

**Electro-Thermal CAD of Power Devices and Circuits with
Fully Physical Time-Dependent Compact Thermal Modelling
of Complex Non Linear 3-Dimensional Systems**

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Abstract

An original, fully analytical, spectral domain decomposition approach is presented for the time-dependent thermal modelling of complex non linear 3-dimensional electronic systems, from metallised power FETs and MMICs, through MCMs, up to circuit board level. This solution method offers a powerful alternative to conventional numerical thermal simulation techniques, and is constructed to be compatible with explicitly coupled electro-thermal device and circuit simulation on CAD timescales. This thermal solution includes the first full treatment in coupled electro-thermal CAD, of thermal non linearity due to temperature dependent diffusivity. In contrast to semi-analytical, frequency space, Fourier solutions involving DFT-FFT, the method presented here is based on explicit, fully analytical, double Fourier series expressions for thermal subsystem solutions in Laplace transform s -space (complex frequency space). It is presented in the form of analytically exact thermal impedance matrix expressions for thermal subsystems. These include double Fourier series solutions for rectangular multilayers, which are an order of magnitude faster to evaluate than existing semi-analytical Fourier solutions based on DFT-FFT. They also include double Fourier series solutions for the case of arbitrarily distributed volume heat sources and sinks, constructed without the use of Green's function techniques, and for rectangular volumes with prescribed fluxes on all faces, removing the adiabatic sidewall boundary condition. This combination allows treatment of arbitrarily inhomogeneous complex geometries, and provides a full description of thermal material non linearities as well as inclusion of position varying and non linear surface fluxes. It provides a fully physical, and near exact, generalised multiport network parameter description of non linear, distributed thermal subsystems, in both the time and frequency domains. In contrast to existing circuit level approaches, it requires no explicit lumped element, RC-network approximation or nodal reduction, for fully coupled, electro-thermal CAD. This thermal impedance matrix approach immediately gives rise to minimal boundary condition independent compact models for thermal systems. Implementation of the time-dependent thermal model as N-port netlist elements within a microwave circuit simulation engine, Transim (NCSU), is described. Electro-thermal transient, single-tone, two-tone and multi-tone harmonic balance simulations are presented for a MESFET amplifier. This thermal model is validated experimentally by thermal imaging of passive grid arrays representative of one form of spatial power combining architecture.

Nomenclature

AC	alternating current
ACPR	adjacent channel power ratio
BCI	boundary condition independent
CAD	computer aided design
CDMA	code division multiple access
DC	direct current
DFT-FFT	discrete Fourier transform, fast Fourier transform
EM	electromagnetic
FDTD	finite difference time domain
FET	field effect transistor
FETD	finite element time domain
HB	harmonic balance
HEMT	high electron mobility transistor
IC	integrated circuit
MCM	multi-chip module
MESFET	metal semiconductor field effect transistor
MMIC	monolithic microwave integrated circuit
MNAM	modified nodal admittance matrix
MOSFET	metal oxide semiconductor field effect transistor
RF	radio frequency
SPICE	simulation programme with integrated circuit emphasis
USE	unsteady surface element
a, b, c	simultaneously piecewise constant functions, Eq. (36)
$A_{mn}, B_{mn}, C_{mn},$ D_{mn}, E_{mn}, F_{mn}	double Fourier series expansion coefficients
C	specific heat
D	z -dimension of rectangular subvolume
D_i	domain corresponding to heating element, i
D_r	z -coordinate of r^{th} interface in N -level multilayer, $r = 1, \dots, N$ (with $D_0 = 0$)
$g, g(x, y, z, \tau)$	rate of heat generation
$\bar{g}(x, y, z; s)$	Laplace transform of $g(x, y, z, \tau)$

$G_{mn}(z)$	source term in 1-dimensional Helmholtz equation for $Z_{mn}(z)$, Eq. (31)
H	surface heat flux coefficient
$H_{0,D}$	surface heat flux coefficient, H , at surfaces $z = 0, D$, in 'radiation' boundary condition, Eq. (7)
i_{ds}	AC drain-source current
I_{ds}	DC drain-source current
I_{mn}^i	double integral over elementary area, D_i , Eq. (20)
I_{mn}^{0i}, I_{mn}^{Dj}	double integrals of the form, Eq. (20), over elementary areas D_i and D_j , on respective faces $z = 0$ and $z = D$, of rectangular subvolume
i, j	integer indices running over elementary heating elements
k	thermal diffusivity, $k = \kappa/\rho C$
k_S	temperature independent thermal diffusivity after time variable transformation
k_r	thermal diffusivity of r^{th} layer in N-level multilayer
L	x -dimension of rectangular subvolume
\mathcal{L}^{-1}	inverse Laplace transformation
l, m, n	summation indices taking values 0, 1, 2, ...
$\underline{\underline{M}}^{(r)}$	2×2 analytically obtained transfer matrix, Eq. (41)
$P_i(\tau)$	time-dependent power dissipation in elementary area, D_i
\overline{P}_i	Laplace transformed power dissipation in elementary area, D_i
$P_j^{(n)}$	values of power dissipation, $P_j(\tau)$, assumed piecewise constant, in element, j , at time step $n\delta\tau$
\underline{P}^a	vector of Laplace transformed power dissipations at active elements, a , of metallised GaAs die, Eq. (44)
$p_{0,D}(x, y, \tau)$	time dependent imposed flux densities at surfaces $z = 0, D$, in 'radiation' boundary condition, Eq. (7)
$\overline{p}_{0,D}(x, y; s)$	Laplace transform of $p_{0,D}(x, y, \tau)$
r	index describing layers of N-level multilayer, $r = 1, \dots, N$
$R_{THij}(s)$	thermal impedance matrix in complex frequency space (N-port impulse response)
$R_{THij}(\tau)$	time domain thermal impedance matrix (N-port step response) $\mathcal{L}^{-1} \{ R_{THij}(s) \frac{1}{s} \}$

$R_{TH_{ii'}}^{00}, R_{TH_{ij}}^{0D}, R_{TH_{ji}}^{D0}, R_{TH_{jj'}}^{DD}$	thermal impedance matrices corresponding to rectangular volume discretised on both top and bottom surfaces, $z = 0, D$, Eq. (43)
$\underline{\underline{R}}_{TH}^{aa}, \underline{\underline{R}}_{TH}^{ai}, \underline{\underline{R}}_{TH}^{ia}, \underline{\underline{R}}_{TH}^{ii}$	thermal impedance matrix, $R_{TH_{ij}}(s)$, for metallised GaAs die, partitioned by active elements, a , and interface elements, i
$\underline{\underline{R}}_{TH}^{DD(n)}$	thermal impedance matrix for the n^{th} piece of surface metallisation, Eq. (43)
$\underline{\underline{R}}_{TH}^{glob}$	global thermal impedance matrix for metallised GaAs die, Eq. (44)
R_i	amplitude in time constant representation, Eq. (26)
s	Laplace transformation variable (complex frequency)
$S_i(x, y)$	2-dimensional step function, equal to 1 in elementary areas, D_i , and 0 otherwise
S_{mn}, C_{mn}	double Fourier series expansion coefficients
t	time
T	temperature
T_S	heatsink mount temperature in Kirchhoff transformation
$u(\tau)$	unit step function
v_{ds}	AC drain-source voltage
V_{ds}	DC drain-source voltage
W	y -dimension of rectangular subvolume
x, y, z	Cartesian coordinates
x_j, y_j	x - and y -coordinates of surface points, ($z = 0$), defining thermal transfer impedance in N-level multilayer solution, Eq. (38)
z_{i1}, z_{i2}	z -coordinates of bounding planes of elementary rectangular power dissipating volume, i , with cross-section, D_i
$Z_{mn}(z)$	z -terms in generalised double Fourier series expansion, Eq. (30)
$z_{mn}(z)$	transformation of $Z_{mn}(z)$ to solve Helmholtz equation, Eq. (31)
$\alpha_{0,D}$	coefficient α equal to 0 (imposed temperature boundary condition) or 1 (imposed flux boundary condition), at surfaces $z = 0, D$, in 'radiation' boundary condition, Eq. (7)
α_{mn}, β_{mn}	coefficients of transfer matrix product, Eq. (39)
$\delta\tau$	elementary interval in transformed time
δ_{mn}	Kronecker delta function
$\overline{\Delta\theta}_i$	Laplace transformed rise of linearised temperature for heating element, i

$\Delta\theta_i^{(m)}$	linearised temperature rise in element, i , at time step $m\delta\tau$
$\underline{\Delta\theta}^a$	vector of Laplace transformed, linearised temperature rises at active elements, a , of metallised GaAs die, Eq. (44)
$\Delta\zeta$	intervals in $\ln \tau$, Fig. 3
γ_{mn}	$(\lambda_m^2 + \mu_n^2 + s/k_S)^{1/2}$, Eq. (12)
Γ_{mn}	$(\lambda_m^2 + \mu_n^2)^{1/2}$, Eq. (24)
$\gamma_{mn}^{(r)}$	γ_{mn} of r^{th} layer in N-level multilayer, $(\lambda_m^2 + \mu_n^2 + s/k_r)^{1/2}$, Eq. (40)
$\kappa(T)$	temperature dependent thermal conductivity
κ_S	temperature independent thermal conductivity after Kirchhoff transformation, $\kappa_S = \kappa(T_S)$
κ_r	thermal conductivity of r^{th} layer in N-level multilayer
λ_m	$m\pi/L$, Eq. (12)
μ_n	$n\pi/W$, Eq. (12)
ω	angular frequency
θ	temperature linearised by Kirchhoff transformation, Eq. (2)
$\bar{\theta}$	Laplace transform of θ
θ_{amb}	ambient temperature
$\bar{\theta}_{av_i}$	Laplace transformed linearised temperature averaged over elementary area, D_i
$\theta(\tau = 0)$	initial temperature distribution
$\bar{\Theta}$	shifted temperature, $s\bar{\theta} - \theta(\tau = 0)$, Eq. (9)
$\theta_{0,D}(x, y, \tau)$	time dependent ambient temperatures ($\alpha_{0,D} \neq 0$), or heatsink mount temperatures ($\alpha_{0,D} = 0$), at surfaces $z = 0, D$, in ‘radiation’ boundary condition, Eq. (7)
$\bar{\theta}_{0av_i}, \bar{\theta}_{Dav_j}$	Laplace transforms of linearised temperatures averaged over elementary areas, D_i , and D_j , on respective faces $z = 0$ and $z = D$, of rectangular subvolume
ρ	density
\sum'_{mn}	sum over all m, n excluding $(m, n) = (0, 0)$, Eq. (24)
τ	transformed time, Eq. (4)
τ_i	time constant, Eq. (26)
ζ	$\ln \tau$, Fig. 3
$\zeta_{mn}(z)$	derivative of $z_{mn}(z)$, dz_{mn}/dz , to reduce Helmholtz equation, Eq. (31), to first order

I. INTRODUCTION

The impact of self-heating and mutual thermal interaction on electronic device and integrated circuit performance is well known, and the electro-thermal simulation problem has been studied for at least 30 years [1]–[3]. The thermal modelling of complex 3-dimensional structures can be achieved by standard numerical techniques, and solutions of the heat diffusion equation for complex 3-dimensional systems are commonly based on finite volume, finite element, finite difference, boundary element or transmission line methods. All of these approaches require construction of a volume or surface mesh. Such solutions have been combined with circuit simulators for joint electro-thermal simulation [4]. However, they are computationally intensive and therefore generally too slow for treatment of large systems, by direct coupling to electronic device and circuit simulators, in the necessarily iterative solution of intrinsically non linear coupled electro-thermal problems. Even numerical solutions optimised for thermal treatment of electronic devices and circuits, *e.g.*, [5], which is based on hierarchical nesting to treat the wide range of length and time scales inherent in the coupled electro-thermal problem, or [6], based on successive node reduction for complex inhomogeneous 3-dimensional structures, are not fast enough for directly coupled electrical and thermal solution. Thus a number of faster thermal descriptions have been developed.

The simplest thermal models for the time-independent case are provided by analytical thermal resistance approaches of varying levels of complexity, *e.g.* [7]–[9]. However, it has been stated repeatedly [10], [11] that the thermal resistance approach is fundamentally approximate and inadequate for detailed description of power devices. Until recently, the state of the art in time-independent thermal simulation of heatsink mounted power FETs and MMICs, for coupled electro-thermal CAD, has been represented by the hybrid finite element Green’s function approach of Bonani *et al.* [12]. This thermal resistance matrix approach treats device structure such as surface metallisation, vias and partial substrate thinning.

The thermal resistance and impedance matrix approach is an example of thermal network extraction. For treatment of complex structures without direct coupling of electrical and thermal simulators, generation of thermal networks for both time-independent and time-dependent electro-thermal co-simulation has been described by, *e.g.*, Lee and Allstot [13], Hefner and Blackburn [14], and Hsu and Vu-Quoc [15]. However, these approaches are all based on spatial discretisation of the thermal system. They can produce thermal networks containing a large number of nodes, are inherently approximate through finite difference [14] or finite element [15] discretisation, and can invoke simplifications to the full 3-dimensional thermal solution, by solving the heat diffusion equation only in a reduced dimensional form and without treating detailed device structure such as die surface metallisation [14].

In contrast, analytical solutions can be fully 3-dimensional and avoid approximations due to spatial discretisation. However, in time-dependent coupled electro-thermal CAD, fast analytical 3-dimensional thermal descriptions have been less able to describe complex structures, and have been limited to simple rectangular multilayers. Analytical thermal

impedance expressions have been presented, for instance, for MOSFETs [16]. Veijola has implemented an approximate thermal impedance description, based on analytical solution for heat-generating spheres, in circuit simulation programme APLAC [17]. Rizzoli has employed a Green's function construction of the thermal resistance matrix in a wide range of circuit level, harmonic balance and transient simulations, but with thermal capacitances described approximately based on an enthalpy formulation [18]. Analytical Green's function and Fourier solutions have been used to describe the time-dependent thermal problem by a number of authors. In particular, Szekely *et al.* have employed a Fourier series method for over 20 years, providing solutions for a variety of ICs, microsystem elements and MCMs [19].

For circuit level electro-thermal simulations, thermal model reduction techniques have been widely employed. Work in this area includes that of Sabry [20], Napieralski [21], Hsu [22] and Szekely [23], and Szekely has combined fast sparse finite difference methods, [6], with lumped element RC network extraction, [23], to provide coupled electro-thermal simulation of complex 3-dimensional systems. The related problem of compact model development is currently an active area of research [24], [25].

The aim of this paper is to describe a new, fully physical and analytical, approach to the non linear time-dependent thermal problem in complex 3-dimensional systems, suitable for explicitly coupled electro-thermal device and circuit simulation on CAD timescales, but requiring essentially no model reduction. To illustrate the advantages of this approach, particular comparison is made with the comprehensive circuit level electro-thermal modelling capability of Szekely *et al.* Fully physical, coupled electro-thermal device simulations for the thermal time-independent and time-dependent cases, have been described by the authors previously [26]–[32]. These were based on coupling of the thermal model presented here, to the quasi-2-dimensional Leeds Physical Model of MESFETs and HEMTs [33]–[36]. Coupling to a microwave circuit simulator Transim (NCSU) [37], has been introduced in [38]–[41].

Generically, the thermal approach presented in this paper, is a fully analytical spectral domain decomposition technique [42]. Simple composite systems have been treated previously by the Unsteady Surface Element (USE) method of Beck *et al.*, [43], and this approach has the advantage that, unlike conventional numerical methods, it only discretises interfaces between subsystems. Like the USE method, the approach presented here discretises only interfaces (along with power dissipating and temperature sensitive elements). It constructs solutions for thermal subvolumes which are fully analytical, with development of double Fourier series solutions for thermal subvolumes by explicit construction of series expansion coefficients. Thus it differs from semi-analytical Fourier approaches for simple rectangular multilayers [19], based on collocation or function sampling, which require numerical manipulation such as DFT-FFT to generate expansion coefficients. As solutions for subvolumes are fully analytical, no volume or surface mesh is required. The method described is a thermal impedance matrix approach. This time-dependent thermal impedance matrix formulation is a natural development of the authors' fully analytical implementation of the thermal resistance matrix approach for the time-independent case, described in [26], [27] and developed fully in [28]. It is shown that, in contrast to previous thermal resistance and impedance approaches, this thermal impedance matrix method can be formulated to provide an

essentially exact solution of the heat diffusion equation in complex 3-dimensional systems. It therefore removes the need to utilise computationally intensive numerical techniques in order to treat complex structures, *e.g.* [4], [20], [21], [44]. Previous Green's function or Fourier approaches have been restricted to simple rectangular homogeneous volumes and multilayers, *e.g.* [3], [18], [23], [45]. The fully analytical model presented here can describe simultaneously all device detail, from surface metallisation, vias and substrate thinning, in power FETs and MMICs, through (actively cooled) MMIC on substrate arrays, up to MCMs and circuit board level. It does this by providing double Fourier series solutions of the heat diffusion equation in thermal subsystems, and then constructing global solutions for complex systems by matching temperature and flux at subsystem interfaces. As the subsystem solutions are matrix expressions, an explicit matrix representation can be obtained for the global thermal impedance matrix of the complex device structure. A particular intended application is the treatment of MMIC arrays for spatial power combining at millimeter wavelengths.

Importantly, this modular thermal solution is constructed to be immediately compatible with explicitly coupled electro-thermal device and circuit simulation on CAD timescales. This is achieved by formulating the analytically exact subsystem solutions in terms of thermal impedance matrices which describe temperature variation with time, only in the vicinity of the power dissipating, temperature sensitive, and interface regions required for coupling of the electrical and thermal problems. No redundant temperature information is generated on the surface or in the body of the subsystem volumes. As these minimal thermal solutions are generated analytically, the thermal impedance matrices are all precomputed, prior to the coupled electro-thermal simulation, purely from structural information. Thermal updates in the coupled electro-thermal problem are therefore rapid.

Fully coupled device level simulation can be implemented by combination of the Leeds thermal impedance matrix model with any thermally self-consistent device model. If the device model includes self-heating effects, then the global thermal impedance matrix will provide an accurate, CAD timescale, description of mutual thermal interaction between power dissipating elements, however complex the thermal system. Coupled electro-thermal solution is achieved by iterative solution of the electrical and thermal problems, with thermal updates provided by small matrix multiplications, and thermal non linearity transferred to the already non linear active device model.

Circuit implementation of this thermal solution, exploits the ability of network based microwave circuit simulators to describe multiport non linear elements in the time domain, and to treat distributed electromagnetic (EM) systems in terms of multiport network parameters [46]. The thermal solution makes use of the close analogy between distributed EM and thermal systems. The magnetic vector potential wave equation in frequency space, is just the Helmholtz equation, as is the time-dependent heat diffusion equation in Laplace transform s -space (complex frequency space), after appropriate exact transformation of thermal non linearity. Double Fourier series thermal solutions resemble analytical EM Green's function solutions (and the same series acceleration techniques can be used in each case). Most importantly, complex EM systems are treated by segmentation [47] and cascading of subsystem solutions by use of network parameter matrices. The transformed (initially non linear) thermal problem is therefore immediately compatible with network based microwave

circuit simulation engines, by interpretation of thermal impedance matrices, for distributed thermal subsystems, in terms of generalised multiport network parameters.

Analytical, s -space solution for thermal subsystems, means that no numerical identification of thermal networks, such as that provided by the NID method [48], is required. It also means that each thermal subsystem can be described directly, in either the time domain or in the frequency domain, by the same analytical solution. The s -space thermal impedance matrix method is therefore more general than time-dependent EM waveguide descriptions, which are conventionally formulated in frequency space (time-dependent harmonic steady-state). In the time domain, the thermal subsystem is treated as a non linear multi-port element, which readily allows non linear matching of transformed temperatures at thermal subsystem interfaces [27], [28]. In the frequency domain, the thermal subsystem is represented by a matrix of complex phasors inserted into the modified nodal admittance matrix (MNAM) for the microwave system, and thermal non linearity is again transferred to the already non linear active device model. This gives coupled electro-thermal harmonic balance and transient solutions on CAD timescales. Coupled electro-thermal circuit level CAD generally requires thermal model reduction, *e.g.*, [20]–[23]. Rapidly convergent, fully analytical and minimal thermal impedance matrix expressions, in both the time and frequency domains, mean that no reduced, lumped element, RC network description, is required in the multiport network parameter approach. The thermal impedance matrices represent boundary condition independent compact models of thermal subsystems [24], [25] for the time-dependent case, and with full treatment of thermal non linearity. Analytical expressions for the multiport network parameters of all thermal subvolumes means that no distinct thermal simulator, separate from the coupled electro-thermal simulation engine, is required to characterise the complex thermal system.

A key aspect of the thermal solution presented here, is application of a generalised ‘radiation’ boundary condition, on the top and bottom surfaces of all thermal subvolumes, in the analytical subsystem solutions. This boundary condition allows analytical subsystem solutions with interface discretisation, and construction of global thermal solutions by vertical matching of temperatures and fluxes at subsystem interfaces. The boundary condition also allows integral treatment of surface radiation and convection in large area systems without approximation such as that invoked in [19]. (Radiation boundary conditions have also been applied, for example, in finite difference solutions for electro-thermal simulation [44], and in analytical solutions at the circuit board level [45].) More comprehensive thermal solutions are also described, which remove the need for an adiabatic sidewall boundary condition in subvolumes, allowing additional horizontal interface matching. Such solutions also allow subvolume embedding for the treatment of inhomogeneous structures. The range of application of this thermal impedance matrix approach is therefore again more general than that of conventional EM waveguide formulations. Analytical solution with prescribed fluxes on all subvolume surfaces allows treatment of complex packages. By dropping the surface convection coefficient from the surface boundary conditions, it includes the case of position varying surface flux, by connection of thermal surface nodes to ambient through thermal resistances describing piecewise constant surface flux defined over the whole subvolume surface.

One aim of this paper is to present explicit analytical solutions for thermal subsystem impedance matrices, allowing global solution for complex systems. Such fully analytical solutions treat arbitrarily complex 3-dimensional thermal systems without the use of volume meshes, uniform or non uniform, thus avoiding all problems with the wide range of length and time scales inherent in the coupled electro-thermal problem [49]. Generation of such solutions requires treatment of thermal non linearity inherent in temperature dependent material parameters. An original treatment of this non linearity, for device and circuit level electro-thermal CAD, is presented first [50]. This is followed by derivation of thermal impedance matrix solutions for a homogeneous MMIC, and for an N-level rectangular multilayer. It is shown how the time-dependent form of the thermal impedance matrix can be expressed in a rapidly convergent forms for all time, t . This is followed by presentation of an original double Fourier series solution to the time-dependent heat diffusion equation with arbitrarily distributed volume heat sources and sinks. This goes beyond previous solutions in the literature, which treat heat dissipating sources as planar, either at the surface or interfaces of rectangular multilayers [19],[27], [28]. Description of complex 3-dimensional structure, and construction of global impedance matrices, are outlined next, followed by discussion of the Leeds thermal impedance matrix approach as a compact model. Implementation of the time-dependent thermal impedance matrix approach, in circuit level CAD, is then illustrated by electro-thermal transient and harmonic balance simulation, particularly demonstrating thermal effects on intermodulation distortion and spectral regrowth. These represent an essential aspect of device optimisation for narrowband digital modulation applications such as CDMA for mobile communications. The ultimate intended application of the modelling capability described here, is study and design of spatial power combining systems for use as high power sources at millimeter wavelengths. The thermal model is therefore validated by thermal imaging of passive grid arrays representative of one form of quasi-optical system architecture.

