]^{0.25}}$$
$$= 8.0864\,\text{nH}. \quad (10.112)$$

Monomial Formula

The data fitted monomial empirical expression for the inductance of the square spiral inductor is [32]

$$L = 0.00162 D^{-1.21} w^{-0.147} \left(\frac{D+d}{2}\right)^{2.4} N^{1.78} s^{-0.03} \text{ (nH)}. \tag{10.113}$$

All dimensions are in μm.

Example 10.14

Calculate the inductance of a square planar spiral inductor with $N = 5$, $d = 60\,\mu\text{m}$, $w = 30\,\mu\text{m}$, $s = 20\,\mu\text{m}$, $h = 20\,\mu\text{m}$, and $\mu_r = 1$ for $\delta > h$. Use the monomial formula.

Solution. The outer diameter is

$$D = d + 2Nw + 2(N-1)s = 60 + 2 \times 5 \times 30 + 2 \times (5-1) \times 20 = 520\,\mu\text{m}. \tag{10.114}$$

The inductance of the square spiral planar inductor for $\delta < a$ is

$$L = 0.00162 D^{-1.21} w^{-0.147} \left(\frac{D+d}{2}\right)^{2.4} N^{1.78} s^{-0.03} \text{ (nH)}$$

$$= 0.00162 \times (520)^{-1.21} w^{-0.147} \left(\frac{520+60}{2}\right)^{2.4} 5^{1.78} 20^{-0.03} = 6.6207\,\text{nH}. \tag{10.115}$$

Jenei's Formula

The total inductance consists of the self-inductance L_{self} and the sum of the positive mutual inductances M^+ and the sum of the negative mutual inductances M^-. The derivation of an expression for the total inductance is given in [34]. The self-inductance of one straight segment is [7]

$$L_{self1} = \frac{\mu_0 l_{seg}}{2\pi} \left[\ln\left(\frac{2l_{seg}}{w+h}\right) + 0.5\right] \tag{10.116}$$

where l_{seg} is the segment length, w is the metal trace width, h is the metal trace thickness. The total length of the conductor is

$$l = (4N+1)d + (4N_i + 1)N_i(w+s) \tag{10.117}$$

where N is the number of turns, N_i is the integer part of N, and s is the spacing between the segments. The total self-inductance of a square planar inductor is equal to the sum of $4N$ self-inductances of all the segments.

$$L_{self} = 4NL_{self1} = \frac{\mu_0 l}{2\pi} \left[\ln\left(\frac{2l}{N(w+h)}\right) - 0.2\right]. \tag{10.118}$$

The antiparallel segments contribute to negative mutual inductance M^-. The sum of all interactions is approximately equal to $4N^2$ average interactions between segments of an average length and an average distance. The negative mutual inductance is

$$M^- = \frac{0.47\mu_0 Nl}{2\pi}. \tag{10.119}$$

The positive mutual inductance is caused by interactions between parallel segments on the same side of a square. The average distance is

$$b = (w + s)\frac{(3N - 2N_i - 1)(N_i + 1)}{3(2N - N_i - 1)}. \tag{10.120}$$

For $N_i = N$,

$$b = \frac{(w + s)(N + 1)}{3}. \tag{10.121}$$

The total positive mutual inductance is

$$M^+ = \frac{\mu_0 l(N - 1)}{2\pi}\left\{\ln\left[\sqrt{1 + \left(\frac{l}{4Nb}\right)^2} + \frac{l}{4Nb}\right] - \sqrt{1 + \left(\frac{4Nb}{l}\right)^2} + \frac{4Nb}{l}\right\}. \tag{10.122}$$

The inductance of a square planar inductors is

$$\begin{aligned} L &= L_{self} + M^- + M^+ \\ &= \frac{\mu_0 l}{2\pi}\left\langle \ln\left[\frac{l}{N(w + h)}\right] - 0.2 - 0.47N \right. \\ &\left. + (N - 1)\left\{\ln\left[\sqrt{1 + \left(\frac{l}{4Nb}\right)^2} + \frac{l}{4Nb}\right] - \sqrt{1 + \left(\frac{4Nb}{l}\right)^2} + \frac{4Nb}{l}\right\}\right\rangle. \end{aligned} \tag{10.123}$$

All dimensions are in m.

Example 10.15

Calculate the inductance of a square planar spiral inductor with $N = N_i = 5$, $d = 60\,\mu\text{m}$, $w = 30\,\mu\text{m}$, $s = 20\,\mu\text{m}$, $h = 20\,\mu\text{m}$, and $\mu_r = 1$ for $\delta > h$. Use Jenei's formula.

Solution. The total length of the conductor is

$$\begin{aligned} l &= (4N + 1)d + (4N_i + 1)N_i(w + s) \\ &= (4 \times 5 + 1) \times 60 \times 10^{-6} + (4 \times 5 + 1)5(30 + 20) \times 10^{-6} \\ &= 6500\,\mu\text{m}. \end{aligned} \tag{10.124}$$

The average distance is

$$\begin{aligned} b &= (w + s)\frac{(3N - 2N_i - 1)(N_i + 1)}{3(2N - N_i - 1)} \\ &= (30 + 20) \times 10^{-6}\frac{(3 \times 5 - 2 \times 5 - 1)(5 + 1)}{3(2 \times 5 - 5 - 1)} = 100\,\mu\text{m}. \end{aligned} \tag{10.125}$$

The inductance of the square spiral planar inductor for $\delta < h$ is

$$\begin{aligned} L = \frac{\mu_0 l}{2\pi}&\left\langle \ln\left[\frac{l}{N(w + h)}\right] - 0.2 - 0.47N \right. \\ &\left. + (N - 1)\left\{\ln\left[\sqrt{1 + \left(\frac{l}{4Nb}\right)^2} + \frac{l}{4Nb}\right] - \sqrt{1 + \left(\frac{4Nb}{l}\right)^2} + \frac{4Nb}{l}\right\}\right\rangle \end{aligned}$$

$$= \frac{4\pi \times 10^{-7} \times 6500 \times 10^{-6}}{2\pi} \left\langle \ln\left[\frac{6500}{5(30+20)}\right] - 0.2 - 0.47 \times 5 + (5-1) \right.$$
$$\times \left\{ \ln\left[\sqrt{1+\left(\frac{6500}{4 \times 5 \times 100}\right)^2} + \frac{6500}{4 \times 5 \times 100}\right] \right.$$
$$\left. \left. - \sqrt{1+\left(\frac{4 \times 5 \times 100}{6500}\right)^2} + \frac{4 \times 5 \times 100}{6500} \right\} \right\rangle$$
$$= 6.95\,\text{nH}. \tag{10.126}$$

Dill's Formula

The inductance of a square planar inductor is [9]

$$L = 8.5 \times 10^{-10} N^{5/3} \text{ (H)}. \tag{10.127}$$

For $N = 5$, $L = 12.427\,\text{nH}$.

Terman's Formula

The inductance of a square planar spiral single-turn inductor is

$$L = 18.4173 \times 10^{-4} D \left[\log\left(\frac{0.7874 \times 10^{-4} D^2}{w+h}\right) - \log(0.95048 \times 10^{-4} D)\right]$$
$$+ 10^{-4} \times [7.3137D + 1.788(w+h)] \text{ (nH)}. \tag{10.128}$$

All dimensions are in μm.

The inductance of a square planar spiral multiple-turn inductor is

$$L = 18.4173 \times 10^{-4} D N^2 \left[\log\left(\frac{0.7874 \times 10^{-4} D^2}{w+h}\right) - \log(0.95048 \times 10^{-4} D)\right]$$
$$+ 8 \times 10^{-4} N^2 [0.914D + 0.2235(w+h)] \text{ (nH)}. \tag{10.129}$$

where the outer diameter is

$$D = d + 2Nw + 2(N-1)s. \tag{10.130}$$

All dimensions are in m.

Example 10.16

Calculate the inductance of a square planar spiral inductor with $N = 5$, $D = 520\,\mu\text{m}$, $w = 30\,\mu\text{m}$, $h = 20\,\mu\text{m}$, and $\mu_r = 1$ for $\delta > h$. Use Terman's formula.

