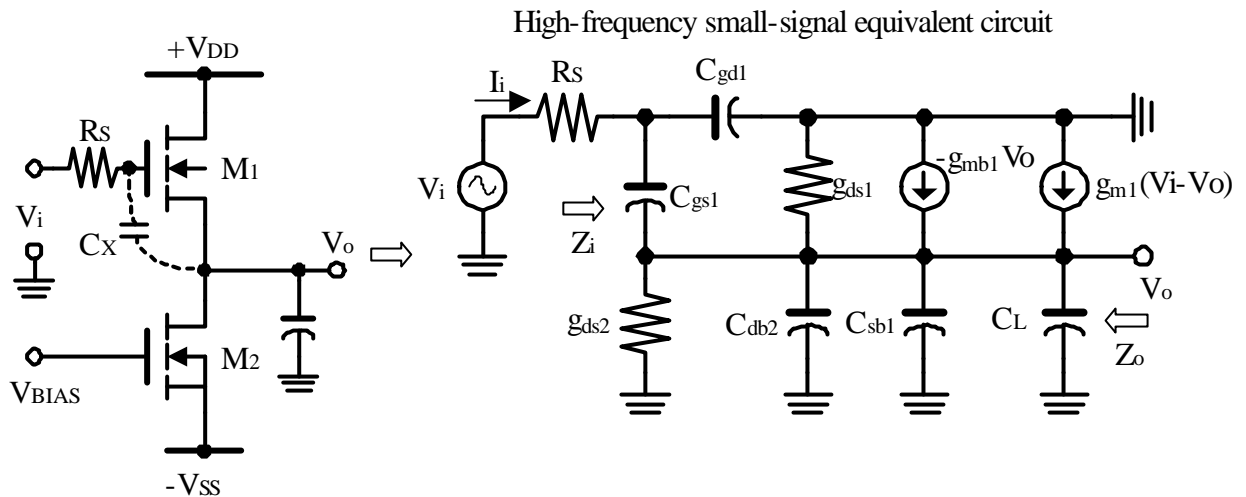


Chapter 6 Frequency Response of MOS Amplifiers

§6-1 Single-Stage Amplifier

§6-1.1 Source follower



$A_v(s)$

$$\equiv \frac{V_o(s)}{V_i(s)} = \frac{sC_{gs1} + g_{m1}}{R_S(C_{gs1}C_{Leq} + C_{gs1}C_{gd1} + C_{gd1}C_{Leq})s^2 + \left(\frac{g_{m1}}{\alpha_1}R_S C_{gd1} + C_{Leq} + C_{gs1}\right)s + g_{m1}/\alpha_1 + G_{Leq}}$$

where $G_{Leq} = g_{ds1} + g_{ds2}$, $C_{Leq} = C_L + C_{sb1} + C_{db2}$

* Left-Half-Plane (LHP) pole: $f_p = \frac{G_{Leq} + g_{m1}/\alpha_1}{2\pi(C_{gs1} + C_{Leq})}$

LHP zero: $f_z = \frac{g_{m1}}{2\pi C_{gs1}}$

In general, $f_p < f_z$ $C_{Leq} > C_{gs1}$

* If $\frac{C_{Leq}}{C_{gs1}} = \left(\frac{1}{\alpha_1} - 1\right) + \frac{G_{Leq}}{g_{m1}}$, we have

$$f_p = f_z \text{ and } A_v(s) \cong \frac{C_{gs1}}{C_{gs1} + C_{Leq}} \cong 1 \text{ indep. of } s.$$

= > Better high frequency response.

How to achieve this?

Adding an extra capacitor C_X such that

$$C_X + C_{gs1} = C_{Leq} \left[\frac{1}{\left(\frac{1}{\alpha_1} - 1 \right) + \frac{G_{Leq}}{g_{m1}}} \right]$$

$$Z_i(s) \equiv \frac{V_i(s)}{I_i(s)} = \left[\frac{1}{C_{gs1}s} + \frac{C_{gs1}s + g_{m1}}{C_{gs1}s(G_{Leq} + g_{mb1} + sC_{Leq})} \right] \parallel (C_{gd1}s)$$

* If $g_{mb1} + G_{Leq} \ll C_{Leq}s$ and C_{gd1} is neglected,

$$Z_i(s) \cong \frac{1}{C_{gs1}s} + \frac{1}{C_{Leq}s} + \frac{g_{m1}}{C_{gs1}C_{Leq}s^2}$$

The input impedance consists of the series connected

C_{gs1} , C_{Leq} , and the negative resistance

$$- \frac{g_{m1}}{C_{gs1}C_{Leq}\omega^2}$$

Thus oscillation is possible.

* If $g_{mb1} + G_{Leq}$ is neglected, the equivalent input capacitances

$$C_{in} = C_{gd1} \parallel C_{in}'$$

$$C_{in}' \cong C_{Leq} \left(\frac{1}{\frac{C_{Leq}}{C_{gs1}} + 1 + \frac{g_{m1}}{C_{gs1}s}} \right)$$

For large g_{m1} , $C_{in}' \ll C_{Leq}$

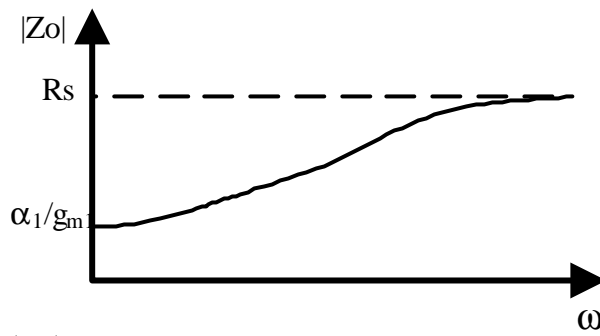
The large load capacitance C_L is well blocked or buffered from the preceding stage.

$$Z_o(s) \equiv \frac{V_o(s)}{I_o(s)} \Big|_{V_i=0} = \frac{1}{G_{Leq} + g_{mb1} + sC_{Leq} + (sC_{gs1} + g_{m1}) \frac{R_s C_{gd1}s + 1}{R_s(C_{gd1}s + C_{gs1}s) + 1}}$$

$$* \text{ If } s = 0, Z_o = R_o = \frac{1}{g_{m1} + g_{mb1}}$$

$$\text{If } s \rightarrow \infty, Z_o'(\text{without } C_{Leq}) \cong R_s \text{ for } R_s < \frac{1}{G_{Leq} + g_{m1}} \text{ and } C_{gs1} \gg C_{gd1}$$

$$\text{Since usually } R_s > \frac{1}{g_{m1} + g_{mb1}}, \text{ we have}$$



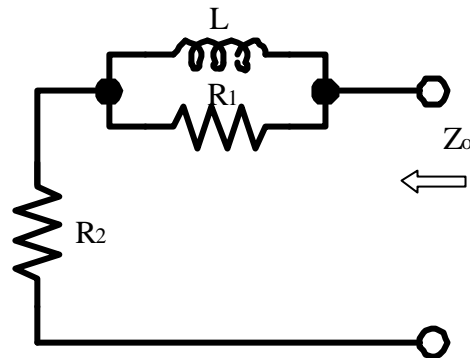
$\Rightarrow |Z_o| \propto \omega \Rightarrow$ Inductive load

$$Z_o(s) \equiv \frac{R_s C_{gs1} s + 1}{g_{m1} / \alpha_1 + C_{gs1} s}$$

$$R_1 = R_s - \frac{\alpha_1}{g_{m1}}$$

$$R_2 = \frac{\alpha_1}{g_{m1}}$$

$$L = \frac{C_{gs1} \alpha_1}{g_{m1}} \left(R_s - \frac{\alpha_1}{g_{m1}} \right)$$



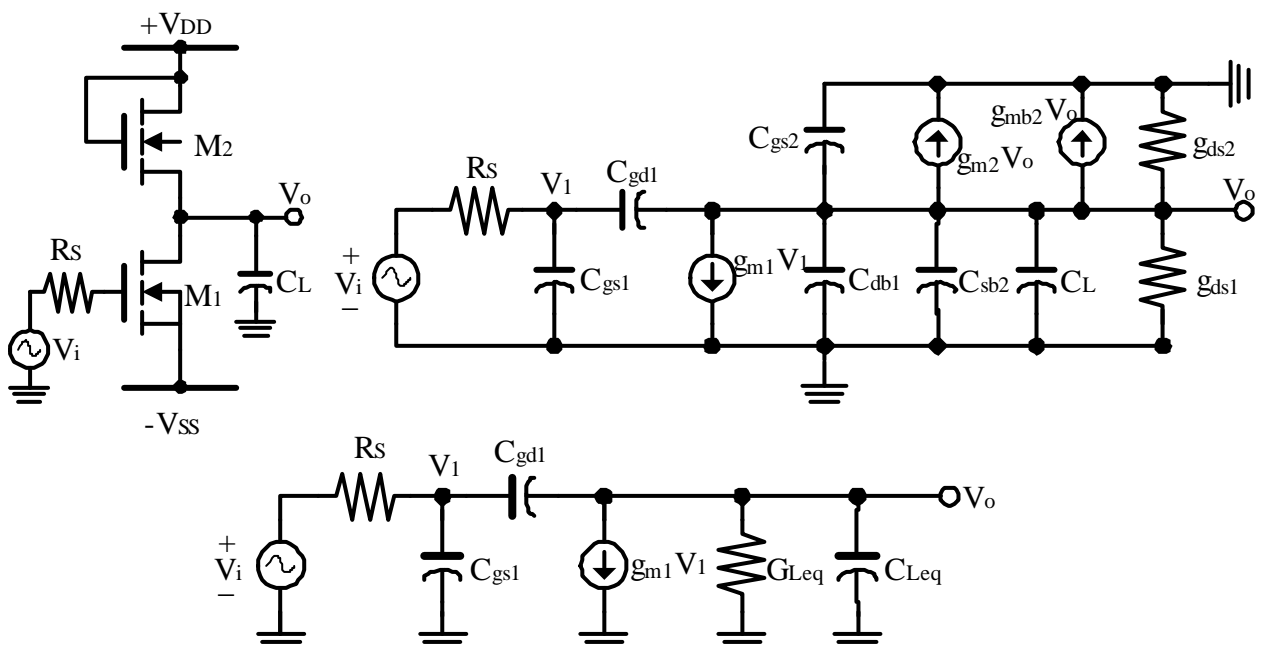
L and C_L causes output signal ringing.

* Two source followers in cascade might cause oscillation because

First SF : L in Z_{o1}

Second SF : $-R$ and $C_{in} Z_{i2}$

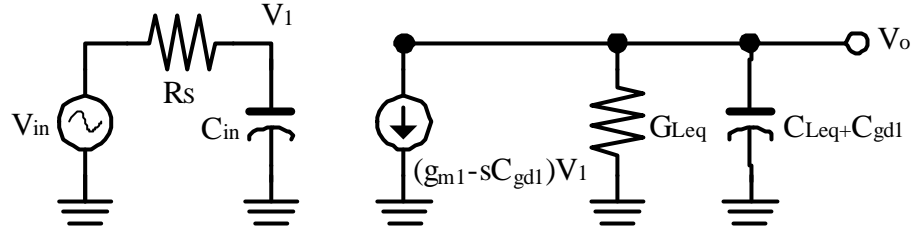
§6-1.2 Enhancement - load NMOS common-source gain stage



$$G_{Leq} = g_{ds1} + g_{ds2} + g_{m2} + g_{mb2}$$

$$C_{Leq} = C_{db1} + C_{gs2} + C_{sb2} + C_L$$

Applying the Miller's theorem, we have



$$C_{in} = C_{gs1} + C_{gd1}(1 + g_{m1}/G_{Leq})$$

$$\Rightarrow A_v(s) \cong \frac{G_s(sC_{gd1} - g_{m1})}{(sC_{in} + G_s)[s(C_{Leq} + C_{gd1}) + C_{Leq}]}$$

* Right-Half-Plane Zero : $S_z = g_{m1}/C_{gd1}$

* Left-Half-Plane Poles : $S_{p1} = -G_s/C_{in}$ (input pole)

$S_{p2} = -G_{Leq}/(C_{Leq} + C_{gd1})$ (output pole).