II. THERMAL NON LINEARITY

The time dependent heat diffusion equation is given by,

$$\nabla \cdot [\kappa(T)\nabla T] + g = \rho C \frac{\partial T}{\partial t}, \quad (1)$$

where T is temperature, t is time, $\kappa(T)$ is temperature dependent thermal conductivity, $g(x, y, z, t)$ is rate of heat generation, ρ is density and C is specific heat. This equation is non linear through the temperature dependence of $\kappa(T)$ (and possibly of ρ and C). To linearise the equation, the Kirchhoff transformation is performed [51],

$$\theta = T_S + \frac{1}{\kappa_S} \int_{T_S}^T \kappa(T) dT, \quad (2)$$

where $\kappa_S = \kappa(T_S)$ and T_S is the heatsink mount temperature. The importance of performing the Kirchhoff transformation has been illustrated, *e.g.*, by Webb [52]. The inverse Kirchhoff transformation is trivial to impose *a posteriori* to solution of the linear heat diffusion equation, by application of a simple analytical formula to the solution temperatures

[28],[53]. The equation for transformed temperature θ becomes,

$$\nabla^2\theta - \frac{1}{k(\theta)} \frac{\partial\theta}{\partial t} = -\frac{g}{\kappa_S}, \quad (3)$$

where diffusivity $k = \kappa/\rho C$. k is now a function of θ so the equation is still non linear.

At this stage it is conventional, in electro-thermal simulations employing the Kirchhoff transformation, to assume that $k(\theta)$ is approximately constant, thus fully linearising the time-dependent heat diffusion equation. However, for typical semiconductor systems this assumption requires further examination and has been discussed by the authors in [50]. It is shown there, that the Kirchhoff transformation does not remove the temperature sensitivity of the material parameters for the time-dependent case.

A further (much less well known) transformation should therefore be applied to fully linearise the heat diffusion equation, by defining a new time variable, τ [54], [55],

$$k_S\tau = \int_0^t k(\theta)dt. \quad (4)$$

The time-dependent heat diffusion equation becomes finally,

$$\nabla^2\theta - \frac{1}{k_S} \frac{\partial\theta}{\partial\tau} = -\frac{g}{\kappa_S}. \quad (5)$$

The fully linearised equation, Eq. (5), can now be solved exactly with general linear boundary conditions. To illustrate the significance of the time variable transformation, Eq. (4), for electro-thermal response [50], an analytical thermal impedance matrix is constructed to describe the response to step power input of 0.4 W, over a central square $0.1L \times 0.1L$, at the surface of a cubic GaAs die, side $L = 400 \mu\text{m}$. Such a configuration is illustrative of, for example, a multi-finger power FET. It is found that total neglect of thermal non linearity leads to a ~ 30 K underestimate of the steady-state temperature rise of ~ 140 K. Including the inverse Kirchhoff transformation, but neglecting the inverse time variable transformation is seen to overestimate the temperature rise by ~ 4 % at any given instant, or equivalently and more importantly, to underestimate the rise time required to reach a given temperature by as much as ~ 35 %.

Another sometimes used approximation, is that of effectively linearising the time-dependent heat diffusion equation about a typical operating point, without employing either the Kirchhoff or the time variable transformation. The error in this approach corresponds to ~ 6 % overestimate of temperature rise, or underestimate of rise time by as much as ~ 60 %. In addition, simply guessing a suitable operating point for linearisation is highly subjective, and for the case of transient thermal variation of large amplitude, easily leads to large errors in the calculated steady-state operating temperatures. In Si systems, temperature dependence of material parameters is even more pronounced than in GaAs [56]. Full linearisation of the time-dependent heat diffusion equation should therefore be implemented to obtain sufficient accuracy in electro-thermal simulations.

III. ANALYTICAL SOLUTIONS

Having described the exact (not small signal) transformation of the non linear time-dependent heat diffusion equation, to produce a fully linear problem, analytical solution of the transformed problem in terms of thermal impedance matrices is now described. The thermal impedance matrix approach reduces to construction of global heat flow functions, for power dissipating and temperature sensitive elements in semiconductor integrated circuits, in the form

$$\overline{\Delta\theta}_i = \sum_j R_{TH_{ij}}(s) \overline{P}_j \quad (6)$$

where $\overline{\Delta\theta}_i$ is the Laplace transformed temperature rise of element i above its initial temperature, $R_{TH_{ij}}(s)$ is the thermal impedance matrix in Laplace s -space and the \overline{P}_j are the transformed time-dependent fluxes due to power dissipation in elements, $j = 1, \dots, i, \dots, M$.

Formulation of the thermal impedance matrix approach in Laplace transform s -space, rather than in the time domain, is chosen for a number of reasons. Firstly, the s -space formulation is a natural development of the thermal resistance matrix approach for the time-independent case, described by the authors previously [26]–[28]. All of the advantageous features of the thermal resistance matrix approach for the coupled electro-thermal description of complex systems, carry over to the time-dependent case in s -space. Secondly, the s -space formulation of the thermal impedance matrix allows immediate incorporation as a multi-port distributed thermal network in circuit level harmonic balance simulators. Finally, Laplace inversion also allows use in circuit level transient simulations, and analytical inversion of s -space expressions readily gives rise to both small-time and large-time results for the thermal response, which are not easily obtained using a direct time domain formulation. However, the thermal impedance matrix approach can also be developed in the time domain using Green’s function techniques. This approach is described in [30].

In the thermal impedance matrix approach presented here, $R_{TH_{ij}}(s)$ is determined in explicit analytical form, purely from structural information. It is independent of temperature and power dissipation, and hence of device bias. Its order is determined only by the number of active device elements, independent of the level of the complexity of the device structure, so is already minimal without any model reduction. Thermal updates in the coupled electro-thermal problem reduce to small matrix multiplications, Eq. (6). This approach therefore offers orders of magnitude speed-up compared to numerical thermal solutions.

A. The homogeneous thermal subsystem

An analytical solution to the linearised heat diffusion equation, Eq. (5), is constructed for the case of a rectangular, homogeneous, generic thermal subvolume, $0 \leq x \leq L$, $0 \leq y \leq W$, $0 \leq z \leq D$, with active device elements $i = 1, \dots, M$ described by surface elementary areas, D_i , and base discretised into elementary areas, D_j , Fig. 1. Adiabatic boundary conditions are assumed on the side faces and a generalised ‘radiation’ boundary condition is imposed on the top and

bottom faces, $z = 0, D$. This can be written,

$$\alpha_{0,D}\kappa_S \frac{\partial \theta}{\partial z} + H_{0,D} (\theta - \theta_{0,D}(x, y, \tau)) + p_{0,D}(x, y, \tau) = 0. \quad (7)$$

Non linear surface flux boundary conditions can be treated in the limit of a sequence of such fully linear problems [27], [57]. Here, imposed flux densities $p_{0,D}(x, y, \tau)$ are time dependent. Coefficients $H_{0,D}$ describe surface fluxes due to radiation and convection. The $\alpha_{0,D}$ equal zero for imposed temperature boundary conditions and unity for imposed flux boundary conditions. The respective ambient temperatures ($\alpha_{0,D} \neq 0$), or heatsink mount temperatures ($\alpha_{0,D} = 0$), are also dependent on time, $\theta_{0,D}(x, y, \tau)$. The generality of this boundary condition allows vertical matching of thermal subsystems, by interface discretisation and thermal impedance matrix manipulation, as well as integral treatment of surface fluxes. However, the adiabatic sidewall boundary condition can be removed, as described later, Sec. III-B and III-F, to allow horizontal interface matching and subvolume embedding.

To solve this problem, the Laplace transform is constructed giving,

$$\nabla^2 \bar{\theta} - \frac{1}{k_S} [s\bar{\theta} - \theta(\tau = 0)] = 0, \quad (8)$$

assuming no volume sources or sinks, and describing surface fluxes by imposed boundary conditions, Eq. (7).

For the case of a uniform initial temperature distribution equal to uniform and time independent ambient temperature, the substitution

$$\bar{\Theta} = s\bar{\theta} - \theta(\tau = 0) \quad (9)$$

is made, giving

$$\nabla^2 \bar{\Theta} - \frac{s}{k_S} \bar{\Theta} = 0. \quad (10)$$

By separation of variables, the general solution for $\bar{\Theta}$ is of the form,

$$\begin{aligned} \bar{\Theta}(s) &= \sum_{mn} \cos \lambda_m x \cos \mu_n y \\ &\quad \times (C_{mn} \cosh \gamma_{mn} z + S_{mn} \sinh \gamma_{mn} z) \end{aligned} \quad (11)$$

where $m, n = 0, 1, 2, \dots$, and

$$\lambda_m = \frac{m\pi}{L}, \quad \mu_n = \frac{n\pi}{W}, \quad \gamma_{mn}^2 = \lambda_m^2 + \mu_n^2 + \frac{s}{k_S}. \quad (12)$$

With the transformation of variable, Eq. (9), the adiabatic side wall boundary conditions retain the same form and the radiation boundary condition on the top and bottom surfaces, $z = 0, D$, becomes

$$\alpha_{0,D}\kappa_S \frac{\partial \bar{\Theta}}{\partial z} + H_{0,D} \bar{\Theta} + s\bar{p}_{0,D}(x, y; s) = 0. \quad (13)$$

Within this framework, the time-dependent problem resembles very closely the time-independent problem [27], [28], thus explicit forms for the expansion coefficients are obtained from,

$$H_0 C_{mn} = -\gamma_{mn} S_{mn} \alpha_0 \kappa_S - \frac{\int_0^L \int_0^W \cos(\lambda_m x) \cos(\mu_n y) \bar{s} \bar{p}_0(x, y; s) dx dy}{\frac{LW}{4} (1 + \delta_{m0})(1 + \delta_{n0})} \quad (14)$$

and

$$\begin{aligned} & C_{mn} [\alpha_D \kappa_S \gamma_{mn} \sinh(\gamma_{mn} D) + H_D \cosh(\gamma_{mn} D)] \\ & + S_{mn} [\alpha_D \kappa_S \gamma_{mn} \cosh(\gamma_{mn} D) + H_D \sinh(\gamma_{mn} D)] \\ & = - \frac{\int_0^L \int_0^W \cos(\lambda_m x) \cos(\mu_n y) \bar{s} \bar{p}_D(x, y; s) dx dy}{\frac{LW}{4} (1 + \delta_{m0})(1 + \delta_{n0})}. \end{aligned} \quad (15)$$

Here δ_{mn} is the Kronecker delta function and the standard result,

$$\int_0^L \cos \lambda_m x \cos \lambda_{m'} x dx = \frac{L}{2} \delta_{mm'} (1 + \delta_{m0} \delta_{m'0}), \quad (16)$$

has been used.

Such fully analytical double Fourier series solutions in Laplace transform s -space have been described previously [3]. They are to be distinguished from semi-analytical Fourier solutions in frequency space, which are based on collocation or function sampling, and require numerical manipulation such as DFT-FFT to obtain expansion coefficients [19].

Explicit forms for the double Fourier series expansion coefficients, C_{mn}, S_{mn} , can only be obtained for the case of constant, $\alpha_{0,D}, H_{0,D}$. Where $H_{0,D}$ varies with position, due to position dependent surface fluxes, a large linear system must be solved for the expansion coefficients [27], [28]. This computational expense can be avoided by dropping the surface flux coefficient, $H_{0,D}$, from the ‘radiation’ boundary condition, Eq. (7), discretising the whole surface, and connecting the thermal subvolume to ambient through a set of external thermal resistances describing piecewise constant surface flux. This approach is discussed further in Sec. V. Similarly, use of a position dependent $\alpha_{0,D}$, for instance in direct construction of a thermal admittance matrix (as opposed to a thermal impedance matrix), can always be avoided by discretisation of the whole surface (as in the boundary element method) and imposition of a corresponding non-mixed boundary condition.

As in the time-independent case for the homogeneous MMIC [27], [28], to illustrate a particular time-dependent form of the thermal impedance matrix, put $\alpha_0 = 1$, $H_0 = 0$ (no radiation from the top surface, $z = 0$) and $\alpha_D = 0$, $H_D = 1$, $p_D(x, y, \tau) = 0$, $\theta_D(x, y, \tau) = \theta(\tau = 0)$ (uniform temperature on the bottom surface, $z = D$, corresponding to heat sink mounting at ambient temperature). Assume a surface power density of the form,

$$p_0(x, y, \tau) = \sum_i S_i(x, y) P_i(\tau), \quad (17)$$

where $S_i(x, y) = 1$ in active device elementary areas D_i , and $S_i(x, y) = 0$ otherwise, then

$$\bar{p}_0 = \sum_i S_i(x, y) \bar{P}_i. \quad (18)$$

The corresponding temperature distribution is given by

$$\begin{aligned} \frac{1}{s} \bar{\Theta}(s) = \bar{\theta}(s) - \frac{\theta(\tau = 0)}{s} = & - \sum_{mn} \cos \lambda_m x \cos \mu_n y \times \\ & \frac{1}{\kappa_s LW} \frac{4}{(1 + \delta_{m0})(1 + \delta_{n0})} \sum_i I_{mn}^i \frac{1}{\gamma_{mn}} \times \\ & (\sinh \gamma_{mn} z - \tanh \gamma_{mn} D \cosh \gamma_{mn} z) \bar{P}_i, \end{aligned} \quad (19)$$

with area integrals I_{mn}^i defined by

$$I_{mn}^i = \iint_{D_i} \cos\left(\frac{m\pi x}{L}\right) \cos\left(\frac{n\pi y}{W}\right) dx dy. \quad (20)$$

Constructing the surface temperatures averaged over elementary areas D_i as,

$$\bar{\Theta}_{av_i} = \frac{\iint_{D_i} \bar{\Theta}|_{z=0} dx dy}{\iint_{D_i} dx dy}, \quad (21)$$

immediately gives the defining equation of the thermal impedance matrix approach, Eq. (6), in the form,

$$\bar{\theta}_{av_i} - \frac{\theta(t = 0)}{s} = \sum_j R_{TH_{ij}}(s) \bar{P}_j, \quad (22)$$

where,

$$R_{TH_{ij}}(s) = \frac{1}{\kappa_s LW} \sum_{mn} \frac{4 \tanh(\gamma_{mn} D)}{\gamma_{mn} (1 + \delta_{m0})(1 + \delta_{n0})} \frac{I_{mn}^i I_{mn}^j}{I_{00}^i}. \quad (23)$$

Extension to treat other realisations of the radiative boundary condition, Eq. (7), is immediate. This allows construction of solutions for large area substrates, with radiation and convection, and generation of series solutions for thermal subsystems with discretised interfaces, permitting vertical matching of thermal subvolume solutions in complex 3-dimensional systems. The expression for $R_{TH_{ij}}(s)$, Eq. (23), can be written in alternative equivalent forms [3], and is readily extended to treat N-level multilayers [3]. The temperature distribution of Eq. (19), and the corresponding thermal impedance matrix of Eq. (23), reduce to the respective steady-state forms [27], [28], in the limit $s/k \rightarrow 0$, giving for the thermal resistance matrix,

$$\begin{aligned} R_{TH_{ij}} = & \\ & \frac{1}{\kappa_s LW} \left[DI_{00}^j + \sum'_{mn} \frac{4 \tanh(\Gamma_{mn} D)}{\Gamma_{mn} (1 + \delta_{m0})(1 + \delta_{n0})} \frac{I_{mn}^i I_{mn}^j}{I_{00}^i} \right] \end{aligned} \quad (24)$$

where now, $\Gamma_{mn}^2 = \lambda_m^2 + \mu_n^2$, and the sum \sum' is over all $m, n = 0, 1, 2, \dots$ excluding $(m, n) = (0, 0)$.

The thermal impedance matrix approach as described here means that generally, temperature will only be calculated in the vicinity of power dissipating and temperature sensitive elements, as required for the coupled electro-thermal solution. No redundant temperature information will be generated on the surface or in the body of the die. However,

the solution of the heat diffusion equation just described, provides analytical expressions for both the thermal impedance matrix and for the corresponding temperature distribution throughout the body of the MMIC. This means that once power dissipations, \overline{P}_i , have been obtained self-consistently, by employing the thermal impedance matrix in the coupled electro-thermal implementation, temperature can be obtained essentially exactly, if required, at any point within the body or on the surface of the MMIC. This is of value for model validation against measured thermal images.

The matrix given by Eq. (23) represents an exact analytical solution for time-dependent 3-dimensional heat flow in a MMIC bearing an arbitrary distribution of power dissipating and temperature sensitive elements. These elements could be transistor fingers or finger subsections, grouped in any fashion, or could represent heat dissipating passive elements or effective thermocouples at metal-substrate contacts [58]. These analytical expressions describe exactly the finite volume of the die and the finite extent of transistor fingers, without making any approximations for infinite volume or finite end effects. If a simpler, single thermal impedance description of device heating is required, the elements of the matrix can be appropriately summed to give the total, area averaged, temperature rise.

The analytical solution, Eq. (23), represents the thermal impulse response of the MMIC. It is frequency dependent as characteristic of distributed systems and contains an infinite number of poles and zeros. It corresponds directly to a multiport thermal network, Fig. 2, which cannot be represented exactly by a finite network of frequency independent primitives, (such as thermal networks generated by numerical mesh descriptions, *e.g.* [13]–[15], [20], [21], [44], which only give an exact thermal description in the limit of infinitely fine mesh discretisation). The multiport network is already minimal, in that it describes nodes corresponding only to power dissipating or temperature sensitive elements (or discretised interface elements). It therefore constitutes a boundary condition independent compact model for the thermal time-dependent case, with 3-dimensional heat flow described exactly by a small number of thermal impedances. The multiport network parameter interpretation presented here, makes the thermal impedance matrix approach immediately compatible with network based electromagnetic and electrical circuit solvers [46], without the need for explicit RC network approximation, or for any model reduction beyond that implicit in summation of infinite series to just a finite number of terms.

The thermal impedance matrix, Eq. (23), can either be used directly in frequency space, for instance in harmonic balance simulations, or Laplace inverted to describe thermal time dependence directly in transient simulations. For the harmonic balance case, the solution for the thermal impedance matrix is just of the s -dependent form, Eq. (23), with $s \rightarrow i\omega$. It takes the form of an array of frequency dependent complex phasors containing phase and amplitude information for the (asymptotic) sinusoidal response to harmonic forcing. This matrix then corresponds to the network parameters of a distributed (originally non linear) multi-port thermal network.

Having obtained transformed temperature in s -space, $\overline{\theta}(s)$, and assuming \overline{P}_j corresponding to simple step inputs of magnitudes P_j , analytical inversion gives the corresponding time domain thermal impedance matrix, $R_{TH_{ij}}(\tau)$,

corresponding to step input,

$$\begin{aligned}
R_{TH_{ij}}(\tau) &= \mathcal{L}^{-1} \left\{ R_{TH_{ij}}(s) \frac{1}{s} \right\} \\
&= \frac{1}{\kappa_S L W} \frac{2}{D} \sum_{lmn} \frac{4}{(1 + \delta_{m0})(1 + \delta_{n0})} \frac{I_{mn}^i I_{mn}^j}{I_{00}^i} \\
&\quad \times \frac{1 - \exp \left\{ -\pi^2 \left[\left(\frac{l+1/2}{D} \right)^2 + \left(\frac{m}{L} \right)^2 + \left(\frac{n}{W} \right)^2 \right] k\tau \right\}}{\pi^2 \left[\left(\frac{l+1/2}{D} \right)^2 + \left(\frac{m}{L} \right)^2 + \left(\frac{n}{W} \right)^2 \right]}
\end{aligned} \tag{25}$$

with $l, m, n = 0, 1, 2, \dots$. Taking the limit $\tau \rightarrow \infty$ and performing the l summation explicitly, the time-independent result is recovered [27], [28]. Using the Watson transformation [59] and the Poisson summation formula [60], series solutions such as Eqns. (23)–(25) can be partially summed explicitly in closed form, and partially accelerated to give even more rapidly evaluated expressions. These results will be presented elsewhere. Such treatment can be particularly important for the description of small elements on large die, which can require summation of large numbers of terms for sufficient resolution. Averaging power dissipations over larger areas can give differences of several decades in calculated time constants, compared to accurate representation of highly localised heating [32], [61]. Accuracy in such descriptions is relevant, for instance, in studies of thermal intermodulation in power HEMTs [32].

Eqns. (23) and (25) give immediately pole-zero or time constant representations for the thermal impedance matrices. Writing [62],

$$R_{TH}(t) = \sum_i R_i (1 - \exp(-t/\tau_i)) \tag{26}$$

with $i \equiv (l, m, n)$, it is apparent from Eq. (25) that R_i and τ_i are obtained in explicit analytical form. Retaining just those terms corresponding to the dominant time constants gives a representation similar to that abandoned by Napieralski *et al.* [21] because it could not be obtained in a simple parameterised form. This description generalises immediately to complex 3-dimensional systems, as described below.

Although it is distributed, the finite thermal system is seen not to be represented by a continuous time constant spectrum, but by a countably infinite number of time constants. However, summing all contributions within range ζ to $\zeta + \Delta\zeta$, where $\zeta = \ln \tau$ [62], essentially continuous spectra are obtained, Fig. 3. These spectra are calculated for a silicon chip considered by Szekely *et al.* [62] and agreement with the calculated results presented in Fig. 23 of that paper is good. Exact agreement is not to be expected, as details of the time constant spectrum depend on the magnitude and placement of intervals $\Delta\zeta$. The two curves shown correspond to division of the calculated 8 decade logarithmic interval, into 40 (solid line) and 80 (dotted line) equal subdivisions, respectively.

Combining tables of standard integrals, *e.g.*, [43], with expressions for the inverse Laplace transform (using properties

of theta functions)[63], gives the equivalent time-domain form of the thermal impedance matrix,

$$\begin{aligned}
& R_{TH_{ij}}(\tau) \\
&= \frac{2}{\kappa_S L W} I_{00}^j \left\{ \sqrt{\frac{k\tau}{\pi}} + 2 \sum_{l=1}^{\infty} (-1)^l \right. \\
&\quad \left. \times \left[\sqrt{\frac{k\tau}{\pi}} e^{-D^2 l^2 / (k\tau)} - D \operatorname{lerfc}\left(\frac{Dl}{\sqrt{k\tau}}\right) \right] \right\} \\
&+ \frac{1}{\kappa_S L W} \sum'_{mn} \frac{4}{(1+\delta_{m0})(1+\delta_{n0})} \frac{I_{mn}^i I_{mn}^j}{I_{00}^i} \frac{1}{\Gamma_{mn}} \times \\
&\quad \left\{ \left[\begin{array}{l} \operatorname{erf}\left(\Gamma_{mn} \sqrt{k\tau}\right) - \sum_{l=1}^{\infty} (-1)^l \times \\ \exp \left\{ \begin{array}{l} 2Dl\Gamma_{mn} + \\ \ln \left[\operatorname{erfc}\left(\frac{Dl}{\sqrt{k\tau}} + \Gamma_{mn} \sqrt{k\tau}\right) \right] \end{array} \right\} \\ - \exp \left\{ \begin{array}{l} -2Dl\Gamma_{mn} + \\ \ln \left[\operatorname{erfc}\left(\frac{Dl}{\sqrt{k\tau}} - \Gamma_{mn} \sqrt{k\tau}\right) \right] \end{array} \right\} \end{array} \right] \right\}.
\end{aligned} \tag{27}$$

This form of the time domain thermal impedance matrix is found to be far more rapidly convergent at very small times. It is an alternative to the explicit time constant form, Eq. (25).

Even though analytical inversion is readily achieved, numerical inversion is highly accurate and algorithmically simple to implement. It requires only evaluation of the Laplace transform and a corresponding weight function, at a small number of real or complex s -points [64]–[67],

$$\mathcal{L}^{-1} \{f(s)\}_{t=\tau_p} = \sum_{\mu} w_{\mu} f(s_{\mu}), \tag{28}$$

with w_{μ} and s_{μ} determined uniquely for a given τ_p . Typically 5 or 6 s -points are adequate so this approach can be computationally much cheaper than analytical inversion.

Fig. 4 shows temperature rise with time at turn-on, calculated using the thermal impedance matrix approach, for cubic GaAs die of side $L = 300, 400$ and $500 \mu\text{m}$, dissipating respectively 0.3, 0.4 and 0.5 W over a central square element of side $0.1L$ on the die surface [68].

