Essential Heat Transfer for Electrical Engineers
Second Edition

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"If your experiment needs statistics, you ought to have done a better experiment."

Ernest Rutherford
(1871-1937)

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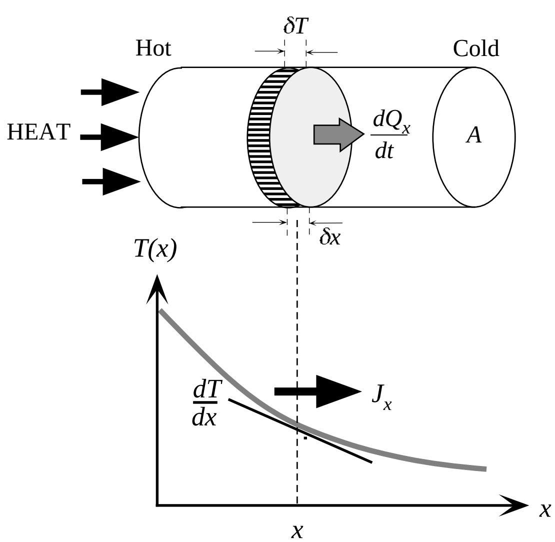
1 Phenomenology of Thermal Conduction

Heat is the amount of energy that is transferred from one system to another (or between the system and its surroundings) as a result of a temperature difference. It is not a new form of energy but rather the *transfer of energy* from one body to another by virtue of the random motions of their molecules. When a hot body is in contact with a cold body, heat is transferred from the hot to the cold body. What is actually transferred is the excess mean kinetic energy of the molecules in the hot body. Molecules in the hot body have higher kinetic energy and vibrate more violently and, as a result of the collisions between the molecules, there is a net transfer of energy from the hot to the cold body until the molecules in both bodies have the same mean kinetic energy; when their temperatures are the same.



Conuction of heat through a solid body from hot to cold regions. Heat flux at a point x is proportional to the temperature gradient at that point.

Figure 1

Heat conduction in materials is generally described by **Fourier's heat conduction law**. Suppose that J_x is the heat flux[1] in the x-direction, defined as the quantity of heat flowing in the x-direction per unit area per unit second; thermal energy flux. Thus, if dQ_x/dt is the rate of heat flow through an area A, then the **heat flux** or the **heat current density** J_x as depicted in Figure 1, is defined by

$$J_x = \frac{1}{A} \cdot \frac{dQ_x}{dt} \qquad \textit{Definition of heat flux} \qquad (1\text{-}1)$$

Fourier's law states that the heat flux at a point in a solid is proportional to the temperature gradient at that point and the proportionality constant depends on the material,

$$J_x = -\kappa \frac{dT}{dx} \qquad \textit{Fourier's Law} \qquad (1\text{-}2)$$

where κ is a constant that depends on the material, called the **thermal conductivity** (W m^{-1} K^{-1} or W m^{-1} °C^{-1}), and $\dfrac{dT}{dx}$ is the temperature gradient. Equation (1-2) is called **Fourier's law**.

The thermal conductivity depends on how the atoms in the solid transfer the energy from the hot region to the cold region. In metals, the energy transfer involves the conduction electrons. In nonmetals, the energy transfer involves lattice vibrations, that is atomic vibrations of the crystal which are described in terms of *phonons*.

Table 1

Typical thermal conductivities of various classes of materials at 25 °C.

Pure metal	Nb	Sn	Fe	Zn	W	Al	Cu	Ag
κ (W m^{-1} K^{-1})	52	64	80	113	178	250	390	420
Metal alloys	Stainless Steel	55Cu-45Ni	Manganin (86Cu-12Mn-2Ni)	70Ni-30Cu	1080 Steel	Bronze (95Cu-5Sn)	Brass (63Cu-37Zn)	Dural (95Al-4Cu-1Mg)
κ (W m^{-1} K^{-1})	12 - 16	19.5	22	25	50	80	125	147
Ceramics and glasses	Glass-borosilicate	Silica-fused (SiO_2)	S_3N_4	Alumina (Al_2O_3)	Magnesia (MgO)	Saphire (Al_2O_3)	Beryllia (BeO)	Diamond
κ (W m^{-1} K^{-1})	0.75	1.5	20	30	37	37	260	1000
Polymers	Polypropylene	Polystyrene	PVC	Polycarbonate	Nylon 6,6	Teflon	Polyethylene low density	Polyethylene high density
κ (W m^{-1} K^{-1})	0.12	0.13	0.17	0.22	0.24	0.25	0.3	0.5

The efficiency of thermal conduction, as gauged by κ, depends on how easily a neighboring cooler region can absorb heat and thus on the heat capacity per unit volume C_v. Since electrons are involved as the energy carriers, κ increases if electrons can travel long distance before being scattered; κ is hence proportional to the mean free path ℓ of the electrons. Increasing the mean speed u of the electrons also increases the rate at which heat is transferred. From the kinetic theory one can derive the following relationship for κ in terms of the heat capacity per unit volume C_v, the mean speed u of the

[1] Flux is flow per unit area per unit time. Particle flux is the number of particles flowing per unit area per unit time. Energy flux is the flow of energy per unit area per unit time, that is, power flow per unit area.

particles (electrons or phonons) involved in the thermal conduction process, and the mean free path ℓ of the particles due to their collisions with each other or with other particles or imperfections:

$$\kappa = \frac{1}{3} C_v u \ell$$

The thermal conductivity, in general, depends on the temperature. Different classes of materials exhibit different κ values and also κ vs. T behavior but we can generalize very roughly as follows:

Most pure metals: $\kappa \approx$ 50 - 400 W m^{-1} K^{-1}. At sufficiently high T, *e.g.* above ~100 K for copper, $\kappa \approx$ constant. In magnetic materials such as iron and nickel, κ decreases with T.

Most Metal alloys: κ less than for pure metals; $\kappa \approx$ 10 – 100 W m^{-1} K^{-1}. κ increases with increasing T.

Most Ceramics: Large range of κ, typically 10 - 200 W m^{-1} K^{-1} with diamond and beryllia being exceptions with high κ. At high T, typically above ~100 K, κ decreases with increasing T and eventually saturates at very high T, *e.g.* ~1000 °C for alumina (Al_2O_3).

Most Glasses: Small κ, typically less than ~5 W m^{-1} K^{-1} and increases with increasing T. Typical examples are borosilicate glasses, window glass, soda-lime plate, fused silica *etc*. Quartz is a SiO_2 *crystal* with κ = 1.5 W m^{-1} K^{-1}, whereas fused silica is *noncrystalline* SiO_2 with κ = 2 W m^{-1} K^{-1}.

Polymers: κ is very small and typically less than 2 W m^{-1} K^{-1} and increases with increasing T. Good thermal insulators.

In the case of metals, conduction electrons are involved in the thermal conduction process. Since the electrical conductivity σ is also determined by these conduction electrons, the Wiedemann-Franz-Lorenz law for *metals* relates electrical and thermal conductivities:

$$\frac{\kappa}{\sigma T} = C_{\mathrm{WFL}} \quad = \text{Constant} = 2.45 \times 10^{-8}\ \text{W}\ \Omega\ \text{K}^{-2}$$

2 Equation of Continuity and the Parabolic Heat Equation

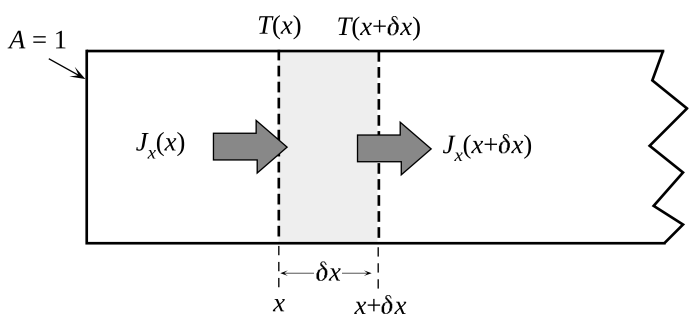


Figure 2

Consider one dimensional heat flow along the x-axis through a rod of uniform cross sectional area as schematically shown in Figure 2. For simplicity take the cross sectional area A = 1. Assume there is a perfect insulator at the cylindrical surface of this rod so that there is no heat escape from the surface. Consider the elemental volume $A\delta x$, or δx, shown in Figure 2 which has its ends at x and $x + \delta x$ at temperatures $T(x)$ and $T(x + \delta x)$ at time t.

The mass of this elemental volume is $\delta x\rho$ where ρ is the density. If c is the specific heat capacity (heat capacity per unit mass), its total heat capacity is $\delta x\rho c$. A temperature rise of δT in δx, will increase the heat content of this volume by $(\delta x\rho c)\delta T$. If this increase δT occurs in a time δt, then

Rate of increase in the heat content of elemental volume = $(\delta x\rho c)\left(\frac{\delta T}{\delta t}\right)$

We now apply the conservation energy law to the volume $A\delta x$ in Figure 2.

