32 ⇒ 0100000

100 0000
101 1111
1
0100000

Input	Output connected to ϕ_1	Output connected to ϕ_2
v_1 = input and $v_2 = V_{CM}$	$\frac{z^{-1}}{1-z^{-1}} \cdot \frac{C_I}{C_F}$	$\frac{z^{-1/2}}{1-z^{-1}} \cdot \frac{C_I}{C_F}$
v_2 = input and $v_1 = V_{CM}$	$\frac{-z^{-1/2}}{1-z^{-1}} \cdot \frac{C_I}{C_F}$	$\frac{-1}{1-z^{-1}} \cdot \frac{C_I}{C_F}$
v_1 and v_2 are both inputs	$\frac{V_1(z) \cdot z^{-1} - V_2(z) \cdot z^{-1/2}}{1-z^{-1}} \cdot \frac{C_I}{C_F}$	$\frac{V_1(z) \cdot z^{-1/2} - V_2(z)}{1-z^{-1}} \cdot \frac{C_I}{C_F}$

Table 31.2 Discrete analog integrator input/output relationships (see also Eqs. [31.136] and [31.137]).

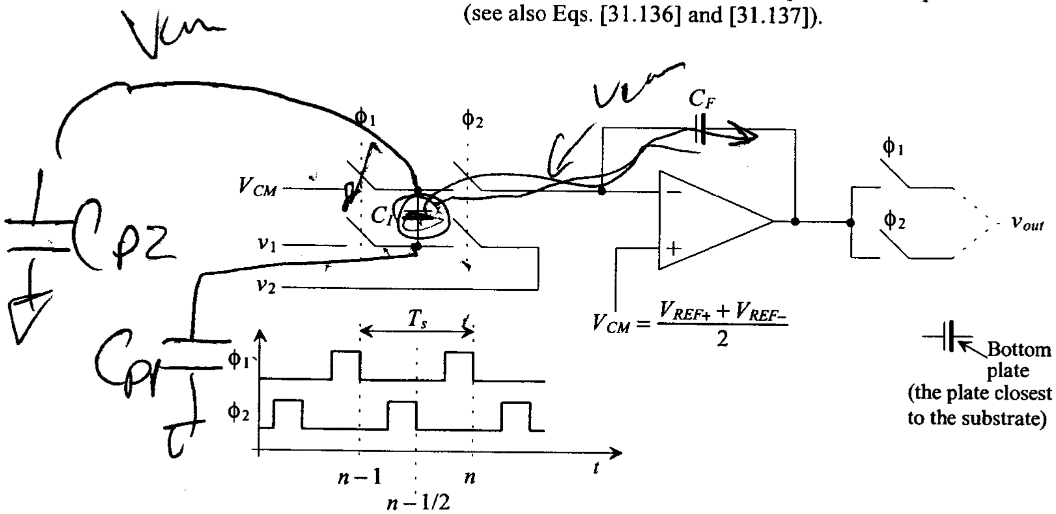
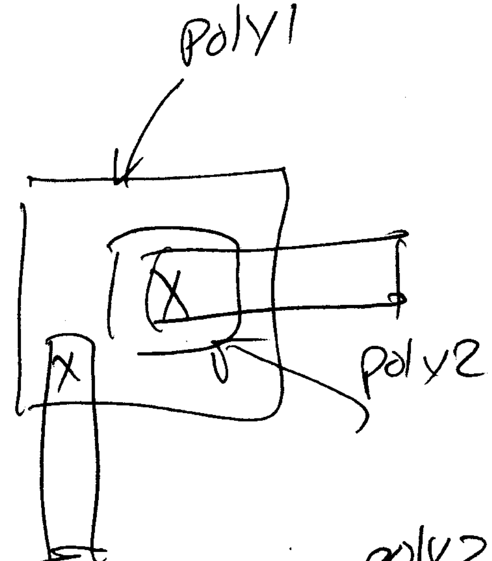
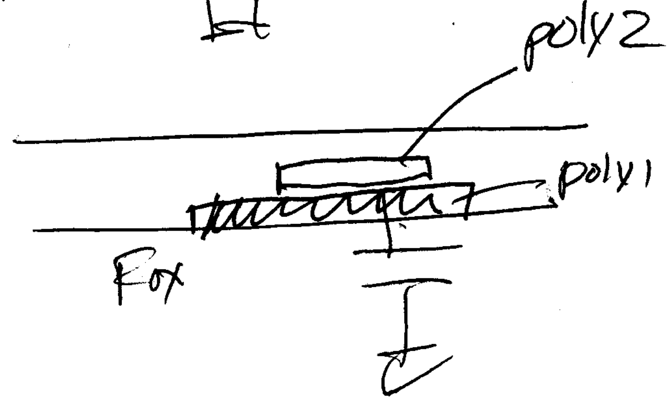
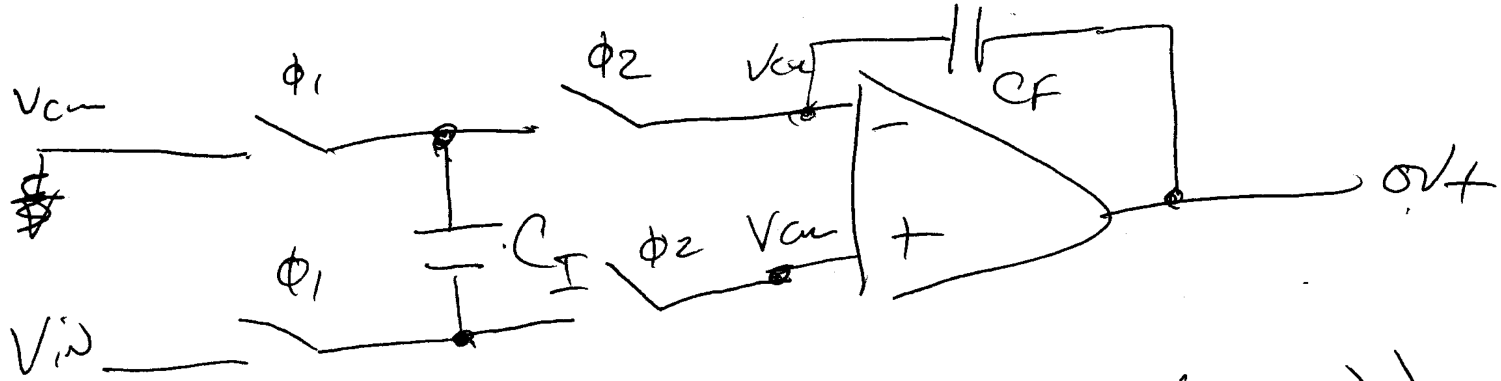