The observed trend in the calculated time constant, with variation in die size, could not be predicted on the basis of commonly used thermal models, which invoke an infinite or semi-infinite substrate approximation. The significance of these results is discussed more fully in [68]. The differences in results obtained by analytical, Eq. (27), and numerical, Eqns. (23) and (28), Laplace inversion respectively, are indistinguishable on the scale of this plot. Numerical Laplace inversion gives by far the fastest solution, and the explicit time constant form, Eq. (25), requires summation of an impractical number of terms for sufficient accuracy at very small times.

B. Volume sources

To construct the time dependent thermal solution with volume heat sources/sinks and arbitrary initial conditions, requires the solution of Helmholtz's equation in Laplace transform s -space. This section presents an original technique

for the generation of double Fourier series solutions, describing arbitrarily distributed volume heat sources and sinks, without the use of Green's functions. It therefore greatly extends the descriptive power of the Fourier approach beyond the surface and interface source terms that have been treated previously [19],[27], [28].

Writing the time-dependent heat diffusion equation with volume heat source in s -space,

$$\nabla^2 \bar{\theta} - \frac{s}{k_S} \bar{\theta} = - \left[\frac{1}{\kappa_S} \bar{g}(x, y, z; s) + \frac{\theta(\tau = 0)}{k_S} \right], \quad (29)$$

and assuming a generalised double Fourier series solution of the form,

$$\bar{\theta}(s) = \sum_{mn} \cos \lambda_m x \cos \mu_n y Z_{mn}(z), \quad (30)$$

gives,

$$\frac{d^2 Z_{mn}}{dz^2} - \gamma_{mn}^2 Z_{mn} = G_{mn}(z), \quad (31)$$

where,

$$G_{mn}(z) = \frac{-4}{(1 + \delta_{m0})(1 + \delta_{n0})LW} \times \int_0^L \int_0^W \cos \lambda_m x \cos \mu_n y \left[\frac{1}{\kappa_S} \bar{g}(x, y, z; s) + \frac{\theta(\tau = 0)}{k_S} \right] dy dx. \quad (32)$$

To solve Eq. (31) define,

$$Z_{mn} = e^{\gamma_{mn} z} z_{mn}, \quad (33)$$

and make the substitution,

$$\zeta_{mn} = \frac{dz_{mn}}{dz}, \quad (34)$$

to reduce Eq. (31) to an equation of 1st-order in ζ_{mn} . This linear 1st-order equation can be solved by use of a simple integrating factor, giving the general solution,

$$Z_{mn} = e^{-\gamma_{mn} z} \int_0^z e^{2\gamma_{mn} z'} \int_0^{z'} e^{-\gamma_{mn} z''} G_{mn}(z'') dz'' dz' + \frac{c_{1mn}}{2\gamma_{mn}} e^{\gamma_{mn} z} + c_{2mn} e^{-\gamma_{mn} z}. \quad (35)$$

This solution of Eq. (31) contains two arbitrary constants so is a general solution, valid for all boundary conditions. It is to be distinguished from a Green's function solution constructed for a δ -function source, with in-built boundary conditions. No constitutive equation with δ -function source has been formulated and no solution of such an equation constructed, so no Green's function has been used in this solution of the time-dependent heat diffusion equation with volume source.

(More generally, the approach of Eqns. (33) and (34) supplies a solution to the problem

$$a \frac{d^2 Z}{dz^2} + b \frac{dZ}{dz} + cZ = G(z), \quad (36)$$

where a, b, c are simultaneously piecewise constant, without the use of Green's functions. It is a constructive solution which avoids the need for trial and error in obtaining a particular integral. With $G(z)$ the Dirac δ -function this method can be used to construct Green's functions. As the integrals of Eq. (35) are easy to construct for simple $G(z)$, such as sums of step functions, this approach offers a 2×2 transfer matrix solution to 1-dimensional numerical problems without the need to formulate large sparse matrices for finite difference operators. Finally, as Eq. (35) represents a general solution with no Green's function containing in-built boundary conditions, it supplies a solution to the 2-point boundary value problem, Eq. (36), with arbitrary non linear boundary conditions treated by a subsidiary non linear problem of order 2.)

These solution techniques are individually implicit in texts such as [69]. However, the authors believe that this double Fourier series method, for treatment of arbitrary volume sources or sinks without use of Green's function techniques, represents an original approach to solution of the time-independent and time-dependent heat diffusion equations. This approach is not discussed in texts such as [43], [70], [71]. The double series solution, Eq. (30), is to be compared with much more computationally expensive triple series solutions obtained using time domain Green's functions. In the time domain, the authors' approach can give both small-time and large-time series solutions, which may not be readily obtainable using such Green's functions. The authors note, however, that identical solutions are often achievable by alternative techniques. In particular, separation of variables for construction of the 3-dimensional Green's function, followed by construction of the Green's function for the resulting 1-dimensional Helmholtz equation, is a widely used technique, *e.g.*, [72] (Appendix A2) for time-independent solution of the Poisson equation. Such an approach might therefore be expected to produce similar results to those described below, though the existence of alternative methods offers the possibility of simplifications in the analysis by appropriate choice of technique.

Using the above approach, the thermal impedance matrix for power dissipating volumes, distributed arbitrarily throughout the body of a heatsink mounted MMIC, Fig. 5, is given by

$$R_{TH_{ij}}(s) = \frac{1}{\kappa_S LW} \sum_{mn} \frac{I_{mn}^i I_{mn}^j}{I_{00}^i} \frac{-4}{(1 + \delta_{m0})(1 + \delta_{n0})} \frac{1}{\gamma_{mn}^2} \times \left[\begin{array}{l} \frac{\sinh \gamma_{mn} z_{i2} - \sinh \gamma_{mn} z_{i1}}{\gamma_{mn}(z_{i2} - z_{i1})} \times \\ \frac{[\cosh \gamma_{mn}(D - z_{i1}) - \cosh \gamma_{mn}(D - z_{i2})]}{\cosh \gamma_{mn} D} \\ + 1 - \frac{\sinh \gamma_{mn}(z_{i2} - z_{i1})}{\gamma_{mn}(z_{i2} - z_{i1})} \end{array} \right]. \quad (37)$$

Here, z_{i1}, z_{i2} are the z -coordinates of the planes bounding heat dissipating volume, i , in the z -direction, and the I_{mn}^i are the area integrals over the $x - y$ cross-sections, D_i , of heat dissipating volumes, i , Eq. (20). This expression is to be compared with the thermal impedance matrix for power dissipating surface areas, Eq. (23). Taking the limit, $z_{i2} \rightarrow z_{i1}$, gives the solution for a die with dissipating areas distributed arbitrarily throughout its volume, of value for instance in describing the buried channels below the semiconductor surface of a multi-gate power FET. Taking the further limit,

$z_{i2}, z_{i1} \rightarrow 0$, reproduces Eq. (23).

By letting the power dissipating volumes tend to power dissipating surface areas, this thermal solution can be used to describe arbitrary surface flux distributions, effectively removing the adiabatic surface boundary condition from all subvolume faces. Similarly, this solution can be used to describe flux distributions over arbitrary internal surfaces, Fig. 6. Such a construction, in combination with the analytical solution to be presented in Sec. III-F, below, can then be used to describe inhomogeneous structures.

This solution also makes possible treatment of the time-dependent problem for other than homogeneous initial conditions. It therefore allows construction of a time-stepping thermal impedance matrix formulation for transient electro-thermal simulations, with repeated analytical resetting of initial conditions for the whole distributed thermal volume. Details will be presented elsewhere.

C. Rectangular N -layer

The simple descriptions of the homogeneous MMIC, presented above, are readily generalised to treat multi-layer systems by use of a transfer matrix, or two-port network, approach [11]. This is based on matching of Fourier components at interfaces, and corresponds to use of the double cosine transform to convert the 3-dimensional partial differential equation, Eq. (5), into a 1-dimensional ordinary differential equation for the z -dependent double Fourier series coefficients. Matching of linearised temperature and flux at the interfaces of a multi-layer structure can then be imposed by use of a 2×2 transfer matrix on the Fourier series coefficients and their derivatives. Arbitrary N -level structures can be treated. Different thermal conductivities can be assumed in each layer allowing treatment of composites like Cu on AlN (both having temperature independent thermal conductivities) and MMICs with conductivities varying from layer to layer due to differences in doping levels (all layers having the same functional form for the temperature dependence of the conductivity).

The corresponding form for the thermal impedance matrix is, Fig. 7,

$$R_{TH_{ij}}(s) = \sum_{mn} \cos \lambda_m x_i \cos \mu_n y_i (\alpha_{mn}/\beta_{mn}) \times \frac{-4}{\kappa_1 L W (1 + \delta_{m0})(1 + \delta_{n0}) \gamma_{mn}^{(1)}} I_{mn}^j, \quad (38)$$

where,

$$\begin{pmatrix} \alpha_{mn} \\ \beta_{mn} \end{pmatrix} = \underline{\underline{M}}^{(1)} \underline{\underline{M}}^{(2)} \dots \underline{\underline{M}}^{(N-1)} \begin{pmatrix} 1 \\ -\coth \gamma_{mn}^{(N)} D_N \end{pmatrix}, \quad (39)$$

with,

$$\gamma_{mn}^{(r)} = \left(\lambda_m^2 + \mu_n^2 + \frac{s}{k_r} \right)^{1/2}, \quad (40)$$

and the layers have thermal conductivity, κ_r , diffusivity, k_r , and interface z -coordinates, D_r , $r = 1, \dots, N$, (with $D_0 = 0$).

The $\underline{\underline{M}}^{(r)}$ are analytically obtained 2×2 matrices, explicitly determined entirely by $\kappa_r, \kappa_{r+1}, \gamma_{mn}^{(r)}, \gamma_{mn}^{(r+1)}$ and D_r ,

$$\underline{\underline{M}}^{(r)} = \cosh \gamma_{mn}^{(r)} D_r \cosh \gamma_{mn}^{(r+1)} D_r \times \left(\begin{array}{cc} 1 - \frac{\gamma_{mn}^{(r+1)} \kappa_{r+1}}{\gamma_{mn}^{(r)} \kappa_r} \tanh \gamma_{mn}^{(r)} D_r \tanh \gamma_{mn}^{(r+1)} D_r & \tanh \gamma_{mn}^{(r+1)} D_r - \frac{\gamma_{mn}^{(r+1)} \kappa_{r+1}}{\gamma_{mn}^{(r)} \kappa_r} \tanh \gamma_{mn}^{(r)} D_r \\ - \tanh \gamma_{mn}^{(r)} D_r + \frac{\gamma_{mn}^{(r+1)} \kappa_{r+1}}{\gamma_{mn}^{(r)} \kappa_r} \tanh \gamma_{mn}^{(r+1)} D_r & \frac{\gamma_{mn}^{(r+1)} \kappa_{r+1}}{\gamma_{mn}^{(r)} \kappa_r} - \tanh \gamma_{mn}^{(r)} D_r \tanh \gamma_{mn}^{(r+1)} D_r \end{array} \right). \quad (41)$$

These transfer matrices become nearly singular at high frequencies, simplifying construction of the multi-layer thermal response.

To illustrate the accuracy and speed of this method [38], the above analytical solution for an N-level multilayer, with $s \rightarrow i\omega$, is used to plot the complex locus of the thermal transfer impedance in Fig. 8. The 4-layer, heatsink mounted device considered, is a structure examined by Szekely *et al.*, ([23] Figs. 5 and 6; [48] Fig. 17). Agreement with the calculations of Szekely seems good. The data for this figure took less than 1s to generate on a 500 MHz Pentium processor and consists of 65 frequency points. This can be compared with Szekely's published steady-state simulation times, [73]. The comparison suggests that despite the speed of the FFT, the need to generate redundant temperature information on a surface mesh in Szekely's semi-analytical function sampling or collocation approach, makes the fully analytical double Fourier series transfer matrix method at least $10\times$ faster for the same number of basis states.

The method can be generalised further, by imposing specified flux discontinuities at the interfaces. The solution then represents, for instance, the case of a MMIC with active device channel buried by a thin layer of semiconductor, as described by Eq. (37) (with $z_{i2} \rightarrow z_{i1}$), but distinguishing the thermal conductivities of the various semiconductor layers. This transfer matrix approach can also be combined with the volume heat source solution of Sec. III-B, to describe rectangular N-layers containing an arbitrary distribution of power dissipating volumes, without the need to introduce artificial interfaces.

Digele has stated [44] that the Kirchhoff transformation is of no value for multilayer structures. However, this is incorrect. The Kirchhoff transformation is exact for N-level multilayers with the same functional form (but different values) for the temperature dependent thermal conductivity in each layer. A single, global Kirchhoff transformation is also a good approximation for multilayers in which the functional form of $\kappa(T)$ differs between layers, so long as an appropriately modified effective value for κ_S is chosen in each layer for which the global transformation is not exact [74], [75].

D. MMIC superstructure

It has been demonstrated that inclusion of surface metallisation is essential for accurate description of thermal effects in power devices [52],[75]–[77]. Comparison with experiment for multi-finger power HBTs shows that the simple thermal description corresponding to the resistance matrix of Eq. (24) is highly accurate when combined with a simple model of heat shunting by an air bridge [78]. The analytical thermal resistance and impedance matrix approach presented here,

has been designed to allow descriptions of surface metallisation and air bridges, and other vertical geometries such as flip chips and solder bumps, as well as MMIC arrays, as outlined below.

The extension to include complex 3-dimensional structure is achieved by solving the heat diffusion equation analytically for thermal sub elements, then combining thermal impedance matrices for subsystems by matching of temperature and flux at discretised interfaces. For illustration, specifying flux on top and bottom surfaces, $z = 0, D$, and assuming no radiative or convective surface losses, ($\alpha_{0,D} = 1, H_{0,D} = 0$), the following relations are obtained for temperatures, $\bar{\theta}_{0\,av_i}$ and $\bar{\theta}_{D\,av_j}$, averaged over elementary areas, D_i , and D_j , on faces $z = 0$ and $z = D$, respectively, Fig. 1,

$$\begin{aligned}\bar{\theta}_{0\,av_i} - \frac{\theta(t=0)}{s} &= \sum_{i'} R_{TH_{ii'}}^{00} \bar{P}_{0i'} + \sum_j R_{TH_{ij}}^{0D} \bar{P}_{Dj}, \\ \bar{\theta}_{D\,av_j} - \frac{\theta(t=0)}{s} &= \sum_i R_{TH_{ji}}^{D0} \bar{P}_{0i} + \sum_{j'} R_{TH_{jj'}}^{DD} \bar{P}_{Dj'}.\end{aligned}\tag{42}$$

Here, \bar{P}_{0i} and \bar{P}_{Dj} are respective imposed fluxes in elementary areas, D_i and D_j , on faces $z = 0$ and $z = D$. The thermal impedance matrices are obtained in the explicit form,

$$\begin{aligned}R_{TH_{ii'}}^{00} &= \frac{1}{\kappa_S LW} \sum_{mn} \frac{4 \coth \gamma_{mn} D}{(1 + \delta_{m0})(1 + \delta_{n0}) \gamma_{mn}} \frac{I_{mn}^{0i} I_{mn}^{0i'}}{I_{00}^{0i}}, \\ R_{TH_{ij}}^{0D} &= \frac{1}{\kappa_S LW} \sum_{mn} \frac{-4 \operatorname{cosech} \gamma_{mn} D}{(1 + \delta_{m0})(1 + \delta_{n0}) \gamma_{mn}} \frac{I_{mn}^{0i} I_{mn}^{Dj}}{I_{00}^{0i}}, \\ R_{TH_{ji}}^{D0} &= \frac{1}{\kappa_S LW} \sum_{mn} \frac{4 \operatorname{cosech} \gamma_{mn} D}{(1 + \delta_{m0})(1 + \delta_{n0}) \gamma_{mn}} \frac{I_{mn}^{Dj} I_{mn}^{0i}}{I_{00}^{Dj}}, \\ R_{TH_{jj'}}^{DD} &= \frac{1}{\kappa_S LW} \sum_{mn} \frac{-4 \coth \gamma_{mn} D}{(1 + \delta_{m0})(1 + \delta_{n0}) \gamma_{mn}} \frac{I_{mn}^{Dj} I_{mn}^{Dj'}}{I_{00}^{Dj}},\end{aligned}\tag{43}$$

where the I_{mn}^{0i} and I_{mn}^{Dj} are area integrals of the form, Eq. (20), over elementary areas D_i and D_j , on faces $z = 0$ and $z = D$, respectively. These expressions are readily inverted analytically or numerically, as described in Sec. III-A, to give (fully parameterised) time domain results. As described in [38], analytical inversion gives rise to explicit expressions for the individual terms in pole-zero or time constant representations and allows direct construction of time constant spectra.

(Resistance matrix expressions for the thermal time-independent case have been given in [28]. Alternative forms for the series constructions given there, can be obtained using closed form summation and series acceleration techniques.)

These series expressions represent generalised multiport Z -parameters for the distributed thermal subsystems. Network parameter descriptions for thermal systems have been described previously, [20]. However, the construction of network parameters described in [20] was neither analytical, nor simple, and required complex numerical manipulations based on the boundary element method. This model also did not treat thermal non linearity, and involved approximate fitting of network parameters and explicit model reduction. Time domain simulation in [20] also required additional

complex manipulation to convert the admittance matrix description to a system of integral equations treatable by the circuit simulator. This is in contrast to direct generation of the time-domain response described in this work, based on Laplace s -space formulation and analytical or numerical Laplace inversion.

Combining the thermal impedance matrices for individual subvolumes, a global thermal impedance matrix for complex 3-dimensional systems can be obtained. This is illustrated in the next section [38] for the case of a metallised MMIC. More generally, thermal subsystems can be represented individually by netlist elements in circuit simulation. Expressing the thermal impedance matrices as non linear elements in the time domain, then allows non linear matching of interface temperatures at subsystem interfaces, in those cases where the functional form of the Kirchoff transformation differs between subvolumes.

The s -space formulation means that no artificial piecewise constant time dependence is assumed for interface fluxes, in contrast to the time-domain USE method [43]. However, the thermal impedance matrix approach can be developed within the USE framework [30], with direct time domain interface matching, where it avoids repeated matrix inversion.

E. Global impedance matrices

Construction of thermal impedance matrices is now described for more complex systems, such as power FETs and MMICs with surface metallisation, Fig. 9.

To illustrate the interface matching approach, the global thermal impedance matrix is constructed for the case of N pieces of rectangular, but otherwise arbitrary, metallisation on the surface of an otherwise homogeneous heatsink mounted MMIC. Multiple levels of metallisation are treated in the same fashion. Matching flux and (linearised) temperature at the interface between metal and MMIC die, the following relation is obtained

$$\underline{\Delta\theta}^a = \underline{\underline{R}}_{TH}^{glob} \underline{P}^a, \quad (44)$$

where \underline{P}^a is the vector of MMIC active device power dissipations, $\underline{\underline{R}}_{TH}^{glob}$ is the global thermal impedance matrix for the coupled GaAs and metal system, and $\underline{\Delta\theta}^a$ is the vector of MMIC active device temperature rises.

The global impedance matrix is given explicitly by

$$\underline{\underline{R}}_{TH}^{glob} = \underline{\underline{R}}_{TH}^{aa} + \underline{\underline{R}}_{TH}^{ai} \underline{R}^{-1} \underline{\underline{R}}_{TH}^{ia}, \quad (45)$$

$$\underline{\underline{R}} = \text{diag} \left(\underline{\underline{R}}_{TH}^{DD(1)}, \dots, \underline{\underline{R}}_{TH}^{DD(n)}, \dots, \underline{\underline{R}}_{TH}^{DD(N)} \right) - \underline{\underline{R}}_{TH}^{ii}. \quad (46)$$

Here, $\underline{\underline{R}}_{TH}$ of Eq. (23) for the MMIC die has been partitioned by active device elementary areas, a , and interface elementary areas between MMIC die and metal, i , and the $\underline{\underline{R}}_{TH}^{DD(n)}$ are thermal impedance matrices for each piece of metallisation, Eq. (43).

Thus, by simple matrix manipulation, the global thermal impedance matrix for the metallised MMIC can be obtained as an explicit matrix expression for any given value of Laplace transform variable, s . Also, using the simple algorithm

for the numerical Laplace inverse, Eq. (28), the value of the global thermal impedance matrix can be evaluated at any time step, $n\delta\tau$, in the time domain. The s -space formulation means that when power dissipation is known *a priori*, temperature can be obtained directly at any required instant, without the need to take consecutive timesteps from $\tau = 0$. In cases where non linear interface matching cannot be neglected, the thermal impedance matrix approach allows formulation of a non linear system of equations for the correctly matched temperatures [27].

The significance of the relation, Eq. (45), should be stressed. It represents an explicit analytical expression for the solution of the time-dependent heat diffusion equation in an arbitrarily complex 3-dimensional volume. In contrast to conventional numerical techniques, such as FDTD or FETD, it requires no volume mesh, discretising only interfaces (and power dissipating and temperature sensitive elements). It is therefore extremely simple to formulate and implement, avoiding the large preparation times of FE simulations, as well as the intricacies of FDTD and FETD code for complex structures. The solution is modular and hierarchical, so once the global impedance matrix has been constructed for a single metallised MMIC, this could then, for example, be used to describe each MMIC in an $N \times N$ MMIC array. The global impedance matrix for the metallised MMIC would only have to be constructed once, to describe all N^2 identical MMICs. It could also be stored for re-use in later coupled electro-thermal simulations, cutting later precomputation time effectively to zero. Finally, there is no restriction on heat loss mechanisms involved in this solution, and for instance, ultimate heat loss from the system could be purely by radiation and convection from the grid array substrate, without any heatsink mounting.

This method therefore avoids all the previous limitations of fully analytical approaches, as listed for instance in [49], and provides a natural solution to the problem of variation in length scale over the whole of an electro-thermal system. Resolution of temperature in each thermal subsystem is defined by its local coordinate system and the corresponding double Fourier series expansion. There is no need for any sort of uniform mesh resolving the finest detail at all length scales, or for imposed non uniform grid construction. Also, by development of closed form and accelerated expressions for the thermal impedance matrices, as indicated earlier and to be presented elsewhere, all series convergence rates are fast and resolution limits are removed within any single thermal subsystem.

The method presented here is immediately compatible with explicitly coupled electro-thermal device and circuit level simulation on CAD timescales. The directly coupled thermal impedance matrix approach represents ‘near exact’ solution of the non linear time dependent heat diffusion equation for the complex 3-dimensional system, at points, or averaged over regions, corresponding to power dissipating and temperature sensitive elements. The only approximations are finite interface discretisation between thermal subsystems; in the time domain, the assumption of piecewise constant time variation and numerical Laplace inversion if employed; and use of a single global Kirchhoff transformation if non linear interface matching between subsystems is not imposed. This ‘near exact’ compact model for arbitrarily complex structures, is to be contrasted with simplified compact component models, based on reduced dimensional solutions of the 3-dimensional heat diffusion equation and neglecting detailed device structure such as die surface metallisation, *e.g.*,

[14].

Finally, the authors' approach allows netlist construction of any thermal system constructable from rectangular subvolumes. This is again in contrast to component models such as those of [14], which require individual analysis and construction of an appropriate discretisation, before they can be entered into the circuit simulator component library.

F. Inhomogeneous Thermal Conductivity

The analytical double Fourier series solution for the thermal impedance matrix can be further generalised to treat, essentially exactly, piecewise uniform, but otherwise arbitrarily inhomogeneous thermal conductivity, such as full and partial thickness vias, and partial substrate thinning in power transistors and MMICs. A computationally much cheaper, but approximate, treatment of vias, based on the the simple equivalence principle method of Bonani *et al.*, [10], [12], [79], has also been implemented within the thermal resistance matrix approach. Construction of such solutions for the time-independent case is described in [28].