Heat flow in – Heat flow out

= Rate of heat accumulation in volume δx

$$J_x(x) - J_x(x+\delta x) = (\delta x\rho c)\left(\frac{\partial T}{\partial t}\right) \quad \textit{Conservation of energy} \quad (2\text{-}1)$$

We have replaced $\delta T/\delta x$ with $\partial T/\partial t$, a partial derivative, as T is a function of both x and t. Suppose that δx is small (and it is, by assumption) and hence the difference $J_x(x + dx)$ and $J_x(x)$ is also small. The flux gradient $\partial J_x/\partial x$ at x at one instant, at time t, can be written as,

$$\frac{\partial J_x}{\partial x} = \frac{J(x+\delta x) - J(x)}{\delta x} \quad \textit{Flux gradient} \quad (2\text{-}2)$$

Using Eq. ((2-1)) and (2-2) we get

$$\frac{\partial J_x}{\partial x} = -\frac{(\delta x\rho c)\left(\frac{\partial T}{\partial t}\right)}{\delta x} = -c\rho\frac{\partial T}{\partial t} \quad \textit{Equation of continuity} \quad (2\text{-}3)$$

Using Fourier's Law in Eq. (1-2) in this we obtain

$$\frac{\kappa}{c\rho}\cdot\frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t} \quad \textit{Parabolic heat equation} \quad (2\text{-}4)$$

We define a new quantity called **thermal diffusivity** D_{th} as follows

$$D_{th} = \frac{\kappa}{c\rho} \quad \textit{Thermal diffusivity} \quad (2\text{-}5)$$

which may be a function of temperature. Thus, Eq. (2-4) is,

$$D_{th}\frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t} \quad \textit{Parabolic heat equation} \quad (2\text{-}6)$$

This is a *linear partial differential equation* only if $D_{th} \approx$ constant. The temperature T is a function of x and t, *i.e.* $T = T(x,t)$.

3 Thermal Resistance, Capacitance and Heat Sinks

1.1 Thermal Resistance

Consider a component of length L that has a temperature difference ΔT between its ends as in Figure 3 (a). The temperature gradient is $\Delta T/L$. Thus, the rate of heat flow, or the "heat current", as determined by Eq. (1-2) means that,

$$Q' = A\kappa\frac{\Delta T}{L} = \frac{\Delta T}{\left(\dfrac{L}{A\kappa}\right)}$$

In analogy with an electrical resistance, we may define a **thermal resistance** θ by

$$Q' = \frac{\Delta T}{\theta} \qquad \textit{Definition of thermal resistance} \qquad (3\text{-}1)$$

where θ, the thermal resistance, for thermal conduction, must be given by

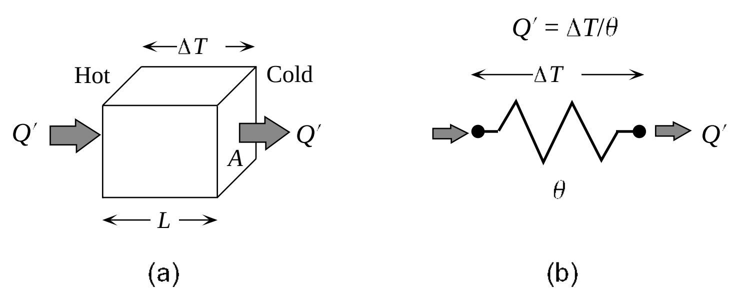
$$\theta = \frac{L}{A\kappa} \qquad \textit{Thermal resistance} \qquad (3\text{-}2)$$

where L is the length of the specimen and A is the cross sectional area as in Figure 3 (a). Equation (3-1) should be compared with Ohm's Law in electrical circuits, *i.e.*

$$I = \frac{\Delta V}{R} \qquad \textit{Ohm's law} \qquad (3\text{-}3)$$

where ΔV is the voltage difference across a conductor of resistance R[2] and I is the current flow. The rate of heat flow, Q', and the temperature difference, ΔT, correspond to the current, I, and potential difference, ΔV, respectively. Thermal resistance is the thermal analog of electrical resistance and its thermal circuit representation is shown in Figure 3 (b).

It should be remarked that Eq. (3-1) is a general definition of thermal resistance θ whatever the heat transfer mechanism. If heat transfer is by thermal conduction, then θ is given by Eq. (3-2). Thermal resistance is a convenient and simple way to relate the heat flow Q' between a temperature difference ΔT to ΔT itself, just like electrical resistance R relates the current to the voltage difference ΔV. The virtue of R in electrical circuits is that the analysis of dc circuits is simplified to solving a set of linear equations because normally R does not depend on the voltage or the current. The same convenience is experienced in thermal analysis if θ can be taken to be temperature independent. Heat transfer equations then become linear. While this is reasonably true for thermal conduction, as apparent from Eq. (3-2), it is not so for heat transfer by convection and radiation. But, even in the latter cases it is still convenient to introduce a thermal resistance.

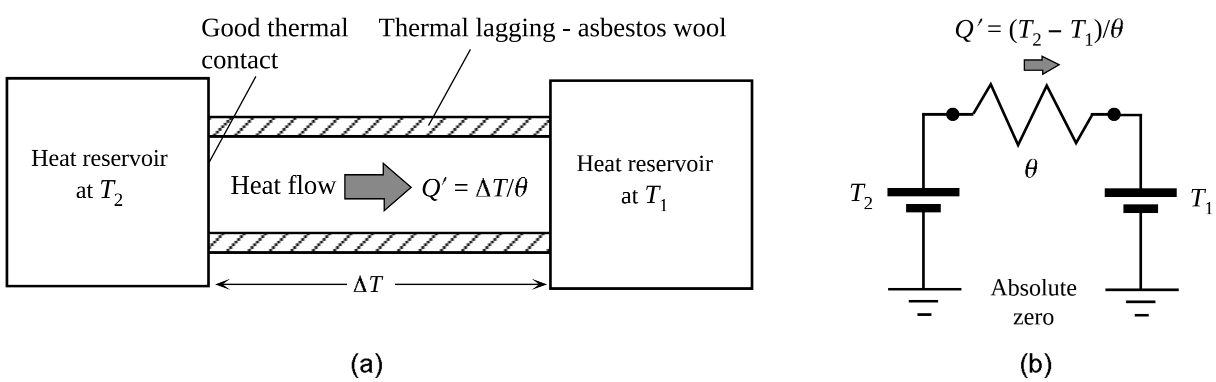


Conduction of heat through a component in (a) can be modeled as a thermal resistance θ shown in (b) where $Q' = \Delta T/\theta$.

Figure 3

[2] The electrical resistance of a conductor $R = L/(A\sigma)$ where σ is the electrical conductivity.

We can further develop electricity-heat analogy by considering a component conducting heat from a constant **temperature bath** at T_2 to a constant temperature bath at T_1 as shown in Figure 4 (a). These constant temperature baths are essentially **hear reservoirs**, that is, they can source or sink heat without changing their temperatures[3]. We neglect heat losses from the surface. This component can be represented by a thermal resistance θ as in Figure 4 (b) The temperature of a temperature bath, by definition, is constant whether one extracts or dumps heat into a such a bath. The electrical analogy is an EMF source, that is a battery with no internal resistance, since the current flowing in or out of such a battery does not change the voltage (internal resistance = 0). We can represent the two temperature baths as two batteries with voltages T_2 and T_1. In electrical circuits, ground is normally taken as the zero-potential. In thermal circuits, the ground corresponds to the absolute temperature, because this is the lowest temperature one can achieve; indeed, the **third law of thermodynamics** states that one never exactly reach zero absolute temperature. Thus, the thermal circuit in Figure 4 (a) can be modeled by the electrical analog in Figure 4 (b).



(a) A conductor between two heat reservoirs T_2 and T_1 , $T_2 > T_1$. There is no heat loss from the surface. Steady state heat conduction through this component is given by $Q' = \Delta T/\theta$. (b) We can model the heat flow using two thermal EMFs and a thermal resistance between them.

Figure 4

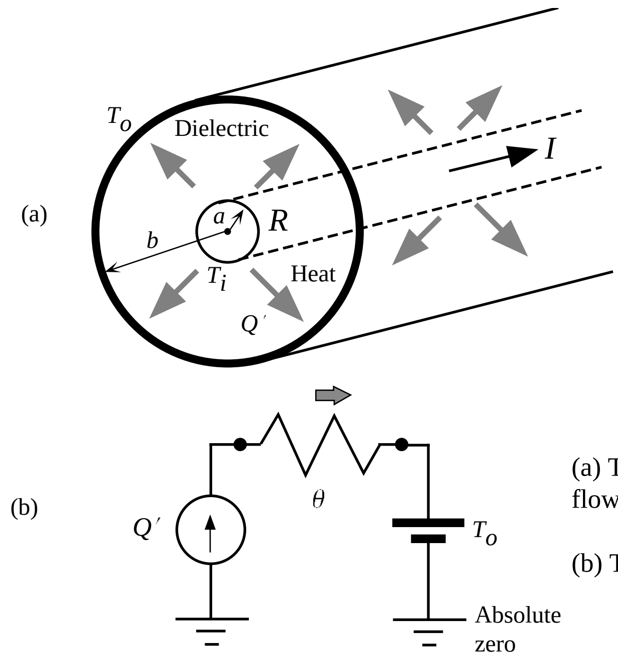
If there is a constant heat flow generated by a component in a system, then we can use a "*constant current supply*" to represent this heat flow. A good electrical engineering example is the flow of heat from the inner conductor to the outer conductor in a coaxial cable carrying a current as shown in Figure 5 (a). The Joule heating of the inner conductor would be I^2R, where I is the current and R is the resistance of the core conductor. This Joule heat has to flow through the dielectric insulation and reach the outer conductor. The outer conductor generally has a thick conducting jacket and it is in good contact with its environment so that its temperature T_o does not change very much; it is approximately the same as the ambient temperature. The heat reaching the outer conductor is quickly removed by convection and conduction. The cable jacket and its environment act as a temperature bath and thus can be represented by a battery with a voltage T_o as shown in Figure 5 (b).

The heat generated by the core conductor can be modeled by using a **heat current generator** as in Figure 5 (b). This heat flows through the thermal resistance θ of the dielectric and reaches the

[3] One can dump or remove heat from a large tank of ice and water bath without changing its temperature. The heat added or removed simply changes the relative amounts of water and ice but not the temperature. The heat added, melts more ice, and the heat removed freezes more water.

temperature bath T_o. The full equivalent circuit has a current generator, resistance and an EMF in series. The temperature of the core-conductor can be found by analyzing the circuit in Figure 5 (b), *i.e.*

$$Q' = \frac{T_i - T_o}{\theta}$$



(a) The joule heat generated in the core conductor flows outwards radially through the dielectric material.