If C_{gd1} and C_{Leq} are small $\Rightarrow S_{p1}$ is the dominant pole.

* If C_L is large, the dominate pole is $S_{p2} \cong (g_{m2} + g_{mb2})/C_L$

* The input impedance can be approximated by

$$Z_{in} \cong \frac{1}{\left[C_{gs1} + (1 + g_{m1} \frac{1}{G_{Leq}}) C_{gd1} \right] s} \text{ near the upper 3dB frequency.}$$

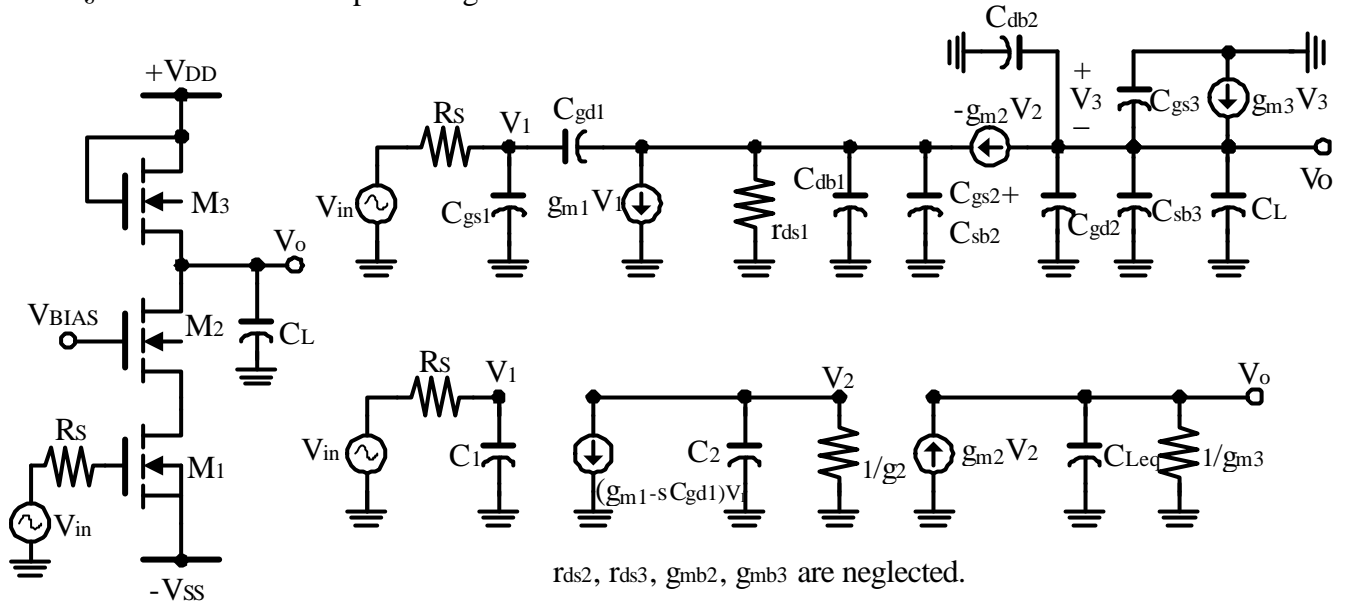
* The exact Z_{in} is

$$Z_{in} \cong C_{gs1} s \parallel \left[\frac{1 + \frac{1}{G_{Leq}} (C_{gd1} + C_{Leq}) s}{C_{gd1} s (1 + g_m \frac{1}{G_{Leq}} + \frac{1}{G_{Leq}} C_{Leq} s)} \right]$$

$$\text{If } \left| \frac{1}{G_{Leq}} (C_{gd1} + C_{Leq}) s \right| \ll 1 \text{ and } \left| \frac{1}{G_{Leq}} C_{gd1} s \right| \ll (1 + g_m \frac{1}{G_{Leq}}),$$

Z_{in} can be approximated by the previous formula.

§ 6-1.3 Cascode amplifier stage



$$g_2 = g_{m2} + \frac{1}{r_{ds1}}$$

$$C_2 = C_{gs1} + \left(1 + \frac{g_{m1}}{g_{m2}}\right)C_{gd1}$$

$$C_2 = C_{gd1} + C_{db1} + C_{gs2} + C_{sb2}$$

$$C_{Leq} = C_L + C_{gd2} + C_{db2} + C_{sb3} + C_{gs3}$$

$$A_v(s) = \frac{-G_s g_{m2} (sC_{gd1} - g_{m1})}{(sC_1 + G_s)(sC_2 + g_2)(sC_{Leq} + g_{m3})}$$

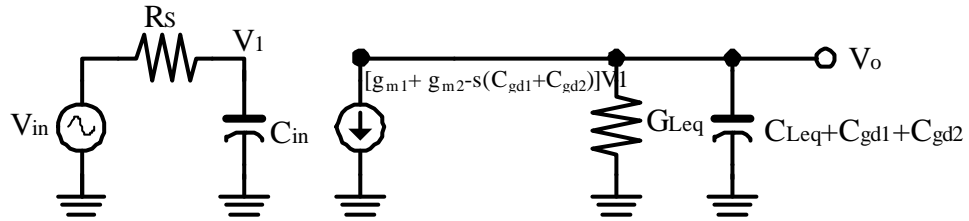
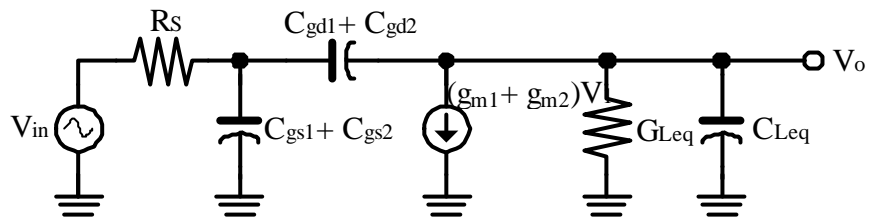
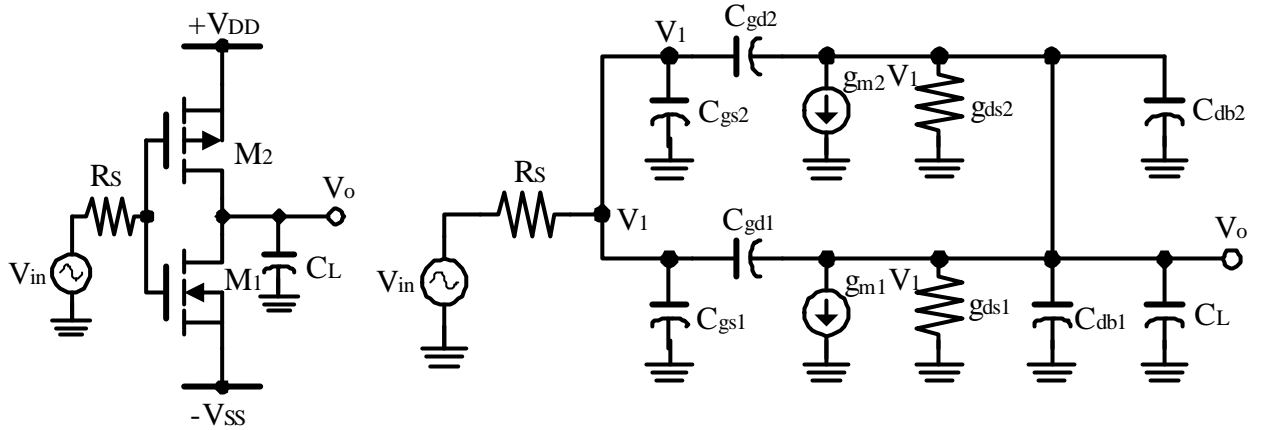
$$\text{RHP Zero: } S_z = \frac{g_{m1}}{C_{gd1}}$$

$$\text{LHP Pole: } S_{p1} = -\frac{G_s}{C_1}; S_{p2} = -\frac{g_2}{C_2}; S_{p3} = -\frac{g_{m3}}{C_{Leq}}$$

S_{p1} usually is the dominant pole.

$$\Rightarrow f_{3dB} \cong \frac{|S_{p1}|}{2\pi} = \frac{G_s}{2\pi C_1}$$

* Typically, $g_{m1} = g_{m2}$, then $C_1 = C_{gs1} + 2C_{gd1}$



$$G_{Leq} = g_{ds1} + g_{ds2} \quad C_{Leq} = C_{db1} + C_{db2} + C_L$$

$$C_{in} = C_{gs1} + C_{gs2} + \left(1 + \frac{g_{m1} + g_{m2}}{G_{Leq}}\right)(C_{gd1} + C_{gd2})$$

$$A_v(s) \cong \frac{G_s [s(C_{gd1} + C_{gd2}) - (g_{m1} + g_{m2})]}{[s(C_{gd1} + C_{gd2} + C_{Leq}) + G_{Leq}](sC_{in} + G_s)}$$

$$\text{RHP Zero: } S_z = \frac{g_{m1} + g_{m2}}{C_{gd1} + C_{gd2}}$$

$$\text{LHP Pole: } S_{p1} = -\frac{G_s}{C_{in}}$$

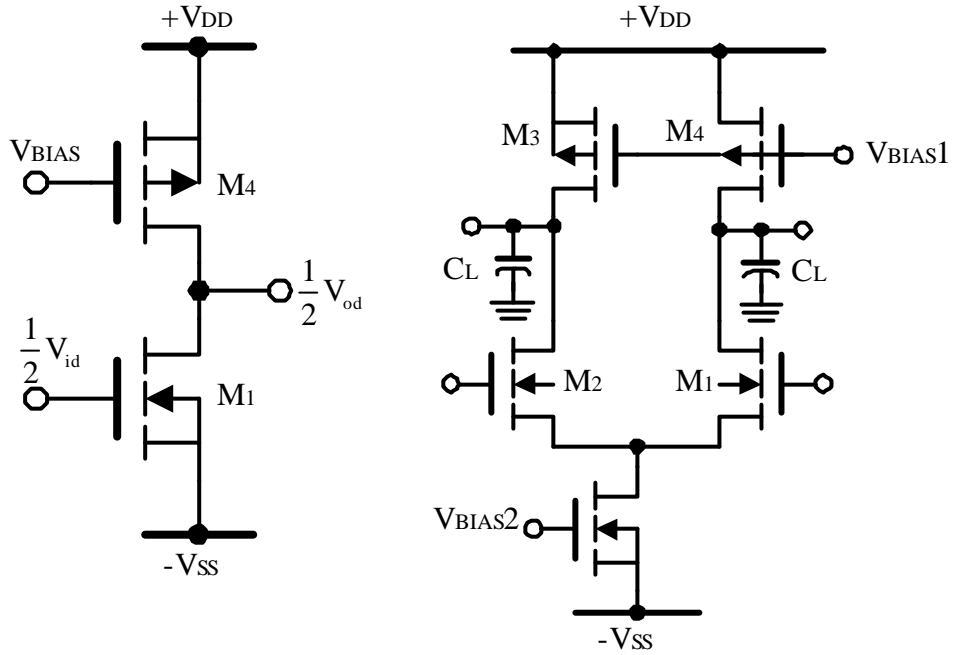
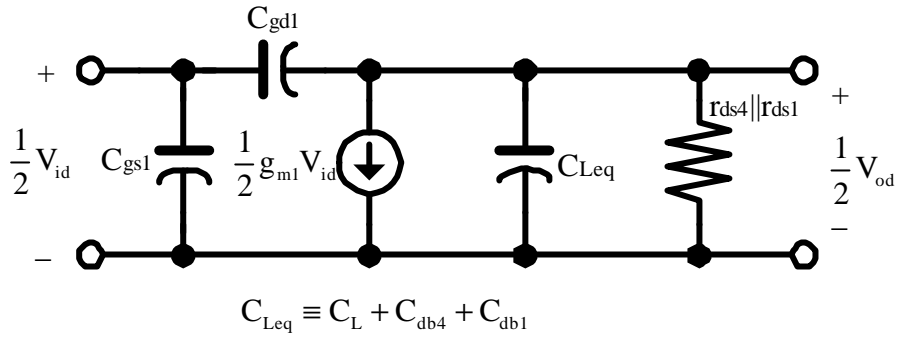
$$S_{p2} = -\frac{G_{Leq}}{C_{gd1} + C_{gd2} + C_{Leq}}$$

If R_s is large enough (R_s is the output resistance of the preceding stage),

$$|S_{p1}| \ll |S_{p2}|$$

S_{p1} is the dominant pole.