Vertical matching obtainable by use of the 'radiation' boundary condition, Eq. (7), can be extended by removal of the adiabatic side wall assumption, Fig. 10. This allows horizontal matching of rectangular subvolumes for which flux boundary conditions are prescribed on all faces. The corresponding double Fourier series solution takes the form,

$$\bar{\theta}(s) = \frac{\theta(\tau = 0)}{s} + \frac{1}{s} \times \sum_{mn} \left\{ \begin{array}{l} \cos \lambda_m x \cos \mu_n y \\ \times (A_{mn} \cosh \gamma_{mn} z + B_{mn} \sinh \gamma_{mn} z) \\ + \cos \mu_m y \cos \gamma_n z \\ \times (C_{mn} \cosh \lambda_{mn} x + D_{mn} \sinh \lambda_{mn} x) \\ + \cos \lambda_m x \cos \gamma_n z \\ \times (E_{mn} \cosh \mu_{mn} y + F_{mn} \sinh \mu_{mn} y) \end{array} \right\}, \quad (47)$$

where,

$$\begin{aligned} \lambda_n &= \frac{n\pi}{L}, \quad \mu_n = \frac{n\pi}{W}, \quad \gamma_n = \frac{n\pi}{D}, \\ \lambda_{mn}^2 &= \mu_m^2 + \gamma_n^2 + \frac{s}{k_S}, \quad \gamma_{mn}^2 = \lambda_m^2 + \mu_n^2 + \frac{s}{k_S}, \\ \text{and } \mu_{mn}^2 &= \lambda_m^2 + \gamma_n^2 + \frac{s}{k_S}. \end{aligned} \quad (48)$$

The expansion coefficients A_{mn}, B_{mn}, \dots , are all obtained as explicit analytical expressions (when surface flux coefficient, H , is excluded from the surface boundary condition). Again, this solution is not described in standard texts such as [43], [70], [71].

Combining the above solution with that for arbitrary distributions of heat sources described in Sec. III-B, gives a fully analytical description of inhomogeneous structures based on small dense matrix manipulation.

IV. COUPLED ELECTRO-THERMAL TRANSIENT

The thermal impedance matrix in s -space can be used directly in coupled electro-thermal harmonic balance simulations. In this case, the matrix of frequency dependent complex phasors corresponds to the network parameters of the distributed multi-port thermal network. It is inserted directly into the MNAM for the microwave system and so does not increase the number of non linear equations describing the coupled solution.

In the coupled electro-thermal transient problem, Laplace transformed active power dissipations, $\bar{P}_j(s)$, are not known explicitly and must be obtained by self-consistent solution. To combine the electrical and thermal descriptions, the corresponding $P_j(\tau)$ must therefore be discretised in time. Dividing the time interval of interest into equal subintervals of length $\delta\tau$, with the $P_j(\tau)$ taking the piecewise constant form (for illustration)

$$P_j(\tau) = P_j^{(n)} \quad \text{for } (n-1)\delta\tau < \tau \leq n\delta\tau, n = 1, \dots, N \quad (49)$$

then gives

$$\bar{P}_j(s) = \sum_n \frac{1}{s} (1 - e^{-s\delta\tau}) e^{-(n-1)s\delta\tau} P_j^{(n)}. \quad (50)$$

Laplace inverting the impedance matrix equation, Eq (22), the temperature rise of element i at time $\tau = m\delta\tau$, $\Delta\theta_i^{(m)}$, is obtained as a function of the $P_j^{(n)}$. Writing $\Delta\theta_i^{(m)} = \Delta\theta_i^{(m)}(P_i^{(m)})$ from the electrical model then gives,

$$\Delta\theta_i^{(m)}(P_i^{(m)}) = \mathcal{L}^{-1} \{ R_{TH_{ij}}(s) \bar{P}_j(s) \}_{\tau=m\delta\tau}, \quad (51)$$

$$= \sum_n \sum_j [u(m-n+1)R_{TH_{ij}}((m-n+1)\delta\tau) - u(m-n)R_{TH_{ij}}((m-n)\delta\tau)] P_j^{(n)}, \quad (52)$$

where $u(\tau)$ is the unit step function.

This corresponds to N systems of equations in M unknowns, where N is the number of discretised time points in the time interval under consideration, and M is the number of power dissipating or temperature sensitive elements. The Laplace inversion, with piecewise constant power dissipation, avoids any explicit convolution operation.

The entire thermal description can therefore be obtained by precomputation of $R_{TH_{ij}}(\tau)$ at timesteps, $\tau = n\delta\tau$, $n = 0, \dots, N$. These precomputed values can be stored for repeated re-use in different electro-thermal simulations. For reduction of precomputation time, the $R_{TH_{ij}}(\tau)$ can be generated at intervals, and interior points obtained accurately by interpolation. This is a time-domain approach equivalent to representation of a frequency space transfer function by a polynomial fit.

Extension to linear, quadratic or higher order interpolation of the active device power dissipations in each subinterval, $\delta\tau$, is immediate, and for sufficiently short step lengths, low orders of interpolation should be required.

After self-consistent electro-thermal solution, and inversion of the Kirchhoff and time variable transformations, physical active device temperatures are finally obtained as a function of physical time, $T_i(t)$, and electrical solutions, DC or RF, are determined.

V. COMPACT MODELS

In the fully analytical approach described above, the global thermal impedance matrix describing a complex 3-dimensional system, such as a packaged electronic component, consists of a minimal number of thermal impedances describing self-heating and mutual thermal interaction at only those sites chosen to be of interest for electro-thermal simulation. This description is minimal in two senses. It contains the smallest number of thermal multiport nodes compatible with coupling to electrical network nodes. It also contains the smallest number of thermal impedances consistent with exact solution of the heat diffusion equation. These thermal impedances do not have a direct interpretation in terms of discretised physical layout, but constitute ‘thermal links’ as defined in [25]. The general form of the equivalent circuit, corresponding to thermal resistance matrix \underline{R}_{TH} for the time independent case, is shown in Fig. 11 [26], [78], and has been employed successfully in SPICE-like circuit simulation [78].

This basic form generalises readily to arbitrary numbers of nodes, unlike thermal networks based on direct physical discretisation of the thermal system, which can grow rapidly more convoluted with increase in size. Generalisation to the time-dependent thermal impedance matrix case is immediate. Implementation of the thermal multiport network in electro-thermal CAD can be achieved in both the time and frequency domains, as described above. Direct use of the solution of the heat diffusion equation, in the form of explicit double Fourier series expressions for thermal impedance matrices describing thermal multiports, avoids the need for lumped element RC network generation, and is already minimal without any node reduction such as that described in [80].

Thermal impedance matrices for the time independent case, and in the time domain, have been described previously by Franke and Froehler [81], for compact model development based on numerical simulation or experiment. Their corresponding equivalent circuit should be compared with Fig. 11. Sabry [20] has suggested the thermal admittance matrix as a compact model for time-dependent electrothermal CAD, as an alternative to demonstrably inadequate star models. Codecasa *et al.* have proposed exact thermal N-port construction, in the form of thermal resistance and impedance matrices [82], [83], though without constructive details of the exact thermal solution for multilayer and complex systems. Thermal impedance matrices and N-ports, for the time-independent and time-dependent cases, have been described by Szekely *et al.* as early as [2], [84], with generation of a minimal thermal impedance matrix at least implicit in compact, lumped element RC network approximation and nodal reduction [80]. Although the concept of the thermal impedance matrix as a compact model for electro-thermal CAD is well explored, the authors’ formulation is unique in providing constructive details for a minimal compact model, obtained by direct analytical solution of the heat diffusion equation for complex structures. It is also unique in its s -space formulation, allowing directly both frequency domain and time domain representations, without explicit realisation as an RC network.

The thermal impedance matrix, which is generated analytically based on the imposed boundary condition, Eq. (7), constitutes a boundary condition independent compact model, as defined broadly by Lasance [24], [25]. Treatment of

the time-dependent case, with description of thermal non linearity, represents a generalisation of the time-independent thermal resistance networks generally defining compact models. The analytically imposed ‘radiation’ boundary condition, Eq. (7), is sufficiently general to include a wide range of boundary condition sets, including free and forced convection, heat sink, cold plate and fluid bath, as well as unbalanced ambient temperatures. Where full layout details are not available, or deviate from nominally specified values, a global thermal impedance matrix expression, such as Eq. (45), provides a fully parameterised relation for compact model optimisation.

Description of electronic packages typically involves position varying surface flux. Where this is described through position dependent surface flux coefficients, such as $H_{0,D}(x, y)$, Eq. (7), the simple, explicit solution of Eqns. (14) and (15) no longer applies. Instead, a large linear system must be solved for the Fourier expansion coefficients [27]. Similarly, even if surface flux coefficients, H , are constant, the generalised double Fourier series of Eq. (47) requires solution of a corresponding linear system. Such solutions are computationally expensive, but not totally intractable, and have been described by the authors for the time independent, double Fourier series treatment of inhomogeneous structures [28]. However, in the time dependent case, where repeated solution of the large dense linear system would be required, this approach is unattractive without reduction of the dense matrix manipulation costs. Such a reduction might be possible by the use of wavelet methods, *e.g.*, [85] (Chapter 4).

This large computational problem is easily avoided by adopting the solutions of Sec. III-B and III-F, with flux prescribed on all free subvolume faces, and with no explicit inclusion of surface flux coefficient, H , in the solution boundary conditions. The magnitude of surface flux from each elementary area of the fully discretised surface can then be determined by external thermal resistances connecting surface thermal nodes to ambient, Fig. 12, as in standard compact model descriptions [25]. This fully analytical approach, with full surface discretisation, also allows immediate treatment of surface flux non linearity [27], [57]. However, the inaccuracy of using just one or two surface nodes to describe position dependent surface fluxes has been noted, for instance by Franke [81]. Surface discretisation in this fashion means that flux is effectively assumed constant over each surface element. In fact, even where surface flux coefficient, H , is constant, flux varies with temperature over each surface element as $H(\theta - \theta_{amb})$. This effect can be included exactly in the fully analytical solution with no explicit surface thermal nodes, when $H_{0,D}$ is constant and is retained explicitly in the radiation boundary condition, Eq. (7).

VI. CIRCUIT SIMULATION

Explicitly coupled electro-thermal circuit level simulation based on the time-dependent thermal impedance matrix approach is now illustrated, by combination with a microwave circuit simulator, Transim (NCSU) [37].

A. *Transim (NCSU)*

Transim has an input format that is similar to the SPICE format with extensions for variables, sweeps, user defined models, and repetitive simulation. The program provides a variety of output data and plots. The addition of new circuit element models and analysis types in Transim is much simpler than in other circuit simulators such as SPICE. For example, new element models are coded and incorporated into the program without modification to the high-level simulator. The circuit analysis types currently available in Transim are DC, AC, harmonic balance (HB) [86], convolution transient [87], wavelet transient [88], and time-marching transient [89]. Some insight into the program architecture is given in [37]. Transim, including the Leeds thermal impedance matrix model, is free software distributed under the GNU license. Further details may be found at the website [90].

Thermal effects were incorporated into the circuit simulator engine by making the thermal model look like an electrical circuit [2], [91], specifically a multi-port network described in either the time or frequency domain. To ensure separate circuits for the electrical and thermal subsystems, a local reference node concept was employed [92]. This concept was initially developed for integrated circuit and EM field analysis of distributed microwave circuits, and guarantees that there is no mixing of electric and thermal currents.

The circuit used in the simulations described below, is shown in Fig. 13. The MESFET was modeled using the Curtice-Ettemberg cubic model with symmetric diodes and capacitances [93]. The extra terminals in the MESFET schematic represent the thermal connections. The thermally distributed power FET die was represented by analytically exact thermal 1-port network parameters (with no explicit lumped element RC network approximation).

Transient analysis of distributed microwave circuits is complicated by the inability of frequency independent primitives to model distributed circuits. Generally, the linear part of a microwave circuit is described in the frequency domain by network parameters, especially where numerical field analysis is used to model a spatially distributed structure. Inverse Fourier transformation of these network parameters yields the impulse response of the linear circuit. This has been used with convolution to achieve transient analysis of distributed circuits. Transim uses a state variable approach for the convolution transient [87]. An algebraic nonlinear system is solved at each time step using a quasi-Newton method. In this analysis the thermal element is treated in the time domain, Eq. (25), along with the nonlinear elements in the circuit. The input power to the thermal system is chosen as the state variable of the element. Then the thermal impedance matrix approach is used to calculate the temperature corresponding to the input power given by the state variable at each iteration to solve the nonlinear system. The Kirchhoff transformation is implemented within the thermal element, so allowing non linear interface matching between thermal subsystems.

The harmonic balance (HB) technique uses a linear combination of sinusoids to approximate the periodic and quasi-periodic signals found in a time-dependent steady-state response. The system of nonlinear differential equations describing the circuit can then be transformed into a nonlinear algebraic system. Details of the implementation of HB

in Transim are given in [86]. In this analysis the thermal element is modeled in the frequency domain, Eq. (23). The elements of the thermal impedance matrix are entered directly into the modified nodal admittance matrix of the circuit at each frequency. The thermal element is then ‘embedded’ in the linear part of the HB formulation and does not increase the size of the nonlinear system of equations. The Kirchhoff transformation for the linear thermal system is transferred into the already non linear active device model by appropriate state variable definition.

No separate thermal simulation is required for the coupled calculation by the electro-thermal simulation engine. All thermal impedance matrices are generated by multiport thermal network elements defined within the electro-thermal circuit simulation engine, Transim. Thermal impedance matrices (in either the time or frequency domain) are precomputed from fully analytical expressions, prior to the coupled electro-thermal simulation. They only have to be generated once, for any specified thermal subsystem, and can be stored for re-use in later simulations.

B. Simulations

Fig. 14 illustrates transient decay in drain-source current I_{ds} , as a result of thermal variation under the influence of a step input in drain-source voltage V_{ds} , for a multi-finger power transistor, calculated using the thermal impedance matrix approach implemented in Transim [38]. 1-tone, 2-tone and multi-tone HB simulations are also presented in Figs. 15 to 19 [39]. These simulations used the simplest 1-port thermal description of the power FET, returning surface average temperature rise as a function of surface average power dissipation over the active regions of the multi-gate device. N-port thermal elements have also been implemented in Transim. After precomputation of thermal impedances, the coupled electro-thermal simulations took a few seconds on a 500 MHz Pentium processor. Figs. 15 and 16 illustrate single-tone HB simulation. Such results raise the question of whether small amplitude thermal oscillations at microwave frequencies could ever act as a source of ‘classical’ thermal noise. Figs. 17 and 18 illustrate two-tone HB, demonstrating intermodulation distortion due to amplifier non linearity. As a result of thermal inertia, thermal response is seen to be much greater at the 1 MHz difference frequency than at the ~ 1 GHz fundamental frequencies. Fig. 19 illustrates the potential of the model for prediction of thermal effects on spectral regrowth and ACPR, by means of multi-tone HB.

VII. THERMAL MODEL VALIDATION

To validate the thermal model, a series of time dependent thermal images of a passive grid array at turn-on, representative of one form of spatial power combining architecture, were obtained using a video capture card interfaced between an Inframetrics PM-280 Thermacam and a PC. Commercial software enabled the capture frame rate to be preset and data output to a file in AVI format. A frame rate of one image per second was chosen and a total of 350 frames recorded. Frame rates of up to 50 Hz are readily available, allowing measurement of thermal time constants in individual MMICs. These showed lateral heat diffusion in related experiments reaching steady state on timescales ~ 1 s.

Experimental thermal images for the FR-4 grid array are plotted in Fig. 20 and corresponding thermal simulations

are shown in Fig. 21. These simulations and measurements of thermal response are for a 10×10 passive grid array, dissipating 2 W over an area $5 \times 5 \text{ cm}^2$. The simulations were based on a simple analytical expression for thermal impedance, returning central temperature as a function of the average power dissipation over the whole area of the central array of heat dissipating resistors [28]. In terms of the corresponding thermal network, the thermal impedance corresponds to a 1-port thermal element with one terminal held at constant ambient temperature. Laplace inversion was performed numerically [64]. Agreement between theory and experiment is good, with just one scalar fitting parameter to determine the ratio of radiative to convective surface flux losses. Adjustment of this parameter essentially fixes the asymptotic value of the surface temperature rise. Data for FR-4 was taken from [94] and (along with geometrical parameters) independently determines the thermal rise times. In all cases, heat loss for the horizontal, essentially free-standing array is purely by surface radiation and convection with no heatsink mounting. Times to steady state for lateral diffusion are seen to be very long with implications for quasi-optical array beam formation. The observed reduction in lateral diffusion in the simulations compared to experiment, is probably due to neglect of the known anisotropy of the FR-4 thermal conductivity. This is approximately 3 times larger in-plane than in the perpendicular direction [95]. This anisotropy is easily treated by simple extension of the analytical model.

VIII. CONCLUSION

An original, fully analytical, spectral domain decomposition approach to the solution of the non linear time-dependent heat diffusion equation, in complex 3-dimensional systems, has been described. It has been illustrated largely for hybrid and monolithic integrated circuits, but is equally applicable to packaged components, multi-chip modules, circuit boards and systems. It is based on fully analytical expressions for solution of the heat diffusion equation in rectangular thermal subvolumes (though other regular geometries, such as cylinders, are readily treated). This fully analytical thermal model is observed to be at least $10 \times$ faster than corresponding semi-analytical Fourier solutions for N-level multilayers, and also treats arbitrarily complex 3-dimensional systems without invoking conventional numerical methods. It requires no volume or surface discretisation, discretising only the interfaces between thermal subsystems. It is compatible with network based EM/electrical circuit simulators via interpretation as a multi-port thermal network, with direct use of essentially exact, generalised multiport network parameters, in either frequency space or the time domain. This approach avoids the need for explicit, lumped element RC network approximation or model reduction, apart from that inherent in truncation of infinite series at a finite number of terms sufficient to ensure convergence (and in numerical Laplace inversion, when employed). It gives rise to minimal boundary condition independent compact models for complex thermal systems in the form of analytically obtained thermal impedance matrices, describing temperature response only at power dissipating and temperature sensitive elements. The method has no power or temperature restrictions, so is not a small signal approximation.

The problem of thermal non linearity, due to temperature dependent thermal diffusivity, has been treated fully for

the first time in electro-thermal CAD. This required application of a time variable transformation, in addition to the well known Kirchhoff transformation for treatment of temperature dependent thermal conductivity. In contrast with many electro-thermal CAD models, which neglect thermal non linearity in order to generate a linear thermal network, the model presented here can treat fully thermal non linearity due to temperature dependence of material parameters, as well as that due to non linear surface fluxes in large area systems.

A range of original thermal solutions have been presented in the form of thermal impedance matrices for electro-thermal subsystems. In particular, an original, Green's function free, approach to the double Fourier series solution of problems with arbitrarily distributed volume heat sources and sinks, has been described. A double Fourier series solution for prescribed flux on all faces of a rectangular volume has also been presented. These two solutions remove the need for an adiabatic sidewall boundary condition. Employing these techniques, construction of global thermal solutions for complex 3-dimensional systems based on the thermal impedance matrix approach, has been outlined, removing the limitations of all previous fully analytical solutions for thermal systems.

This thermal impedance matrix method has been illustrated by generation of thermal responses for test systems in both the frequency and time domains, and compared against published results. Agreement was found to be good. The model was validated experimentally for the case of a passive grid array at turn-on, by high resolution time-dependent thermal imaging. Simulations were performed for the horizontal, essentially free-standing array, assuming heat loss purely by surface radiation and convection with no heatsink mounting.

Coupled electro-thermal simulation has been demonstrated by implementation of the Leeds thermal impedance matrix model in a microwave circuit simulator, Transim (NCSU). Simulated transient, 1-tone, 2-tone and multi-tone HB results were presented for a power MESFET.

Future development of the Leeds thermal impedance matrix model in Transim will include extension of the approach to generate explicitly, heat transfer coefficient, H , describing surface flux losses. It will explore wavelet techniques for reduction of large, dense, $N \times N$, matrix eigenvalue and inversion problems, from $O(N^3)$ operation to $O(N)$ operation processes, with particular application to the economical description of inhomogeneous thermal systems. It will include integration of the rapid Leeds Physical Model of MESFETs and HEMTs, into circuit simulator, Transim, to produce fully physical, coupled electro-thermal, circuit level CAD. Finally, it will include development of circuit simulation techniques to treat more fully the huge range of time constants inherent in fully coupled electro-thermal simulations of large systems.

The modelling capability described here will be applied to the study and design of spatial power combining architectures for use as high power sources at millimeter wavelengths.

REFERENCES

- [1] R. C. Joy and E. S. Schlig, 'Thermal properties of very fast transistors,' *IEEE Trans. Electron Devices*, vol. ED-17, no. 8, pp. 586–594, 1970.
- [2] V. Szekely and K. Tarnay, 'Accurate algorithm for temperature calculation of devices in nonlinear-circuit-analysis programs,' *Electron. Lett.*, vol. 8, no. 19, pp. 470–472, 1972.
- [3] A. G. Kokkas, 'Thermal analysis of multiple-layer structures,' *IEEE Trans. Electron Devices*, vol. ED-21, no. 11, pp. 674–681, 1974.
- [4] S. Wunsche, C. Clauss, P. Schwarz and F. Winkler, 'Electro-thermal circuit simulation using simulator coupling,' *IEEE Trans. VLSI Systems*, vol. 5, no. 3, pp. 277–282, 1997.
- [5] J. S. Wilson, P. E. Raad and D. C. Price, 'Transient adaptive thermal simulation of microwave integrated circuits,' *Proc. 14th IEEE SEMI-THERM Symp.*, pp. 1–7, 1998.
- [6] V. Szekely, A. Pahi and M. Rencz, 'SUNRED, a new field solving approach,' *Proc. Symp. Design, Test and Microfab. of MEMS and MOEMS*, SPIE vol. 3680, no. 1, pp. 278–288, 1999.
- [7] H. Fukui, 'Thermal resistance of GaAs field-effect transistors,' *Tech. Dig. IEEE Int. Electron Devices Meeting*, Washington, pp. 118–121, December 1980.
- [8] H. F. Cooke, 'Precise technique finds FET thermal resistance,' *Microwaves and RF*, pp. 85–87, August 1986.
- [9] R. Anholt, 'Electrical and Thermal Characterization of MESFETs, HEMTs, and HBTs,' Artech, Norwood, 1994.
- [10] F. Bonani, G. Ghione, M. Pirola and C. U. Naldi, 'Large-scale, computer-aided thermal design of power GaAs integrated devices and circuits,' *Proc. IEEE GaAs IC Symposium*, pp. 141–144, 1994.
- [11] J.-M. Dorkel, P. Tounsi and P. Leturcq, 'Three-dimensional thermal modeling based on the two-port network theory for hybrid or monolithic integrated power circuits,' *IEEE Trans. Components, Packaging and Manufacturing Technology – Part A*, vol. 19, no. 4, pp. 501–507, 1996.
- [12] F. Bonani, G. Ghione, M. Pirola and C. U. Naldi, 'Thermal CAD for power III-V devices and MMICs,' *Proc. SBMO/IEEE MTT-S Internat. Microwave and Optoelectronics Conf.*, vol. 1, pp. 352–357, 1995.
- [13] S.-S. Lee and D. J. Allstot, 'Electrothermal simulation of integrated circuits,' *IEEE J. Solid-State Circuits*, vol. 28, no. 12, pp. 1283–1293, 1993.
- [14] A. R. Hefner and D. L. Blackburn, 'Thermal component models for electrothermal network simulation,' *IEEE Trans. Components, Packaging, and Manufacturing Technology – Part A*, vol. 17, no. 3, pp. 413–424, 1994.
- [15] J. T. Hsu and L. Vu-Quoc, 'A rational formulation of thermal circuit models for electrothermal simulation – part I: finite element method,' *IEEE Trans. Circ. Sys. – I Fundamental Theory and Applications*, vol. 43, no. 9, pp. 721–732, 1996.
- [16] J. S. Brodsky, R. M. Fox, D. T. Zweidinger and S. Veeraraghavan, 'A physics-based, dynamic thermal impedance model for SOI MOSFET's,' *IEEE Trans. Electron Devices*, vol. 44, no. 6, pp. 957–964, 1997.
- [17] T. Veijola, L. Costa and M. Valtonen, 'An implementation of electrothermal component models in a general purpose circuit simulation programme,' *Proc. THERMINIC '97*, pp. 96–100, Cannes, September 21–23, 1997.
- [18] V. Rizzoli, A. Lipparini, A. Costanzo and V. Frontini, 'Three-dimensional computation of the thermal parameters of multiple-gate power FETs,' *Proc. European Microwave Conf.*, vol. 23, pp. 698–700, 1993.