(b) The equivalent circuit

Figure 5

Example 3.1: Steady state heat flow in a coaxial cable

Consider a coaxial cable operating under steady state conditions when the current flow through the inner conductor generates Joule heat at a rate $P = I^2R$ as shown in Figure 6. The heat generated per second, I^2R, by the core conductor flows through the dielectric. Thus, the rate of heat flow $Q' = I^2R$. Consider a thin cylindrical shell of thickness dr. The temperature difference across dr is dT. The surface area of this shell is $2\pi rL$. Thus, from Fourier's law,

$$Q' = -(2\pi rL)\kappa\frac{dT}{dr}$$

which we can integrate with respect to r from $r = a$ where $T = T_i$ to $r = b$ where $T = T_o$,

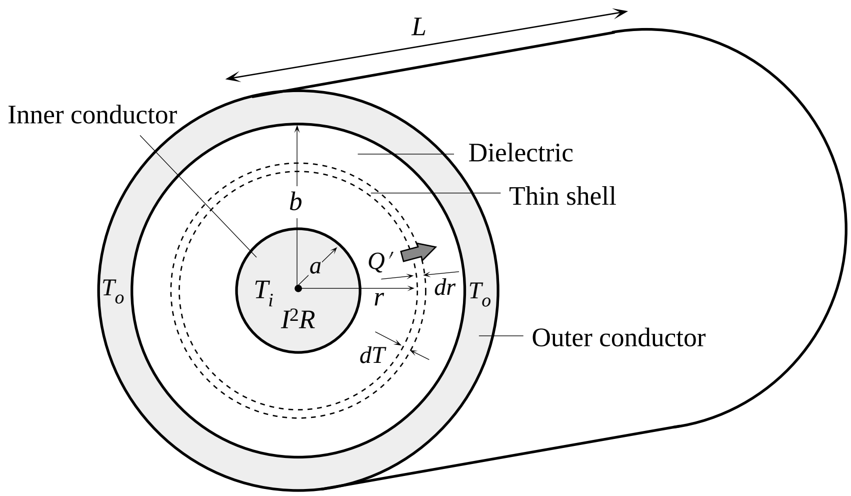
$$Q'\int_a^b \frac{dr}{r} = -2\pi L\kappa\int_{T_i}^{T_o} dT$$

i.e.

$$Q' = (T_i - T_o)\frac{2\pi\kappa L}{\ln\left(\frac{b}{a}\right)}$$

Thus the thermal resistance of the hollow cylindrical insulation is

$$\theta = \frac{(T_i - T_o)}{Q'} = \frac{\ln\left(\frac{b}{a}\right)}{2\pi\kappa L} \qquad \textit{Thermal resistance of hollow cylinder} \qquad (3\text{-}4)$$



Thermal resistance of a hollow cylindrical shell. Consider an infinitesimally thin cylindrical shell of radius r and thickness dr in the dielectric and concentrically around the inner conductor. The surface area is $2\pi rL$.

Figure 6

Consider a coaxial cable that has an aluminum core conductor and polyethylene (PE) as the dielectric insulation with the following properties: Core conductor resistivity ρ = 27 nΩ m, core radius, a = 5 mm, dielectric thickness, $b - a$ = 3 mm, thermal conductivity of PE κ = 0.3 W m^{-1} K^{-1}. The outside temperature T_o is 20 °C. It is carrying a current of 500 A. What is the temperature of the inner conductor?

The actual length of the conductor does not affect the calculations as long as the length is sufficiently long that there is no heat transfer along the length; heat flows radially from the inner to the outer conductor. We consider a portion of length L of a very long cable and we set L = 1 m so that calculations are per unit length. The joule heating per unit second (power) generated by the current I through the core conductor is

$$Q' = I^2\frac{\rho L}{\pi a^2} = (500^2)\frac{(27\times10^{-9})(1)}{\pi(5\times10^{-3})^2} = \; 85.9\text{ W}$$

The thermal resistance of the insulation is

$$\theta = \frac{\ln\left(\frac{b}{a}\right)}{2\pi\kappa L} = \frac{\ln\left(\frac{(5+3)\times10^{-3}}{5\times10^{-3}}\right)}{2\pi(0.3)(1)} = 0.25\text{ °C/W}$$

Thus, the temperature difference ΔT due to Q' flowing through θ is,

$$\Delta T = Q'\theta = (85.9\text{ W})(0.25\text{ °C/W}) = 21.5\text{ °C}.$$

The inner temperature is therefore

$T_i = T_o + \Delta T = 20 + 21.5 = 41.4$ °C.

Note that for simplicity we assumed that the inner conductor resistivity ρ and thermal conductivity κ are constant (do not change with temperature). The thermal conductivity of many polymers increases with temperature so the assumption that κ remains constant is only approximately true. The κ value used is the average κ in the temperature range T_o to T_i.

3.2 Thermal Capacitance

Consider heat flow into a body that has infinite thermal conductivity so that its temperature is uniform everywhere. As heat is pumped into this body, the temperature difference ΔT between the body and its environment at T_o increases as shown in Figure 7 (a). An amount of heat δQ pumped into the body in time δt increases the temperature difference by $\delta\Delta T$, where

$$\delta Q = C\delta\Delta T$$

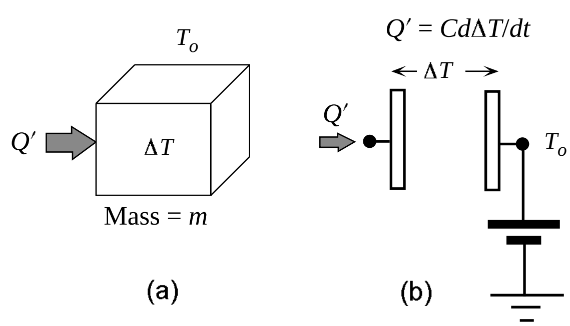
where C is thermal capacitance of the body, *i.e.* if c is the specific heat capacity and m is the mass of the body then $C = mc$. We can divide both side of the above equation by δt to relate the rate of heat flow to the rate of increase in the temperature difference,

$$Q' = C\frac{d\Delta T}{dt} \qquad \textit{Thermal capacitance} \qquad (3\text{-}4)$$

This expression is analogous to a current flow into a capacitor across which the voltage difference is ΔV. Thus, we can use a capacitor analog as in Figure 7 (b) to represent this type of capacitive thermal behavior. Suppose that we touch the tip of a 15 W soldering iron to a solder blob of mass 1 gram. The specific heat capacity of solder is about 0.17 J K^{-1} g^{-1}. Thus, $C = 0.17$ J K^{-1}. The time Δt it takes for this solder to reach 183 °C, its melting temperature, from 20 °C is

$$Q' = 10\ \text{W} = C\frac{d\Delta T}{dt} = (0.17)\frac{183-20}{\Delta t}$$

solving, $\Delta t = 1.8$ seconds. However, additional heat, heat of fusion (ΔH_f), still needs to be supplied to actually melt the solder at 183°C. (We also assumed that all the heat flows into the solder blob.)



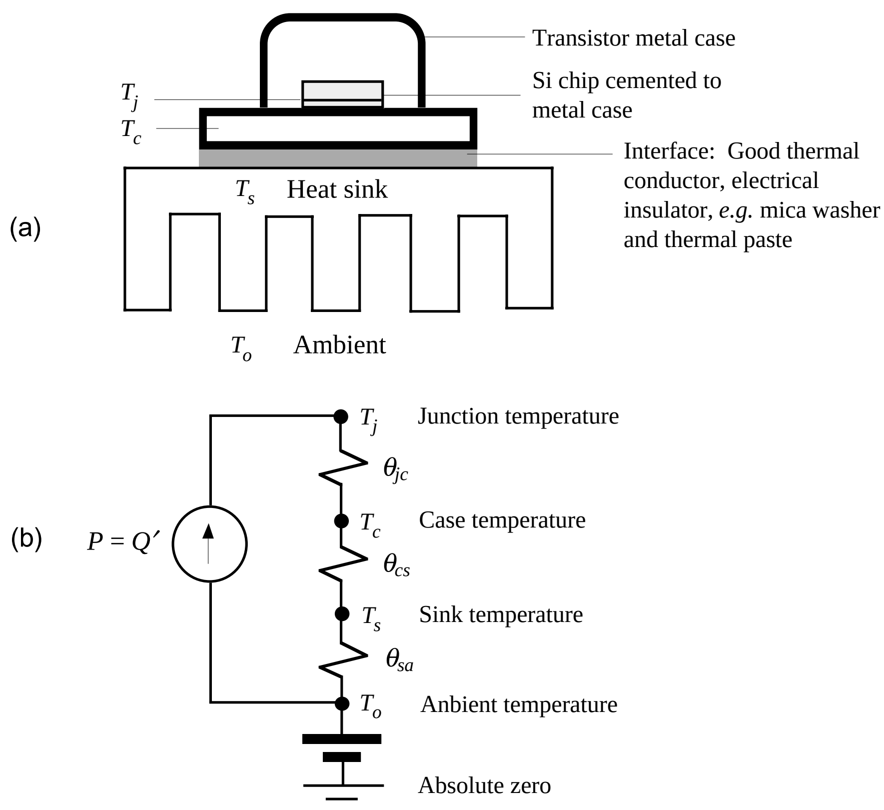
(a) When heat flows into a body, its temperature increases. The rate of increase in the temperature difference ΔT between the body and its invironment is determined by the heat current flowing into the body, that is, Q'. (b) Heating of a body uniformly can be represented as a thermal capacitance C into which a heat current flows and changes the temperature difference ΔT across C.