1. Differential-mode half circuit

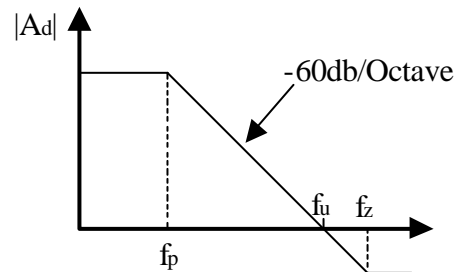


$$A_d = \frac{V_{od}}{V_{id}} = H(s) = -\frac{g_{m1}}{g_{ds4} + g_{ds1}} \left[\frac{1 - s \frac{C_{gd1}}{g_{m1}}}{1 + \left(\frac{C_{Leq} + C_{gd1}}{g_{ds4} + g_{ds1}} \right) s} \right]$$

RHP Zero: $f_z = \frac{g_{m1}}{2\pi C_{gd1}}$ $f_z > f_p$

LHP Pole: $f_p = \frac{g_{ds4} + g_{ds1}}{2\pi(C_{db4} + C_{db1} + C_L + C_{gd1})}$

f_u $A_0 f_p = \frac{g_{m1}}{2\pi(C_{db4} + C_{db1} + C_L + C_{gd1})}$



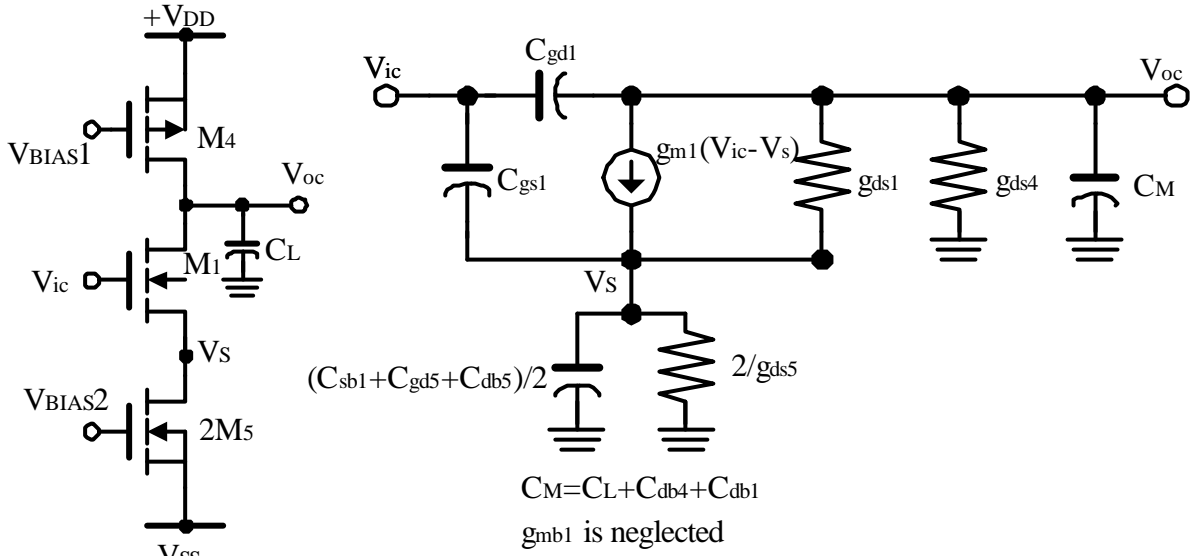
2. Common-mode half circuit:

$$(g_{ds4} + sC_M)V_{oc} + g_{ds1}(V_{oc} - V_s) + g_{m1}(V_{ic} - V_s) + C_{gd1}s(V_{oc} - V_{ic}) = 0$$

$$g_{ds1}(V_s - V_{oc}) + V_s \left(\frac{1}{2r_{ds5}} + \frac{C_{gd5} + C_{db5} + C_{sbl}}{2} s \right) - g_{m1}(V_{ic} - V_s) - C_{gs1}s(V_{ic} - V_s) = 0$$

$$V_s \left[\frac{1}{2r_{ds5}} + \frac{1}{2}(C_{gd5} + C_{db5} + C_{sbl})s + C_{gs1}s \right] = -[g_{ds4} + sC_M + sC_{gd1}]V_{oc} + (C_{gs1}s + C_{gd1}s)V_{ic}$$

$$V_s = - \frac{[g_{ds4} + sC_M + sC_{gd1}]V_{oc} - (C_{gs1}s + C_{gd1}s)V_{ic}}{\left[\frac{1}{2r_{ds5}} + \frac{C_{gd5} + C_{db5} + C_{sbl}}{2} s + C_{gs1}s \right]}$$

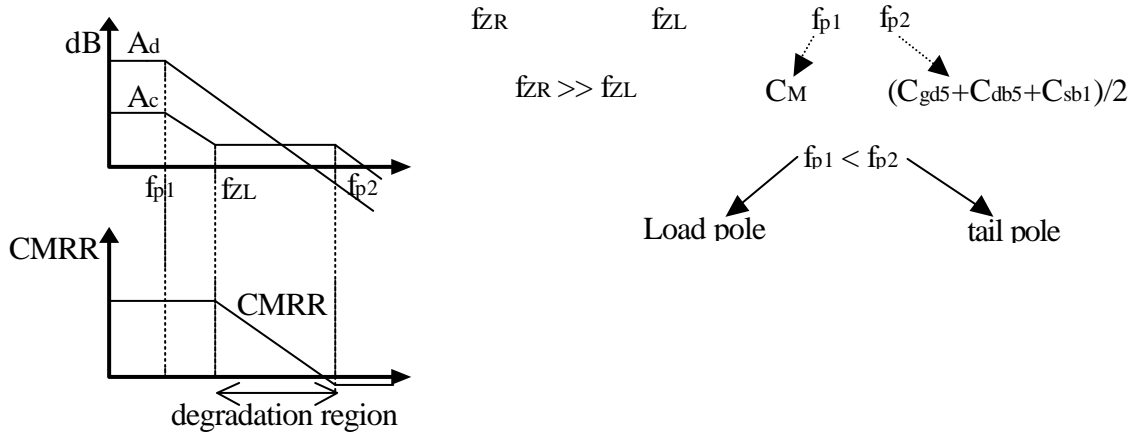


$$\Rightarrow A_c(s) = \frac{V_{oc}}{V_{ic}} = - \frac{-C_{gd1} \left(\frac{C_{gd5} + C_{db5} + C_{sbl}}{2} + C_{gs1} \right) s^2 + \left[\left(\frac{C_{gd5} + C_{db5} + C_{sbl}}{2} + C_{gs1} \right) g_{m1} \right]}{\left(\frac{C_{gd5} + C_{db5} + C_{sbl}}{2} + C_{gs1} \right) (C_M + C_{gd1}) s^2 + \left[\left(\frac{C_{gd5} + C_{db5} + C_{sbl}}{2} + C_{gs1} \right) \right.}$$

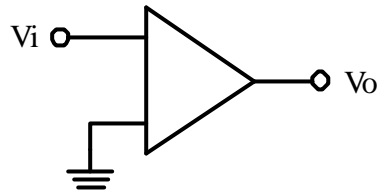
$$\left. - \frac{1}{2r_{ds5}} C_{gd1} - (g_{ds1} + g_{m1})(C_{gs1} + C_{gd1}) \right] s + \frac{1}{2r_{ds5}} g_{m1}}$$

$$(g_{ds4} + g_{ds1}) + \frac{C_M + C_{gd1}}{2r_{ds5}} + (C_M + C_{gd1})(g_{ds1} + g_{m1}) \Big] s + \frac{g_{ds4} + g_{ds1}}{2r_{ds5}} + (g_{ds1} + g_{m1})g_{ds4}$$

Solve the pole-zero position : \Rightarrow 1 RHP zero, 1 LHP zero, 2 LHP poles



§ 6-1.6 CMOS differential-input-to-single-ended output converter



$$V_i = V_{id} + V_{ic} \quad V_o = V_{od} + V_{oc}$$

* The half-circuit method cannot be used in the high frequency analysis.

* Two unequal signal paths to the output

⇒ Load path and tail path

⇒ Both C_s and C_E appears in the $A_d(s)$ expression.

* There are two dominate poleo in A_d .

$$\text{Output pole } W_{p1} \cong \frac{g_{ds1} + g_{ds4}}{C_{Leq}}$$

$$\text{Mirror pole } W_{p2} \cong \frac{g_{m34}}{C_E}$$

$$\text{Tail path: } A_1(s) = \frac{A_0}{1 + \frac{s}{W_{p1}}}$$

$$\text{Load path: } A_2(s) = \frac{A_0}{(1 + \frac{s}{W_{p1}})(1 + \frac{s}{W_{p2}})}$$

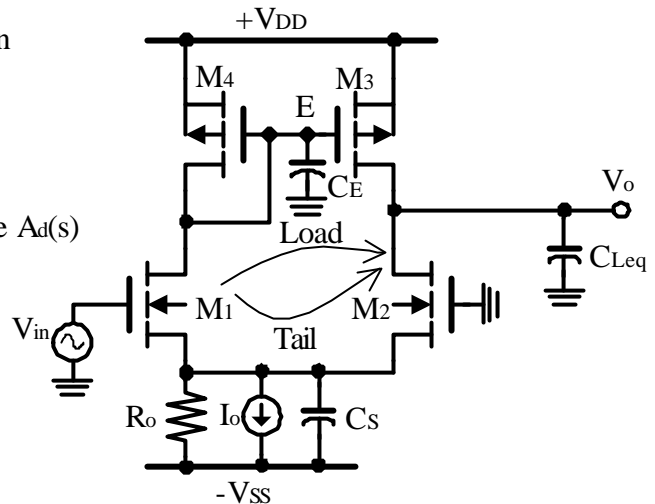
$$A_d(s) = A_1(s)A_2(s) = \frac{A_0(2 + \frac{s}{W_{p2}})}{(1 + \frac{s}{W_{p1}})(1 + \frac{s}{W_{p2}})}$$

$$\text{LHP zero: } W_{z1} \cong \frac{2g_{m34}}{C_E} = 2W_{p2}$$

* Approximate analysis:

The dominant pole of $A_d(s)$ is $S_{p1} = -\frac{g_{ds1} + g_{ds4}}{C_{Leq}}$ (output pole)

$$A_d(s) \cong \frac{g_{m1}}{sC_{Leq} + (g_{ds1} + g_{ds4})}$$



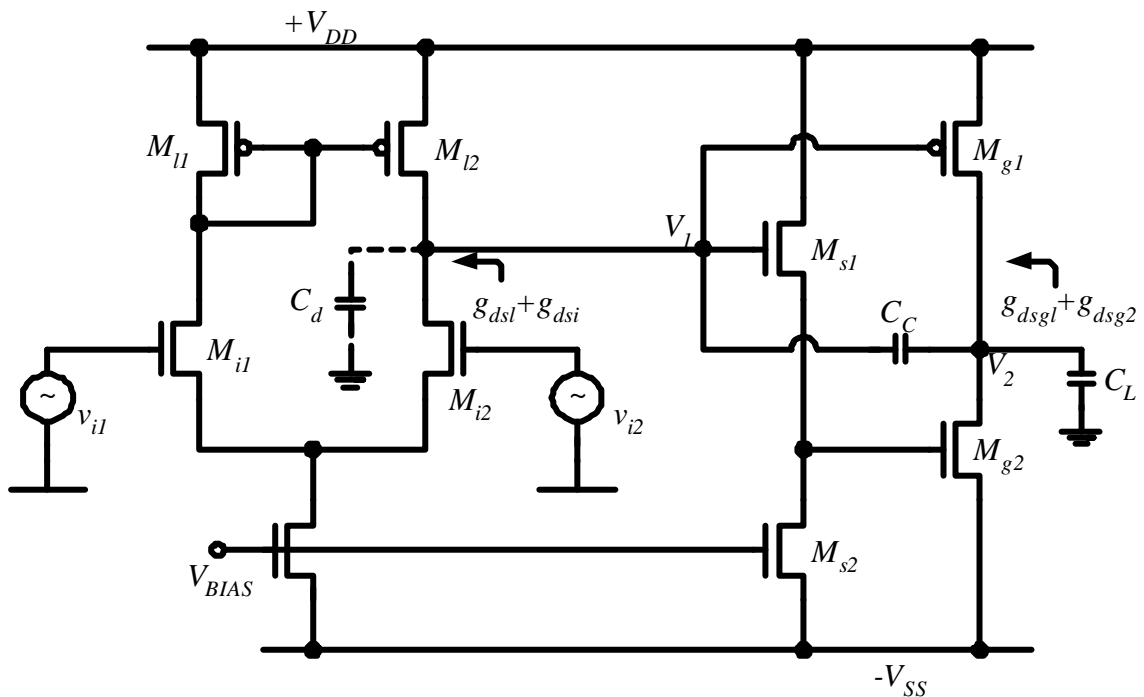
The $A_c(s)$ can be written as $A_c(s) \cong -\frac{g_{ds1}}{2g_{m4}} \frac{(\frac{1}{R_0}) + sC_s}{sC_{Leq} + (g_{ds1} + g_{ds4})}$

The dominant pole of $A_c(s)$ is $S_{p1} = -\frac{g_{ds1} + g_{ds4}}{C_{Leq}}$

But the left-half-plane zero is $S_{zL} = -\frac{1}{R_0}(\frac{1}{C_s})$

The CMRR ($\equiv \frac{A_d}{A_c}$) is degraded by 20dB/decade at high frequency.

§6-2 Frequency Compensations

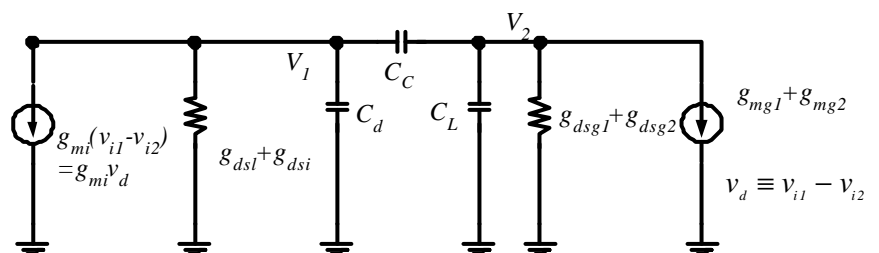


Without C_c

$$S_{p1} = -\frac{1}{C_d}(g_{ds1} + g_{dsi}) , \quad S_{p2} = -\frac{1}{C_L}(g_{ds1} + g_{ds2})$$

equivalent
circuit

P



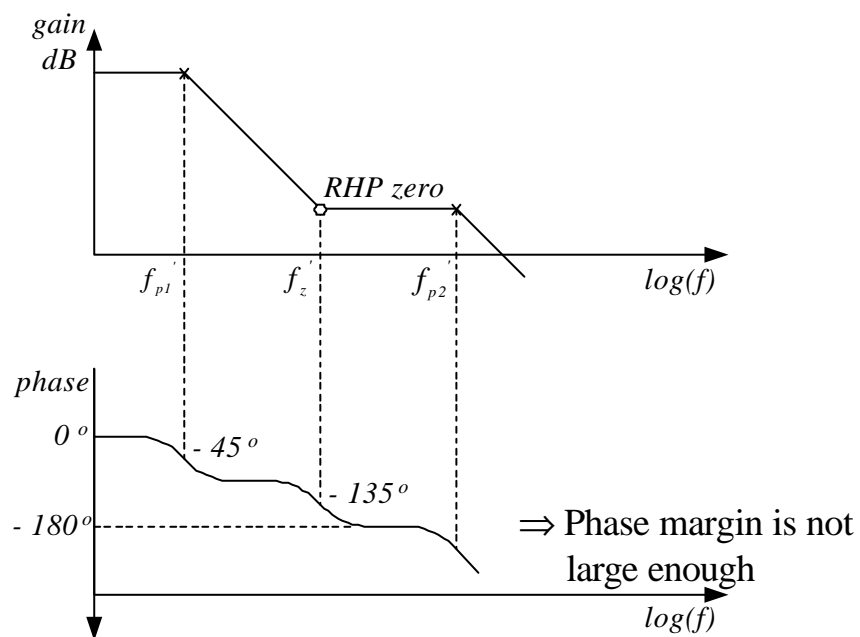
$$H(s) = \frac{v_2}{v_{i1} - v_{i2}}$$

$$= \frac{+g_{m1}(g_{m1} + g_{m2})R_d R_o \left(1 - \frac{sC_c}{g_{m1} + g_{m2}}\right)}{1 + s[(C_L + C_c)R_o + (C_c + C_d)R_d + C_c(g_{m1} + g_{m2})R_o R_d] + (C_c C_L + C_c C_d + C_d C_L)R_o R_d s^2}$$

where $R_o \equiv -\frac{1}{g_{ds1} + g_{ds2}}$ $R_d \equiv -\frac{1}{g_{ds1} + g_{ds2}}$

$$\Rightarrow S_{p1}' \approx -\frac{1}{(g_{m1} + g_{m2})R_o R_d C_c} \quad S_{p2}' \approx -\frac{C_c(g_{m1} + g_{m2})}{C_o C_L + C_d C_L + C_d C_c}$$

$$S_z' \approx \frac{g_{m1} + g_{m2}}{C_c} \rightarrow \text{RHP Zero}$$



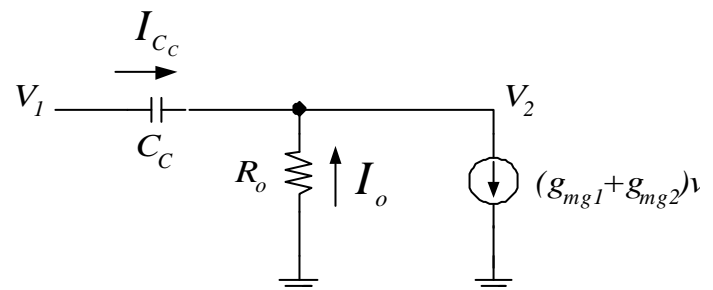
* Feedforward effect on C_c

How to solve this problem ?

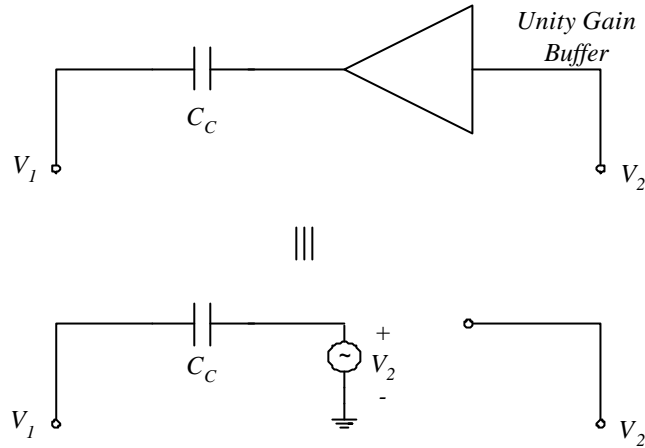
If $I_{C_c} = (g_{m1} + g_{m2})V_d$,

$$I_o = 0 \quad \text{and} \quad V_2 = 0$$

\Rightarrow A zero is formed.



§6-2.1 Using a unity-gain buffer in the feedback path



- * Isolate node 1 from node 2 to prevent feedforward.
- * Keep the Miller effect unchanged.
- * Source follower can act as a unity gain buffer.

$$g_{mi}V_d + \frac{V_1}{R_d} + C_d s V_1 + (V_1 - V_2)C_c s = 0 \quad \text{----- (1)}$$

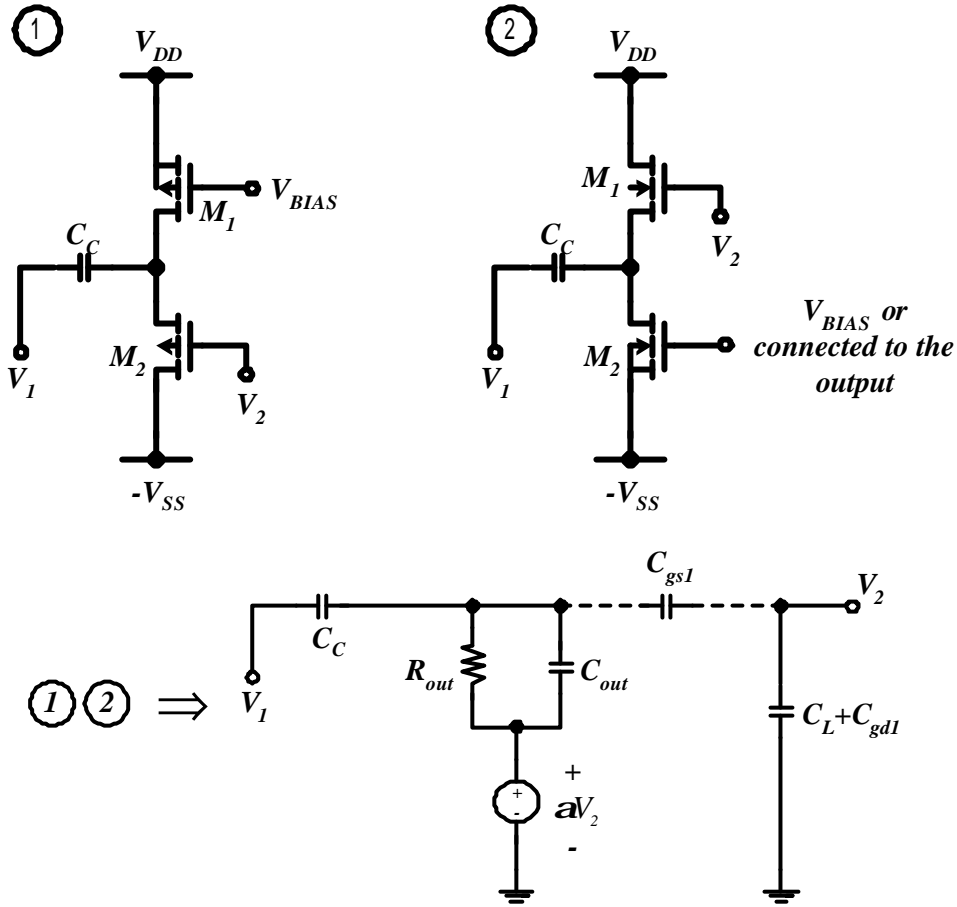
$$(g_{mg1} + g_{mg2})V_1 + \frac{I}{R_o}V_2 + C_L s V_2 = 0 \quad \text{----- (2)}$$

$$H(s) = \frac{V_2}{V_d} = \frac{g_{mi}(g_{mg1} + g_{mg2})}{1 + s[R_o C_c + R_d(C_d + C_c) + C_c(g_{mg1} + g_{mg2})R_o R_d] + (C_c C_L + C_d C_L)R_o R_d s}$$

$$S_{p1}' \approx -\frac{I}{(g_{mg1} + g_{mg2})R_o R_d C_c} \quad \text{(unchanged)}$$

$$S_{p2}' \approx -\frac{C_c(g_{mg1} + g_{mg2})}{C_c C_L + C_d C_L} \quad \text{RHP Zero has be eliminated.}$$

Actual Circuits :



* C_{gs1} may introduce a RHP zero. But usually this RHP zero is large.