Solution. The inductance of a square planar spiral multiple-turn inductor is

$$L = 18.4173 \times 10^{-4} D N^2 \left[\log\left(\frac{0.7874 \times 10^{-4} D^2}{w+h}\right) - \log(0.95048 \times 10^{-4} D)\right]$$
$$+ 8 \times 10^{-4} N^2 [0.914D + 0.2235(w+h)] \text{ (nH)}$$

$$= 18.4173 \times 10^{-4} \times 520 \times 5^2 \left[\log\left(\frac{0.7874 \times 10^{-4} \times 520^2}{30 + 20}\right)\right.$$
$$\left. - \log(0.95048 \times 10^{-4} \times 520)\right]$$
$$+ 8 \times 10^{-4} 5^2 [0.914 \times 520 + 0.2235(30 + 20)]$$
$$= 32.123 \text{ (nH)}. \quad (10.131)$$

Rosa's, Wheeler's, Grover's, Bryan's, Jenei's, and monomial formulas give similar values of inductance. Geenhouse's, Cranin's, Terman's formulas give higher values of inductance than those calculated from the former group of formulas.

10.14.3 Inductance of Hexagonal Spiral Inductors

Wheeler's Formula

The inductance of a hexagonal spiral inductor is given by modified Wheeler's formula [3, 32]

$$L = 1.165\mu_0 N^2 \frac{D + d}{1 + 3.82\frac{D - d}{D + d}} \text{ (H)} \quad (10.132)$$

where

$$D = d + 2Nw + 2(N - 1)s. \quad (10.133)$$

All dimensions are in m.

Rosa's Formula

The equation for the inductance of hexagonal planar spiral inductors is expressed as [1, 8, 32]

$$L = 0.2725\mu N^2 (D + d) \left\{ \ln\left[\frac{2.23(D + d)}{D - d}\right] + 0.17\left(\frac{D - d}{D + d}\right)^2 \right\} \text{ (H)}. \quad (10.134)$$

All dimensions are in m.

Example 10.17

Calculate the inductance of a hexagonal planar spiral inductor with $N = 5$, $d = 60\,\mu\text{m}$, $w = 30\,\mu\text{m}$, $s = 20\,\mu\text{m}$, $h = 20\,\mu\text{m}$, and $\mu_r = 1$ for $\delta > h$. Use Rosa's formula.

Solution. The outer diameter is

$$D = d + 2Nw + 2(N - 1)s = 60 + 2 \times 5 \times 30 + 2 \times (5 - 1) \times 20 = 520\,\mu\text{m}. \quad (10.135)$$

The inductance of the square spiral planar inductor for $\delta < h$ is

$$L = 0.2725\mu N^2 (D + d) \left\{ \ln\left[\frac{2.23(D + d)}{D - d}\right] + 0.17\left(\frac{D - d}{D + d}\right)^2 \right\} \text{ (H)}.$$

$$= 0.2725 \times 4\pi \times 10^{-7} \times 5^2 (520 + 60) \times 10^{-6}$$
$$\times \left\{ \ln \left[\frac{2.23(520 + 60)}{520 - 60} \right] + 0.17 \left(\frac{520 - 60}{520 + 60} \right)^2 \right\}$$
$$= 5.6641\,\text{nH}. \tag{10.136}$$

Grover's Formula

The inductance of a hexagonal planar inductor is [7]

$$L = \frac{2\mu l}{\pi} \left[\left(\frac{l}{6r} \right) + 0.09848 \right] \text{ (H)}. \tag{10.137}$$

All dimensions are in μm.

Monomial Formula

The data fitted monomial empirical expression for the inductance of the hexagonal spiral inductor is [32]

$$L = 0.00128 D^{-1.24} w^{-0.174} \left(\frac{D + d}{2} \right)^{2.47} N^{1.77} s^{-0.049} \text{ (nH)}. \tag{10.138}$$

All dimensions are in μm.

10.14.4 Inductance of Octagonal Spiral Inductors

Wheeler's Formula

The inductance of an octagonal spiral inductor is given by a modified Wheeler's formula [3, 32]

$$L = 1.125 \mu_0 N^2 \frac{D + d}{1 + 3.55 \dfrac{D - d}{D + d}} \text{ (H)} \tag{10.139}$$

where

$$D = d + 2Nw + 2(N - 1)s. \tag{10.140}$$

All dimensions are in m.

Rosa's Formula

. The inductance of an octagonal planar spiral inductors is expressed as [1, 8, 32]

$$L = 0.2675 \mu N^2 (D + d) \left\{ \ln \left[\frac{2.29(D + d)}{D - d} \right] + 0.19 \left(\frac{D - d}{D + d} \right)^2 \right\} \text{ (H)}. \tag{10.141}$$

All dimensions are in m.

Grover's Formula

The inductance of an octagonal planar inductor is [7]

$$L = \frac{2\mu l}{\pi}\left[\left(\frac{l}{8r}\right) - 0.03802\right] \text{ (H)}. \quad (10.142)$$

All dimensions are in m.

Monomial Formula

The data fitted monomial empirical expression for the inductance of an octagonal spiral inductor is [32]

$$L = 0.00132D^{-1.21}w^{-0.163}\left(\frac{D+d}{2}\right)^{2.43}N^{1.75}s^{-0.049} \text{ (nH)}. \quad (10.143)$$

All dimensions are in μm.

The inductance of octagonal inductors is almost the same as that of hexagonal inductors.

10.14.5 Inductance of Circular Spiral Inductors

Rosa's Formula

The inductance of a circular planar spiral inductors is expressed as [1, 8, 32]

$$L = 0.25\mu N^2 (D+d)\left\{\ln\left[\frac{2.46(D+d)}{D-d}\right] + 0.19\left(\frac{D-d}{D+d}\right)^2\right\} \text{ (H)}. \quad (10.144)$$

All dimensions are in m.

Example 10.18

Calculate the inductance of a circular planar spiral inductor with $N = 5$, $d = 60\,\mu\text{m}$, $w = 30\,\mu\text{m}$, $s = 20\,\mu\text{m}$, and $\mu_r = 1$ for $\delta > h$. Use Rosa's formula.

Solution. The outer diameter is

$$D = d + 2Nw + 2(N-1)s = 60 + 2\times 5\times 30 + 2\times(5-1)\times 20 = 520\,\mu\text{m}. \quad (10.145)$$

The inductance of the circular spiral planar inductor for $\delta < a$ is

$$\begin{aligned} L &= 0.25\mu N^2 (D+d)\left\{\ln\left[\frac{2.46(D+d)}{D-d}\right] + 0.19\left(\frac{D-d}{D+d}\right)^2\right\} \text{ (H)} \\ &= 0.25\times 4\pi\times 10^{-7}\times 5^2(520+60)\times 10^{-6} \\ &\quad \times\left\{\ln\left[\frac{2.46(520+60)}{520-60}\right] + 0.19\left(\frac{520-60}{520+60}\right)^2\right\} \\ &= 5.7\,\text{nH}. \end{aligned} \quad (10.146)$$

Wheeler's Formula

The inductance of a circular planar inductor is [2]

$$L = 31.33\mu N^2 \frac{a^2}{8a + 11h} \text{ (H)}. \quad (10.147)$$

All dimensions are in m.

Schieber Formula

The inductance of a circular planar inductor is [15]

$$L = 0.874\pi \times 10^{-5} DN^2 \text{(H)}. \quad (10.148)$$

All dimensions are in m.

Spiral IC inductors have high parasitic resistances and high shunt capacitances, resulting in a low Q_{Lo} and low self-resonant frequencies. It is difficult to achieve $Q_{Lo} > 10$ and f_r greater than a few GHz for planar inductors due to the loss in the substrate and metalization. The RF MEMS technology has the potential to improve the performance of RF IC inductors by removing the substrate under the planar spiral via top-side etching, which decouples the RF IC inductor performance from substrate characteristics.

10.15 Multimetal Spiral Inductors

Multimetal planar spiral inductors (called stacked inductors) are also used to achieve compact high-inductance magnetic devices. A double layer spiral inductor can be implemented using metal 1 and metal 2 layers, as shown in Figure 10.11 [39, 40]. An equivalent circuit of a two-layer inductor is shown in Figure 10.12.