Figure 7

3.3 Heat Sinking in Electronics

A. Heat Sink Thermal Resistance

In electronic applications using semiconductor transistors and diodes, the junction temperature generally must be kept less than a certain maximum temperature; for Si based devices this is typically 110 − 180 °C. A maximum power rating on any semiconductor device by itself is a meaningless parameter. The significant parameters are the maximum operating **junction temperature** and **thermal resistance** from the junction to the transistor metal case. The mechanism of heat removal from the junction of almost all semiconductor devices is by heat conduction through the leads, and not convection. What is important in electronic design using a semiconductor device is that the junction temperature T_j inside the semiconductor chip does not reach or exceed the maximum value permitted.



(a) A heat sinked transistor. The transistor has a metal case and is mounted on a heat sink using an insulating washer and thermal paste (grease). Note: The heat sink is normally placed with the fins facing up, upside down to that shown, to allow heated air to rise and set a convective flow. This sketch is for convenience only.)
(b) The equivalent steady state circuit.

Figure 8

In a bipolar junction transistor (BJT) operating under normal conditions, the Joule heat is essentially generated in collector junction. If this heat is not removed then the junction temperature T_j will rise and eventually reach the breakdown temperature. Suppose that the **power dissipated** at the collector junction is P_d, for example, 15 W (as in a power transistor in a power amplifier). The semiconductor crystal is cemented to the metal case. The latter obviously warms up as it receives this heat flow from the semiconductor. The metal case is typically fastened (by screws) to a heat sink. However, a mica insulation (an insulating washer) has to be used between the transistor case and the heat sink since the transistor case is normally one of the transistor terminals (such as the collector), as depicted in Figure 8 (a).

The "heat current" flowing from the collector junction to the ambient is the power dissipated by the collector junction so that $Q' = P_d$. Thermal conduction path for the heat current is:

junction → case → mica → sink → ambient.

An electronics designer would calculate the required thermal resistance for the heat sink so that the junction temperature T_j does not exceed the maximum rated value, *e.g.* 110 °C. Based on the thermal conduction path in the heat sinked transistor example in Figure 8 (a) there are four temperatures to consider:

T_j is the *junction temperature* of the semiconductor chip where heat is being dissipated.

T_c is the *case temperature*; the case contains the semiconductor chip and may be metal or plastic.

T_s is the *sink temperature*, that of the heat sink, usually the maximum temperature closest to the device.

T_o is the *ambient temperature*, usually the air temperature in which the whole system is housed.

We can identify three thermal resistances:

θ_{jc} is the thermal resistance between the collector junction and the metal case.

θ_{cs} is the thermal resistance between the metal case and the heat sink. This is usually called the *interface resistance*. For the heat sinked transistor in Figure 8 (a) θ_{cs} is the resistance of the mica washer and the thermal paste (grease) between the transistor and the sink.

θ_{sa} is the thermal resistance between the heat sink and the ambient, *i.e.* thermal resistance of the heat sink. Heat sink data sheets typically provide values for θ_{sa}. The latter depends on the temperature difference between the sink and ambient and the nature of convective cooling that removes heat from the sink, *i.e.* natural convection or forced air flow.

All three resistances are in series as shown in the equivalent circuit of the heat sinked transistor in Figure 8 (b). Thus the total thermal resistance from the junction to the ambient is

$$\theta_{ja} = \theta_{jc} + \theta_{cs} + \theta_{sa}$$

so that

$$P_d = Q' = \frac{T_j - T_o}{\theta_{ja}} = \frac{T_j - T_o}{\theta_{jc} + \theta_{cs} + \theta_{sa}} \tag{3-5}$$

Given P_d we can thus calculate θ_{sa}. We can use some real values, for example, for a BJT (2N 3716) power transistor. Suppose that the power dissipated P_d is 15 W. From data sheets, this transistor has θ_{jc} = 1.17 °C/W and the maximum allowed junction temperature is 110 °C. The mica washer has $\theta_{\text{mica}} = \theta_{jc}$ = 0.5 °C/W. If the ambient temperature is 25 °C, the total θ_{ja} we need is

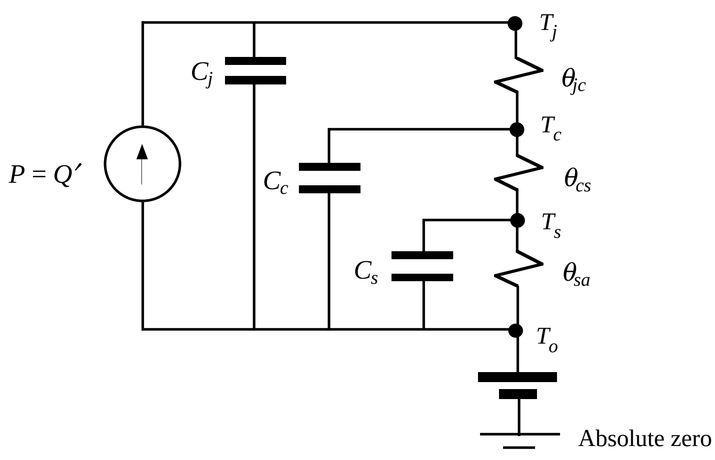
$$\theta_{ja} = \frac{110\ \text{°C} - 25\ \text{°C}}{15\ \text{W}} = 5.67\ \text{°C/W}$$

Thus,

$$\theta_{sa} = \theta_{ja} - \theta_{jc} - \theta_{cs} = 5.67 - 1.17 - 0.5 = 4\ \text{°C/W}$$

We can now go and look up in data books what heat sinks have this thermal resistance θ_{sa} and choose the right type of heat sink with the right dimensions.

Figure 9 shows the thermal capacitances of the various components that make up the heat sinked transistor in Figure 8 (a). C_j is the thermal capacitance of the semiconductor chip and its j*unction*, C_c is thermal capacitance of the *case* and C_s is the thermal capacitance of the heat *sink*. These capacitances are useful in examining the temperature transients, for example, when the transistor is turned on and off. For example, C_j is very small compared with C_c and C_s, so that the thermal time constant $C_j\theta_{jc}$ that determines the rise time of the junction temperature is short, typically, a few milliseconds, and the junction temperature rises in a few milliseconds. Thus, without a heat sink, we can only exceed the temperature rating of the transistor if we keep the duration of large signal changes to less than $C_j\theta_{jc}$, a few milliseconds.



The equivalent circuit of a heat sinked transistor with its thermal resistances and capacitances.

Figure 9

B. Power Derating

Examination of the thermal equivalent model in Figure 8 for a heat sinked transistor shows that the power dissipated P_d at the collector junction flows from the junction to the case temperature. Thus,

$$P_d = \frac{T_j - T_c}{\theta_{jc}}$$

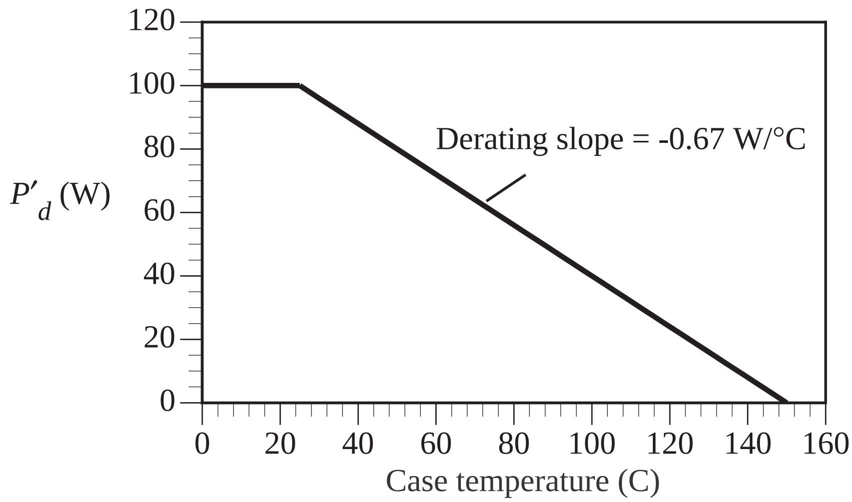
Many data specifications quote the allowed power dissipation P_d for a given case temperature T_c. This effectively means a quote for the junction-to-case thermal resistance θ_{jc}. For example, a particular transistor may have a maximum collector junction temperature of 150 °C and a maximum of 100 W for P_d when the case temperature T_c = 25 °C. Thus, its junction-to-case thermal resistance is

$$\theta_{jc} = (150 - 25\ °\text{C})/(100\ \text{W}) = 1.25\ °\text{C/W}.$$

It is apparent that if a transistor is operated with a case temperature above 25 °C, then the allowed poser dissipation P_d has to be decreased, that is **derated**. This is called the **power dissipation derating curve**. Suppose that the new power dissipation is P_d' (derated power), then

$$P_d' = P_d - \frac{(T_c - 25)}{\theta_{jc}} \qquad \textit{Derated power} \qquad (3\text{-}6)$$

where the case temperature T_c is in °C. Figure 10 shows the derated power as a function of the case temperature. It is apparent that the maximum allowed power dissipation is determined by the case temperature which itself is determined by the thermal resistance of the heat sink.



A power dissipation derating curve as a function of transistor case temperature

Figure 10

Example 3.2: Transistor specifications

A particular power transistor is carrying a current of 2 A when the collector voltage is 12 V. Maximum allowed collector junction temperature is 150 °C. The junction-to-case thermal resistance is 2 °C/W. Find the thermal resistance required from the case to the ambient.