C_{gs1} is very small.

$$* R_{out} \approx \left(\frac{1}{g_{m1}} \parallel \frac{1}{g_{m2}} \right)$$

$$g_{m1} V_d + \frac{V_1}{R_d} + C_d s V_1 + (V_1 - a V_2) \left(\frac{1}{C_c s} + \frac{1}{\frac{1}{R_{out}} + C_{out} s} \right)^{-1} = 0$$

$$\text{If } \frac{1}{R_{out}} \geq C_{out} s$$

$$\Rightarrow \left(\frac{1}{C_c s} + \frac{1}{\frac{1}{R_{out}} + C_{out} s} \right)^{-1} \approx \frac{C_c s}{C_c R_{out} s + 1}$$

The numerator of $H(s) = \frac{V_2(s)}{V_d(s)}$ is $g_{mi}(g_{mg1} + g_{mg2})(C_c R_{out}s + 1)$

$$\Rightarrow \text{LHP Zero : } -\frac{1}{C_c R_{out}}$$

If R_{out} is large, LHP Zero may form a pole-zero doublet with S_{p1} or S_{p2} ,

\Rightarrow very slow slew rate !!

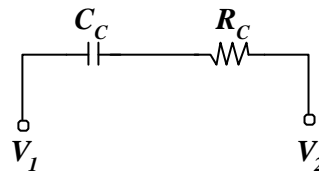
If R_{out} is small, too large g_{m1} or g_{m2} is required.

\Rightarrow (large area, large power)

\Rightarrow large C_{out} . Freq. Resp.

- * Somehow difficult to design.
- * Also the power dissipation of the buffer is large. (additional power dissipation)

§6-2.2 Adding R_c in series with C_c .



$H(s) = \frac{v_2}{v_d}$ can be solved.

Low frequency gain : $A_{dm} = g_{mi}(g_{mg1} + g_{mg2})R_o R_d$

LHP Poles : $S_{p1} \cong \frac{-1}{(g_{mg1} + g_{mg2})R_o R_d C_c}$ (unchanged)

$$S_{p2} \cong -\frac{(g_{mg1} + g_{mg2})C_c}{C_d C_L + C_c C_L + C_d C_c} \quad (\text{unchanged})$$

$$S_{p3} \cong -\frac{C_d C_c + C_d C_L + C_c C_L}{R_c C_d C_L C_c}$$

$$\left(\begin{matrix} LHP \\ RHP \end{matrix} \right) \text{Zero : } S_z = -\frac{g_{mg1} + g_{mg2}}{C_c [R_c (g_{mg1} + g_{mg2}) - 1]}$$

1. If $R_C = \frac{I}{g_{mg1} + g_{mg2}}$ or $R_C = \frac{I}{g_{m2}}$ g_{m2} : second-stage transconductance

$S_Z \rightarrow \pm\infty$ No effect on the frequency response of the OP.

$$S_{P1} \text{ dominant pole} \Rightarrow A_d(s) \cong \frac{A_{dm}}{\frac{s}{S_{P1}} + 1} = \frac{A_{do} S_{P1}}{s + S_{P1}}$$

$$\text{For } \omega \gg S_{P1} \quad A_d(j\omega) = \frac{A_{dm} S_{P1}}{j\omega}, |A_d(j\omega)| = \frac{A_{dm} S_{P1}}{\omega}$$

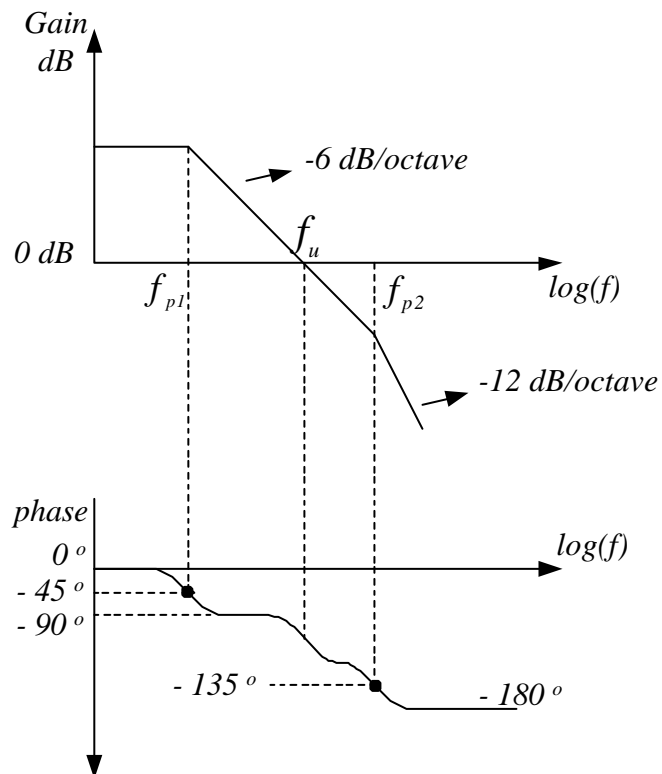
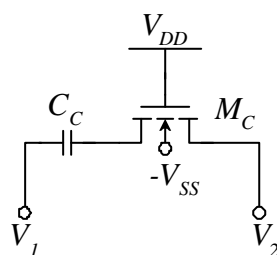
$$\text{At } \omega_u, |A_d(j\omega_u)| = 1 \Rightarrow \omega_u = A_{dm} S_{P1} = \frac{g_{mi}}{C_c}$$

$$\text{Large } C_L \Rightarrow S_{P2} \approx -\frac{g_{mg1} + g_{mg2}}{C_L}$$

$$\text{For phase margin } 45^\circ \sim 60^\circ \Rightarrow \frac{S_{P2}}{\omega_u} \cong 2 \sim 4, \frac{C_c}{C_L} \frac{g_{mg1} + g_{mg2}}{g_{mi}} = 2 \sim 4$$

$$\text{If } \frac{g_{mi}}{g_{mg1} + g_{mg2}} \cong 2 \sim 4, C_L \cong C_c \text{ stable}$$

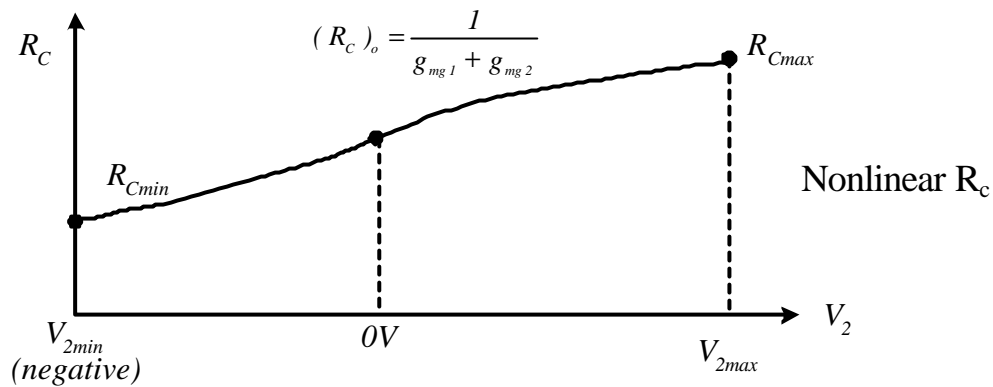
1) NMOS Realization :



$$I_{DS} = \frac{m_n C_{ox}}{2} \frac{W}{L} [2(V_{DD} - V_2 - V_{TH})V_{DS} - V_{DS}^2],$$

$$R_c = \left(\frac{\partial I_{DS}}{\partial V_{DS}} \right)^{-1} \bigg|_{V_{DS}=0} = \frac{I}{\frac{m_n C_{ox}}{2} \frac{W}{L} [2(V_{DD} - V_2 - V_{TH})]}$$

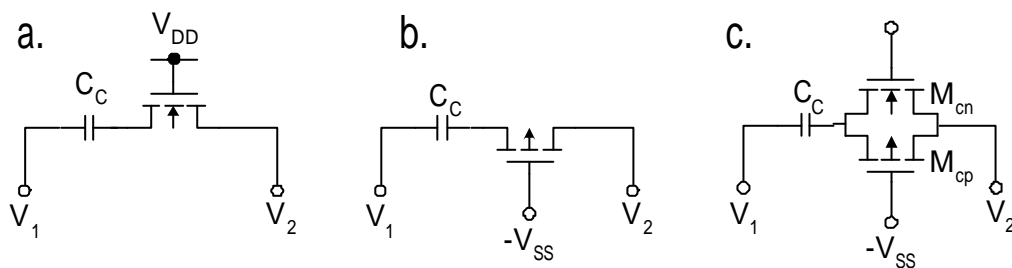
$V_2 \uparrow \quad V_{TH} \uparrow$ body effect



Design R_c : (1) Design R_c , s.t. $(R_c)_{V_2=0V} = \frac{I}{g_{m2}} \left(\frac{I}{g_{mgl} + g_{mg2}} \right)$

(2) At $R_c = R_{cmax}$ or R_{cmin} ,
 S_z must be large enough ! Otherwise, frequency performance will be degraded.