- [19] A. Csendes, V. Szekely, M. Rencz, 'An efficient thermal simulation tool for ICs, microsystem elements and MCMs: the μ S-THERMANAL,' *Microelectron. J.*, vol. 29, 241–255, 1998.
- [20] M.-N. Sabry, 'Static and dynamic thermal modelling of ICs,' *Microelectron. J.*, vol. 30, pp. 1085–1091, 1999.
- [21] M. Furmanczyk, A. Napieralski, K. Szaniawski, W. Tylman and A. Lara, 'Reduced electro-thermal models for integrated circuits,' *Proc. 1st Internat. Conf. on Modeling and Simulation of Semiconductor Microsystems*, pp. 139–144, 1998.
- [22] J. T. Hsu and L. Vu-Quoc, 'A rational formulation of thermal circuit models for electrothermal simulation – part II: model reduction techniques,' *IEEE Trans. Circ. Sys. – I*, vol. 43, no. 9, pp. 733–744, 1996.
- [23] V. Szekely, 'THERMODEL: a tool for compact dynamic thermal model generation,' *Microelectron. J.*, vol. 29, pp. 257–267, 1998.
- [24] C. J. M. Lasance, 'Two benchmarks for the study of compact thermal modelling phenomena,' *Proc. 6th THERMINIC Workshop*, pp. 235–243, Budapest, September 2000.
- [25] C. J. M. Lasance, D. den Hertog and P. Stehouwer, 'Creation and evaluation of compact models for thermal characterisation using dedicated optimisation software,' *Proc. 15th IEEE SEMI-THERM Symp.*, pp. 189–200, 1999.
- [26] W. Batty, A. J. Panks and C. M. Snowden, 'Fully coupled electro-thermal simulation of MMICs and MMIC arrays based on a physical model,' *IEEE MTT-S Internat. Microwave Symp. Dig.*, vol. 2, Paper TU4C-5, pp. 693–696, 1999.
- [27] W. Batty, A. J. Panks, R. G. Johnson and C. M. Snowden, 'Electro-thermal modelling and measurement for spatial power combining at millimeter wavelengths,' *IEEE Trans. Microwave Theory Tech.*, vol. 47, no. 12, pp. 2574–2585, 1999.
- [28] W. Batty, A. J. Panks, R. G. Johnson and C. M. Snowden, 'Electro-thermal modelling of monolithic and hybrid microwave and millimeter wave IC's,' *VLSI Design*, vol. 10, no. 4, pp. 355–389, 2000.
- [29] R. G. Johnson, W. Batty, A. J. Panks and C. M. Snowden, 'Fully physical, coupled electro-thermal simulations and measurements of power FETs,' *IEEE MTT-S Internat. Microwave Symp. Dig.*, Paper TUIF-17, vol. 1, pp. 461–464, Boston, June 2000.
- [30] S. David, W. Batty, A. J. Panks, R. G. Johnson and C. M. Snowden, 'Fully physical coupled electro-thermal modelling of transient and steady-state behaviour in microwave semiconductor devices,' *Proc. 8th Gallium Arsenide Application Symp. (GAAS 2000)*, paper GAAS/P1 (3), Paris, October 2000.
- [31] S. David, W. Batty, A. J. Panks, R. G. Johnson and C. M. Snowden, 'Electro-thermal modelling of microwave transistors and MMICs for optimised transient and steady-state performance,' *Proc. 8th IEEE Internat. Symp. Electron. Dev. for Microwave and Optoelectron. Appl. (EDMO 2000)*, Glasgow, November 2000.
- [32] S. David, W. Batty, A. J. Panks, R. G. Johnson and C. M. Snowden, 'Thermal transients in microwave active devices and their influence on intermodulation distortion,' *IEEE MTT-S Internat. Microwave Symp. Dig.*, Arizona, May 2001.
- [33] C. M. Snowden and R. R. Pantoja, 'Quasi-two-dimensional MESFET simulations for CAD,' *IEEE Trans. Electron Dev.*, vol. 36, pp. 1564–1574, 1989.
- [34] C. G. Morton, J. S. Atherton, C. M. Snowden, R. D. Pollard and M. J. Howes, 'A large-signal physical HEMT model,' *IEEE MTT-S Internat. Microwave Symp. Dig.*, pp. 1759–1762, 1996.
- [35] R. G. Johnson, C. M. Snowden and R. D. Pollard, 'A physics-based electro-thermal model for microwave and millimetre wave HEMTs,' *IEEE MTT-S Internat. Microwave Symp. Dig.*, vol. 3, Paper TH3E-6, pp. 1485–1488, 1997.

- [36] L. Albasha, R. G. Johnson, C. M. Snowden and R. D. Pollard, 'An investigation of breakdown in power HEMTs and MESFETs utilising an advanced temperature-dependent physical model,' *Proc. IEEE 24th Internat. Symp. Compound Semiconductors*, pp. 471–474, San Diego, 1997.
- [37] C. E. Christoffersen, U. A. Mughal, M. B. Steer, 'Object oriented microwave circuit simulation,' *Int. Journal of RF and Microwave CAE*, vol. 10, no. 3, pp. 164–182, 2000.
- [38] W. Batty, C. E. Christoffersen, S. David, A. J. Panks, R. G. Johnson, C. M. Snowden and M. B. Steer, 'Steady-state and transient electro-thermal simulation of power devices and circuits based on a fully physical thermal model,' *Proc. 6th Internat. Workshop on Thermal Investigations of ICs and Systems (THERMINIC 2000)*, Budapest, pp. 125–130, September 2000.
- [39] W. Batty, C. E. Christoffersen, S. David, A. J. Panks, R. G. Johnson, C. M. Snowden and M. B. Steer, 'Predictive microwave device design by coupled electro-thermal simulation based on a fully physical thermal model,' *Proc. 8th IEEE Internat. Symp. Electron. Dev. for Microwave and Optoelectron. Appl. (EDMO 2000)*, Glasgow, November 2000.
- [40] W. Batty, C. E. Christoffersen, S. David, A. J. Panks, R. G. Johnson, C. M. Snowden and M. B. Steer, 'Fully physical time-dependent compact thermal modelling of complex non linear 3-dimensional systems for device and circuit level electro-thermal CAD,' *Proc. 17th Annual IEEE Semiconductor Thermal Measurement and Management Symp. (SemiTherm XVII)*, San Jose, March 2001.
- [41] W. Batty, C. E. Christoffersen, S. David, A. J. Panks, R. G. Johnson, C. M. Snowden and M. B. Steer, 'Global electro-thermal CAD of complex non linear 3-d systems based on a fully physical time-dependent compact thermal model,' *IEEE MTT-S Internat. Microwave Symp. Dig.*, Arizona, paper WE1C-2, May 2001.
- [42] D. Gottlieb and S. A. Orszag, 'Numerical analysis of spectral methods: theory and applications,' CBMS-NSF Regional Conference Series in Applied Mathematics, SIAM, Philadelphia, 1977.
- [43] J. V. Beck, K. Cole, A. Haji-Sheikh and B. Litkouhi, 'Heat conduction using Green's functions,' Hemisphere Publishing Corp., Washington DC, 1992.
- [44] G. Digele, S. Lindenkreuz and E. Kasper, 'Fully coupled dynamic electro-thermal simulation,' *IEEE Trans. VLSI Systems*, vol. 5, no. 3, pp. 250–257, 1997.
- [45] G. N. Ellison, 'Thermal analysis of microelectric packages and printed circuit boards using an analytic solution to the heat conduction equation,' *Advances in Engineering Software*, vol. 22, no. 2, pp. 99–111, 1995.
- [46] M. B. Steer, J. F. Harvey, J. W. Mink, M. N. Abdulla, C. E. Christoffersen, H. M. Gutierrez, P. L. Heron, C. W. Hicks, A. I. Khalil, U. A. Mughal, S. Nakazawa, T. W. Nuteson, J. Patwardhan, S. G. Skaggs, M. A. Summers, S. Wang and A. B. Yakovlev, 'Global modeling of spatially distributed microwave and millimeter-wave systems,' *IEEE Trans. Microwave Theory Tech.*, vol. 47, pp. 830–839, 1999.
- [47] K. C. Gupta, 'Emerging trends in millimeter-wave CAD,' *IEEE Trans. Microwave Theory Tech.*, vol. 46, pp. 747–755, 1998.
- [48] V. Szekely, 'Identification of RC networks by deconvolution: chances and limits,' *IEEE Trans. Circ. Sys. I*, vol. 45, no. 3, pp. 244–258, 1998.
- [49] P. E. Raad, J. S. Wilson and D. C. Price, 'Adaptive modeling of the transients of submicron integrated circuits,' *IEEE Trans. Comp., Packag. and Manufact. Technol.*, Part A, vol. 21, no. 3, pp. 412–417, 1998.
- [50] W. Batty and C. M. Snowden, 'Electro-thermal device and circuit simulation with thermal non linearity due to temperature dependent diffusivity,' *Electron. Lett.*, vol. 36, no. 23, pp. 1966–1968, 2000.

- [51] W. B. Joyce, 'Thermal resistance of heat sinks with temperature-dependent conductivity,' *Solid-State Electronics*, vol. 18, pp. 321–322, 1975.
- [52] P. W. Webb and I. A. D. Russell, 'Thermal simulation of transients in microwave devices,' *IEE Proc.*, vol. 138, pt. G, no. 3, pp. 329–334, 1991.
- [53] L. L. Liou, J. L. Ebel and C. I. Huang, 'Thermal effects on the characteristics of AlGaAs/GaAs heterojunction bipolar transistors using two-dimensional numerical simulation,' *Trans. Electron Dev.*, vol. 40, no. 1, pp. 35–43, 1993.
- [54] V. Kadambi and B. Dorri, 'Solution of thermal problems with nonlinear material properties by the boundary integral method,' in C. A. Brebbia, ed., *BETECH 85* (Springer, Berlin, 1985).
- [55] L. C. Wrobel and C. A. Brebbia, 'The dual reciprocity boundary element formulation for nonlinear diffusion problems,' *Comp. Meth. Appl. Mech. Eng.*, vol. 65, pp. 147–164, 1987.
- [56] Y. S. Touloukian, Ed., 'Thermophysical Properties of Matter, The TPRC Data Series (Thermophysical Properties Research Center, Purdue University),' vol. 10, IFI/Plenum, 1973.
- [57] R. M. S. da Gama, 'A linear scheme for simulating conduction heat transfer problems with nonlinear boundary conditions,' *Appl. Math. Modelling*, vol. 21, no. 27, pp. 447–454, 1997.
- [58] V. Szekely, A. Poppe, A. Pahi, A. Csendes, G. Hajas and M. Rencz, 'Electro-thermal and logi-thermal simulation of VLSI designs,' *IEEE Trans. VLSI Systems*, vol. 5, no. 3, pp. 258–269, 1997.
- [59] A. D. Wunsch, 'Complex Variables with Applications,' 2nd Edition, (Appendix to Chapter 6), Addison-Wesley, 1994.
- [60] H. Dym and H. P. McKean, 'Fourier Series and Integrals,' Academic Press, New York, 1972.
- [61] M. N. Sabry, W. Fikry, Kh. Abdel Salam, M. M. Awad and A. I. Nasser, 'A lumped transient thermal model for self-heating in MOSFETs,' *Proc. 6th Internat. Workshop on Thermal Investigations of ICs and Systems (THERMINIC 2000)*, Budapest, pp. 137–144, September 2000.
- [62] V. Szekely, A. Poppe and M. Rencz, 'Algorithmic extension of thermal field solvers: time constant analysis,' *Proc. 16th IEEE SEMI-THERM Symposium*, pp. 99–106, 2000.
- [63] F. Oberhettinger and L. Badii, 'Tables of Laplace Transforms,' Springer-Verlag, 1973.
- [64] H. Stehfest, 'Algorithm 368 Numerical inversion of Laplace transforms [D5],' *Comm. ACM*, vol. 13, no. 1, pp. 47–49 and 624, 1970.
- [65] P. Satravaha and S. Zhu, 'An application of the LTDRM to transient diffusion problems with nonlinear material properties and nonlinear boundary conditions,' *Appl. Math. Comp.*, vol. 87, no. 2–3, pp. 127–160, 1997.
- [66] K. Singhal and J. Vlach, 'Computation of time domain response by numerical inversion of the Laplace transform,' *J. Franklin Inst.*, vol. 299, no. 2, pp. 109–126, 1975.
- [67] R. Griffith and M. S. Nakhla, 'Mixed frequency/time domain analysis of nonlinear circuits,' *IEEE Trans. CAD*, vol. 11, no. 8, pp. 1032–1043, 1992.
- [68] W. Batty, A. J. Panks, S. David, R. G. Johnson and C. M. Snowden, 'Electro-thermal modelling and measurement of thermal time constants and natural convection in MMIC grid arrays for spatial power combining,' *IEEE MTT-S Internat. Microwave Symp. Dig.*, vol. 3, pp. 1937–1940, 2000.
- [69] Rektorys Karel, 'Survey of Applicable Mathematics,' Iliffe, London, 1969.
- [70] H. S. Carslaw and J. C. Jaeger, 'Conduction of Heat in Solids,' 2nd Edition, Oxford University Press, 1959.

- [71] M. N. Ozisik, 'Heat Conduction,' Wiley, New York, 1980.
- [72] M. N. Sabry, 'An integral method for studying the onset of natural convection,' *Eur. J. Mech., B/Fluids*, vol. 12, no. 3, pp. 337–365, 1993.
- [73] V. Szekely, M. Rencz and B. Courtois, 'Simulation, testing and modeling of the thermal behaviour and electro-thermal interactions in ICs, MCMs and PWBs,' *Proc. 1999 SouthWest Symp. on Mixed-Signal Design*, pp. 168–173, 1999.
- [74] F. Bonani and G. Ghione, 'On the application of the Kirchhoff transformation to the steady-state thermal analysis of semiconductor devices with temperature-dependent and piecewise inhomogeneous thermal conductivity,' *Solid-State Electronics*, vol. 38, no. 7, pp. 1409–1412, 1995.
- [75] P. W. Webb, 'Thermal modeling of power gallium arsenide microwave integrated circuits,' *IEEE Trans. Electron. Dev.*, vol. 40, no. 5, pp. 867–877, 1993.
- [76] P. W. Webb and I. A. D. Russell, 'Thermal resistance of gallium-arsenide field-effect transistors,' *IEE Proc.*, vol. 136, pt. G, no. 5, pp. 229–234, 1989.
- [77] P. W. Webb, 'Thermal design of gallium arsenide MESFETs for microwave power amplifiers,' *IEE Proc.-Circuits Devices Syst.*, vol. 144, no. 1, pp. 45–50, 1997.
- [78] C. M. Snowden, 'Large-signal microwave characterization of AlGaAs/GaAs HBT's based on a physics-based electrothermal model,' *IEEE Trans. Microwave Theory Tech.*, vol. 45, no. 1, pp. 58–71, 1997.
- [79] F. Bonani, G. Ghione, M. Pirola and C. U. Naldi, 'A large-scale, self-consistent thermal simulator for the layout optimization of power III-V field-effect and bipolar transistors,' *IEEE GaAs '94, Proc. European Gallium Arsenide and Related III-V Compounds Applications Symp.*, pp. 411–414, 1994.
- [80] M. Rencz, V. Szekely, A. Pahi and A. Poppe, 'An alternative method for electro-thermal circuit simulation,' *Proc. 1999 SouthWest Symp. on Mixed-Signal Design*, pp. 168–173, 1999.
- [81] T. Franke and U. Froehler, 'Thermal modelling of semiconductor packages,' *Proc. 6th Internat. Workshop on Thermal Investigations of ICs and Systems (THERMINIC 2000)*, Budapest, pp. 193–198, September 2000.
- [82] L. Codecasa and M. Santomauro, 'A new approach to model self-heating of electric circuits through thermal networks,' *Proc. European Conf. Circuit Theory and Design*, vol. 1, pp. 563–566, 1999.
- [83] L. Codecasa, D. D'Amore and P. Maffezzoni, 'Electro-thermal networks for the analysis of power devices,' *Proc. 6th Internat. Workshop on Thermal Investigations of ICs and Systems (THERMINIC 2000)*, Budapest, pp. 131–136, September 2000.
- [84] V. Szekely, 'Accurate calculation of device heat dynamics: a special feature of the TRANS-TRAN circuit-analysis programme,' *Electron. Lett.*, vol. 9, no. 6, pp. 132–134, 1973.
- [85] G. Erlebacher, M. Y. Hussaini and L. M. Jameson, Eds. 'Wavelets: theory and applications,' Oxford University Press, New York, Oxford, 1996.
- [86] C. E. Christoffersen, M. B. Steer and M. A. Summers, 'Harmonic balance analysis for systems with circuit-field interactions,' *IEEE Int. Microwave Symp. Dig.*, pp. 1131–1134, June 1998.
- [87] C. E. Christoffersen, M. Ozkar, M. B. Steer, M. G. Case and M. Rodwell, 'State-variable-based transient analysis using convolution,' *IEEE Trans. Microwave Theory Tech.*, vol. 47, no. 6, pp. 882–889, 1999.
- [88] C. E. Christoffersen and M. B. Steer, 'State-variable microwave circuit simulation using wavelets,' to be published in the *IEEE Microwave and Guided Waves Letters*, April 2001.

- [89] C. E. Christoffersen, *Global modeling of nonlinear microwave circuits*, Ph.D. Dissertation, North Carolina State University, December 2000.
- [90] Transim software at <http://www.ece.ncsu.edu/erl/transim/>.
- [91] H. Gutierrez, C. E. Christoffersen and M. B. Steer, 'An integrated environment for the simulation of electrical, thermal and electromagnetic interactions in high-performance integrated circuits,' *Proc. IEEE 6th Topical Meeting on Electrical Performance of Electronic Packaging*, Sept. 1999.
- [92] C. E. Christoffersen and M. B. Steer 'Implementation of the local reference concept for spatially distributed circuits,' *Int. J. of RF and Microwave Computer-Aided Eng.*, vol. 9, no. 5, 1999.
- [93] Compact Software, *Microwave Harmonica Elements Library*, 1994.
- [94] Thermal Wizard, MAYA Heat Transfer Technologies Ltd, <http://www.mayahtt.com/tmwiz>.
- [95] F. Sarvar, N. J. Poole and P. A. Witting, 'PCB glass-fiber laminates – thermal-conductivity measurements and their effect on simulation,' *J. Electronic Materials*, vol. 19, no. 12, pp.1345–1350, 1990.

IX. CAPTIONS

Fig. 1: Generic thermal subvolume for analytical construction of the thermal impedance matrix.

Fig. 2: Thermal N-port generated directly from analytical solution of the heat diffusion equation, and represented by thermal impedance matrix, $R_{TH_{ij}}(s)$.

Fig. 3: Time constant spectra obtained from Eqns. (25) and (26) for a Si chip considered by Szekely *et al.* [62].

Fig. 4: Temperature rise with time at turn-on, in the immediate vicinity of the device active region in a central square, $0.1L \times 0.1L$, on the surface of cubic die, side $L = 300, 400, 500 \mu\text{m}$, dissipating 0.3, 0.4, 0.5 W [68].

Fig. 5: Thermal subvolume with arbitrary distribution of power dissipating volume sources.

Fig. 6: Illustration of power dissipation over an internal surface, described by the analytical solution of Sec. III-B.

Fig. 7: Illustrative N-level multilayer thermal subvolume for analytical construction of thermal transfer impedance.

Fig. 8: Complex locus of the thermal transfer impedance, calculated using an analytical series expression, for a 4-layer, heatsink mounted structure examined by Szekely *et al.* [23], [48].

Fig. 9: Surface metallisation of a power MESFET as depicted by Webb [77].

Fig. 10: Illustration of power dissipation over the whole external surface of a rectangular subvolume, described by the analytical solution of Sec. III-F.

Fig. 11: Thermal equivalent circuit corresponding to resistance matrix $R_{TH_{ij}}$ [26], [78].

Fig. 12: Illustration of the treatment of position varying surface heat flux coefficient in the analytical thermal impedance matrix description of electronic packages.

Fig. 13: Schematic of the simulated amplifier with thermal circuit [38], [39].

Fig. 14: Drain-source current I_{ds} (\leftarrow , solid line) and drain-source voltage V_{ds} (dashed line, \rightarrow) for a 5-finger power transistor, from transient electro-thermal analysis [38], [39].

Fig. 15: Drain-source current i_{ds} (\leftarrow , solid line) and drain-source voltage v_{ds} (dashed line, \rightarrow) from single-tone HB analysis with fundamental frequency 10 MHz [39].

Fig. 16: Power dissipation (\leftarrow , solid line) and temperature variation (dashed line, \rightarrow) for the 5-finger power transistor, from single-tone HB analysis with fundamental frequency 10 MHz [39].

Fig. 17: Power dissipation in the frequency domain for a 50-finger power transistor, from two-tone HB analysis with fundamental frequency 1 GHz and difference frequency 1 MHz [39].

Fig. 18: Temperature variation in the frequency domain for the 50-finger power transistor, from two-tone HB analysis with fundamental frequency 1 GHz and difference frequency 1 MHz [39].

Fig. 19: Current response from multi-tone HB analysis with 11 fundamentals at frequency 0.5 GHz and difference frequency steps of 0.5 MHz [39]. Circles: without thermal effects. Crosses: with thermal effects.

Fig. 20: Time dependent measurements of 10×10 passive grid array: FR-4, $5 \times 5 \text{ cm}^2$, dissipating 2 W. Cooling

purely by surface radiation and convection (no heatsink mounting).

Fig. 21: Time dependent thermal simulations of 10×10 passive grid array: FR-4, $5 \times 5 \text{ cm}^2$, dissipating 2 W. Cooling purely by surface radiation and convection (no heatsink mounting).

X. FOOTNOTES

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XI. BIOGRAPHIES

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Microwaves and Photonics (IMP), at the University of Leeds, in the field of electro-thermal modelling.

Christopher M. Snowden received the BSc(Hons), MSc and PhD degrees from the University of Leeds, England. He is currently Joint Chief Executive of Filtronic plc and Professor of Microwave Engineering at the University of Leeds. He is a Fellow of the IEEE, Fellow of the Royal Academy of Engineering and Fellow of the IEE. He is currently a Distinguished Lecturer for the IEEE (Electron Devices Society). He was awarded the 1999 Microwave Prize of the IEEE Microwave Theory and Techniques Society. After graduating in 1977 he worked as an Applications Engineer for Mullard near London, England (now part of Philips). His PhD studies were later conducted in association with Racal-MESL and were concerned with the large-signal characterisation and design of MESFET microwave oscillators. He has held the personal Chair of Microwave Engineering at the University of Leeds since 1992. During the period 1995-98 he was Head of the Department and subsequently Head of the School of Electronic and Electrical Engineering. He was the first Director of the Institute of Microwaves and Photonics located in the School. He was a Consultant to M/A-COM Inc., from 1989 to 1998. In 1998 he joined Filtronic as Director of Technology. His main personal research interests include semiconductor device and circuit modelling (CAD), microwave, millimetre-wave and optoelectronic circuit technology, compound semiconductor device modelling, microwave, terahertz and optical nonlinear subsystem design and advanced semiconductor devices. He has written 8 books, over 250 refereed journal and conference papers, and many other articles.