The impedance of a two-layer inductor is given by

$$Z = j\omega(L_1 + L_2 + 2M) = j\omega(L_1 + L_2 + 2k\sqrt{L_1 L_2}) \approx j\omega(L_1 + L_2 + 2\sqrt{L_1 L_2}) \quad (10.149)$$

where the coupling coefficient $k \approx 1$. If both inductors are equal,

$$Z \approx j\omega(L + L + 2L) = j\omega(4L) = j\omega L_s. \quad (10.150)$$

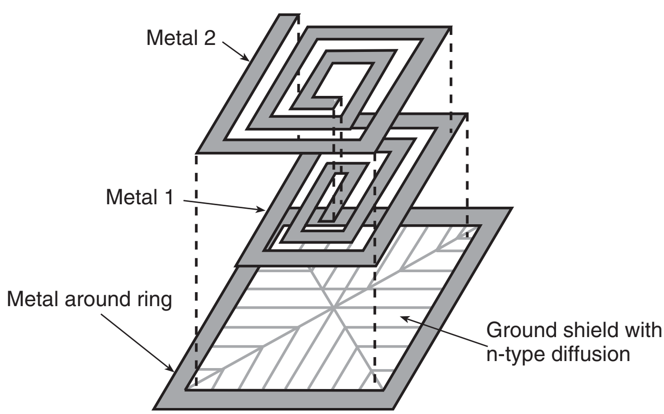

Figure 10.11 Multimetal spiral inductor that uses metal 1 and metal 2.

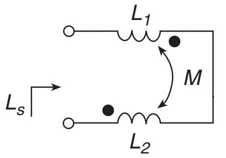

Figure 10.12 Equivalent circuit of two-layer inductor.

Thus, the total inductance L_s for a two-layer inductor increases nearly four times due to mutual inductance. For an m-layer inductor, the total inductance is increased by a factor of m^2 as compared to a self-inductance of a single-layer spiral inductor. Modern CMOS technologies provide more than five metal layers and stocking inductors or transformers gives large inductances in a small chip area.

A patterned ground shield reduces eddy currents. Many RF IC designs incorporate several inductors on the same die. Since these structures are physically large, substrate coupling can cause significant problems, such as feedback and oscillations.

The spiral inductors have several drawbacks. First, the size is large compared with other inductors for the same number of turns N. Second, the spiral inductor requires a lead wire to connect the inside end of the coil to the outside, which introduces a capacitance between the conductor and the lead wire, and this capacitance is the dominant component of the overall stray capacitance. Third, the direction of the magnetic flux is perpendicular to the substrate, which can interfere with the underlying circuit. Fourth, the quality factor is very low. Fifth, the self-resonant frequency is low.

10.16 Planar Transformers

Monolithic transformers are required in many RF designs. The principle of planar transformers relies on lateral magnetic coupling. They can be used as narrowband or wideband transformers. An interleaved planar spiral transformer is shown in Figure 10.13. The coupling coefficient of theses transformers is $k \approx 0.7$. Figure 10.14 shows a planar spiral transformer, in which the turns ratio $n = N_p : N_s$ is not equal to 1. The coupling coefficient k is low in this transformer, typically $k = 0.4$. Stacked transformers use multiple metal layers.

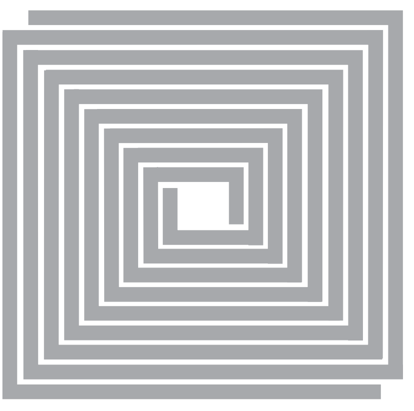

Figure 10.13 Interleaved planar spiral transformer.

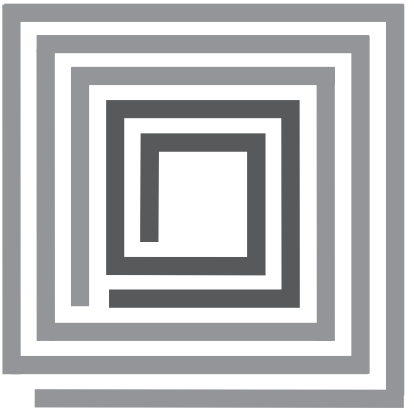

Figure 10.14 Planar spiral transformer in which the turns ratio $n = N_p : N_s$ is not equal to 1.

10.17 MEMS Inductors

Microelectromechanical system (MEMS) technology can improve the performance of integrated inductors MEMS inductors usually are solenoids fabricated using surface micromachining techniques and polymer/metal multilayer process techniques. Figure 10.15 show an integrated solenoid inductor [29]. These inductors may have air cores or electroplated Ni-Fe Permalloy cores. The winding is made up of electroplated copper layers. There is an air gap between the metal winding and the substrate. This geometry gives small inductor size, low stray capacitance C_S, high self-resonant frequency f_r, low power loss, and high quality factor. However, it requires a 2D design approach.

The magnetic field intensity inside the solenoid is uniform and given by

$$B = \frac{\mu NI}{l_c} \tag{10.151}$$

where l_c is the length of the core and N is the number of turns. The magnetic flux is

$$\phi = A_c B = \frac{\mu NIA_c}{l_c} \tag{10.152}$$

where A_c is the cross-sectional area of the core. The magnetic linkage is

$$\lambda = N\phi = \frac{\mu N^2 IA_c}{l_c}. \tag{10.153}$$

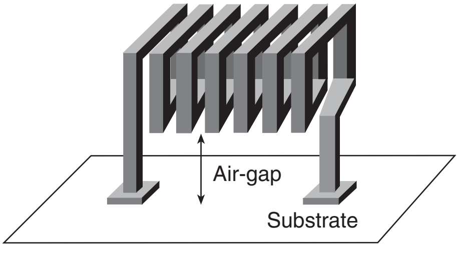

Figure 10.15 Integrated MEMS solenoid inductor [29].

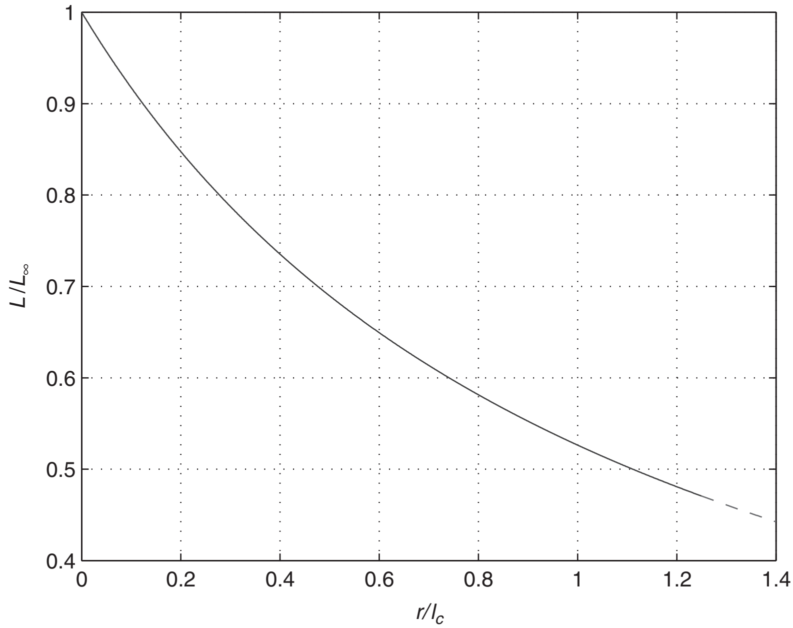

Figure 10.16 Ratio L/L_∞ versus r/l_c.

The inductance of the infinitely long MEMS solenoid inductor is given by

$$L = \frac{\lambda}{I} = \frac{\mu N^2 A_c}{l_c}. \tag{10.154}$$

The inductance of a short solenoid is lower that of an infinitely long one. The inductance of a round solenoid of radius r and length l_c can be determined from Wheeler's equation [2]

$$L = \frac{L_\infty}{1 + 0.9\dfrac{r}{l_c}} = \frac{\mu A_c N^2}{l_c\left(1 + 0.9\dfrac{r}{l_c}\right)}. \tag{10.155}$$

For a square solenoid, we can use an equivalent cross-sectional area of the solenoid

$$A = h^2 = \pi r^2 \tag{10.156}$$

yielding

$$r = \frac{h}{\sqrt{\pi}}. \tag{10.157}$$

Figure 10.16 show a plot of L/L_∞ as a function of r/l_c.

Example 10.19

Calculate the inductance of a MEMS solenoid made of $N = 10$ turns of square wire with thickness $h = 8\,\mu$m and space between the turns $s = 8\,\mu$m. The solenoid has a square shape with width $w = 50\,\mu$m.