1) CMOS Realizations :



Consider the case in c.:

$$I_{DSn} = \frac{m_n C_{ox}}{2} \frac{W_n}{L_n} [2(V_{DD} - V_2 - V_{THn})V_{DS} - V_{DS}^2]$$

$$R_{cn} = \frac{I}{\frac{m_n C_{ox}}{2} \frac{W_n}{L_n} [2(V_{DD} - V_2 - V_{THn})]}$$

$$I_{DSp} = \frac{\mathbf{m}_p C_{ox}}{2} \frac{W_p}{L_p} [2(V_2 + V_{SS} - V_{THp})V_{DS} - V_{DS}^2]$$

$$R_{cp} = \frac{I}{\frac{\mathbf{m}_p C_{ox}}{2} \frac{W_p}{L_p} [2(V_2 + V_{SS} - V_{THp})]}$$

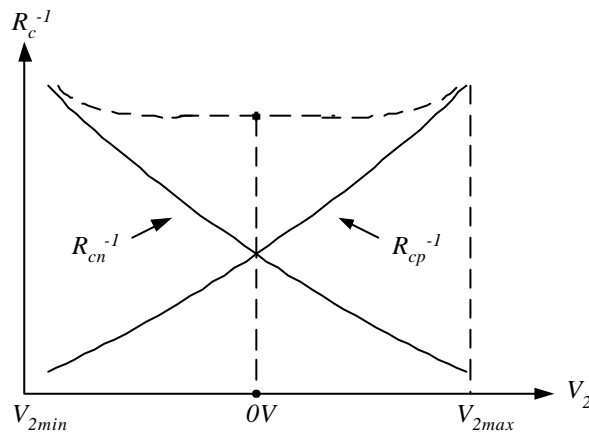
$$R_c^{-1} = (R_{cn} // R_{cp})^{-1} = R_{cn}^{-1} + R_{cp}^{-1}$$

$$= \frac{\mathbf{m}_n C_{ox}}{2} \frac{W_n}{L_n} [2(V_{DD} - V_2 - V_{THn})] + \frac{\mathbf{m}_p C_{ox}}{2} \frac{W_p}{L_p} [2(V_2 + V_{SS} - V_{THp})]$$

If $\frac{\mathbf{m}_n C_{ox}}{2} \frac{W_n}{L_n} = \frac{\mathbf{m}_p C_{ox}}{2} \frac{W_p}{L_p} = \mathbf{b}$

$$R_c^{-1} = \mathbf{b} [2V_{DD} - 2V_{THn} + 2V_{SS} - 2V_{THp}] \text{ nearly indep. Of } V_2$$

$$R_c^{-1} \Big|_{V_2=0V} = g_{mg1} + g_{mg2}$$



2. If $R_c = \frac{I + (C_d + C_L)/C_c}{g_{mg1} + g_{mg2}}$

$S_z = S_{p2}$ and pole-zero cancellation occurs.

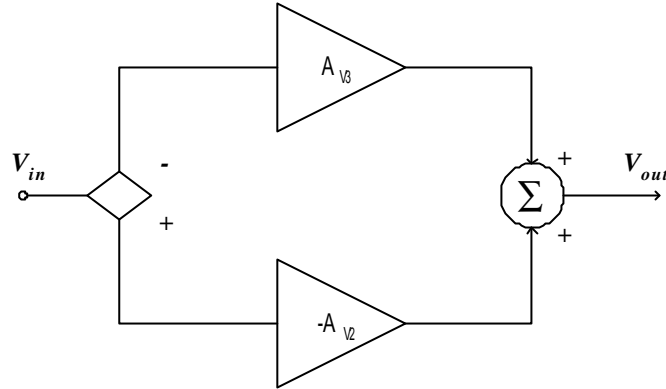
$$\Rightarrow S_{p3} \gg S_{p1} \Rightarrow A_{dm} S_{p1} < S_{p3} \Rightarrow \text{stable}$$

However, if the cancellation is not complete

\Rightarrow pole-zero doublet occurs ! \Rightarrow slow slew rate.

§6-2.2 Feedforward compensation

A_{v3} is the gain of the source follower



$$A_{v3} = \frac{A_{v3}(0) \left(1 + \frac{s}{z_3}\right)}{\left(1 + \frac{s}{p_3}\right)}$$

1 LHP zero
1 LHP pole

$$A_{v2} = \frac{A_{v2}(0) \left(1 - \frac{s}{z_1}\right) \left(1 + \frac{s}{z_2}\right)}{\left(1 + \frac{s}{p_1}\right) \left(1 + \frac{s}{p_2}\right)}$$

2 LHP poles
1 RHP zero (C_C)
1 LHP zero

z_3 & z_2 are generated from the C_{gs} of the source follower.

$$\frac{V_{out}}{V_{in}} = A_{TOT}(s) = A_{v2}(s) + A_{v3}(s)$$

$$= [A_{v2}(0) + A_{v3}(0)] \frac{\left(1 + \frac{s}{z_1'}\right) \left(1 + \frac{s}{z_2'}\right) \left(1 + \frac{s}{z_3'}\right)}{\left(1 + \frac{s}{p_1}\right) \left(1 + \frac{s}{p_2}\right) \left(1 + \frac{s}{p_3}\right)}$$

p_1' : dominant pole

z_1', z_2', z_3' : LHP Zeros

Design consideration : Any zeros below the unity-gain frequency must be placed as close as possible to their matching poles.

This prevents the formation of any doublet !

$z_1' = p_2$ by adding CB1 and CB2(3.8pF) to control $C_{gs9} + C_{gs11}$

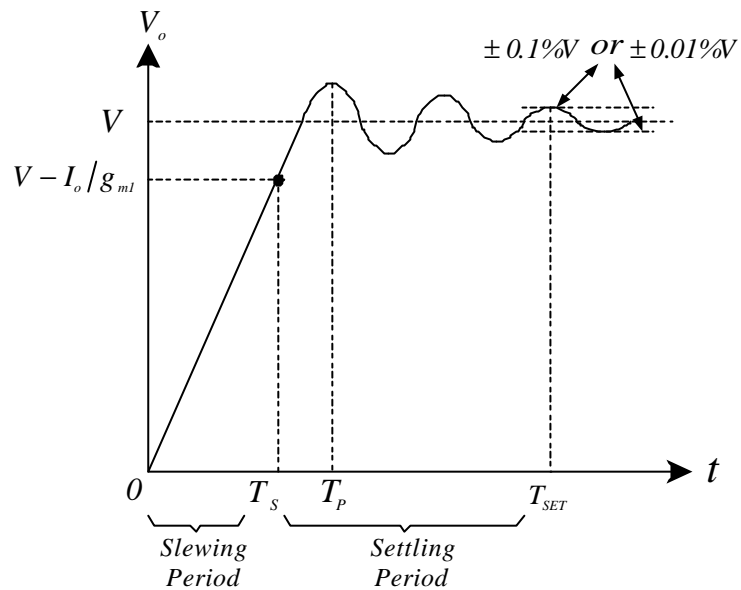
Ref:IEEE JSSC , col SC-14, no.6 pp.1070-1077 , DEC.1979

Feedforward + Miller(direct)

Ref:IEEE JSSC , col SC-15, no.6 pp.921-928 , DEC.1980

Feedforward + Unity gain buffer + Miller

§ 6-3 Settling Behavior



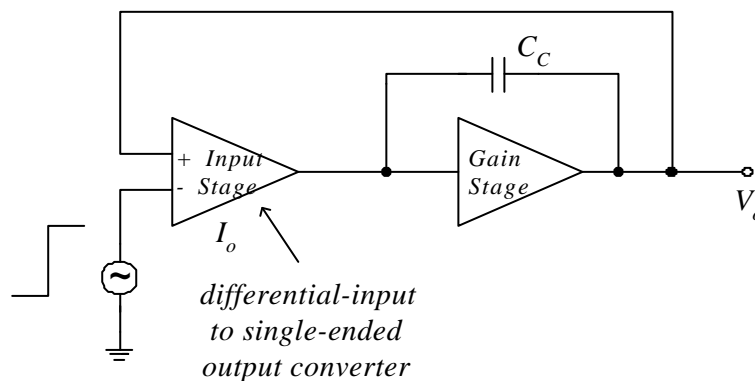
Slewing Period (T_s): V_o from 0V to $V - I_o/g_{m1}$ under voltage follower connection and worse case loading.(nonlinear operation)

Settling Period ($T_{SET} - T_s$):

V_o from $(V - I_o/g_{m1})$ to $\pm 0.1\%V$ or $\pm 0.01\%V$ (quasi-linear operation)

Settling Time (T_{SET}): $T_s + (T_{SET} - T_s)$ = slewing period + settling period.

§ 6-3.1 Single-pole case



Slew rate:

$$SR \equiv \frac{dV_o}{dt} \Big|_{max} = \frac{I_o}{C_c}$$

$$\mathbf{w}_u = \frac{g_{mi}}{C_c} \leftarrow \text{single-pole case}$$

$$SR = \frac{I_o \mathbf{w}_u}{g_{mi}} = \mathbf{w}_u \sqrt{\frac{I_o}{2 \frac{u C_{ox}}{2} \left(\frac{W}{L} \right)_i}}$$

§ 6-3.1 Two-pole case

Ref ; IEEE JSSC vol.SC-17, no.1 pp.74-80, Feb. 1982

$$Ts = -\frac{1}{\mathbf{w}_l} \ln \left[1 - \frac{g_{ml}}{I_o a_o} \left(V - \frac{I_o}{g_{ml}} \right) \right] \quad \text{Fig.2}$$

approximation : $e^{-\mathbf{w}_l Ts} \cong 1 - \mathbf{w}_l Ts \Rightarrow \text{eq.(19) conventional expression}$

After Ts : $V_o = V - I_o/g_{ml}$ Input voltage = $V - (V - I_o/g_{ml}) = I_o/g_{ml}$

\Rightarrow enter the linear (or quasi-linear) region

Feedback Function for unity-gain voltage-follower connection

$$\Rightarrow A(s) = \frac{a(s)}{1 + a(s)} \quad \text{eq.(20)-(23)}$$

$$\text{two poles : } S = -\mathbf{x} \mathbf{w}_n \pm \sqrt{\mathbf{x}^2 - 1} \mathbf{w}_n \quad \text{eq.(24)}$$

$$\boxed{\xi = \frac{\omega_1 + \omega_2}{2\omega_n}}$$

(double negative real poles)

damping ratio

$\mathbf{x} = 1$ critically damped

$\mathbf{x} < 1$ underdamped

$\mathbf{x} > 1$ overdamped

(complex conjugate poles)

(real and negative pole)

$$\mathbf{x} = \frac{\mathbf{w}_l + \mathbf{w}_2}{2\mathbf{w}_n} \cong \frac{\sqrt{\mathbf{w}_2}}{2\sqrt{a_o \mathbf{w}_l}} = \frac{\sqrt{\mathbf{w}_2}}{2\sqrt{\mathbf{w}_u}} = \frac{\sqrt{g_{m2}/c_2}}{2\sqrt{g_{m1}/c_c}} \quad (C_c, C_2 \gg C_1)$$

$$\mathbf{X} \leq 1 \Rightarrow C_c \leq 4\left(\frac{g_{m1}}{g_{m2}}\right)C_2 \quad (C_c, C_2 \gg C_1)$$

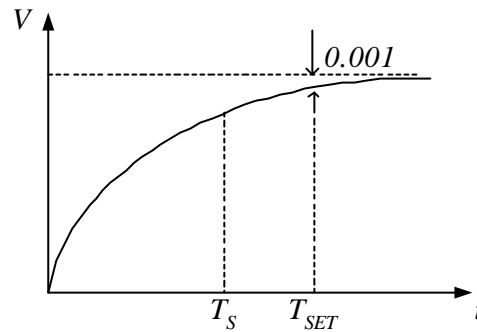
$$\Rightarrow \mathbf{w}_2 \leq 4\mathbf{w}_u \Leftrightarrow \frac{\mathbf{w}_2}{\mathbf{w}_u} = 2 \sim 4 \quad \begin{array}{ll} \omega_2 < 4\omega_u & \text{underdamped} \\ \omega_2 > 4\omega_u & \text{overdamped} \end{array}$$

(1) Underdamped: T_S eq. (14) or (19) max.overshoot: eq.(36)

T_P eq. (35), (33) settling time: eq.(40),(39)

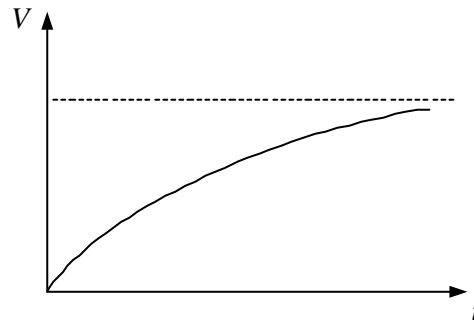
(2) Critically Damped $V_o(t)$: eq.(41)