Solution

The ambient temperature is normally taken as 25 °C. Thus,

$$P_d = IV = \frac{T_j - T_o}{\theta_{ja}}$$

i.e $\theta_{ja} = \dfrac{150\ \text{C} - 25\ \text{C}}{(2\ \text{A})(12\ \text{V})} = 5.2\ °\text{C/W}$

Thus, $\theta_{ca} = \theta_{ja} - \theta_{jc} = 5.2 - 2 = 3.2\ °\text{C/W}$

Example 3.3: Derated power

A power transistor has a specification of power dissipation of 20 W at a case temperature of 25 °C. The maximum collector junction resistance is 150 °C. The transistor is mounted on a heat sink that has a thermal resistance of 5 °C/W. Neglect the thermal resistance of the washer and determine the maximum power that this transistor can dissipate.

Solution

The thermal resistance from the junction-to-case is

$$\theta_{jc} = (T_j - T_c) / P_d = (150 - 25\ ^\circ\text{C})/(20\ \text{W}) = 6.25\ ^\circ\text{C/W}.$$

With the transistor mounted on the heat sink, the maximum power P'_d flows from the junction to the ambient. Suppose that T_o = 25 °C. The case-to-ambient thermal resistance θ_{ca} is the heat sink resistance and is given as 5 °C /W. Thus, the derated power is,

$$P'_d = \frac{(T_j - T_a)}{\theta_{jc} + \theta_{ca}} = \frac{(150 - 25)}{6.25 + 5} = 11.1\ \text{W}.$$

This heat sink choice limits the allowed dissipation to half the rated value. The case temperature, by the way, is 80.5 °C. (Why?)

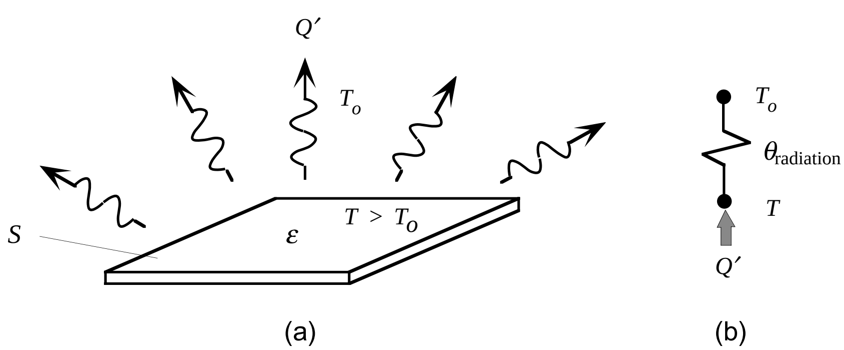
---

4 Heat Transfer by Radiation

All surfaces of objects emit electromagnetic (EM) radiation. All surfaces also absorb EM radiation incident on them. When a body in thermal equilibrium with its environment, then there is no net radiation of EM energy from the surface as the emitted radiation just balances the absorbed radiation from the environment. The net rate of radiation of EM energy from a surface is described by **Stefan's Law**. When a surface is heated to a temperature T above the ambient temperature T_o, as depicted in Figure 11 (a) then it radiates net EM energy at a rate given by

$$P_{\text{radiated}} = \varepsilon\sigma_S S(T^4 - T_o^4) \qquad \textit{Stefan's law} \qquad (4\text{-}1)$$

where σ_S is Stefan's constant (= 5.67×10^{-8} W m^{-2} K^{-4}), ε is the emissivity of the surface, S is the surface area emitting the radiation and T_o is the ambient temperature. The net rate of emission is only finite if the temperature T of the surface is above the ambient temperature T_o. If the surface is cooler than the ambient, T is below T_o then the surface absorbs radiation energy from the ambient.



(a) Radiation loss from a surface area S that an emissivity of ε.(b) Equivalent circuit representation using an effective thermal resistance.

Figure 11

A black body is a hypothetical (an ideal) body that absorbs all the electromagnetic radiation falling onto it and therefore appears to be black at all wavelengths. When heated, a black body emits the maximum possible radiation at that temperature. A small hole in the wall of a cavity maintained at a uniform temperature emits radiation that approximately corresponds to that from a black body .For an ideal black body, the emissivity would be unity. Indeed duller surfaces tend to have higher emissivities than polished surfaces. The emissivity ε of a surface is difficult to determine exactly because it depends strongly on the surface condition of the body. For example, polished cast iron has an ε of about 0.2 whereas cast iron with surface oxidized has an ε in the range 0.6 - 0.7 as apparent in Table 2.

The emissivity in Eq. (4-1) must be viewed as some average emissivity over all the wavelengths and over the temperature range of interest. Emissivity normally depends on the temperature. This ε vs. T dependence can be quite strong. For example polished Cr has an ε of 0.08 at 38 °C but an ε of 0.36 at 1093 °C.

Table 2

Typical emissivity. Actual values may vary substantially.

Surface	ε	Surface	ε	Surface	ε
Al, foil	0.04	Black paint, nonglossy	0.8–0.96	Iron, cast polished	0.21
Al, oxidized at 316 °C	0.05	Brass polished	0.03–0.1	Iron, cast oxidized	0.60
Al, anodized	0.2–0.8	Brick, red rough	0.93	Iron, completely rusted	0.69
Al, oxidized, black, heat sink	0.9	Brick, building	0.45	Iron, wrought, oxidized	0.94
Asbestos	0.93–0.96	Concrete tile	0.63	Tungsten filament	0.35–0.39
Beryllium	0.16–0.30	Gold coating	0.05–0.1	White paint, nonglossy	0.8–0.9

For many problems it is useful to represent the radiative heat flow from the object's surface to the ambient by using an *effective thermal resistance* as in Figure 11 (b). If Q' is the radiative heat current from the surface at T to the ambient at T_o then the effective thermal resistance between the surface and ambient is

$$\theta_{\text{radiation}} = \frac{T - T_o}{Q'} = \frac{T - T_o}{\varepsilon\sigma_S S(T^4 - T_o^4)}$$

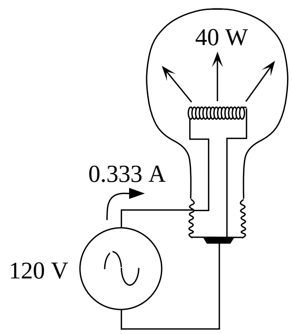
or

$$\theta_{\text{radiation}} = \frac{1}{\varepsilon\sigma_S S(T + T_o)(T + T_o^2)} \qquad \textit{Effective thermal resistance} \qquad (4\text{-}2)$$

which depends on both the surface and ambient temperature, T and T_o. It should be apparent to the reader that the whole concept of using constant thermal resistances to conveniently solve heat transfer problems becomes somewhat useless; it is like solving electric circuits in which resistances depend on voltages. Nonetheless. $\theta_{\text{radiation}}$ can sometimes be quite helpful in approximate calculations. For a 10×10 cm^2 black body at $T = T_o = 20$ °C, $\theta_{\text{radiation}} = 11.77$ °C/W.

Example 4.1: Temperature of a light bulb

Consider a 40 W, 120 V incandescent General Electric light bulb. The tungsten filament is of length 0.381 m and diameter 33 μm. Its resistivity at room temperature is 5.65×10^{-8} Ω m. Given that the resistivity of the tungsten filament varies as $T^{1.2}$, estimate the temperature of the filament when the bulb is operated at the rated voltage, *i.e.* it is lit directly from a mains outlet as schematically shown in Figure 12. Note that the bulb dissipates 40 W at 120 V. The emissivity of the tungsten surface is 0.35. Calculate the temperature of the filament using the temperature dependence of the resistivity and also using Stefan's radiation law and the compare the two values.



Power radiated from a light bulb at 2408 °C is equal to the electrical power dissipated in the filament.

Figure 12

Solution

First, we find the current through the bulb at 100 W and 120 V.

$$\text{Current} = \frac{\text{Power}}{\text{Voltage}}$$

or $$I = \frac{P}{V} = \frac{40\ \text{W}}{120\ \text{V}} = 0.333\ \text{A}$$

From Ohm's law the resistance R of the filament operating at 40 W of power dissipation is

$$R = V/I = (120\ \text{V})/(0.333\ \text{A}) = 360\ \Omega$$

The values for length of the filament (L = 0.381 m) and diameter of the filament ($2r$ = 33 μm) at the operating temperature are given. Using these values we can find the resistivity ρ of the filament when the bulb is on, that is,

$$R = \frac{\rho L}{\pi r^2}$$

i.e. $$\rho = \frac{R\pi r^2}{L} = \frac{(360\ \Omega)\pi\left(\frac{33}{2}\times 10^{-6}\ \text{m}\right)^2}{(0.381\ \text{m})} = 8.08\times 10^{-7}\ \Omega\ \text{m}$$

This is the resistivity of the filament at the operating temperature T (K). We can use,

$$\rho = \rho_o\left[\frac{T}{T_o}\right]^n;\ n = 1.2$$

to find T, given ρ at T, ρ_o at T_o, *i.e.*

$$T = T_o\left[\frac{\rho}{\rho_o}\right]^{\frac{1}{n}} = (298\ \text{K})\left[\frac{8.08\times10^{-7}\ \Omega\ \text{m}}{5.65\times10^{-8}\ \Omega\ \text{m}}\right]^{\frac{1}{1.2}}$$

i.e. T = 2736 K or **2462 °C** (melting temperature of W is 3680 K)

We can also find the operating temperature of the filament by using Stefan's law. Under steady state conditions:

Rate of electrical energy converted to heat

= Rate of heat lost from the filament by radiation

or $$P = \varepsilon\sigma_S S\left(T^4 - T_o^4\right)$$

To use Stefan's law, we need the surface area S of the tungsten filament. Since it is cylindrical in shape,

$$S = L(2\pi r) = (0.381\ \text{m})(\pi 33\times10^{-6}\ \text{m}) = 3.95\times10^{-5}\ \text{m}^2$$

The temperature T at which P = 40 W, can be found from,

$$40\ \text{W} = (0.35)\left(5.67\times10^{-8}\ \text{W m}^{-2}\ \text{K}^{-4}\right)\left(3.95\times10^{-3}\ \text{m}^2\right)\left[T^4 - (293\ \text{K})^4\right]$$

solving for T, we find T = 2673 K or **2375 °C**.