Solution. The cross-sectional area of the MEMS solenoid is

$$A_c = w^2 = (50 \times 10^{-6})^2 = 25 \times 10^{-10}\ \mathrm{m}^2. \tag{10.158}$$

The length of the solenoid is

$$l_c = (N-1)(h+s) = (10-1)(8+8) \times 10^{-6} = 144 \times 10^{-6}\ \mathrm{m} = 144\,\mu\mathrm{m}. \tag{10.159}$$

The inductance of the solenoid is

$$L = \frac{\mu N^2 A_c}{l_c} = \frac{4\pi \times 10^{-7} \times 10^2 \times 25 \times 10^{-10}}{144 \times 10^{-6}} = 2.182\,\mathrm{nH}. \tag{10.160}$$

Brooks Inductor

The maximum inductance with a given length of wire is obtained for $a/c = 3/2$ and $b = c$, where a is the coil mean radius, b is the coil axial thickness, and c is the coil radial thickness. The inductance in this case is [16, 17]

$$L = 1.353\mu a N^2. \tag{10.161}$$

10.18 Inductance of Coaxial Cable

The inductance of a coaxial cable is

$$L = \frac{\mu l}{2\pi} \ln\left(\frac{b}{a}\right) \tag{10.162}$$

where a is the inner radius of the dielectric, b is the outer radius of the dielectric, and l is the length of the coaxial cable.

10.19 Inductance of Two-wire Transmission Line

The inductance of a two-parallel-wire transmission line is

$$L = \frac{\mu l}{\pi}\left[\ln\left(\frac{d}{2a}\right) + \sqrt{\left(\frac{d}{2a}\right)^2 - 1}\,\right] \tag{10.163}$$

where a is the of the wires and d is the distance between the centers of the wires, and l is the length of the transmission line.

10.20 Eddy Currents in Integrated Inductors

The performance degradation of planar spiral inductors is caused by the following effects:

(1) trace resistance due to its conductivity ($\sigma < \infty$);

(2) eddy-current losses due to magnetic field penetration into the substrate and adjacent traces;

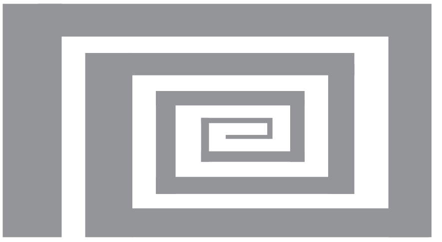

Figure 10.17 RF IC inductor with a variable trace width.

(3) substrate ohmic losses;

(4) capacitance formed by metal trace, silicon oxide, and substrate.

When a time-varying voltage is applied between the terminals of the inductor, a time-varying current flowing along the inductor conductor induces a magnetic field. The magnetic field induces eddy current in the substrate.

A broken guard ring made up of n^+ or p^+ diffusion regions surrounding the coil and connected to ground is often used. The purpose of the guard ring is to provide a low-impedance path to ground for currents induced in the substrate, reducing the substrate loss and increasing the inductor quality factor.

Another method for reducing the loss in the substrate is to insert a high-conductivity ground shield between the spiral and the substrate. The ground shield is patterned with slots to reduce the paths for eddy currents.

If an n-well is fabricated beneath the oxide, the p-substrate and the n-well form a pn junction. If this junction is reverse biased, it represent a capacitance C_J. The oxide capacitance C_{ox} and the junction capacitance C_J are connected in series, reducing the total capacitance. Therefore, the current flowing from the metal trace to the substrate is reduced, reducing substrate loss and improving the quality factor.

Various structures may be used to block eddy currents. Eddy currents flow in paths around the axis of the spiral. This can be accomplished by inserting narrow strips of n^+ regions perpendicular to the eddy current flow to create blocking pn junctions.

The magnetic field reaches the maximum value near the axis of the spiral inductor. The current density is higher in the outer turns than that in ihe inner turns. Therefore, the series resistance and the power loss can be reduced if the outer turns are wider than the inner turns, as illustrated in Figure 10.17. Also, the spacing between the turns can be increased the center of the spiral to the outer turns.

10.21 Model of RF Integrated Inductors

A two-port π-model of RF IC planar spiral inductors is shown in Figure 10.18 [25]. The components L_s, R_s, and C_s represent the inductance, resistance, and capacitance of the metal trace. The capacitance C_{ox} represents the metal-oxide-substrate capacitance. The components R_{si} and C_{Si} represent the resistance and capacitance of the substrate. The inductance L is given by one of the equation given in Section 10.14. The ac resistance is given by (10.17).

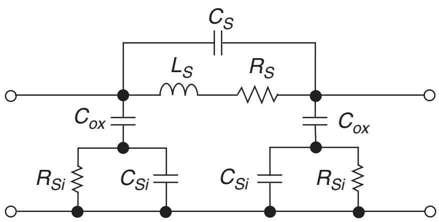

Figure 10.18 Lumped-parameter physical model of an RF IC spiral inductors.

The shunt stray capacitance is formed by the overlap of the underpass and the spiral inductor turns. It is given by

$$C_S = \epsilon_{ox} \frac{Nw^2}{t_{ox}}. \tag{10.164}$$

The turn-to-turn capacitances C_{tt} are small even for very small spacing s between the turns because the thickness of the metal trace h is small, resulting in a small vertical area. The lateral turn-to-turn capacitances are connected in series and therefore the equivalent interturn capacitance C_{tt}/N is small. The adjacent turns sustain a very small voltage difference. The electric energy stored in turn-to-turn capacitances is proportional to the square of voltage. Also, the displacement current through the turn-to-turn capacitance $i_C = C_{tt}\, \mathrm{d}v/\mathrm{d}t \approx C_{tt} \Delta v/\delta t$ is very low. Therefore, the effect of the turn-to-turn capacitances is negligible. The effect of the overlap capacitance is more significant because the potential difference between the underpass and the spiral turns is higher. The stray capacitance allows the current to flow directly from the input port to the output port without passing through the inductance. The current can also flow between one terminal and the substrate through capacitances C_{ox} and C_{Si}.

The capacitance between the metal trace and the silicon substrate is

$$C_{ox} = \epsilon_{ox} \frac{A}{2t_{ox}} = wl \frac{\epsilon_{ox}}{2t_{ox}} \tag{10.165}$$

where w is the trace width, l is the trace length, $A = wl$ is the area of the metal trace, t_{ox} is the thickness of the silicon dioxide SiO_2, and $\epsilon_{ox} = 3.9\epsilon_0$ is the permittivity of the silicon dioxide. The typical value of t_{ox} is 1.8×10^{-8} m.

The capacitance of the substrate is

$$C_{Si} \approx \frac{wlC_{sub}}{2} \tag{10.166}$$

where $C_{sub} = 10^{-3}$ to 10^{-2} fF/μm^2.

The resistance representing the substrate dielectric loss is given by

$$R_{Si} = \frac{2}{wlG_{sub}} \tag{10.167}$$

where G_{sub} is the substrate conductance per unit area, typically $G_{sub} = 10^{-7}$ S/μm^2.

10.22 PCB Integrated Inductors

A printed circuit board (PCB) inductor is shown in Figure 10.19. The types of inductors that can be used for printed circuit boards technology are

- coreless inductors;

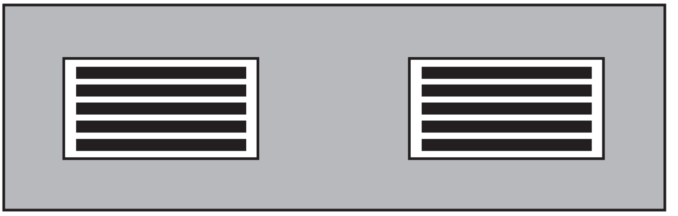

Figure 10.19 PCB inductor

- inductors with planar windings and magnetic plates;
- closed-core structure.

The simplest structure of a PCB inductor is a coreless winding. The maximum inductance is achieved by fitting the largest number of winding turns in each of the PCB layers. Typically, there are six layers in a PCB. The maximum power capability of the coreless PCB inductor is limitted by the maximum temperature rise.

The magnetic plates on either side of the planar winding structure increase the inductance. The inductance also increases when the plate thickness increases. PCB inductors with magnetic plates offer better EMI performance than coreless inductors.

The closed-core structure of a PCB inductor consists of a core area provided by magnetic holes. The inductance increases with the inner core radius and the number of winding turns.

The advantages of planar inductors are:

- low profile;
- high power density;
- excellent repeatability;
- low leakage inductance;
- high magnetic coupling;
- reduced winding ac resistance and power losses.

10.23 Summary

- Inductors are key components of radio transmitters and other wireless communication circuits.
- The family of integrated inductors includes planar inductors, meander inductors, bondwire inductors, and MEMS inductors.
- Planar inductors are compatible with integrated circuit technology.
- planar inductors are compatible with integrated circuit technology.
- The range of integrated inductances is from 0.1 to 20 nH.
- Integrated inductors require a large amount of chip area.
- Intergated inductors have a very low quality factor.