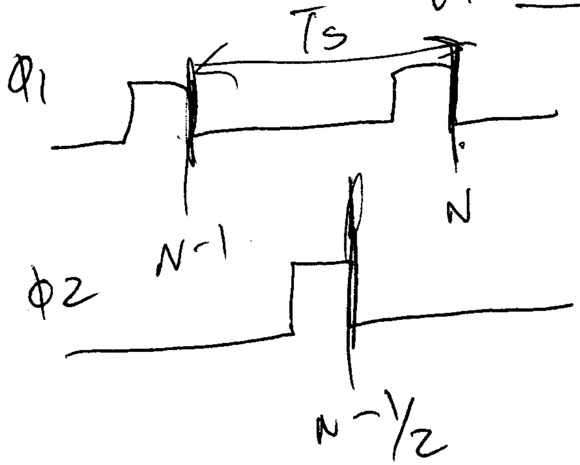
Figure 31.78 Schematic diagram of a discrete analog integrator (DAI).





1)





$$Q_I^{\phi_1} = C_I \cdot \left(V_{cm} - V_{IN}((N-1)T_s)\right)$$

$$Q_I^{\phi_2} = 0$$

$$V_{out}(N-1/2) = V_{out}(N-3/2)T_s + \frac{Q_I^{\phi_1}}{C_F}$$

$$V_{out}(N-1/2) = V_{out}((N-3/2)T_s) + \frac{C_I}{C_F}\left(-V_{IN}(N-1)T_s\right)$$

$$z^{-1/2} V_{out}(z) = V_{out} z^{-3/2} + \frac{C_I}{C_F} z^{-1} V_{IN}(z)$$

$$V_{out}\left(z^{-1/2} - z^{-3/2}\right) = \frac{C_I}{C_F} z^{-1} V_{in}(z)$$

$$\frac{V_{out}(z)}{V_{in}(z)} = \frac{C_I}{C_I} \frac{z^{-1} \cdot z^{1/2}}{z^{-1/2} - z^{-3/2} \cdot z^{1/2}}$$

$$\boxed{= \frac{C_I}{C_F} \frac{z^{-1/2}}{1-z^{-1}}}$$

2)

$$\frac{1}{1-z^{-1}}, \quad \frac{z^{-1}}{1-z^{-1}}$$

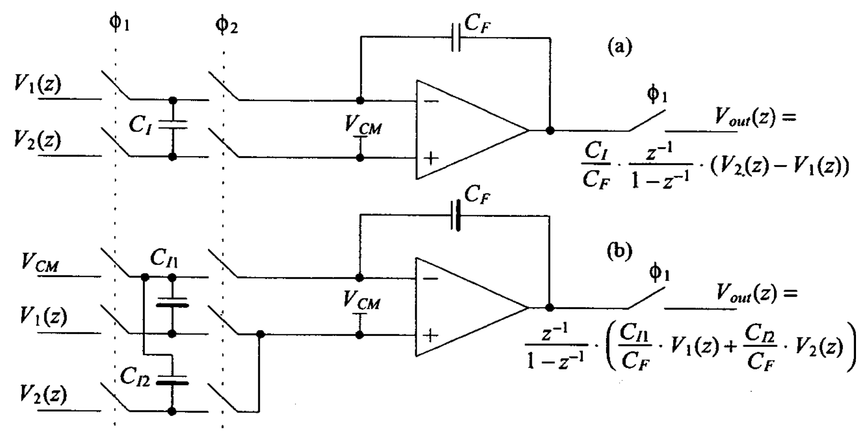


Figure 31.80 Other forms of DAIs.

3)

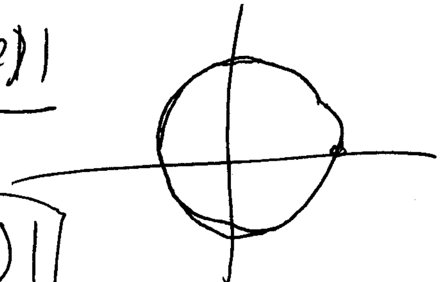
Name: Key

EE 515 CMOS Mixed-Signal IC Design Midterm exam Spring 2003

1) If a digital system's input, $X(z)$, is z how are the magnitude of system's output, $Y(z)$, and transfer function, $H(z)$, related in the frequency domain? (How are $Y(f)$ and $H(f)$ related?) (10 points)

$$\frac{Y(z)}{X(z)} = H(z) \qquad \text{If } X(z) = z \qquad X(f) = 1\angle 2\pi\frac{f}{f_s}$$

$$x(f) \rightarrow \boxed{H(f)} \rightarrow Y(f) \qquad |H(f)| = \frac{|Y(f)|}{1}$$



varying f

$$\boxed{|H(f)| = |Y(f)|}$$

2) Show that a discrete integrator frequency response is the same as an analog integrator's response if $f << f_s$ (where f is the frequency of our input sinusoids and f_s is the clock frequency). That is, show

$$\frac{z^{-1}}{1-z^{-1}} \approx \frac{1}{j2\pi \cdot \frac{f}{f_s}} = \frac{f_s}{s} \text{ where } s = j\omega \qquad \text{(15 points)}$$

$$\frac{1}{z-1} \qquad z = e^{j\theta} = \cos\theta + j\sin\theta \qquad \theta = 2\pi\frac{f}{f_s}$$

$$e^{x}\Big|_{x \text{ small}} \simeq 1 + x \qquad \frac{1}{\cos\theta + j\sin\theta - 1} \simeq \frac{1}{2\pi f/f_s}$$