The difference from the resistivity method is less than 3%.

Example 4.2: How long does it take to light a bulb?

Consider a 40 W, 120 V incandescent General Electric light bulb. The tungsten filament is of length 0.381 m and diameter 33 μm. Its resistivity at room temperature is 5.7×10^{-8} Ω m. Tungsten has a density d = 19300 kg m^{-3}, specific heat capacity c = 130 J kg^{-1} K^{-1}; temperature coefficient of resistivity (TCR) α = 0.005 °C^{-1}. What is the time it takes for the filament to reach the operating temperature?

As the filament is in an evacuated bulb, the only heat loss is by radiation from the surface of the filament which obeys Stefan's law. During a time interval dt, the electrical energy released in the filament as heat is $(V/R^2)dt$. Part of this energy escapes from the surface by radiation (Stefan's Law) and the remaining energy increases the heat content (internal energy) of the specimen, *i.e* it increases the temperature by an amount dT determined by the mass m and specific heat capacity c of the specimen. Thus, energy balance during dt requires,

Electrical energy dissipated = Increase in heat content of filament

+ Energy radiated from filament surface

i.e. $$\left(\frac{V^2}{R}\right)dt = mcdT + \varepsilon\sigma_S S\left(T^4 - T_o^4\right)dt$$ *Energy balance* (4-3)

The resistivity can be written as $\rho = \rho_o[1 + \alpha_o(T - T_o)]$ where α is the thermal coefficient of resistivity and T_o is the reference temperature, say 20 °C, the temperature at which the filament starts heating. Further, the density is given by $d = m/(\pi r^2 L)$ where L is the length and r is the radius of the filament. Thus, Eq. (4-3) can be integrated to find the time t_f it takes to reach the final temperature T_f,

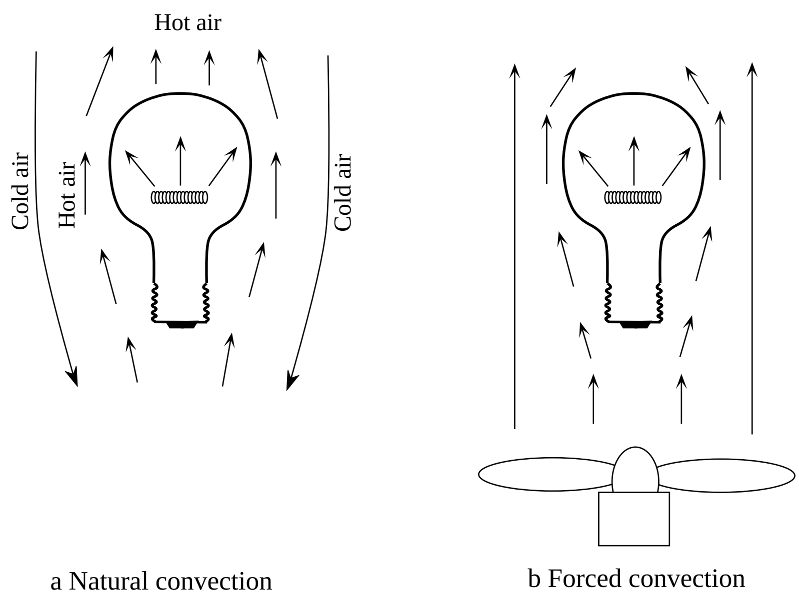
$$t_f = cdL\pi r^2 \int_{T_o}^{T_f} \frac{dT}{\dfrac{\pi r^2 V^2}{L\rho_o[1+\alpha_o(T-T_o)]} - 2\pi L r\varepsilon(T^4 - T_o^4)}$$ *Time to turn on a light bulb* (4-4)

The above equation can be integrated numerically with respect to T to find t_f. From Example 4.2, T_f = 2673 K. Substituting all the filament properties given, L = 0.381 m, $r = 16.5 \times 10^{-6}$ m, $\rho_o = 5.65 \times 10^{-8}$ Ω m, α_o = 0.005 °K^{-1}, d = 19300 kg m^{-3}, c = 130 J kg^{-1} K^{-1}, ε = 0.35, and carrying out the numerical integration from 298 K to 2673 K, we find, 0.042 seconds. This is probably an underestimation because heat will also be thermally conducted away from the filament to its end electrodes holding it. Initially this thermal conduction will be important and will delay the temperature rise.

5 Heat Transfer by Convection

Heat from the surface of a body can also be removed by the flow of a fluid adjacent to the surface. Transfer of heat by motions of fluid atoms is generally termed **convection**. Figure 13 (a) shows the transfer of heat from the hot glass surface by the motion of air (fluid) around the light bulb. The air next to bulb warms up, becomes less dense and hence and rises. Hot air flows "upwards". Air from cooler surrounding regions must replace the hot air that has moved up. Thus, a **convection current** is set up in which hot air next to the bulb flows up, carrying the heat away from the bulb, and cooler air from the surroundings flows down to replace to lost hot air. This type of fluid flow that depends on the changes in the density of the fluid with temperature is called **natural** or **free convection**. The flow is controlled not only by the changes in the density, that is properties of the fluid, but also the external resistance to the flow such as other objects in the flow path.

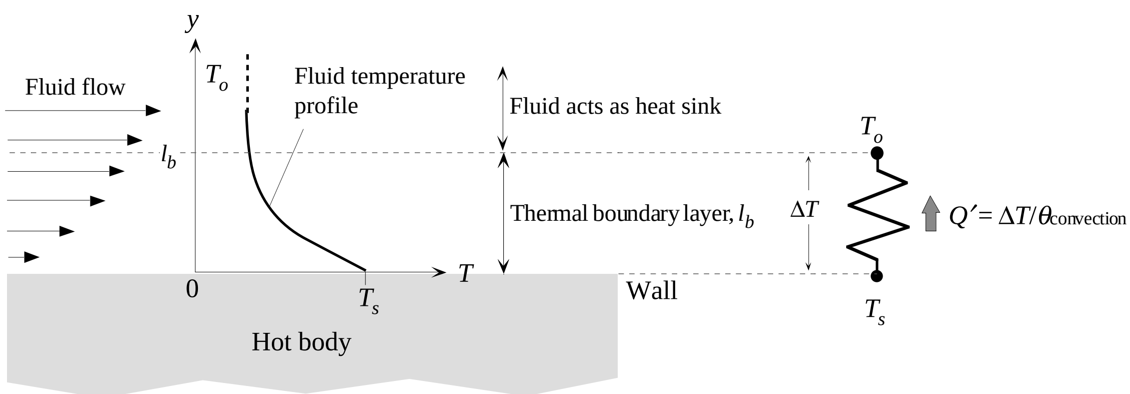
When heat is removed from a the hot surface by artificially inducing a fluid flow past the hot object as in Figure 13 (b), the heat transfer mechanism is called **forced convection**.



Convection is the transfer of heat from a hot surface by the motion of a fluid in contact with the surface. (a) natural convection. (b) forced convection.

Figure 13

The rate of heat transfer from a hot wall, that is, the surface of a body, to an adjacent fluid depends on the temperature difference between the wall and the fluid temperature away from the surface. Suppose that the temperature of the fluid far away from the surface is T_o, which is the **bulk fluid temperature**. Suppose also that the wall temperature is T_s. Due to the flow of the fluid over the body's surface, the temperature decays from T_s to T_o with distance y away from the wall as shown in Figure 14. The fluid velocity next to surface is nearly zero but increases with distance y as we move away from the surface as also depicted in Figure 14. One can define a **thermal boundary layer** over which the majority of the temperature drop occurs from T_s to T_o; perhaps 99% of ($T_s - T_o$). The temperature at the end of boundary layer at l_b is 99% of the bulk fluid temperature T_o.



Heat transfer by convection from a hot surface to a cold fluid.

Figure 14

All experiments show that the rate of heat flow Q' from the wall by convection *increases* with the temperature difference, $T_s - T_o$, and *increases* with the surface area S of the wall in contact with the fluid. Thus,

$$Q' = hS(T_s - T_o) \qquad \textit{Convection heat transfer rate} \qquad (5\text{-}1)$$

where h is the **coefficient of convective heat transfer**. Equation (5-1) is called **Newton's law of cooling**. Equation (5-1), as stated, appears to suggests that h is constant for a given wall and thermal convection system.

In general, h may depend on the temperature difference, $\Delta T = (T_s - T_o)$, which means that Q' is *not* linearly proportional to ($T_s - T_o$). In many cases, the ΔT-dependence is not strong and may be neglected in approximate calculations. Further, h depends on the geometry of the body, the type of flow, *i.e.* laminar or turbulent flow, whether cooling is natural or forced convection, fluid properties, and a number of other factors. The calculation of h for various thermal problems is one of the most challenging areas in engineering. Table 3 lists some typical ranges of values for h in familiar fluids: air and water. Notice that h for a thin plate depends on whether the plate is vertical or horizontal, and on its dimensions. For a temperature difference ΔT of 100 °C, a vertical plate has an h that depends only on the vertical dimension and is independent of the horizontal length. For a height of 1 m, h is about ~5 W m^{-2} °C^{-1}. Forced convection can increase h dramatically, and in such cases h typically increases as the square root of the flow velocity, $h \sim \sqrt{v_{\text{flow}}}$. Increasing the flow velocity four times, doubles h.