- Planar spiral RF IC inductors are the most widely used inductors.
- Meander inductors require only one metal layer.
- Meander inductors have a low inductance-to-surface area ratio.
- Planar inductors usually require two metal layers.
- Planar spiral inductors have a high inductance per unit area.
- The direction of magnetic flux is perpendicular to the substrate and therefore can interfere with the circuit.
- In square planar inductors, the total self-inductance is equal to the sum of self-inductances of all straight segments.
- In square planar inductors, the the mutual inductance is only present between parallel segments.
- The mutual inductance between parallel segments is positive if the currents in parallel conductors flow in the same direction.
- The mutual inductance between parallel segments is negative if the currents in parallel conductors flow in the opposite directions.
- The inductance of planar inductors is approximately proportional to the trace length l.
- An increase in the conductor width w and spacing s reduces the inductance and the resistance.
- The conductor thickness does not affect the inductance value and significantly reduces the resistance.
- The magnetic flux of RF IC inductors penetrates the substrate, where it induces high-loss eddy currents.
- Patterned ground shield can be used to reduce eddy current loss.
- MEMS solenoid inductors are more complex to fabricate than the planar spiral inductors.
- MEMS inductors have a higher quality factor than the planar integrated inductors.
- The predictability of bondwire inductors is low.
- The parasitic capacitance of integrated planar inductors can be minimized by implementing the spiral with the topmost metal layer, increasing the self-resonant frequency.

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10.25 Review Questions

10.1 List the types of integrated inductors.

10.2 What is the range of integrated inductances?

10.3 Is it easy to achieve good performance of integrated inductors?

10.4 What kind of integrated inductors are the most widely used?

10.5 What is the main disadvantage of planar integrated inductors?

10.6 How many metal layers are usually used in RF IC planar inductors?

10.7 What is underpass in spiral planar inductors?

10.8 What is air bridge in integrated inductors?

10.9 When is the mutual inductance between two conductors zero?

10.10 When is the mutual inductance between two conductors positive and when is it negative?

10.11 Is the quality factor high for integrated inductors?

10.12 What is the self-resonant frequency of an inductor?

10.13 What is the main disadvantage of bondwire inductors?

10.14 What is the advantages and disadvantage of planar inductors?

10.15 What are the advantages and disadvantage of MEMS inductors?

10.16 How can eddy current losses be reduced?

10.17 What are the components of the model of planar spiral inductors?

10.26 Problems

10.1 Calculate the skin depth at $f = 1\,\text{GHz}$ for (a) copper, (b) aluminum, (c) silver, and (d) gold.

10.2 Calculate the dc resistance of a copper trace with $l = 50\,\mu\text{m}$, $w = 1\,\mu\text{m}$, and $h = 1\,\mu\text{m}$. Find the ac resistance of the trace at 10 GHz. Calculate F_R.

10.3 Calculate the inductance of a straight trace with $l = 100\,\mu\text{m}$, $w = 1\,\mu\text{m}$, $h = 1\,\mu\text{m}$, and $\mu_r = 1$.

10.4 Calculate the inductance of a meander inductor with $N = 10$, $w = 40\,\mu\text{m}$, $s = 40\,\mu\text{m}$, $a = 40\,\mu\text{m}$, $h = 100\,\mu\text{m}$, and $\mu_r = 1$.

10.5 Calculate the inductance of a round straight inductor with $l = 2\,\text{mm}$ and $\mu_r = 1$ for $\delta \gg h$. Find the inductance of the above copper round straight conductor for $a = 20\,\mu\text{m}$ at $f = 2.4\,\text{GHz}$.

10.6 Calculate the inductance of a bondwire with $l = 1\,\text{mm}$ and $a = 1\,\mu\text{m}$.

10.7 Find the inductance of the spiral planar inductor with $N = 10$ and $r = 100\,\mu\text{m}$.

10.8 Calculate the inductance of a square planar spiral inductor with $N = 10$, $s = 20\,\mu\text{m}$, $w = 30\,\mu\text{m}$, and $d = 40\,\mu\text{m}$. Use Bryan's formula.

10.9 Calculate the square planar spiral inductance with $N = 10$, $s = 20\,\mu\text{m}$, $w = 30\,\mu\text{m}$, and $d = 40\,\mu\text{m}$. Use Wheeler's formula.

10.10 Calculate the square planar spiral inductance with $N = 15$, $s = 20\,\mu\text{m}$, $w = 30\,\mu\text{m}$, $h = 20\,\mu\text{m}$, and $d = 40\,\mu\text{m}$. Use Greenhouse's formula.

10.11 Calculate the square planar spiral inductance with $N = 10$, $s = 20\,\mu\text{m}$, $w = 30\,\mu\text{m}$, $h = 20\,\mu\text{m}$, and $d = 40\,\mu\text{m}$. Use Rosa's formula.

10.12 Calculate the square planar spiral inductance with $N = 10$, $s = 20\,\mu\text{m}$, $w = 30\,\mu\text{m}$, $h = 20\,\mu\text{m}$, and $d = 40\,\mu\text{m}$. Use Cranin's formula.

10.13 Calculate the square planar spiral inductance with $N = 10$, $s = 20\,\mu\text{m}$, $w = 30\,\mu\text{m}$, $h = 20\,\mu\text{m}$, and $d = 40\,\mu\text{m}$. Use the monomial formula.

10.14 Calculate the square planar spiral inductance with $N = N_i = 10$, $s = 20\,\mu\text{m}$, $w = 30\,\mu\text{m}$, $h = 20\,\mu\text{m}$, and $d = 40\,\mu\text{m}$. Use Jenei's formula.

10.15 Calculate the square planar spiral inductance with $N = 10$, $w = 30\,\mu\text{m}$, $h = 20\,\mu\text{m}$, and $D = 1000\,\mu\text{m}$. Use Terman's formula.

10.16 Calculate the hexagonal planar spiral inductance with $N = 10$, $s = 20\,\mu\text{m}$, $w = 30\,\mu\text{m}$, $h = 20\,\mu\text{m}$, and $d = 40\,\mu\text{m}$. Use Rosa's formula.

10.17 Calculate the octagonal planar spiral inductance with $N = 5$, $s = 20\,\mu\text{m}$, $w = 30\,\mu\text{m}$, $h = 20\,\mu\text{m}$, and $d = 60\,\mu\text{m}$. Use Rosa's formula.

10.18 Calculate the circular planar spiral inductance with $N = 15$, $d = 40\,\mu\text{m}$, $w = 30\,\mu\text{m}$, and $s = 20\,\mu\text{m}$. Use Wheeler's formula. Use Wheeler's formula.

10.19 Calculate the inductance of a MEMS solenoid with $N = 20$. The thickness of the square wire is $h = 10\,\mu\text{m}$ and the separation between the turns is $s = 10\,\mu\text{m}$. The solenoid has a square shape with $w = 100\,\mu\text{m}$.

Appendices

A

SPICE Model of Power MOSFETs

Figure A.1 shows a SPICE large-signal model for n-channel enhancement MOSFETs. It is a model for integrated MOSFETs, which can be adopted to power MOSFETs. SPICE parameters of the large-signal model of enhancement-type n-channel MOSFETs are given in Table A.1. The diode currents are

$$i_{BD} = IS\left(e^{\frac{v_{BD}}{V_T}} - 1\right) \tag{A.1}$$

and

$$i_{BS} = IS\left(e^{\frac{v_{BS}}{V_T}} - 1\right). \tag{A.2}$$

The junction capacitances in the voltage range close to zero are

$$C_{BD} = \frac{(CJ)(AD)}{\left(1 - \frac{v_{BD}}{PB}\right)^{MJ}} \quad \text{for} \quad v_{BD} \le (FC)(PB) \tag{A.3}$$

and

$$C_{BS} = \frac{(CJ)(AS)}{\left(1 - \frac{v_{BS}}{PB}\right)^{MJ}} \quad \text{for} \quad v_{BS} \le (FC)(PB) \tag{A.4}$$

where CJ is the zero-bias junction capacitance per unit area, AD is the drain area, AS is the source area, PB is the built-in potential, and MJ is the grading coefficient.