Michael B. Steer received his B.E. and Ph.D. in Electrical Engineering from the University of Queensland, Brisbane, Australia, in 1976 and 1983 respectively. Currently he is Professor of Electrical and Computer Engineering at North Carolina State University. His research work has been closely tied to solving fundamental problems in modeling and implementing RF and microwave circuits and systems. Up to 1996 Dr. Steer was the founding librarian of the IBIS consortium which provides a forum for developing behavioral models. A converter written by his group to automatically develop behavioral models from a Spice netlist (spice2ibis) is being used throughout the digital design community. Currently one of his interests is in global modeling of the physical layer of RF, microwave and millimeterwave electronic systems in support of multifunctional, adaptive RF front ends. He is doing research in technology integration as well as signal integrity and circuit modeling in general. Professor Steer is a Fellow of the Institute of Electrical and Electronic Engineers cited "for contributions to the computer aided engineering of non-linear microwave and millimeter-wave circuits." He is active in the Microwave Theory and Techniques (MTT) Society. In 1997 he was Secretary of the Society and from 1998 to 2000 was an Elected Member of its Administrative. In 1999 and 2000 he was Professor in the School of Electronic and Electrical Engineering at the University of Leeds where he held the Chair in Microwave and Millimeterwave Electronics. He was also Director of the Institute of Microwaves and Photonics at the University of Leeds. He has organized many workshops and taught many short courses on Signal Integrity, Wireless, and RF Design. He has authored or co-authored more than 200 refereed papers and book chapters on topics related to high speed digital design

and to RF and microwave design methodology. He is coauthor of the book Foundations of Interconnect and Microstrip Design, John Wiley, 2000. He is a 1987 Presidential Young Investigator (USA) and in 1994, and again in 1996, he was awarded the Bronze Medallion by U.S. Army Research for “Outstanding Scientific Accomplishment.”

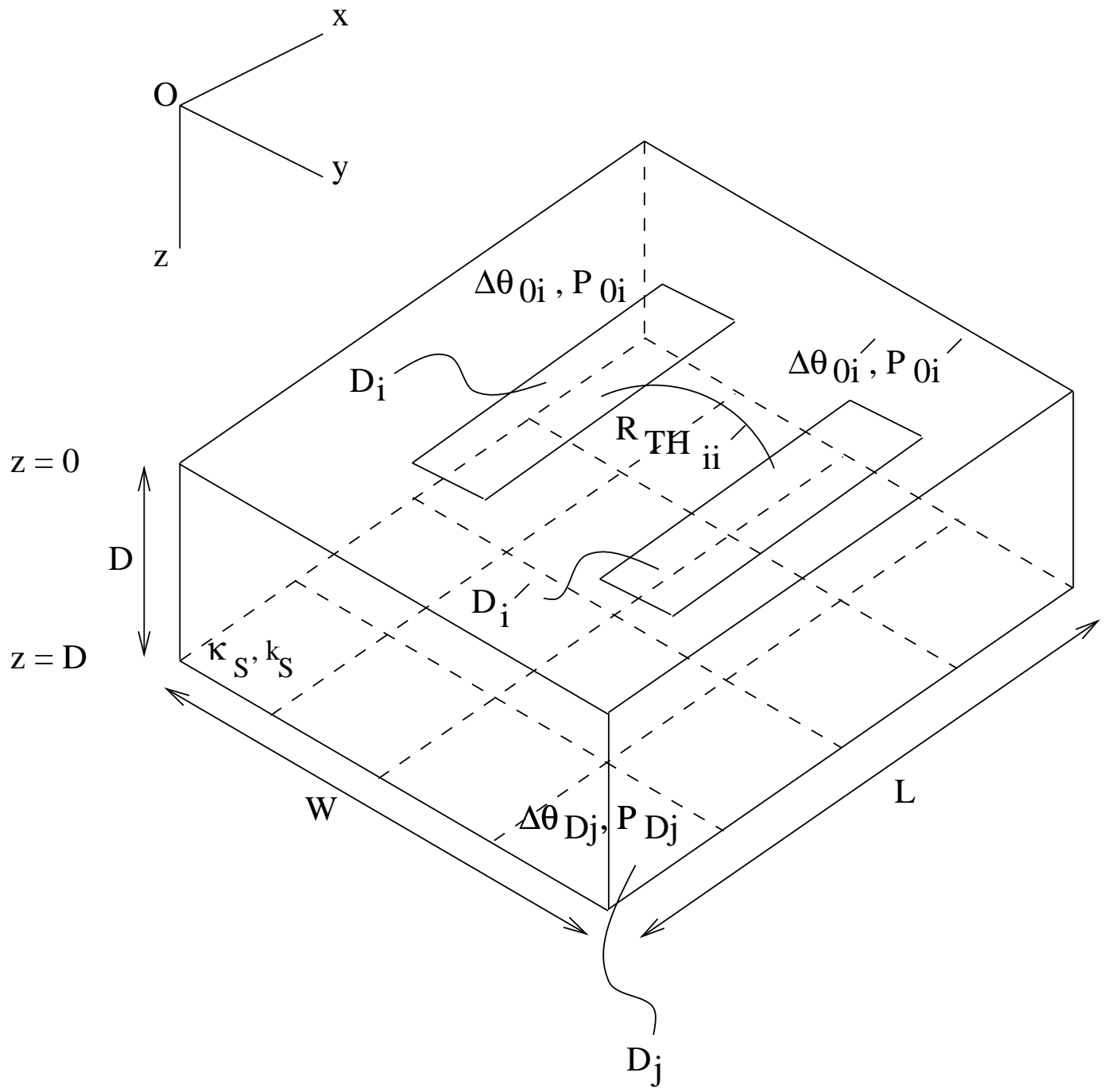


Fig. 1.

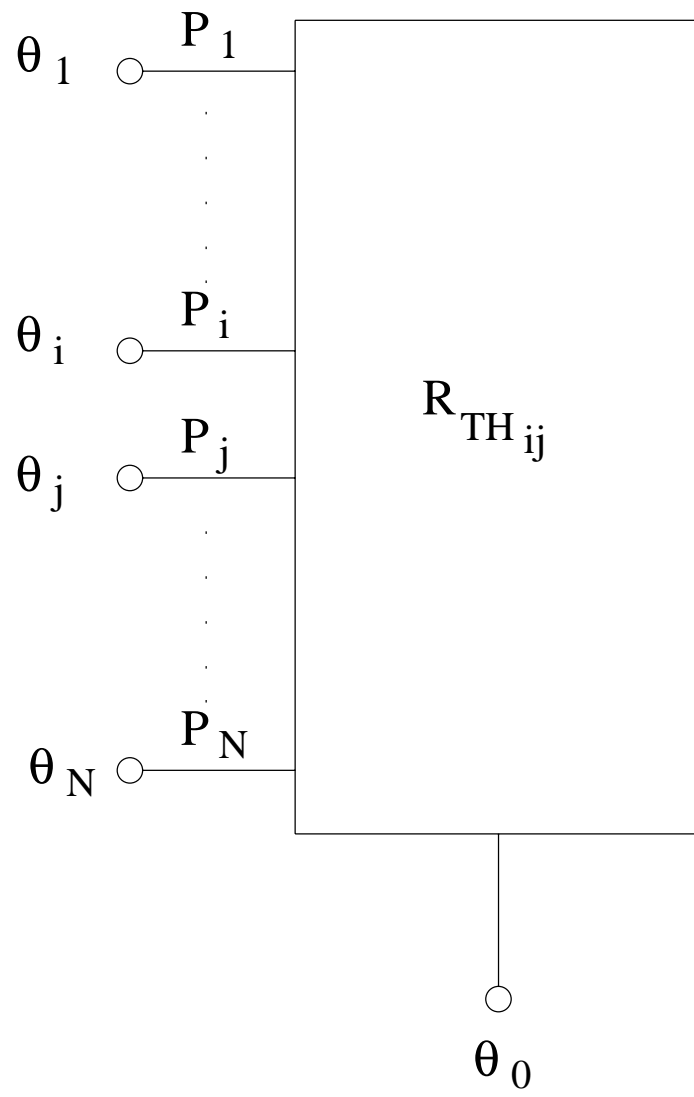


Fig. 2.

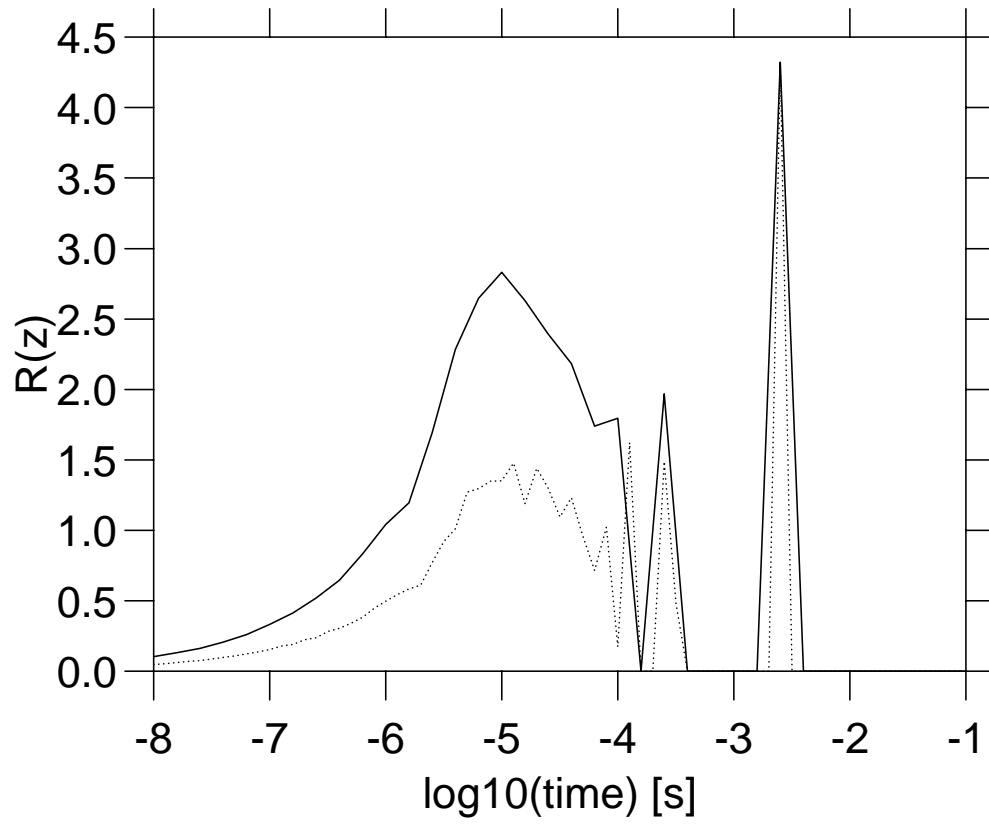


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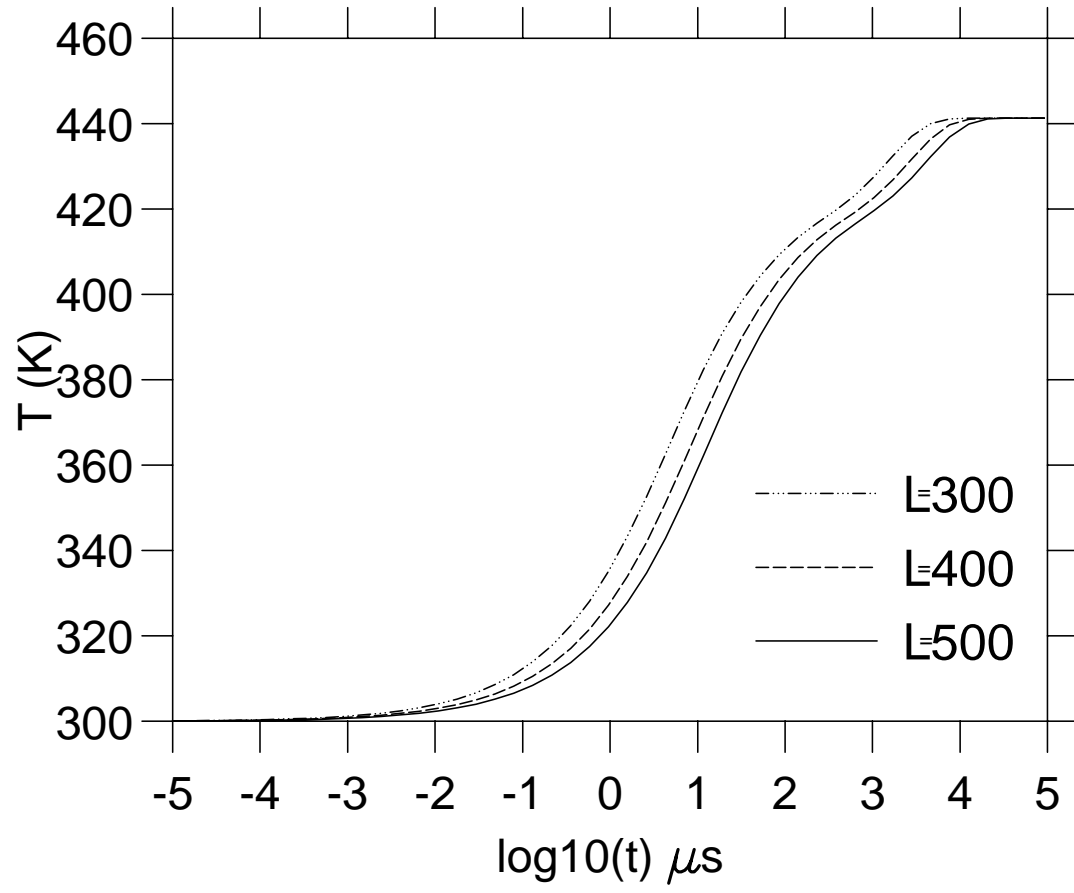


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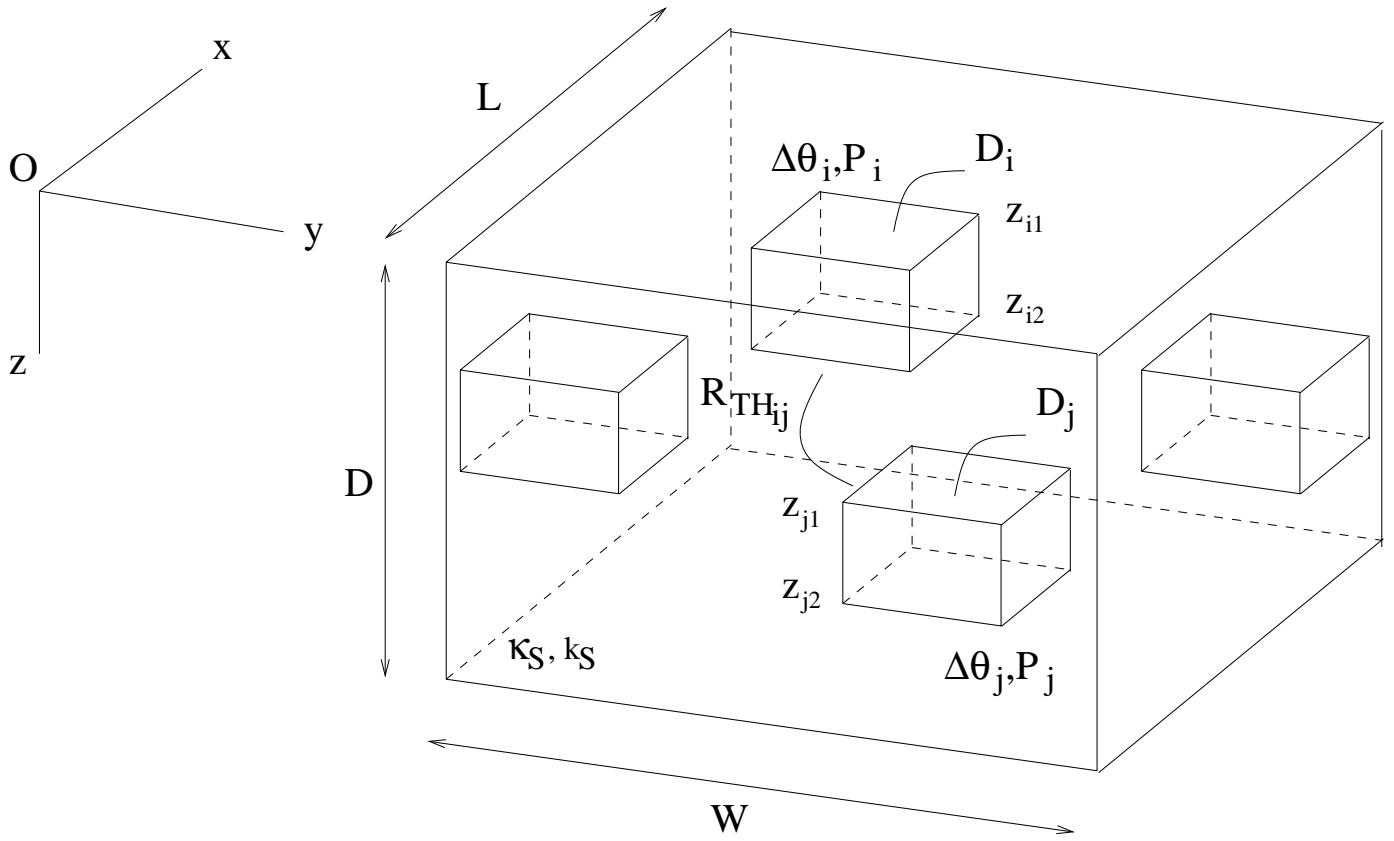


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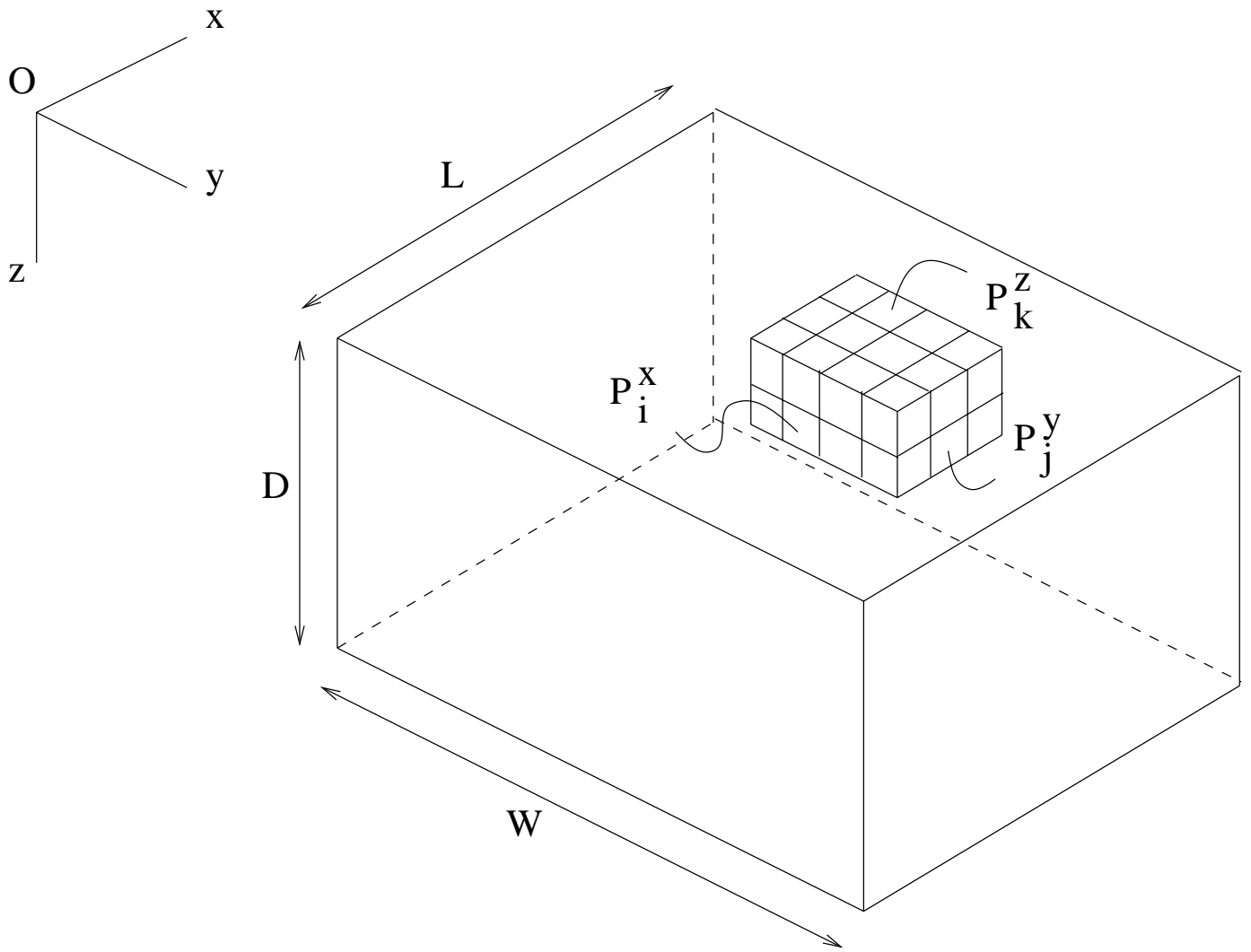


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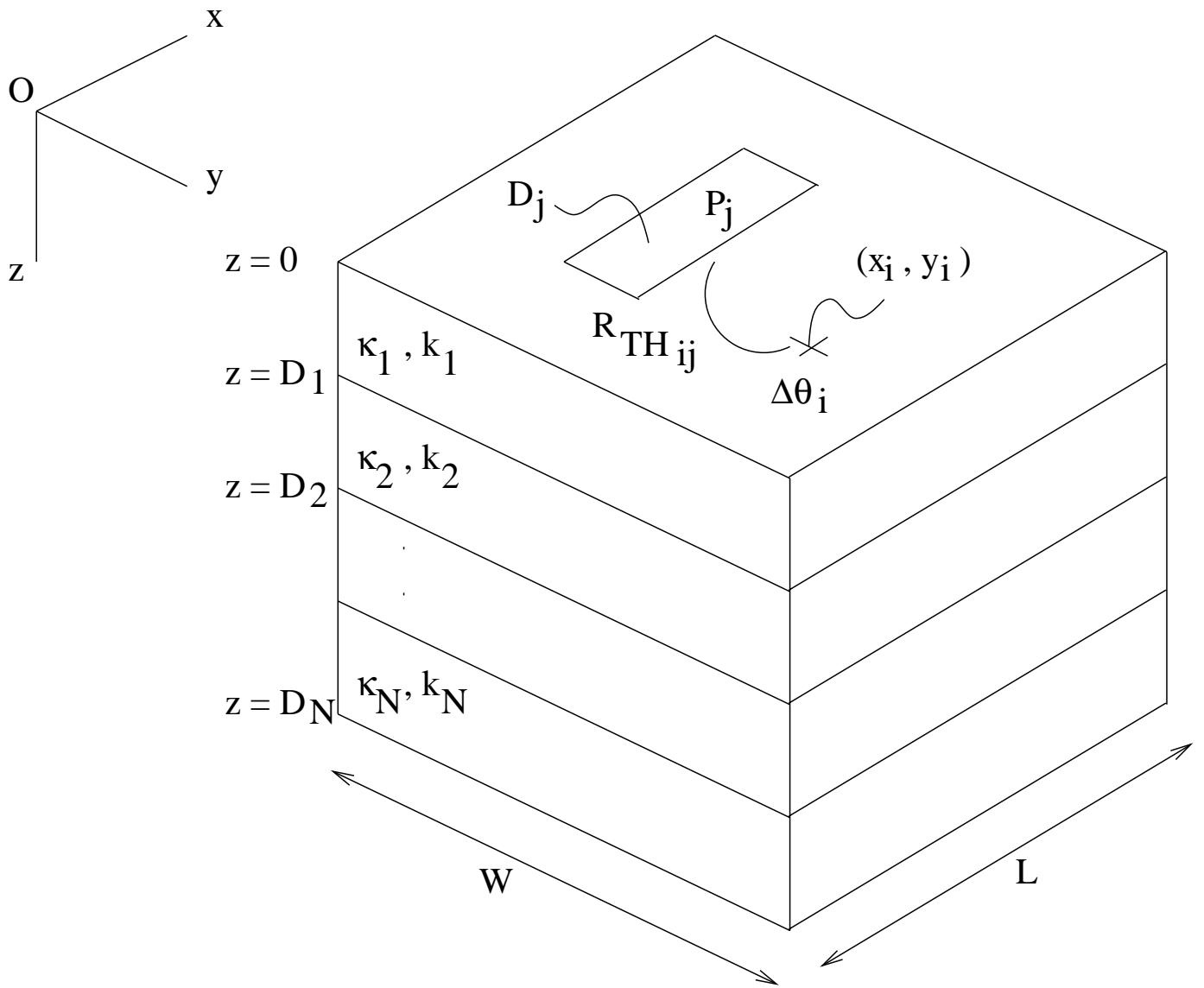