For may thermal problems it is convenient to model convective heat transfer by using an equivalent thermal resistance $\theta_{\text{convection}}$ between the surface at T_s and the bulk fluid at T_o as indicated in Figure 14. We are thus representing the thermal boundary layer in terms of an effective thermal resistance $\theta_{\text{convection}}$ such that,

$$Q' = \frac{\Delta T}{\theta_{\text{convection}}} \quad \text{where} \quad \theta_{\text{convection}} = \frac{1}{hS} \qquad \textit{Effective thermal resistance} \qquad (5\text{-}2)$$

Table 3

Convection coefficient h for various modes of fluid flow. L_{ch} is a parameter called the *characteristics length* and depends on the geometry of the object.

Free convection	***h*** (W m^{-2} °C^{-1})
Atmospheric air	5 - 25
Water	400 - 1000
Forced convection	***h*** (W m^{-2} °C^{-1})
Air	15 - 500
Water	100 - 15,000
Engine oil	1000 - 2000
Flow system in natural Cooling	***h*** (W m^{-2} °C^{-1})
Vertical thin plate of height d_{vertical}.	$h = 1.51(\Delta T/d_{\text{vertical}})^{1/4}$
	$1.012(\Delta T/d_{\text{vertical}})^{0.35}$; $d_{\text{vertical}} < 0.1$
Horizontal thin plate, heated face up.	$h = 1.38(\Delta T/L_{ch})^{1/4}$; $L_{ch} = WL/[2(W+L)]$
	W = width; L = length of plate
Horizontal thin plate, heated face down.	$h = 0.69(\Delta T/L_{ch})^{1/4}$; $L_{ch} = WL/[2(W+L)]$
	W = Width; L = Length of plate
Flow system in forced cooling	***h*** (W m^{-2} °C^{-1})
Forced air flow (laminar flow) along a	$h = 3.9[v_{\text{flow}}/d_{\text{flow}}]^{0.5}$
surface with length d_{flow} along the flow.	d_{flow} = Length along the flow direction (m)
	v_{flow} = Flow velocity (m/s)

Example 5.1: Convective heat transfer

Consider the electric convection heater shown in Figure 15 that is operating under steady state conditions. The electrical power P generated by the heater inside the cylindrical metal case is circulated by oil to the whole body. This heat is then carried from the surface of the heater to the ambient by air convection. Suppose that the dimensions of a particular 750 W heater is 1 m × 0.75 m. Using an average h of about 6 W m^{-2} °C^{-1} for this particular thermal system, estimate the temperature of the heater wall.

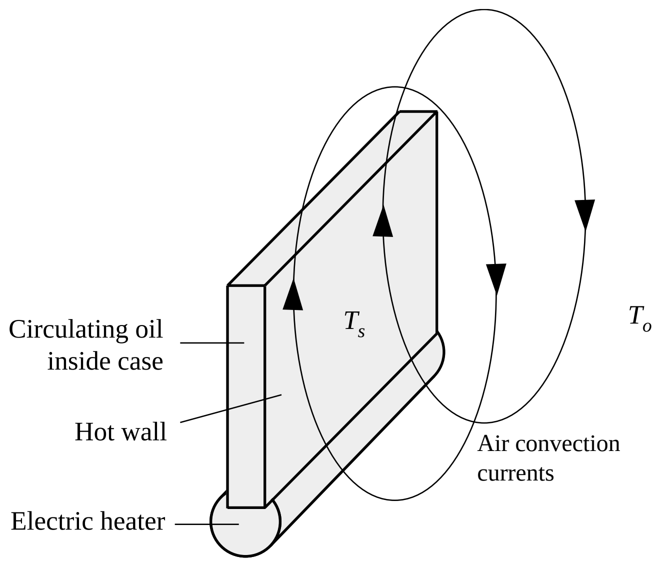
Solution

Under steady state,

$$P = Q' = hS\Delta T$$

so that $$\Delta T = \frac{P}{hS} \approx \frac{(750\ \text{W})}{(6\ \text{W m}^{-2}\ \text{C}^{-1})[2(1\ \text{m}\times 0.75\ \text{m})]} = 83.3\ ^\circ\text{C}$$

where the surface area S is twice 1 m × 0.75 m (Figure 15), assuming that both surfaces of the heater in contact with the ambient. The wall temperature $T_o + \Delta T$ is thus 25 °C + 83.3 °C or 108.3 °C. The calculation is only very approximate, since h for this system is only an estimate.



Heat transfer by convection from an electric heater to the ambient.

Figure 15

Example 5.2: Convective heat transfer coefficient *h*

The convective heat transfer coefficient h for a vertical plate with a vertical dimension (height) d_{vertical}, cooled by natural convection is usually given by

$$h_{\text{vertical}} = 1.51(\Delta T/d_{\text{vertical}})^{0.25}$$

If the plate is small, $d_{\text{vertical}} < 0.1$ m, a better formula is given by

$$h_{\text{vertical}} = 1.012(\Delta T/d_{\text{vertical}})^{0.35}$$

Consider a temperature difference ΔT of 100 °C. Then for a large dimension such as $d_{\text{vertical}} = 1$ m, $h_{\text{vertical}} = 4.8$ W m^{-2} C^{-1}, from he first equation. For a short height, $d_{\text{vertical}} = 0.10$ m, the second equation predicts $h_{\text{vertical}} = 11.3$ W m^{-2} C^{-1} (whereas the first equation underestimates h as 8.5 W m^{-2} C^{-1}).

For a horizontal thin plate with the heated surface facing up,

$$h_{\text{horizontal}} = 1.38(\Delta T/L_{ch})^{1/4}\ ;\ L_{ch} = WL/[2(W+L)]$$

L and W are the length and width of the plate and L_{ch} is a parameter called the *charactertic length*; the thickness is assumed to be very thin.

Taking a plate that is 1 m × 1 m, $h_{\text{horizontal}} = 6.85$ W m^{-2} C^{-1}.

Consider forced cooling in forced air flow. Suppose that the flow velocity is v_{flow} and d_{flow} is the dimension, or length, of any surface *along* the flow direction. Then, assuming laminar flow, the air-flow forced h is approximately given by

$$h_{\text{air-flow}} \approx 3.9(v_{\text{flow}}/d_{\text{flow}})^{0.5}$$

Notice that h does not depend on ΔT which makes the problem linear! Taking v_{flow} = 1 m/s, and a length along flow that is 0.1 m, we find,

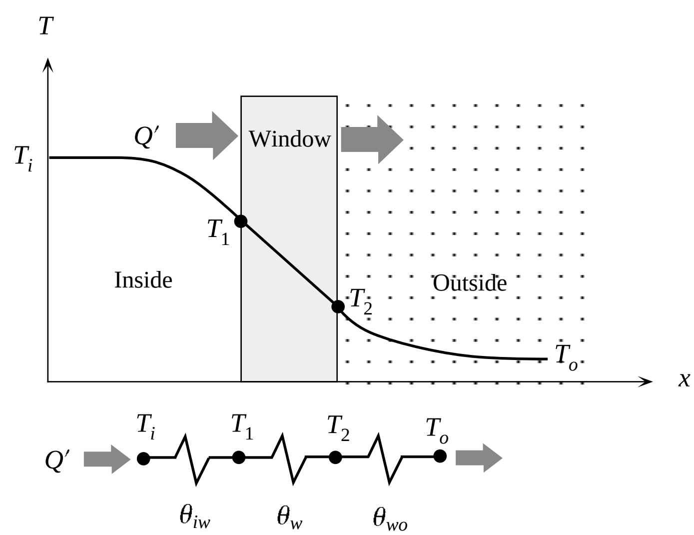
$$h_{\text{air-flow}} \approx 3.9[(1)/(0.10)]^{0.5} = 12.3 \text{ W m}^{-2} \text{ C}^{-1}.$$

If the flow velocity increases to v_{flow} = 4 m/s, then $h_{\text{air-flow}}$ = 24.7 W m^{-2} C^{-1}.

There are two conclusions. Since h depends on the temperature difference ΔT, the effective thermal resistance θ will also depend on ΔT and hence the thermal problem will loose its simple linear beauty; the relationship between Q' and ΔT will not be linear. Secondly, h can change substantially depending on the nature of convective cooling, whether natural or forced convection. Further, the type of flow, whether laminar flow or turbulent flow, also affects h; we only considered laminar flow in this example.

Example 5.3: Convective and conductive heat transfer

Consider the escape of heat from a perfectly insulated room to the cold whether outside through a glass window as shown Figure 16. The temperature on the inside face of the window glass is not at room temperature; a fact easily verified by touching the glass surface with our finger (it always feels colder than the room temperature whenever the outside is cold). The heat flows by convection to the glass window, by conduction through the glass and by convection to the outside ambient as shown Figure 16. Consider a 1 m × 0.75 m window with a glass of thickness 10 mm. Window glass has κ = 0.76 W m^{-1} °C^{-1}. Suppose that the inside and outside ambient temperatures are T_i = 25 °C and T_o = –40 °C. Taking h_i ≈ 15 and h_o ≈ 25 W m^{-2} °C^{-1}, calculate the temperature of the inside and outside surfaces of the window and also the rate of heat escape.



Heat transfer transfer from a heated room to the cold outside through a glass window.

Figure 16

Let the ambient temperature inside the room be T_i, temperature of the outside ambient be T_o and the temperatures on the inside and outside surfaces of the window glass be T_1 and T_2 respectively. Let Q' be the rate of heat transfer, heat current, under steady state conditions. . The problem is most easily solved by considering the equivalent thermal circuit shown in Figure 16.