The junction capacitances in the voltage range far from zero are

$$C_{BD} = \frac{(CJ)(AD)}{(1 - FC)^{1+MJ}}\left[1 - (1 + MJ)FC + MJ\frac{v_{BD}}{PB}\right] \quad \text{for} \quad v_{BD} \ge (FC)(PB) \tag{A.5}$$

and

$$C_{BS} = \frac{(CJ)(AS)}{(1 - FC)^{1+MJ}}\left[1 - (1 + MJ)FC + MJ\frac{v_{BD}}{PB}\right] \quad \text{for} \quad v_{BS} \ge (FC)(PB). \tag{A.6}$$

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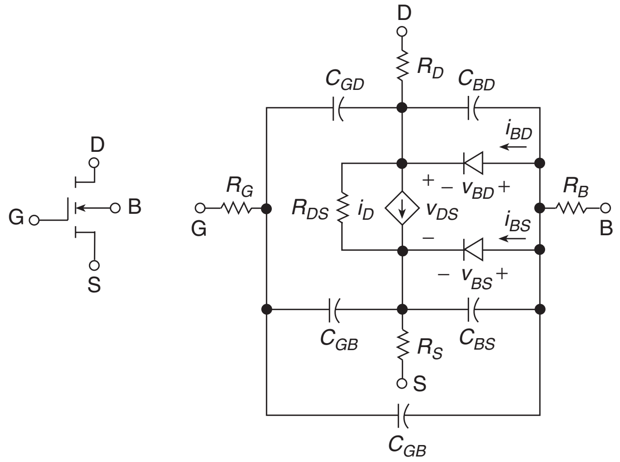

Figure A.1 SPICE large-signal model for n-channel MOSFET.

Table A.1 Selected SPICE Level 1 NMOS large-signal model parameters.

Sym.	SPICE S.	Model parameter	Default value	Typical value
V_{to}	VTO	Zero-bias threshold voltage	0 V	0.3–3 V
μC_{ox}	KP	Process constant	2×10^{-5} A/V^2	20 to 346 μA/V^2
λ	Lambda	Channel-length modulation	0 V^{-1}	0.5–10^{-5} V^{-1}
γ	Gamma	Body-effect V_t parameter	0 V$^{\frac{1}{2}}$	0.35 $V^{\frac{1}{2}}$
$2\phi_F$	PHI	Surface potential	0.6 V	0.7 V
R_D	RD	Drain series resistance	0 Ω	0.2 Ω
R_S	RS	Source series resistance	0 Ω	0.1 Ω
R_G	RG	Gate series resistance	0 Ω	1 Ω
R_B	RB	Body series resistance	0 Ω	1 Ω
R_{DS}	RDS	Drain-source shunt R	∞	1 MΩ
R_{SH}	RSH	Drain-source diffusion sheet R	0	20 Ω/Sq.
I_S	IS	Saturation current	10^{-14} A	10^{-9} A
M_j	MJ	Grading coefficient	0.5	0.36
C_{j0}	CJ	Zero-bias bulk junction C/m^2	0 F/m^2	1 nF/m^2
V_{bi}	PB	Junction potential	1 V	0.72 V
M_{jsw}	MJSW	Grading coefficient	0.333	0.12
C_{j0sw}	CJSW	Zero-bias junc. perimeter C/m	0 F/m	380 pF/m
V_{BSW}	PBSW	Junction sidewell potential	1 V	0.42 V
C_{GDO}	CGDO	Gate-drain overlap C/m	0 F/m	220 pF/m
C_{GSO}	CGSO	Gate-source overlap C/m	0 F/m	220 pF/m
C_{GBO}	CGBO	Gate-bulk overlap C/m	0 F/m	700 pF/m
F_C	FC	Forward-biased C_J coefficient	0.5	0.5
t_{ox}	TOX	Oxide thickness	∞	4.1–100 nm
μ_{ns}	UO	Surface mobility	600 cm^2/Vs	600 cm^2/Vs
n_{sub}	NSUB	Substrate doping	0 cm^{-3}/Vs	0 cm^{-3}/Vs

The typical values:
$C_{ox} = 3.45 \times 10^{-5}$ pF/μm
$t_{ox} = 4.1 \times 10^{-3}$ μm
$\epsilon_{ox(SiO_2)} = 3.9\epsilon_0$
$C_{j0} = 2 \times 10^{-4}$ F/m^2
$C_{jsw} = 10^{-9}$ F/m
$C_{GBO} = 2 \times 10^{-10}$ F/m
$C_{GDO} = C_{GSO} = 4 \times 10^{-11}$ F/m

SPICE NMOS Syntax

Mxxxx D G S B MOS-model-name L = xxx W = yyy
Example:
M1 2 1 0 0 M1-FET L = 0.18um W = 1800um

SPICE NMOS Model Syntax

.model model-name NMOS (parameter=value ...)
Example:
.model M1-FET NMOS (Vto = 1VKp = E-4)

SPICE PMOS Model Syntax

.model model-name PMOS (parameter=value ...)

SPICE Subcircuit Model Syntax

xname N1 N2 N3 model-name
Example:
x1 2 1 0 IRF840
Copy and paste the obtained device model.
.SUBCKT IRF840 1 2 3
and the content of the model.

B

Introduction to SPICE

SPICE is an abbreviation for *Simulation Program for Integrated Circuits Emphasis*. PSPICE is the PC version of SPICE. Analog and digital electronic circuit designs are verified widely by both industry and the academia using PSPICE. It is used to predict the circuit behavior.

Passive Components: Resistors, Capacitors, and Inductors

Rname N+ N- Value [IC = TC1]
Lname N+ N- Value [IC = Initial Voltage Condition]
Cname N+ N- Value [IC = Initial Current Condition]
Examples:
R1 1 2 10K
L2 2 3 2M
C3 3 4 100P

Transformer

Lp Np+ Np- Lpvalue
Ls Ns+ Ns- Lsvalue
Kname Lp Ls K
Example:
Lp 1 0 1mH
Ls 2 4 100uH
Kt Lp Ls 0.999

Temperature

.TEMP list of temperatures
Example:
.TEMP 27 100 150

Independent DC Sources

Vname N+ N- DC Value
Iname N+ N- DC Value
Examples:
Vin 1 0 DC 10V
Is 1 0 DC 2A

DC Sweep Analysis

.DC Vsource-name Vstart Vstop Vstep
Example:
.DC VD 0V 0.75V 1mV

Independent Pulse Source for Transient Analysis

Vname N+ N- PULSE (VL VH td tr tf PW T)
Example:
VGS 1 0 PULSE(0 1E-6 0 1 1 10E-6 100e-6)

Transient Analysis

.TRAN time-step time-stop
Example:
.TRAN 0.1ms 100ms 0ms 0.2ms

Independent AC Sources for Frequency Response

Vname N+ N- AC Vm Phase
Iname N+ N- AC Im Phase
Example:
Vs 2 3 AC 2 30
Is 2 3 AC 0.5 30

Independent Sinusoidal AC Sources for Transient Analysis

Vname N+ N- SIN (Voffset Vm f T-delay Damping-Factor Phase-delay)
Iname N+ N- SIN (Ioffset Im f T-delay Damping-Factor Phase-delay)
Examples:
Vin 1 0 SIN (0 170V 60Hz 0 -120)
Is 1 0 SIN (0 2A 120Hz 0 45)

AC Frequency Analysis

.AC DEC points-per-decade fstart fstop
Example:
.AC DEC 100 20 20kHz

Operating Point

.OP

Getting Started the SPICE Program

1. Open the PSpice A/D Lite window (**Start** > **Programs** > **Orcad9.2 Lite Edition** > **PSpice AD Lite**).
2. Create a new text file (**File** > **New** > **Text File**).
3. Type the example code.
4. Save the file as fn.cir (for example, Lab1.cir), file type: all files, and simulate by pressing the appropriate icon.
5. To include the Spice code of a commercial device model, visit the web site, e.g., http://www.irf.com, http:www.onsemi.com, or http://www.cree.com. For example, for IRF devices, click on (**Design** > **Support** > **Models** > **Spice Library**).

Example Program

```
Diode I-V Characteristics
*Joe Smith
VD 1 0 DC 0.75V
D1N4001 1 0 Power-Diode
.model Power-Diode D (Is=195pA n=1.5)
.DC VD 0V 0.75V 1mV
.TEMP 27C 50C 100C 150C
.probe
.end
```

C

Introduction to MATLAB

MATLAB is an abbreviation for MATrix LABoratory. It is a very powerful mathematical tool used to perform numerical computation using matrices and vectors to obtain two and three-dimensional graphs. MATLAB can also be used to perform complex mathematical analysis.