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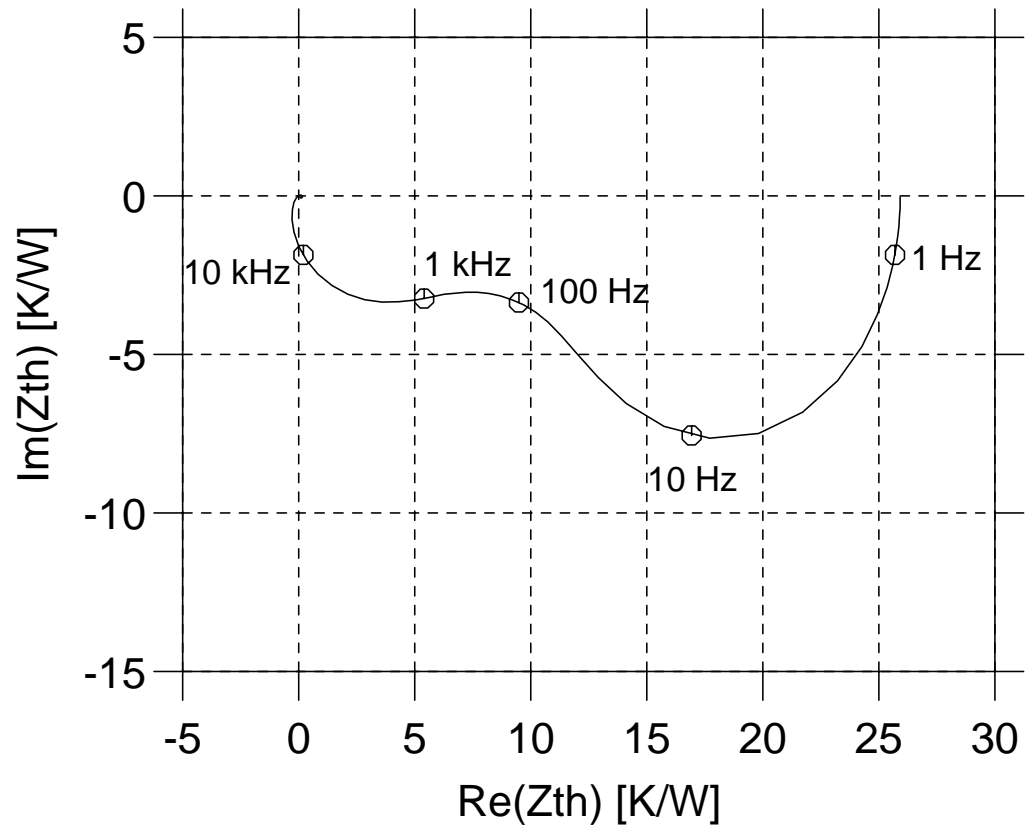


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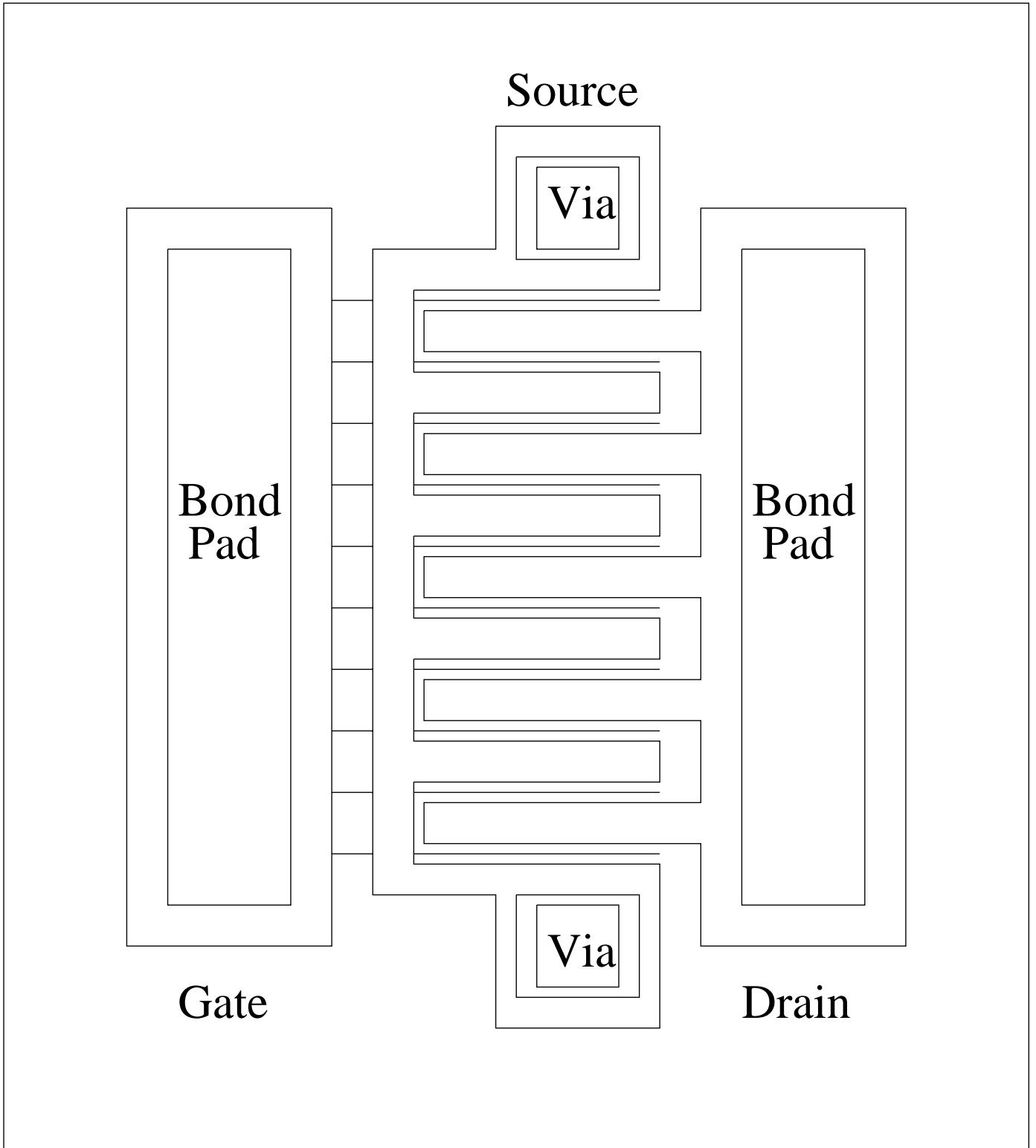


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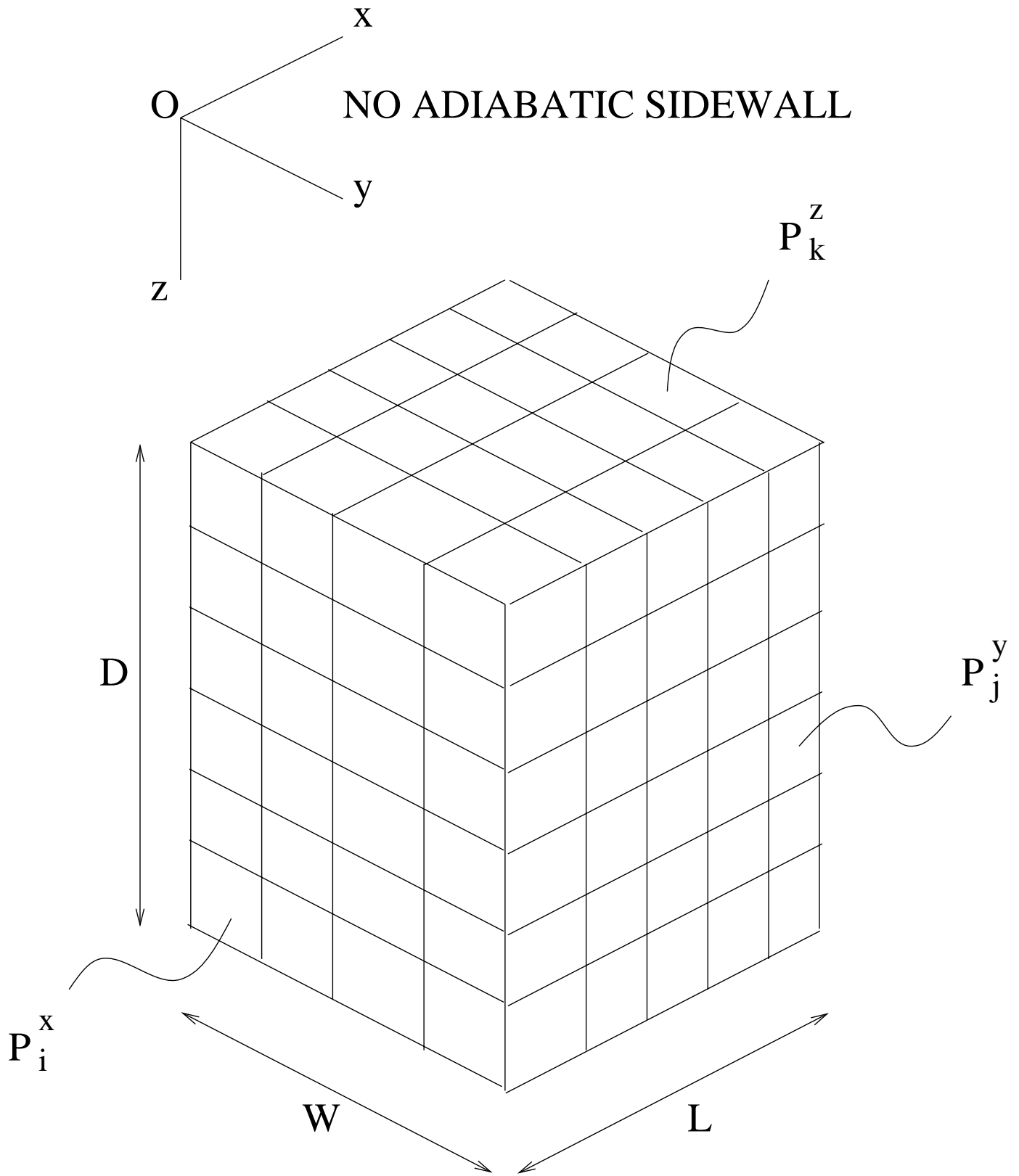


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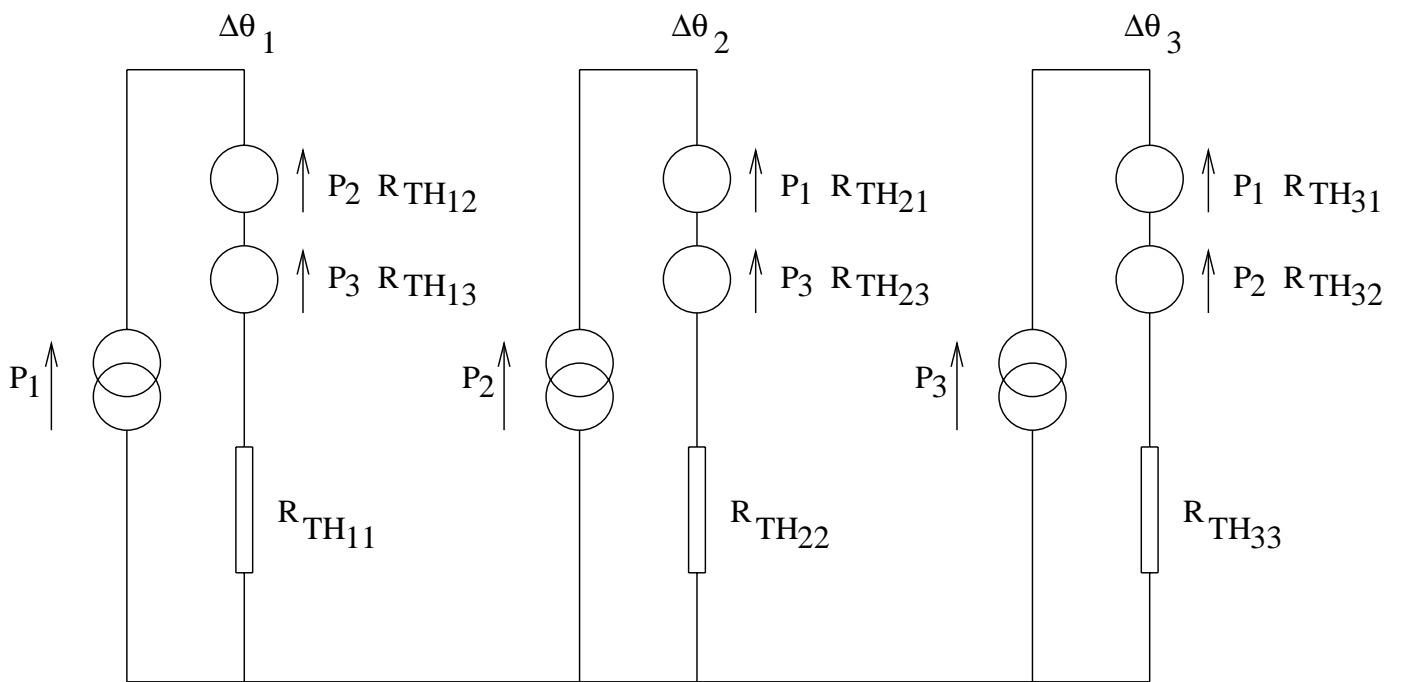


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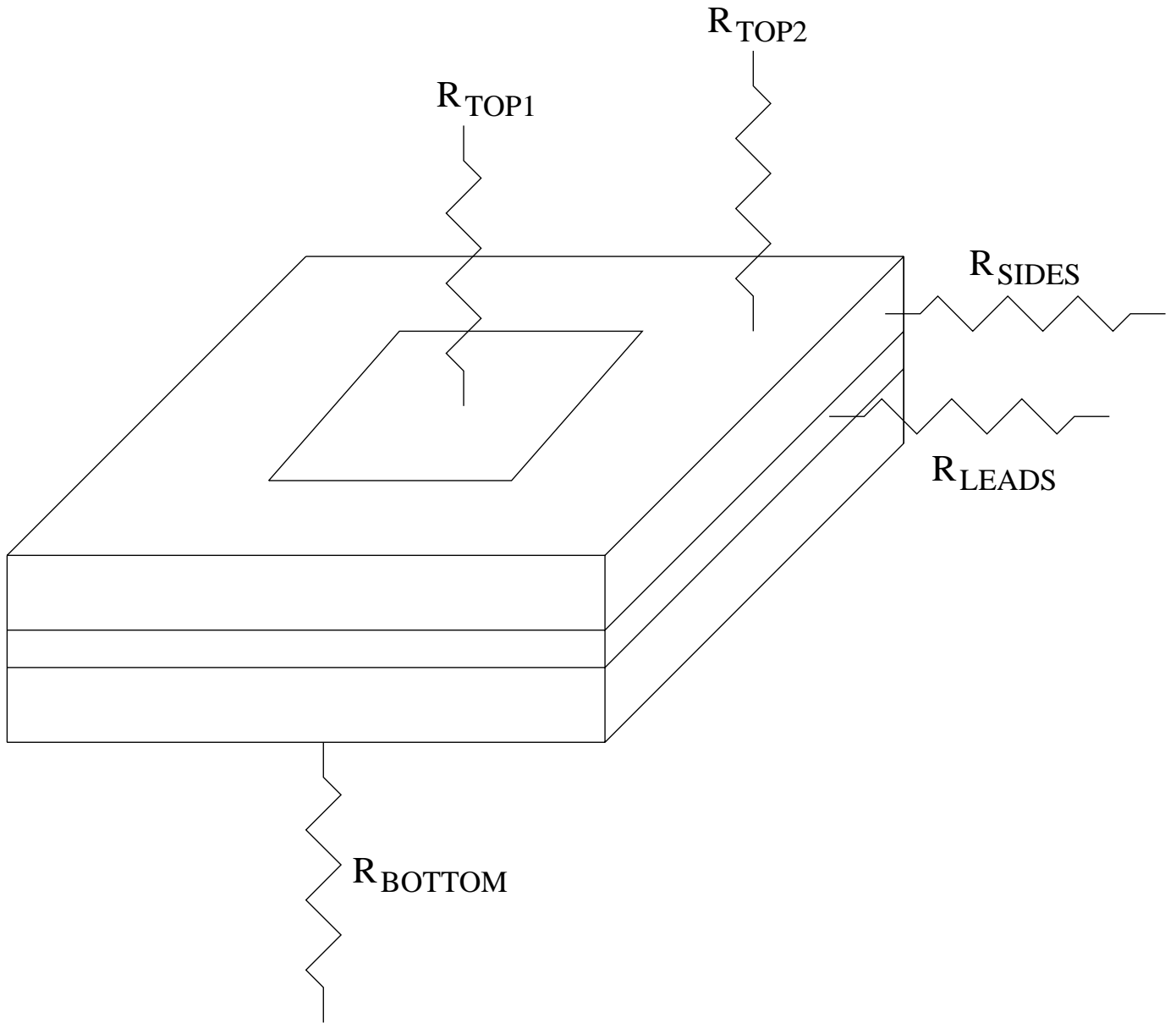


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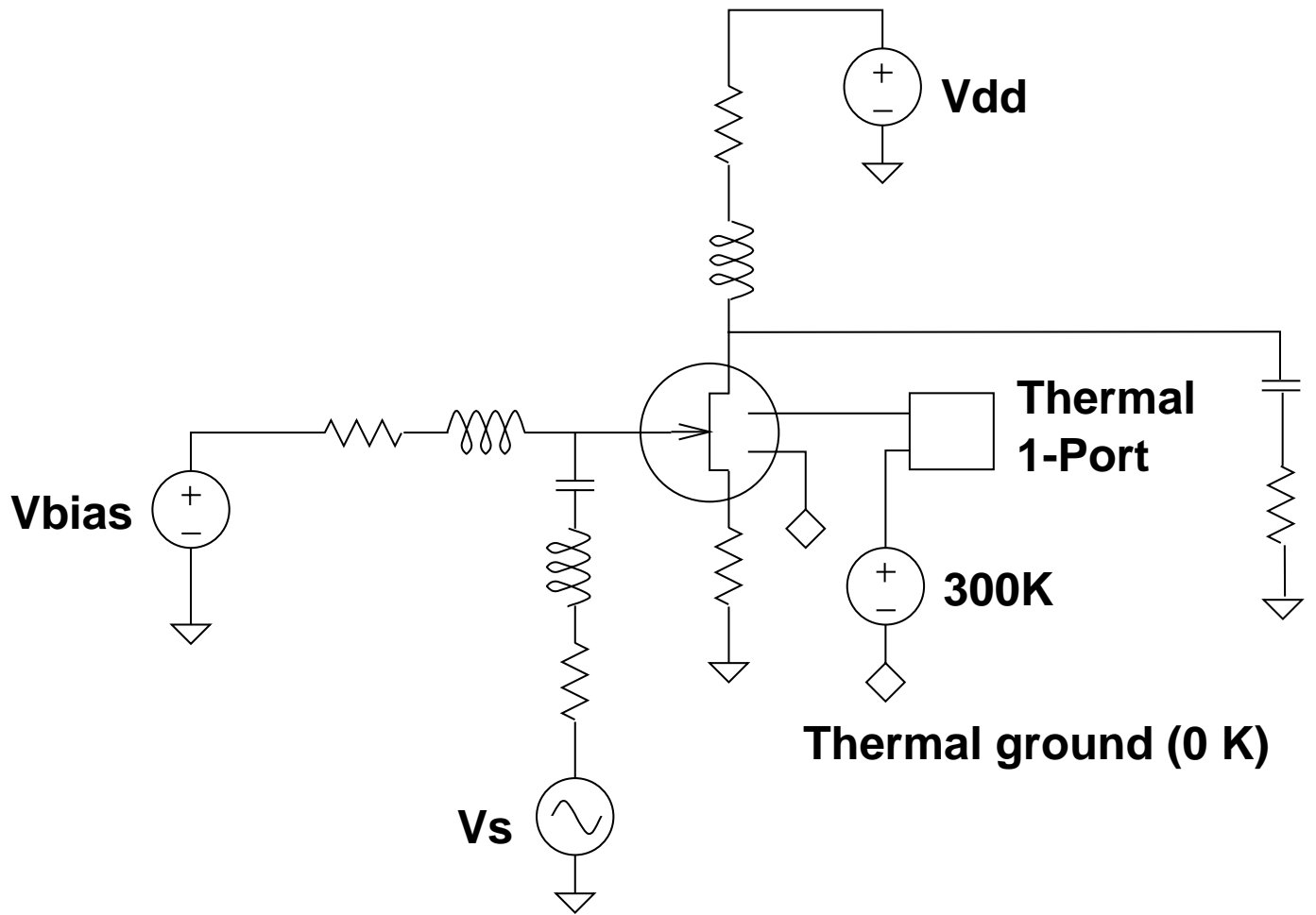


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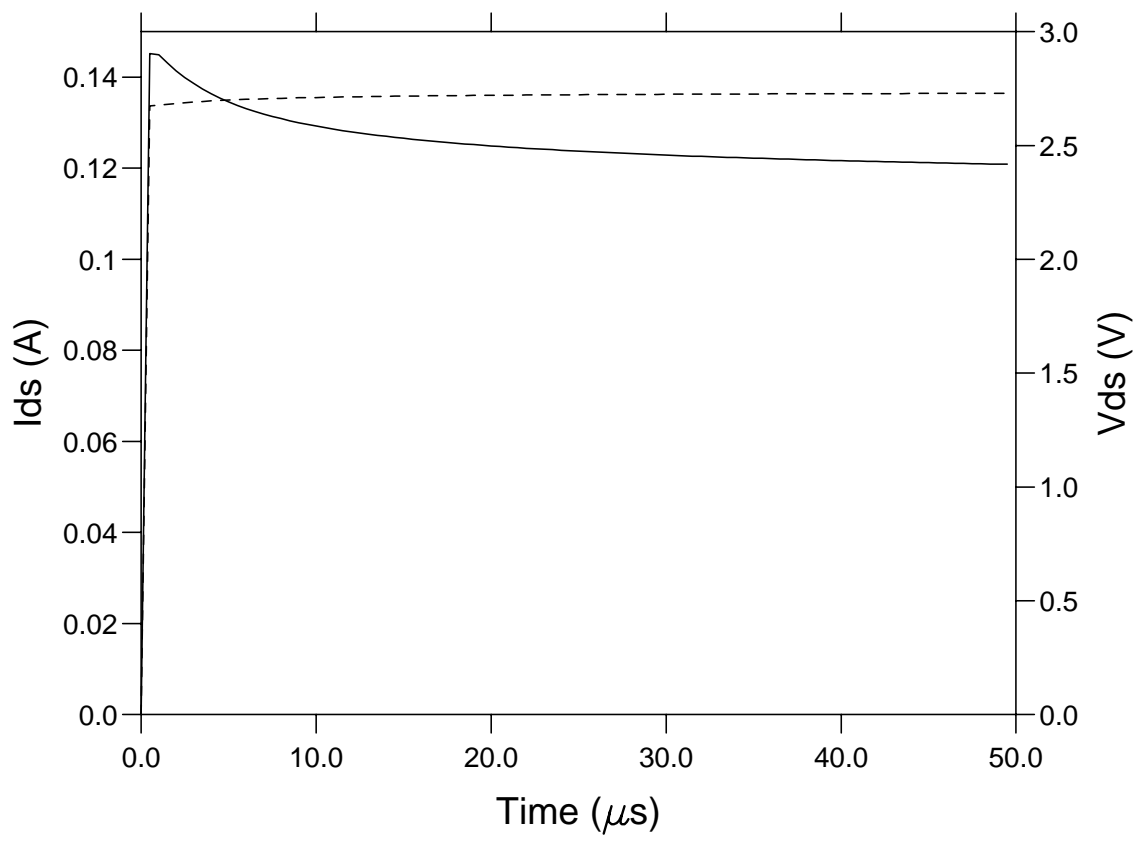