T_{SET} : eq.(43)



(3) Overdamped T_{SET} : eq.(47)

Simulation & Calculation : Fig.7, Fig.8



Further references:

(1) *IEEE JSSC*, vol. SC-18, pp.389-394, Aug. 1983

(2) *IEEE JSSC*, vol. SC-21, pp.478-483, June. 1986

§ 6-4 Slew rate of CMOS OP AMPs

§ 6-4.1 Two-stage OP AMPs

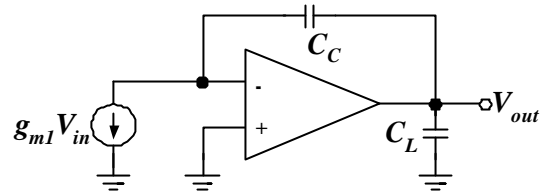
Two poles: $S_{p1}, S_{p2}, |S_{p1}| \ll |S_{p2}|$

If $|S_{p1}| \ll \omega_u \ll |S_{p2}|, V_{out}(s) = g_{mi} V_{in}(s) / s C_c$

$$\frac{V_{out}(j\omega)}{V_{in}(j\omega)} = \frac{g_m}{j\omega C_c}$$

At $\omega = \omega_u, \frac{V_{out}}{V_{in}} = 1$

$$\Rightarrow \omega_u = \frac{g_{mi}}{C_c} \text{ or } C_c = g_{mi} / \omega_u$$

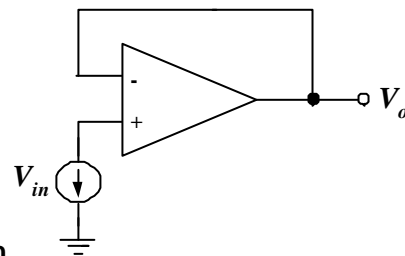
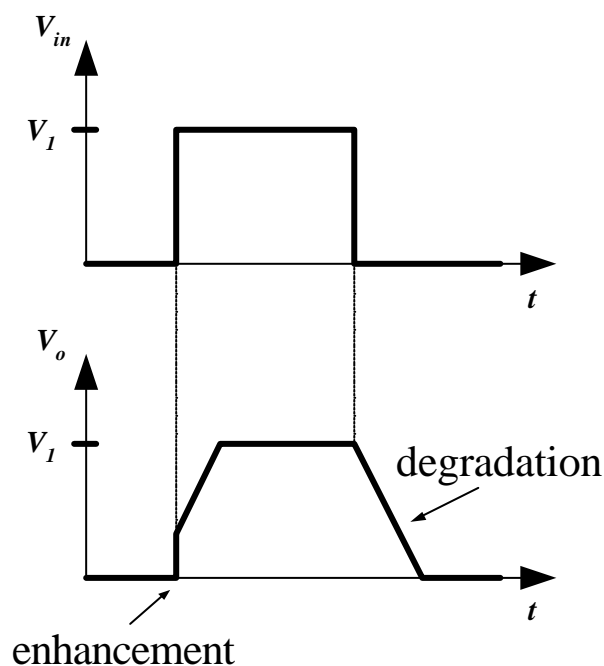


The slew rate $SR = \left. \frac{dV_{out}}{dt} \right|_{max} = I_o / C_c = \frac{I_c \omega_u}{g_{mi}} = \omega_u \sqrt{\frac{I_o}{2\mu C_{ox}(W/L)_i}}$

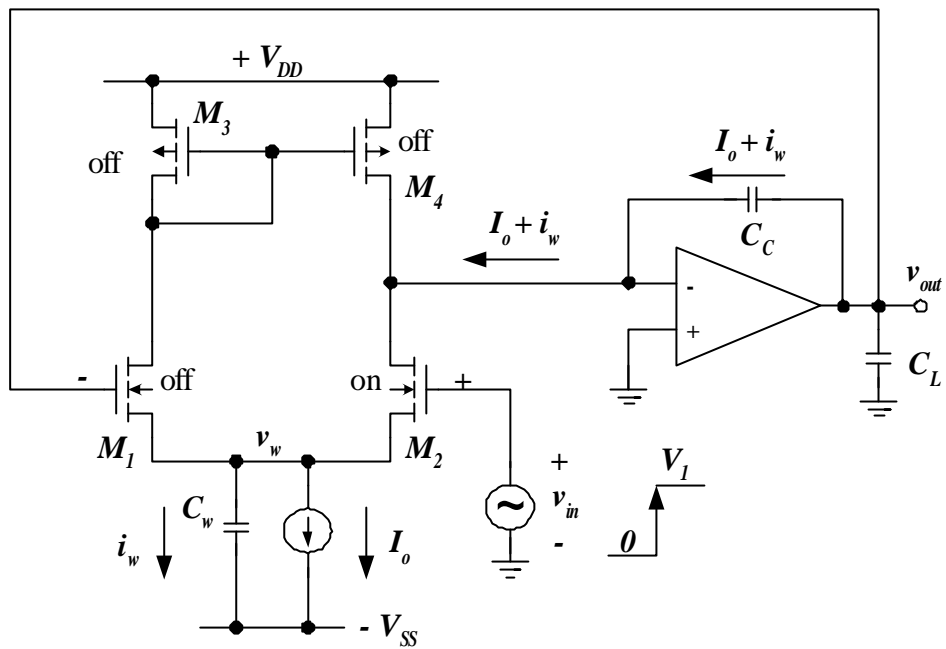
$$\omega_u \uparrow, I_o \uparrow, (W/L)_i \downarrow \Rightarrow SR \uparrow$$

$$* I_o / C_L \geq I_o / C_c \text{ or } C_L \frac{dV_{out}}{dt} \leq C_c \frac{dV_{out}}{dt} (= I_o)$$

Slew rate enhancement and degradation



(1) Positive step

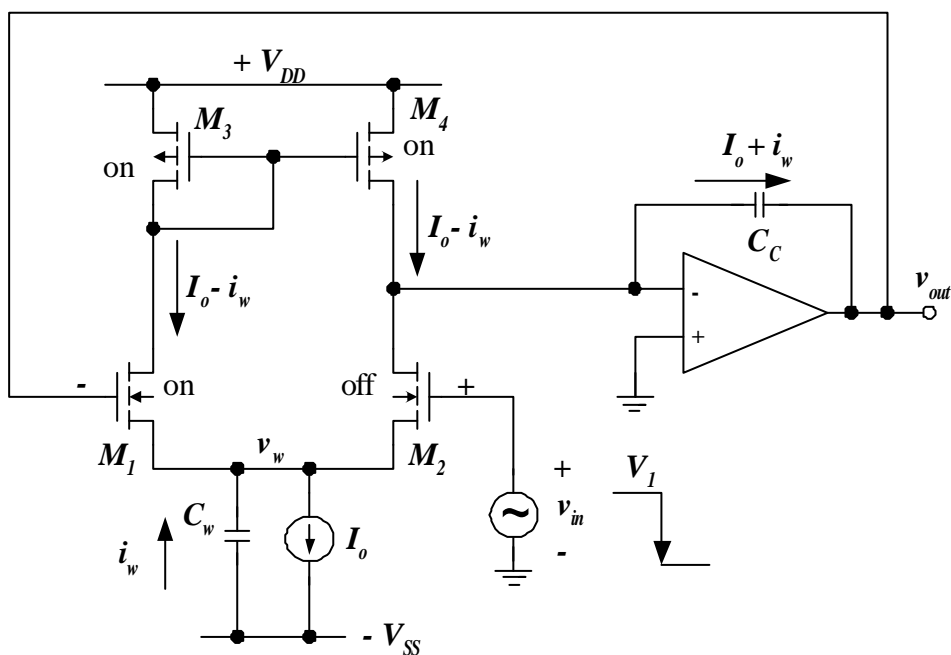


$$i_w(t) = C_w \frac{dv_w(t)}{dt} \cong C_w \frac{dv_{in}(t)}{dt}$$

$$v_{out}(t) = \frac{1}{C_C} \int_0^t (I_o + i_w) dt = \frac{I_o}{C_C} t + \frac{C_w}{C_C} \int_0^t \frac{dv_{in}}{dt} dt$$

$$= \frac{I_o}{C_C} t + \frac{C_w}{C_C} V_I u(t)$$

(2) Negative step



$$v_{out} \cong v_w$$

$$\frac{d}{dt}v_{out} = -\frac{I_o - i_w}{C_c} = \frac{dv_w}{dt} = -\frac{i_w}{C_w} \Rightarrow i_w = \frac{I_o C_w}{(C_c + C_w)}$$

$$\frac{dv_{out}}{dt} = -\frac{I_o}{C_c + C_w} \quad \text{slew degradation}$$

§ 6-4.2 Single-stage OP AMPs

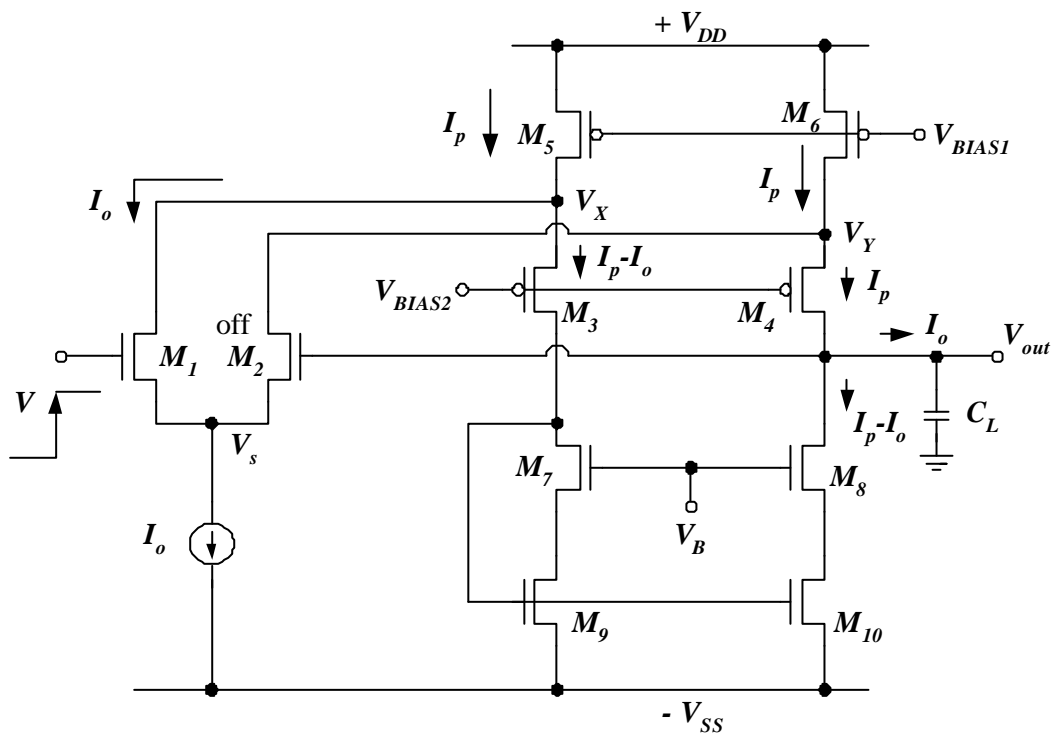
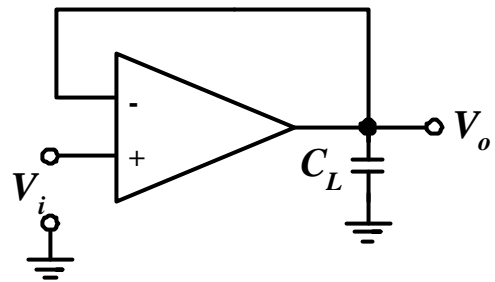
$$SR = \frac{I_o}{C_L}$$

Different phase margins

\Rightarrow different settling behavior.