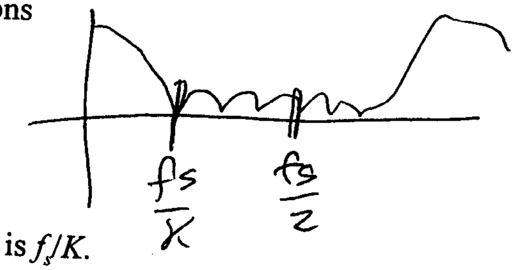
$$f << f_s$$

$$\cos\theta \simeq 1$$

$$\frac{1}{1 + j2\pi\frac{f}{f_s} - 1} = \frac{1}{j2\pi\frac{f}{f_s}} \qquad \sin 2\pi\frac{f}{f_s} \simeq 2\pi\frac{f}{f_s}$$

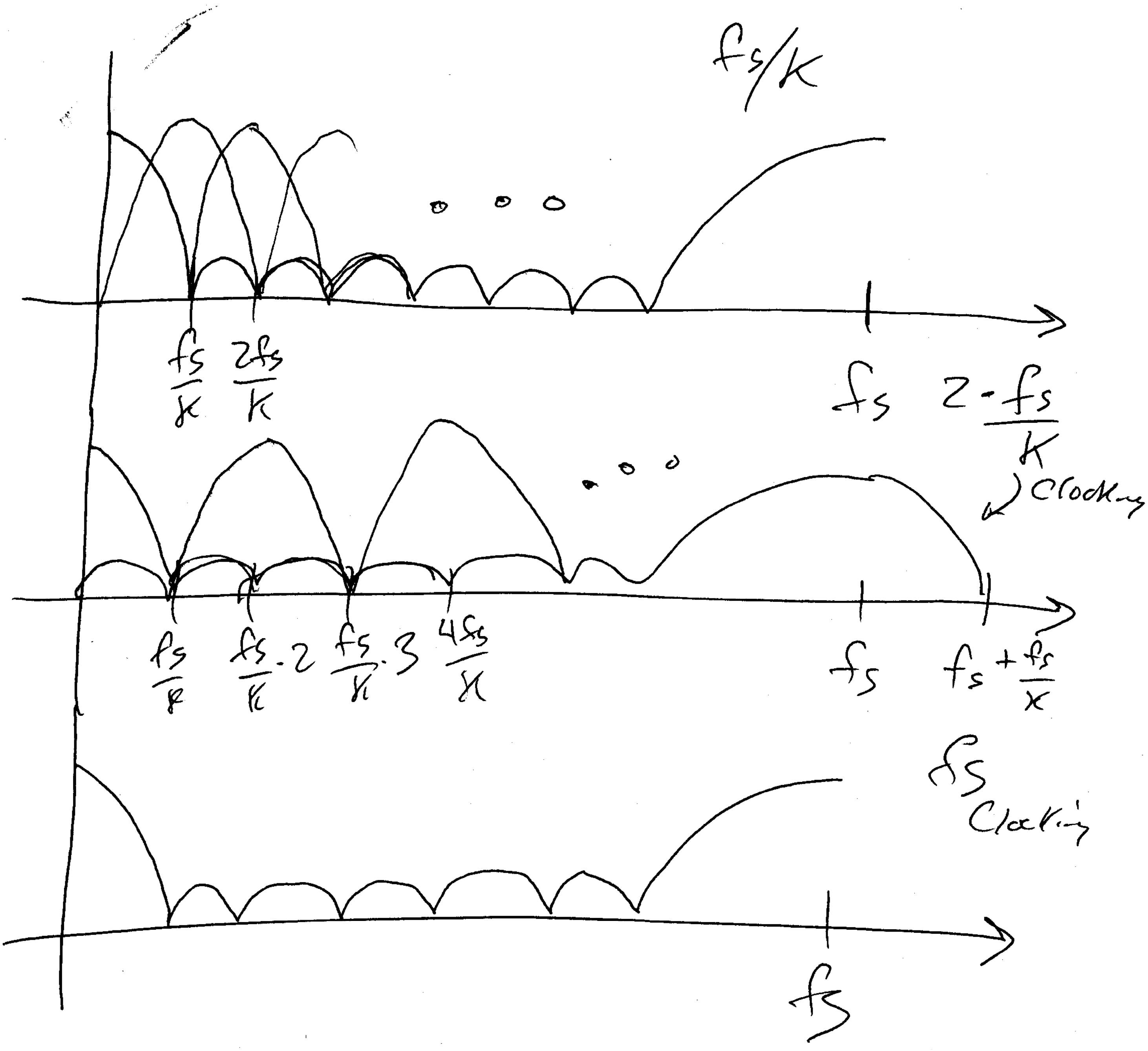
3. (25 points) Consider three digital filters with transfer functions