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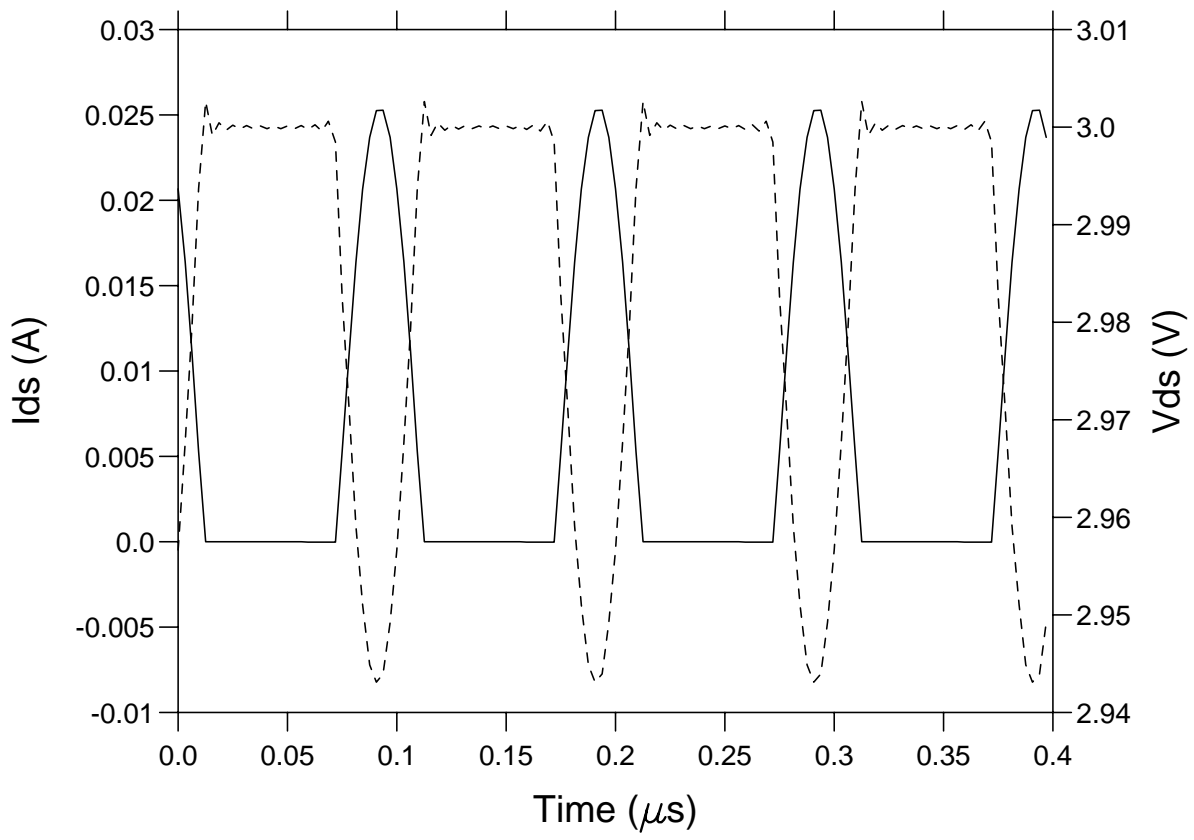


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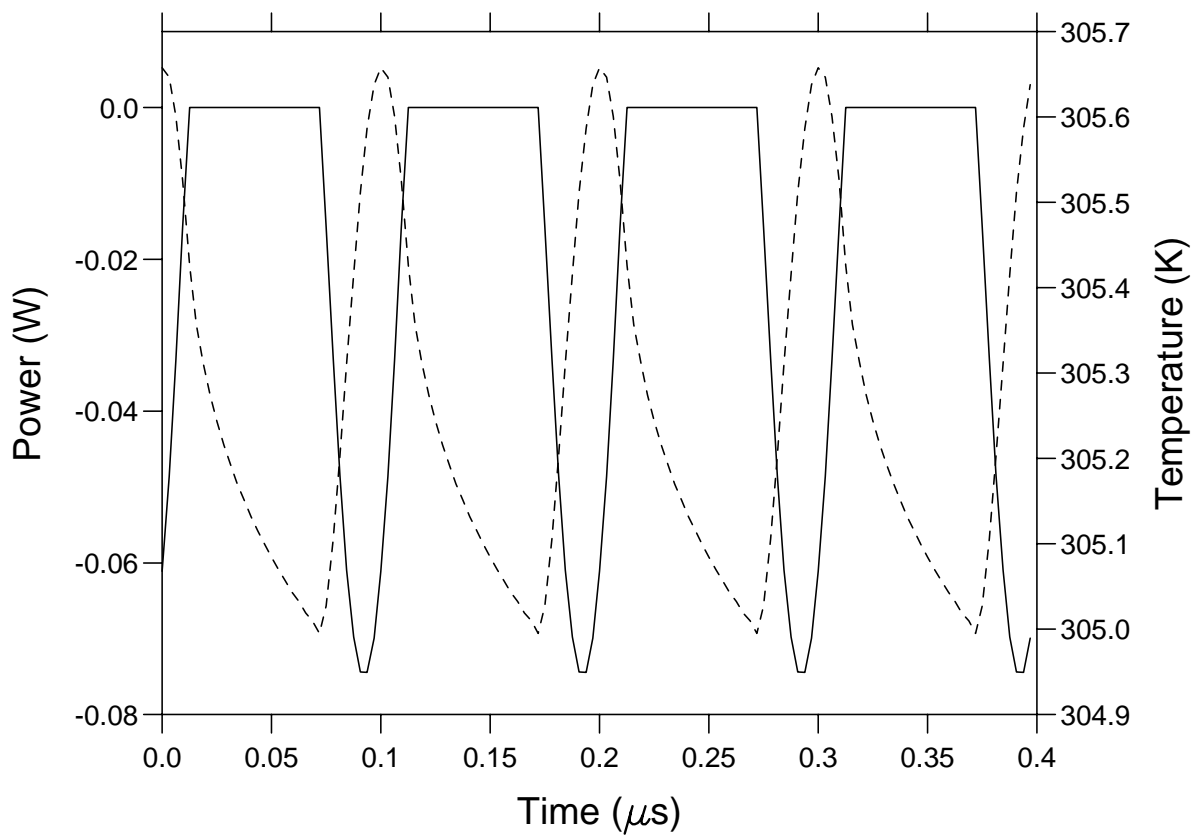


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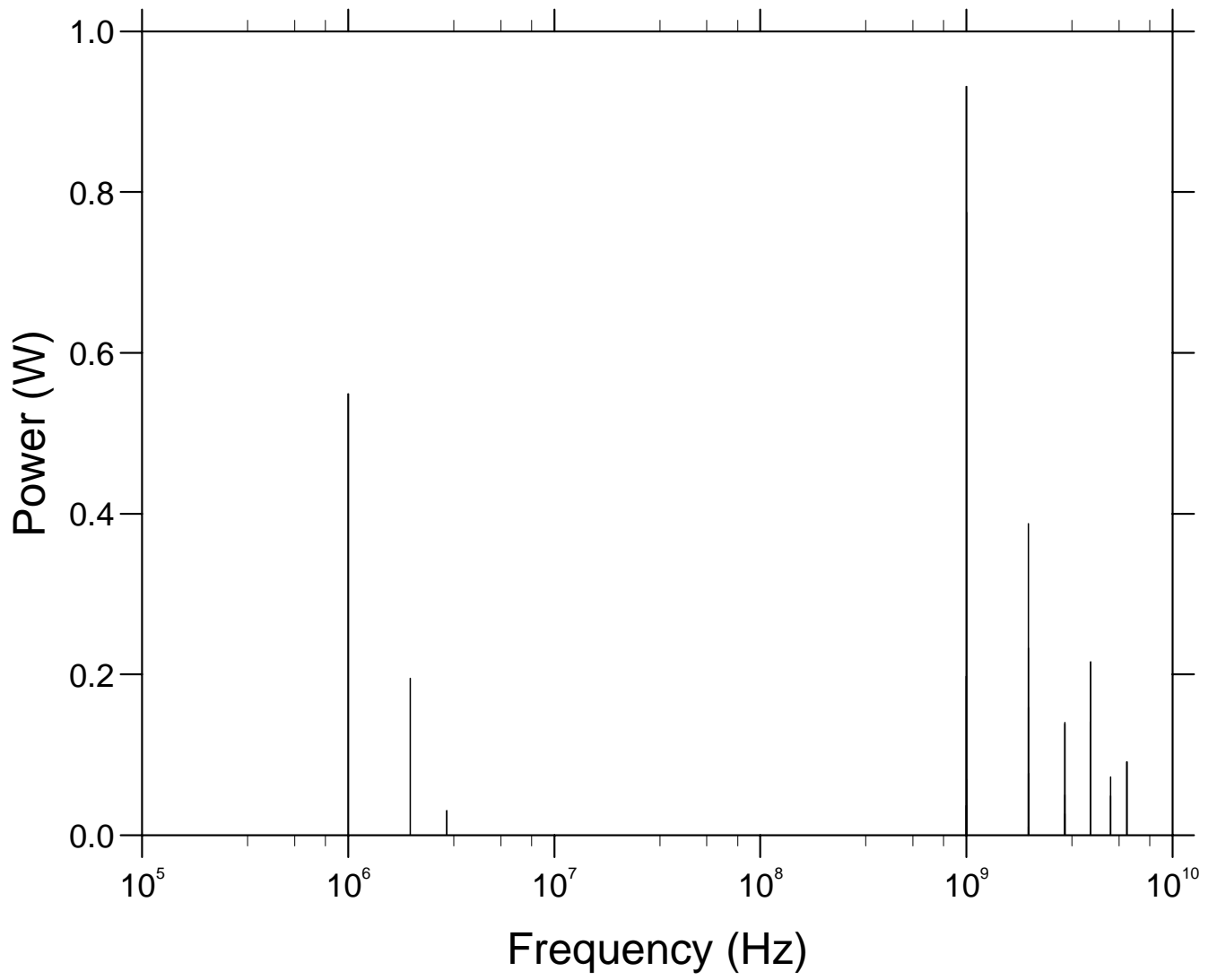


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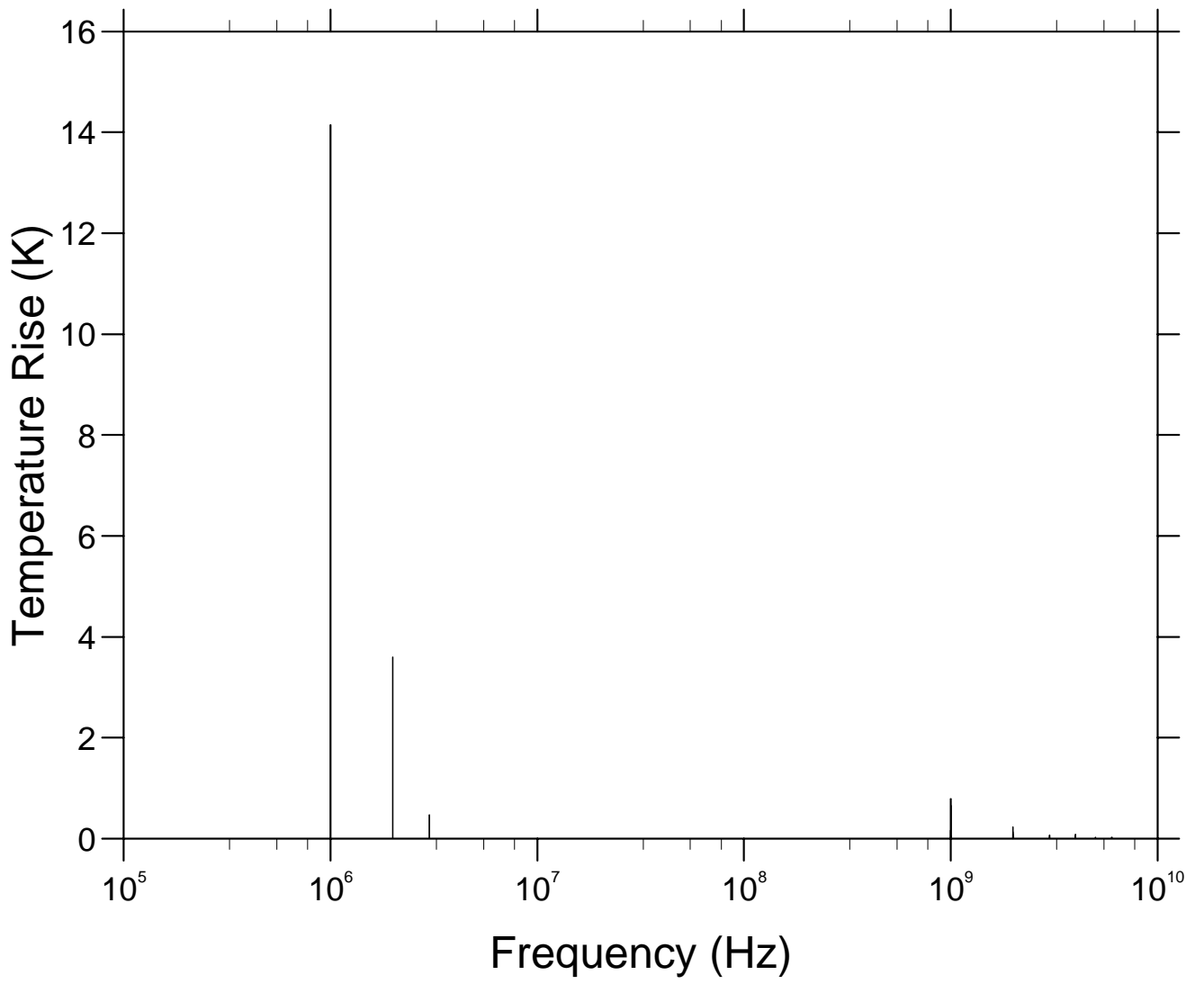


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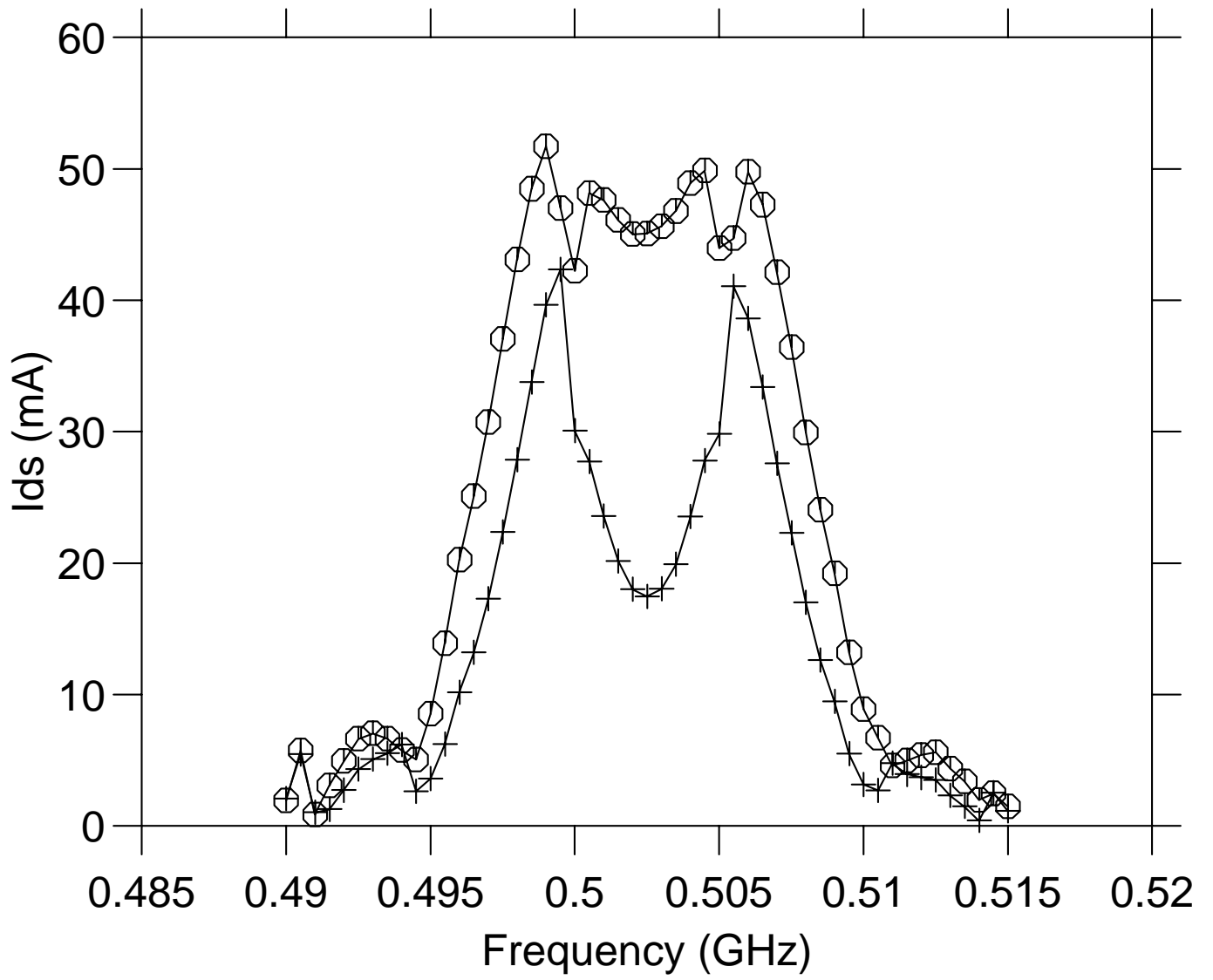
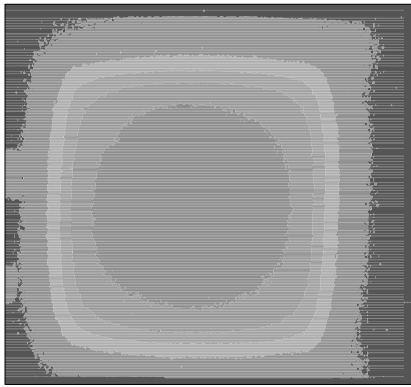
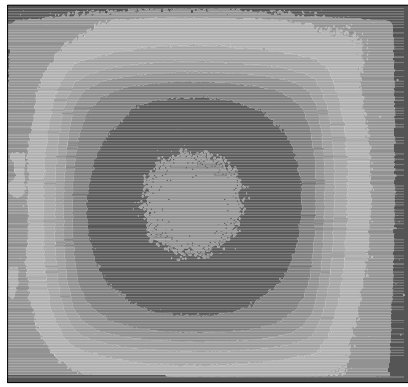


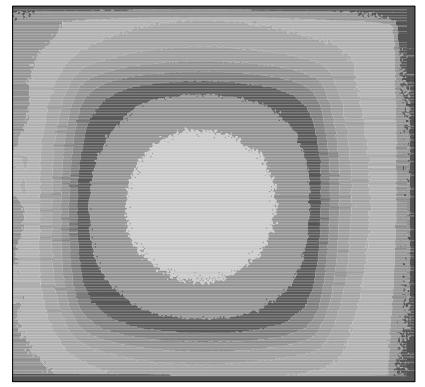
Fig. 19.



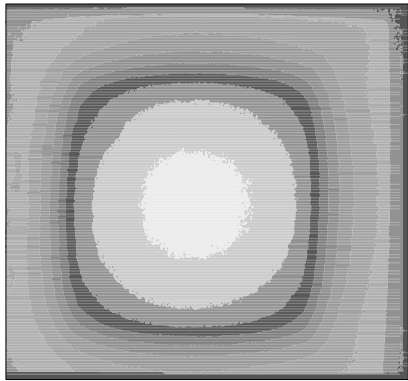
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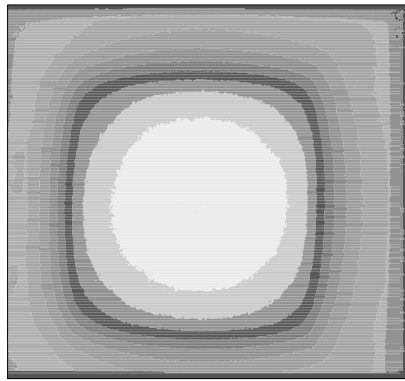
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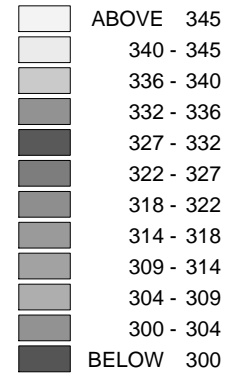
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$t = 200$ s

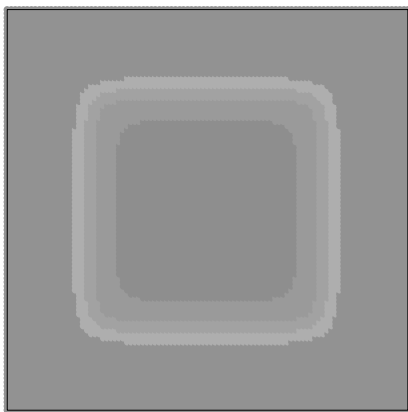


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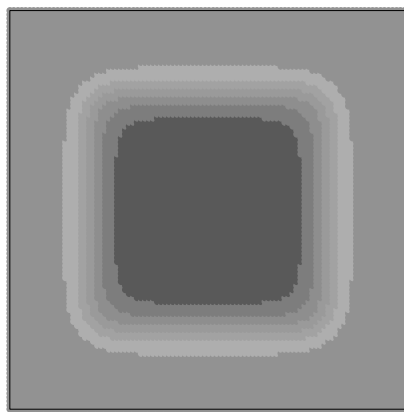


Temperature (K)

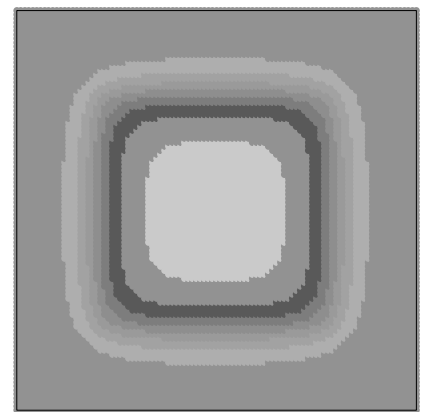
Fig. 20.



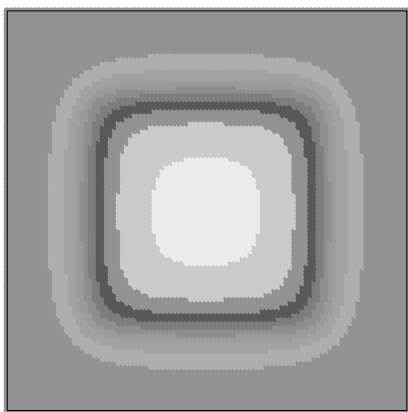
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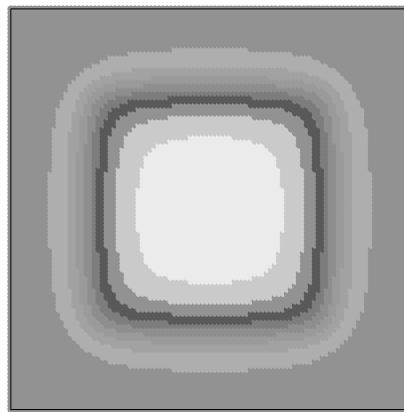
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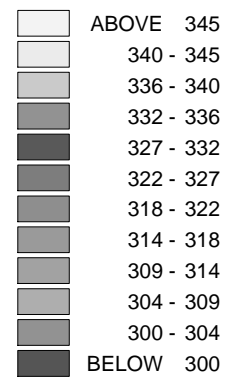
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$t = 200$ s



$t = 250$ s



Temperature (K)

Fig. 21.