I_o : First-stage bias current

SR of the folded cascode OP AMPs



$$SR = \frac{I_o}{C_L}$$

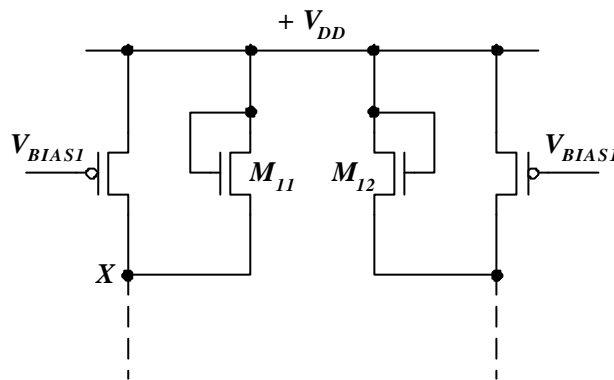
* If $I_p = I_o$, we can keep M_5 , M_1 and I_o current source in saturation.

The change of V_x is not significant because the gain of the common-source amplifier M_1 is nearly equal to -1 . When M_2 is turned on, the recovery time of V_x is very short.

- * If $I_p < I_o$, the current source I_o is forced to linear region and $V_s \downarrow$
 $V_x \downarrow$. The decrease of V_x is large. Thus the recovery time of V_x
when M_2 is turned on is very long, \Rightarrow The settling is slow down.

How to solve this problem?

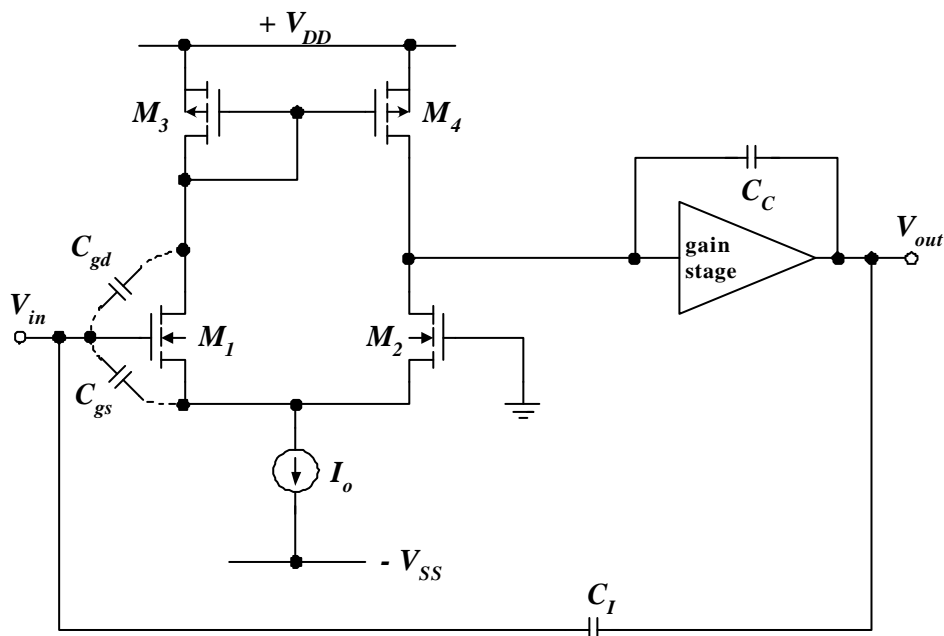
- (1) Keep $I_p = I_o$ as the optimal design.
- (2) Add clamping devices between V_{DD} and $V_x(V_Y)$



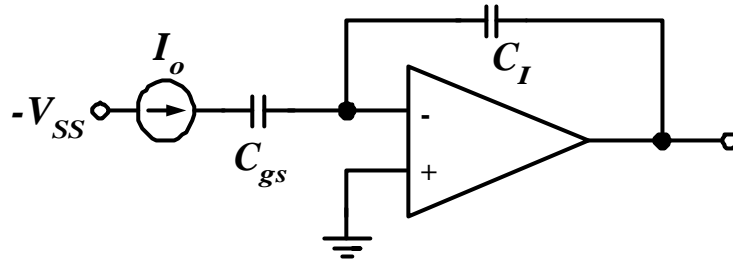
In normal operation, M_{11} and M_{12} are turned off by setting $V_{DD} - V_x < V_{TH11}, V_{TH12}$.

§ 6-5 Power supply rejection ratio (PSRR)

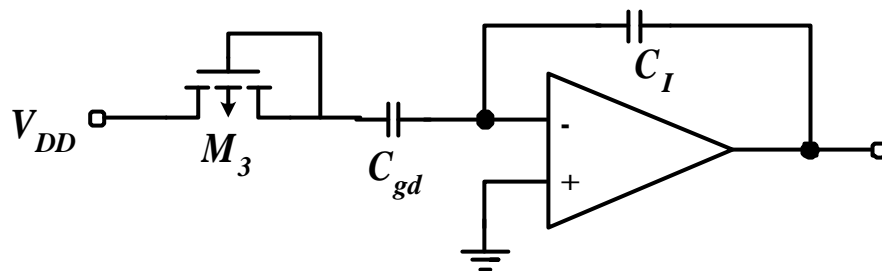
§ 6-5.1 Low frequency analysis for integrators



$$\frac{\partial V_{out}}{\partial V_{SS}} \cong \frac{C_{gs}}{C_I} \left[\frac{\partial I_o}{\partial V_{SS}} \frac{1}{2g_{m1}} + \frac{\partial V_{GS1}}{\partial V_{SS}} \right] + \frac{C_{gd}}{C_I} \frac{1}{2g_{m3}} \frac{\partial I_o}{\partial V_{SS}}$$



$$\frac{\partial V_{out}}{\partial V_{DD}} \cong -\frac{C_{gd}}{C_I} \left[1 - \frac{\partial I_o}{\partial V_{DD}} \frac{1}{2g_{m3}} \right] + \frac{C_{gs}}{C_I} \frac{1}{2g_{m1}} \frac{\partial I_o}{\partial V_{DD}}$$

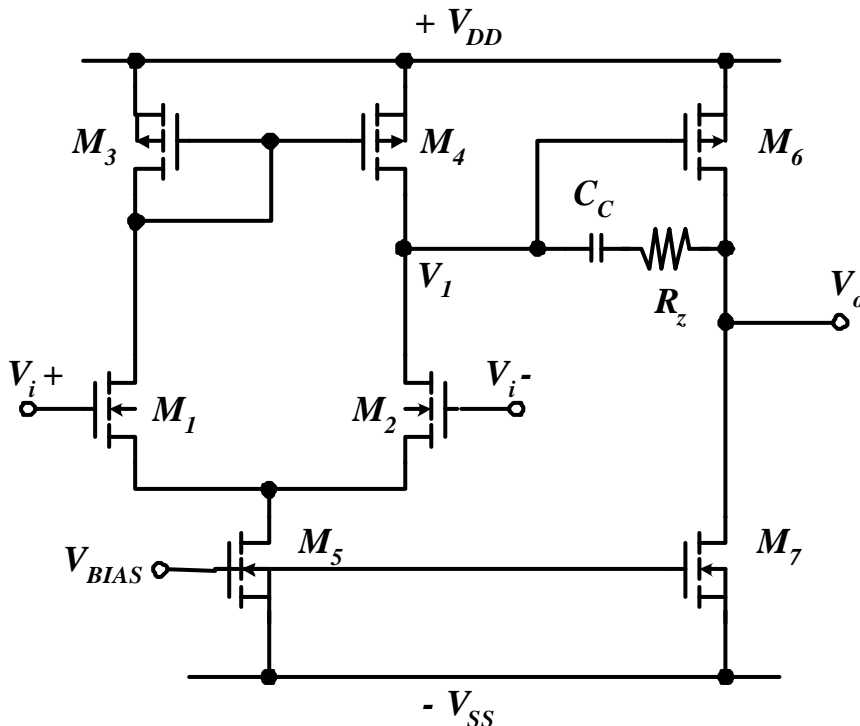


Ref. : IEEE , JSSC , vol.SC-15 , pp.929-938 , Dec. 1980.

- * C_{gs}/C_I and C_{gd}/C_I have a strong effect on $PSRR^+$ and $PSRR^-$.
- * Small $C_I \Rightarrow$ chip area but $PSRR$.

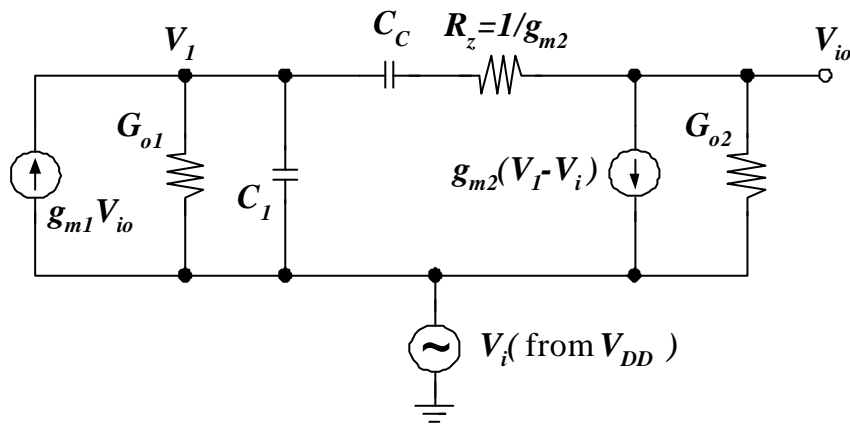
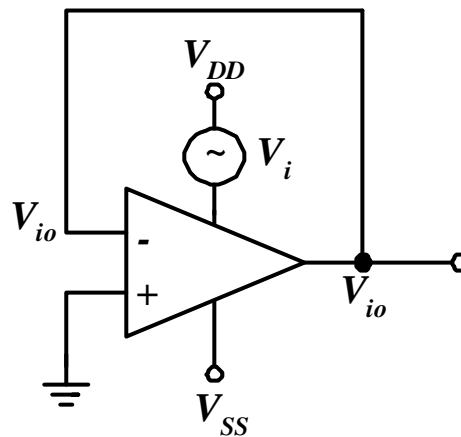
§ 6-5.2 High frequency analysis for OP AMP's

Ref. : IEEE JSSC , vol. sc-19 , pp. 919-925 , Dec. 1984.



$$* \quad PSRR^+ \equiv \frac{\left| \frac{\partial V_o}{\partial V_i} \right|}{\left| \frac{\partial V_o}{\partial V_{DD}} \right|} = \frac{\left| \frac{\partial V_o}{\partial V_i} \right|_{V_o=0}}{\left| \frac{\partial V_o}{\partial V_{DD}} \right|_{V_o=0}} = \left[\frac{\partial V_{io}}{\partial V_{DD}} \right]_{V_o=0}^{-1}$$

How to calculate $\frac{\partial V_{io}}{\partial V_{DD}}$?



$$PSRR^+(s) \cong \frac{s + G_{o1}G_{o2}/(g_{m2}C_c)}{s + g_{m1}/C_c}$$

where $G_{o1}=g_{o4}$ (g_{o2} is connected to the drain of M_5 which is open-circuited, i.e. $r_{ds5} \rightarrow \infty$)

$$G_{o2}=g_{o6} \quad (r_{ds7} \rightarrow \infty)$$

$$G_{o1}G_{o2}/(g_{m2}C_c) < g_{m1}/C_c$$

\Rightarrow Low-frequency LHP zero degrades the $PSRR$.

- * To improve $PSRR$, C_c must be decoupled from the gate of M_6 to eliminate the LHP zero .