Getting Started

1. Open MATLAB by clicking **Start** > **Programs** > **MATLAB** > **R2006a** > **MATLAB R2006a**.
2. Open a new M-file by clicking **File** > **New** > **M-File**.
3. Type the code in the M-File.
4. Save the file as fn.m (e.g., Lab1.m).
5. Simulate the code by doing one of the following:

(a) Click on **Debug** > **Run**.
(b) Press F5
(c) On the tool bar, click the icon **Run**.
Use **HELP** by pressing F1.
Use % at the beginning of a line for comments.

1. Generating an x-axis Data

x=Initial-Value: Increment:Final-Value;
Example:
x=1:0.001:5;
or
x=[list of all the values];
Example:
x = [1, 2, 3, 5, 7, 10];

or
x = linspace(start-value, stop-value, number-of-points);
Example:
x = linspace(0, 2*pi, 90);
or
x = logspace(start-power, stop-power, number-of-points);
Example:
x = logspace(1, 5, 1000);

1. Semilogarithemic Scale

semilogx(x-variable, y-variable);
grid on

1. Log-log Scale

loglog(x, y);
grid on

1. Generate an y-axis Data

y = f(x);
Example:
y = cos(x);
z = sin(x);

Multiplication and Division

A dot should be used in front of the operator for matrix multiplication and division.
c = a.*b;
or
c = a./b;

1. Symbols and Units

Math symbols should be in italic. Math signs (like ( ), =, and +) and units should not be in italic. Leave one space between a symbol and a unit.

1. x-axis and y-axis Labels

xlabel('{ \it x} (unit) ')
ylabel('{ \it y} (unit) ')
Example:
xlabel('{\it v_{GS}} (V)')
ylabel('{\it i_{DS} } (A)')
set(gca, 'ylim', [1, 10])
set(gca, 'ytick', [0:2:10])

Greek Symbols

Type: \alpha , \beta , \Omega , \omega , \pi , \phi , \psi , \gamma , \theta , and \circ to obtain: α, β, Ω, ω, π, ϕ, ψ, γ, θ, and $\circ$.

Plot Commands

plot (x, y, '.-', x, z, '- -')

```
set(gca, 'xlim', [x1, x2]);
set(gca, 'ylim', [y1, y2]);
set(gca, 'xtick', [x1:scale-increment:x2]);
text(x, y, '{\it symbol} = 25 V');
```

Examples:

```
set(gca, 'xlim', [4, 10]);
set(gca, 'ylim', [1, 8]);
set(gca, 'xtick', [4:1:10]);
text(x, y, '{\it V} = 25 V');
```

3 D Plot Commands

```
plot3(x1, y1, z1);
```

Example:

```
t = linespace(0, 9*pi);
xlabel('sin(t)')
ylabel('cos(t)')
zlabel('t')
plot(sin(t), cos(t), t)
```

Bode Plots

```
f = logspace(start-power, stop-power, number-of-points)
NumF = [a1 a2 a3];    %Define the numerator of polynomial in s-domain.
DenF = [a1 a2 a3];    %Define the denominator of polynomial in s-domain.
[MagF, PhaseF] = bode(NumF, DenF, (2*pi*f));
figure(1)
semilogx(f, 20*log10(MagF))
F = tf(NumF, DenF)    %Converts the polynomial into transfer function.
[NumF, DenF] = tfdata(F)    %Converter transfer function into polynomial.
```

Step Response

```
NumFS = D*NumF;
t = [0 : 0.000001 : 0.05];
[x, y] = step(NumFS, DenF, t);
figure(2)
plot(t, Initial-Value + y);
```

To Save Figure

Go to File, click Save as, go to EPS file option, type the file name, and click Save.

Example Program

```
clear all
clc
x = linspace(0, 2*pi, 90);
y = sin(x);
z = cos(x);
```

```
grid on
xlabel('{\it x}')
ylabel(' {\it y }, {\it z }')
plot(x , y, '-.', x, z, '- -')
```

Polynomial Curve Fitting

```
x = [00.51.01.52.02.53.0];
y = [101216243037 51];
p = polyfit(x, y, 2)
yc = polyval(p, x);
plot(x, y, 'x', x, yc)
xlabel('x')
ylabel('y'), grid
legend('Actual data', 'Fitted polynomial')
```

Answers to Problems

CHAPTER 1

1.1 $THD = 1\,\%$.

1.2 $h_a = 3.125\,\text{cm}$.

1.3 $P_{AM} = 11.25\,\text{kW}$.

1.4 $m_{max} = 91.67\,\%$.

1.5 (a) $k_{IMP} = 5$. (b) $k_{IMP} = 5$.

1.6 $P_O = 10\,\text{kW}$.

1.7 $m_f = 2$.

1.8 $BW = 90\,\text{kHz}$.

1.9 $BW = 210\,\text{kHz}$.

1.10 $P_{AV} = 100\,\text{W}$.

CHAPTER 2

2.1 (a) $\eta_D = 12.5\,\%$. (b) $\eta_D = 40.5\,\%$.

2.2 $P_I = 10\,\text{W}$.

2.3 $L = 0.0224\,\text{nH}$, $C = 196.2\,\text{pF}$, $L_f = 2.24\,\text{nH}$, $I_m = 0.3846\,\text{A}$, $I_{DM} = 0.7692\,\text{A}$, $V_{DSM} = 2.8\,\text{V}$, $P_{LS} = 0.6545\,\text{W}$, $\eta = 17.375\,\%$, $W/L = 13061$, $W = 4.5713\,\text{mm}$.

2.4 $C_1 = 6.631\,\text{pF}$, $C_2 = 4.872\,\text{pF}$, $L = 1.4117\,\text{nH}$.

CHAPTER 3

3.1 $R = 4.805\,\Omega$, $V_m = 3.1\,\text{V}$, $I_m = 0.66\,\text{A}$, $I_I = 0.42\,\text{A}$, $I_{DM} = 1.32\,\text{A}$, $P_D = 0.386\,\text{W}$, $P_I = 1.386\,\text{W}$, $\eta_D = 72.15\,\%$, $L = 0.03117\,\text{nH}$, and $C = 141\,\text{pF}$.

3.2 $R = 50\,\Omega$, $V_m = 47\,\text{V}$, $I_m = 0.94\,\text{A}$, $I_I = 0.7116\,\text{A}$, $I_{DM} = 1.7527\,\text{A}$, $P_D = 12.1568\,\text{W}$, $P_I = 34.1568\,\text{W}$, $\eta_D = 64.41\,\%$, $L = 0.8841\,\text{nH}$, and $C = 35.36\,\text{pF}$.

3.3 $R = 19.22\,\Omega$, $V_m = 3.1\,\text{V}$, $I_m = 0.161\,\text{A}$, $I_I = 0.08976\,\text{A}$, $I_{DM} = 0.4117\,\text{A}$, $P_D = 0.0462\,\text{W}$, $P_I = 0.2962\,\text{W}$, $\eta_D = 84.4\,\%$, $L = 12.74\,\text{nH}$, and $C = 0.345\,\text{pF}$.

3.4 $R = 10\,\Omega$, $V_m = 11\,\text{V}$, $I_m = 1.1\,\text{A}$, $I_I = 0.5849\,\text{A}$, $I_{DM} = 3.546\,\text{A}$, $P_D = 1.0188\,\text{W}$, $P_I = 7.0188\,\text{W}$, $\eta_D = 85.48\,\%$, $Q_L = 10$, $L = 0.0663\,\text{nH}$, and $C = 66.31\,\text{pF}$.

CHAPTER 4

4.1 $L = 2.215\,\text{nH}$, $C = 11.437\,\text{pF}$, $I_m = 1.057\,\text{A}$, $V_{Cm} = V_{Lm} = 14.71\,\text{V}$, $\eta = 90.7\,\%$.

4.2 $f_0 = 1\,\text{MHz}$, $Z_o = 529.2\,\Omega$, $Q_L = 2.627$, $Q_o = 365$, $Q_{Lo} = 378$, $Q_{Co} = 10584$.

4.3 $Q = 65.2\,\text{VA}$, $P_O = 24.8\,\text{W}$.

4.4 $V_{Cm} = V_{Lm} = 262.7\,\text{V}$, $I_m = 0.4964\,\text{A}$, $Q = 65\,\text{VA}$, and $f_o = 1\,\text{MHz}$.

4.5 $\eta_r = 99.2\,\%$.

4.7 $V_{SM} = 400\,\text{V}$.

4.8 $f_{Cm} = 230.44\,\text{kHz}$, $f_{Lm} = 233.87\,\text{kHz}$.