$$H(z) = \frac{1 - z^{-K}}{1 - z^{-1}}$$



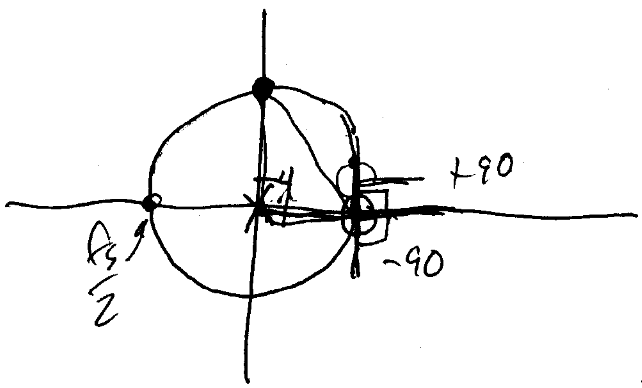
In all three filters the input digital word rate is f_s.

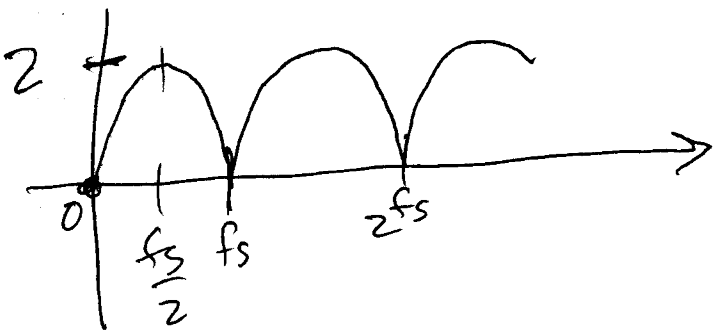
In the first filter, an accumulate and dump, the output word rate is f_s/K.
In the second filter the output word rate is $2f_s/K$.
In the third filter the output word rate is f_s.
Sketch the frequency response showing aliasing, if potentially present, for each filter. Comment on the benefits and drawbacks of each filter.



4.(25 points) Sketch the z-plane pole/zero plot, magnitude response, and phase reponse for the following transfer function:

$$H(z) = z^{-3} - z^{-4} = \frac{z-1}{z^4}$$

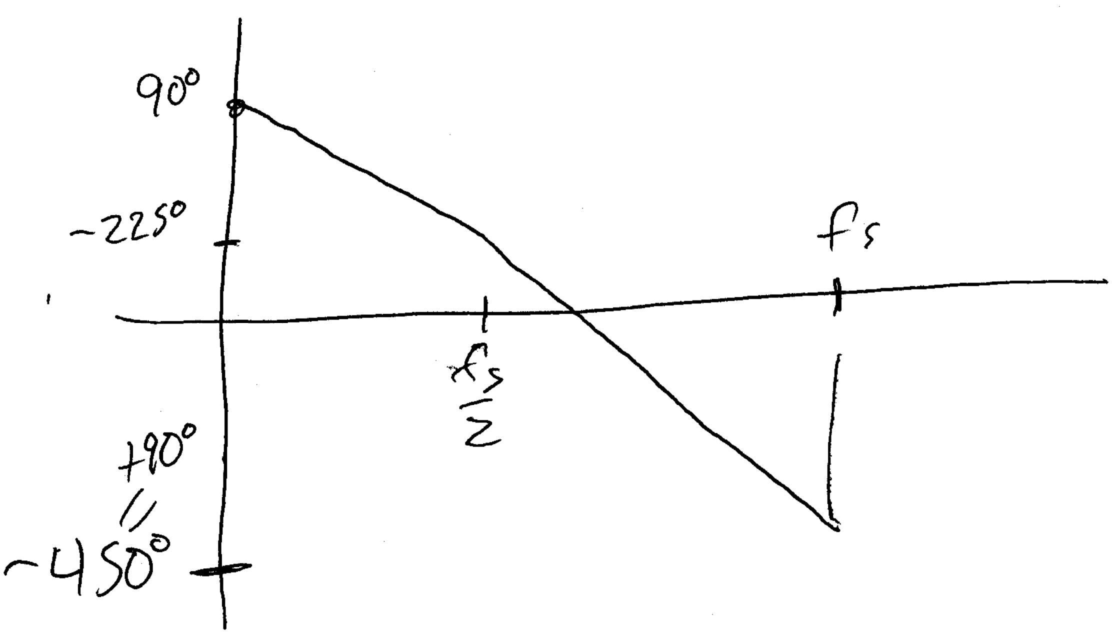




$\angle = \angle \text{zeroes} - \angle \text{poles}$

at DC $\quad \angle = 90 - 4 \cdot 0 = 90^\circ$

at $f_s/4$ $\quad \angle = 135 - 4 \cdot 90 = -225^\circ$

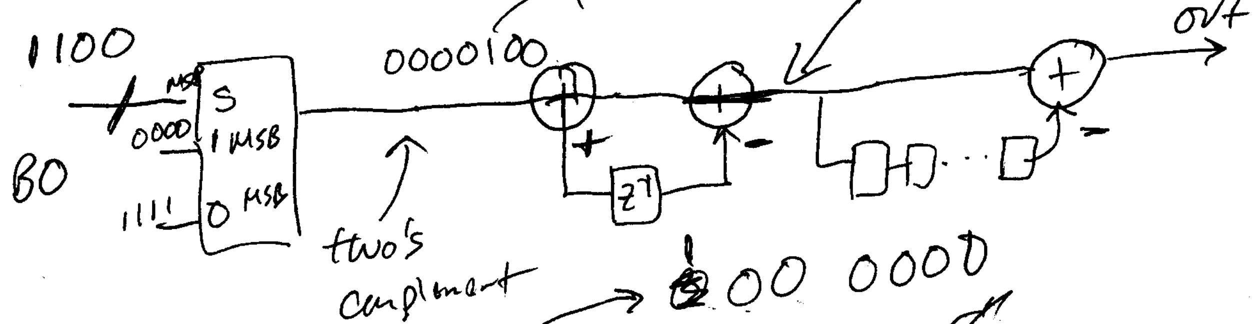


5. (25points) Show, using two's complement numbers, that the following filter can be implemented without overflow concerns. Assume a 4 bit offset binary word of 1100 (12) is applied to the filter.

$$H(z) = \frac{1-z^{-8}}{1-z^{-1}}.$$

32 0010000 t.c.

(Hint: the output word size is 7-bits and ...why?) 4



→ 1 00 0000

~~56~~ 56

60

N = ~~2~~ ~~1~~

N	X'	
N=1	4	000 0100
2	8	
3	12	
4	16	
5	20	
6	24	
7	28	0011100
8	32	
9	36	0100100
10	40	
11	44	
12	48	
13	~~48~~ 52	

63
0111111

X'	
N = 14	
→N = 15	
N = 16	64
17	
18	
19	
20	
21	
22	
23	
24	
25	
26	
27	
28	
29	
30	

0100100 (36)
1111011 (4)
1

~~0100000~~

0111100 (60)
−0011100 (28)

0111100
1100011
1

~~0100000~~

120