4.9 $R = 162.9\,\Omega$, $L = 675\,\mu\text{H}$, $C = 938\,\text{pF}$, $I_I = 0.174\,\text{A}$, $I_m = 0.607\,\text{A}$, $f_o = 200\,\text{kHz}$, $V_{Cm} = V_{Lm} = 573\,\text{V}$.

4.10 $P_I = 88.89\,\text{W}$, $R = 82.07\,\Omega$, $L = 145.13\,\mu\text{H}$, $C = 698.14\,\text{pF}$.

CHAPTER 5

5.1 $R = 6.281\,\Omega$, $C_1 = 4.65\,\text{pF}$, $L = 5\,\text{nH}$, $C = 6.586\,\text{pF}$, $L_f = 43.56\,\text{nH}$, $V_{SM} = 11.755\,\text{V}$, $I_{SM} = 0.867\,\text{A}$, $V_{Cm} = 13.629\,\text{V}$, $V_{Lm} = 17.719\,\text{V}$, $V_{Lfm} = 8.455\,\text{V}$, $\eta = 97.09\,\%$.

5.2 $R = 6.281\,\Omega$, $L = 5\,\text{nH}$, $C_1 = 4.65\,\text{pF}/12\,\text{V}$, $C_2 = 20.99\,\text{pF}/10\,\text{V}$, $C_3 = 8.4\,\text{pF}/3\,\text{V}$.

5.3 $R = 10.63\,\Omega$, $I_{SM} = 7.44\,\text{A}$, $V_{SM} = 170.976\,\text{V}$, $L = 4.23\,\mu\text{H}$, $C_1 = 1.375\,\text{nF}$, $L_f = 36.9\,\mu\text{H}$.

5.4 $V_{SM} = 665\,\text{V}$.

5.5 $V_{SM} = 1274.5\text{V}$.

5.7 $f_{max} = 0.95\,\text{MHz}$.

5.8 $R = 8.306\,\Omega$, $C_1 = 1.466\,\text{pF}$, $L = 5.51\,\text{nH}$, $V_{SM} = 11.755\,\text{V}$, $I_{SM} = 0.867\,\text{A}$, $L_f = 24$ nH, $C = 0.9024\,\text{pF}$, $V_{Cm} = 114\,\text{V}$, $V_{Lm} = 128.79\,\text{V}$, and $\eta = 96.15\,\%$.

5.9 $R = 8.306\,\Omega$, $C_1 = 1.466\,\text{pF}$, $L = 5.51\,\text{nH}$, $C_2 = 1.21\,\text{pF}$, $C_3 = 2.97\,\text{pF}$, and $V_{C2m} = 85.07\,\text{V}$.

CHAPTER 6

6.1 $R = 1.3149\,\Omega$, $L_1 = 1.2665\,\text{nH}$, $C = 16.8\,\text{pF}$, $L = 0.862\,\text{nH}$, $V_{SM} = 42.931\,\text{V}$, $I_I = 0.66\,\text{A}$, $I_{SM} = 2.351\,\text{A}$, $I_m = 3.9\,\text{A}$, $R_{DC} = 22.49\,\Omega$, $f_{o1} = 1.32\,\text{GHz}$, $f_{o2} = 805.63$ MHz.

6.2 $V_{SM} = 973.1\,\text{V}$.

6.3 $R = 7.57\,\Omega$, $L = 4.25\,\mu\text{H}$, $L_1 = 32.8\,\mu\text{H}$, $C = 21\,\text{nF}$, $I_{SM} = 4.95\,\text{A}$, $V_{SM} = 515.2\,\text{V}$.

6.4 $R = 9.23\,\Omega$, $V_I = 125.7\,\text{V}$, $V_{SM} = 359.7\,\text{V}$.

6.5 $R = 11.7\,\Omega$, $L_1 = 10.1\,\mu\text{H}$, $C = 3.02\,\text{nF}$, $L = 0.383\,\mu\text{H}$, $V_{SM} = 286.2\,\text{V}$, $I_I = 0.5\,\text{A}$, $I_{SM} = 1.78\,\text{A}$, $I_m = 2.92\,\text{A}$, $R_{DC} = 200\,\Omega$.

CHAPTER 7

7.1 $R = 1.266\,\Omega$, $C_1 = 5\,\text{pF}$, $C = 3.73\,\text{pF}$, $L = 0.504\,\text{nH}$, $f_o = 3.67\,\text{GHz}$, $P_I = 1.0638\,\text{W}$, $I_I = 0.212\,\text{A}$, $V_m = 1.591\,\text{V}$, $I_{Smax} = 1.257\,\text{A}$, $V_{SM} = 5\,\text{V}$, and $V_{Cmax} = 13.41\,\text{V}$.

7.2 $C_s = 10\,\text{pF}$, $C_{s(ext)} = 8\,\text{pF}$.

CHAPTER 8

8.1 $V_m = 133.875\,\text{V}$, $V_{m3} = 14.875\,\text{V}$, $R = 89.613\,\Omega$, $V_{DSmax} = 240\,\text{V}$, $I_m = 1.494\,\text{A}$, $I_{DM} = 2.988\,\text{A}$, $I_I = 0.951\,\text{A}$, $P_I = 114.12\,\text{W}$, $P_D = 14.12\,\text{W}$, $\eta_D = 87.63\,\%$, and $R_{DC} = 125.12\,\Omega$.

8.2 $V_m = 137.409\,\text{V}$, $V_{m3} = 22.902\,\text{V}$, $R = 94.406\,\Omega$, $V_{DSmax} = 240\,\text{V}$, $I_m = 1.4555\,\text{A}$, $I_{DM} = 2.911\,\text{A}$, $I_I = 0.9266\,\text{A}$, $P_I = 111.192\,\text{W}$, $P_D = 11.192\,\text{W}$, $\eta_D = 89.93\,\%$, and $R_{DC} = 128.392\,\Omega$.

8.3 $V_m = 6.26\,\text{V}$, $V_{m2} = 1.565\,\text{V}$, $R = 19.59\,\Omega$, $V_{DSmax} = 13.33\,\text{V}$, $I_m = 0.319\,\text{A}$, $I_{DM} = 0.501\,\text{A}$, $I_I = 0.25\,\text{A}$, $P_I = 1.25\,\text{W}$, $P_D = 0.25\,\text{W}$, $\eta_D = 80\,\%$, and $R_{DC} = 18.7\,\Omega$.

CHAPTER 9

9.1 $A_{p1} = 1 \quad \text{for } 0 < v_s \leq 1\,\text{V}$, $A_{p2} = 3 \quad \text{for } 1\,\text{V} < v_s \leq 2\,\text{V}$,

9.2 $A_{f1} = 9.89$, $A_{f2} = 9.836$, $A_{f3} = 9.6774$.

9.3 $v_o = v_s/\beta + \sqrt{v_o/A}$

9.4 $v_o = 100[v_s - \beta v_o + (v - \beta v_o)^2/2]$.

CHAPTER 10

10.1 $\delta_{Cu} = 2.0897\,\mu\text{m}$, $\delta_{Al} = 2.59\,\mu\text{m}$, $\delta_{Ag} = 2.0069\,\mu\text{m}$, $\delta_{Au} = 2.486\,\mu\text{m}$.

10.2 $R_{dc} = 1.325\,\Omega$, $R_{ac} = 2.29\,\Omega$, $F_R = 1.728$.

10.3 $L = 0.1022$ nH.

10.4 $L = 1.134$ nH.

10.5 $L = 0.1$ nH, $\delta = 1.348$ µm, $L_{HF} = 0.0134$ nH.

10.6 $L = 1.37$ nH.

10.7 $L = 39.47$ nH.

10.8 $D = 1000$ µm, $L = 42.64$ nH.

10.9 $D = 1000$ µm, $L = 43.21$ nH.

10.10 $D = 1500$ µm, $L = 354.8$ nH.

10.11 $D = 1000$ µm, $L = 45$ nH.

10.12 $D = 1000$ µm, $l = 21060$ µm, $L = 59.13$ nH.

10.13 $D = 1000$ µm, $L = 41.855$ nH.

10.14 $l = 22140$ µm, $b = 183.33$ µm, $L = 54.59$ nH.

10.15 $L = 495.679$ nH.

10.16 $D = 1000$ µm, $L = 36.571$ nH.

10.17 $D = 520$ µm, $L = 5.75$ nH.

10.18 $D = 1000$ µm, $L = 122.38$ nH.

10.19 $l_c = 380$ µm, $L = 13.228$ nH.

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