The thermal resistance θ_{iw} from T_i to T_1 is the thermal resistance of the thermal boundary layer inside next to the window

$$\theta_{iw} = \frac{1}{h_i S} = \frac{1}{(15)(1\times 0.75)} = 0.089 \text{ °C/W}$$

The thermal resistance θ_w of the window glass is

$$\theta_w = \frac{\ell}{\kappa A} = \frac{10\times 10^{-3}}{(0.76)(1\times 0.75)} = 0.018 \text{ °C/W}$$

The thermal resistance θ_{wo} from T_2 to T_o is the thermal resistance of the thermal boundary layer outside next to the window (from the window surface to the outside ambient),

$$\theta_{wo} = \frac{1}{h_o S} = \frac{1}{(25)(1\times 0.75)} = 0.053 \text{ °C/W}$$

The total thermal resistance is

$$\theta = \theta_{iw} + \theta_w + \theta_{wo} = 0.089 + 0.018 + 0.053 = 0.16 \text{ °C/W} .$$

The heat current flowing through is

$$Q' = \frac{(T_i - T_o)}{\theta} = \frac{25-(-40)}{0.16} = 406.25 \text{ W.}$$

The temperature drop ΔT_{wo} from the window to the outside or ambient is

$$T_2 - T_o = Q'\theta_{wo} = (406.25 \text{ W})(0.053 \text{ °C/W}) = 21.5 \text{ °C}$$

giving $T_2 = -40 + 21.5 = -18.5$ °C

The temperature drop $\Delta T_{wo\backslash}$ across the window is

$$T_1 - T_2 = Q'\theta_w = (406.25 \text{ W})(0.018 \text{ °C/W}) = 7.31 \text{ °C}$$

giving $T_1 = -18.5 + 7.31 = -11.2$ °C

The temperature drop ΔT_{iw} from the inside to the window is

$$T_i - T_1 = Q'\theta_{iw} = (406.25 \text{ W})(0.089 \text{ °C/W}) = 36.16 \text{ °C}$$

giving $T_1 = 25 - 36.16 = -11.2$ °C

It is also possible to solve the problem without using thermal resistances. Under steady state,

Convection heat current from inside ambient to window glass surface

= Conduction heat current through the window glass

= Convection heat current flow from outside glass surface to outside ambient

i.e.
$$Q' = h_i S(T_i - T_1) = \kappa S\frac{(T_1 - T_2)}{\ell} \qquad (5\text{-}3)$$

and $$Q' = \kappa S\frac{(T_2 - T_1)}{\ell} = h_o S(T_2 - T_o) \quad (5\text{-}4)$$

where h_i and h_o are the convective heat transfer coefficients inside and outside the window, ℓ is the thickness of the window glass and κ is the thermal conductivity of window glass. Equations (5-3) and (5-4) can be solved together to find T_1 and T_2. For example,

$$T_1 = \frac{\kappa(T_i h_i + T_o h_o) + \ell T_i h_i h_o}{\kappa(h_i + h_o) + \ell h_i h_o}$$

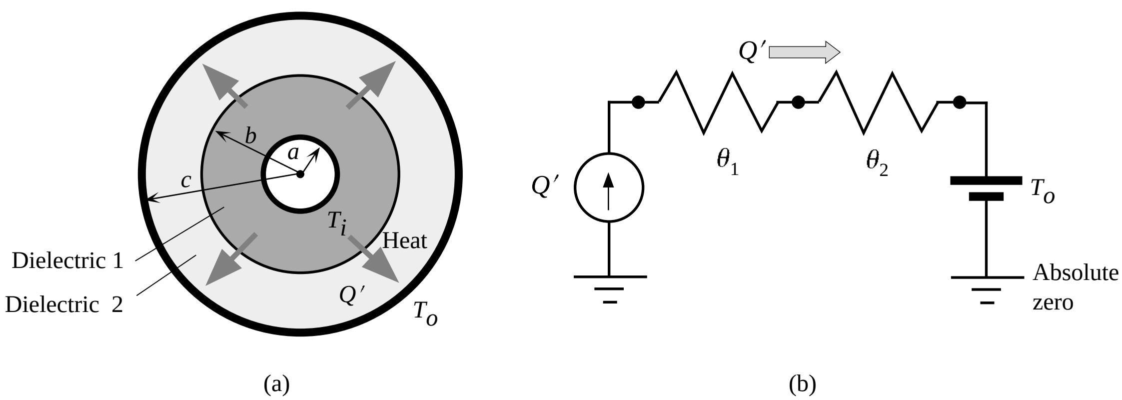
From T_1 we can calculate T_2 and also Q'. For example,

$$T_1 = \frac{\kappa(T_i h_i + T_o h_o) + \ell T_i h_i h_o}{\kappa(h_i + h_o) + \ell h_i h_o} = \frac{(0.75)[(25)(15) + (-40)(25)] + [(0.01)(25)(15)(25)]}{(0.75)(15 + 25) + (0.01)(25)(15)}$$

i.e. $T_1 = -11.1$ °C.

It is apparent that the thermal resistance approach has the benefit of clarity and mathematical tractability.

Example 5.4: Steady state heat flow in a coaxial cable with double dielectric insulation In a coaxial cable with a homogeneous dielectric layer, the maximum field occurs at $r = a$, right next to the inner conductor in Figure 5 (a). Due to the Joule heating (I^2R) of the inner conductor, $r = a$ is also where the temperature is highest and the **dielectric breakdown field** $E_{\text{breakdown}}$ is lowest; $E_{\text{breakdown}}$ decreases with increasing temperature. The maximum field can be shifted away from $r = a$ into the dielectric where the temperature is lower by using two different dielectric layers as in Figure 17 (a). If the relative permittivity of the first layer is sufficiently larger than the second layer, the maximum field is shifted away from $r = a$ to the interface between the layers at $r = b$.



(a) A coaxial cable with two layers of dielectric insulation. (b) Equivalent thermal circuit.

Figure 17

Consider a coaxial cable that has a copper core conductor with a resistivity $\rho = 18$ nΩ m. The core radius is 5 mm. The dielectric insulation consists of two different concentric layers of polymer insulation as indicated in Figure 17 (a). First polymer layer next to the inner core has a thickness of 1.5 mm and the second polymer layer, between the first layer and the outside conductor, has a thickness of 2 mm. The equivalent thermal circuit under state operation is shown in Figure 17 (b). The thermal conductivities of the first and the second layers are of 0.3 W m^{-1} K^{-1} and 0.25 W m^{-1} K^{-1} respectively.

The outside (ambient) temperature T_o is 20 °C. Ambient convection is sufficiently strong (h very large, *i.e.* $h \rightarrow \infty$) to maintain T_o at 20 °C. The coaxial cable is carrying a current of 700 A. What is the temperature of the inner conductor? Sketch the temperature vs. distance profile from the inner conductor to the outside conductor. (Note: this is a sketch not a plot.) How would you modify your calculation to include a finite convection from the surface of the cable given a convective heat transfer coefficient h of 25 W m^{-2} K^{-1}?

Solution

The coaxial cable with double insulation is shown in Figure 17 (a). The equivalent thermal circuit under state operation is shown in Figure 17 (b). To solve this problem we make the following assumptions:

a. The operation has reached steady state so that the temperature anywhere inside the cable does not change with time.

b. The cable is sufficiently long so that there is no heat transfer along the cable, that is, the temperature does no vary along the cable. Thus, we only need to consider a portion of length L of a very long cable. For simplicity we can set L = 1 m.

c. The thermal conductivity of the core conductor is *very much greater* than the thermal conductivity of the region outside the core (the dielectric) so that for all practical purposes, the inner conductor is at a uniform temperature, T_i. (This is justified by the fact that κ_{Cu} = 400 W m^{-1} °C^{-1} for Cu and $\kappa_{\text{dielectric}} \approx$ 0.3 W m^{-1} °C^{-1} for the dielectric insulation, which means $\kappa_{\text{Cu}}/\kappa_{\text{dielectric}} \approx$ 1,300; three orders of magnitude!).

d. Assume that the thermal conductivities do not change with temperature. This assumption is not entirely true as κ for polymers increases with T.

e. We neglect the change in the electrical resistivity of the inner conductor with temperature

The thermal resistance of a hollow cylindrical component, as for the first dielectric layer in Figure 17 (a), is

$$\theta = \frac{(T_i - T_o)}{Q'} = \frac{\ln\left(\frac{b}{a}\right)}{2\pi\kappa L}$$

Thus for dielectric 1, the thermal resistance θ_1 is,

$$\theta_1 = \frac{\ln\left(\frac{b}{a}\right)}{2\pi\kappa_1 L} = \frac{\ln\left(\frac{(5+1.5)\times10^{-3}}{5\times10^{-3}}\right)}{2\pi(0.3)(1)} = 0.139\ °\text{C/W}$$

For dielectric 2, the thermal resistance θ_2 is,

$$\theta_2 = \frac{\ln\left(\frac{c}{b}\right)}{2\pi\kappa_2 L} = \frac{\ln\left(\frac{(5+1.5+2)\times10^{-3}}{(5+1.5)\times10^{-3}}\right)}{2\pi(0.25)(1)} = 0.171\ °\text{C/W}$$

Total thermal resistance from the inner conductor to the outer conductor is

$$\theta = \theta_1 + \theta_2 = (0.139\ °\text{C/W}) + (0.171\ °\text{C/W}) = 0.310\ °\text{C/W}$$

The joule heating per unit second (power) generated by the current I through the core conductor is

$$Q' = I^2 \frac{\rho L}{\pi a^2} = (700^2)\frac{(18\times10^{-9})(1)}{\pi(5\times10^{-3})^2} = \;\; 112.5 \text{ W}$$

Thus, the temperature difference ΔT due to the heat current Q' flowing through $\theta = \theta_1 + \theta_2$ in Figure 17 (b) is,

$$\Delta T = Q'\theta = (112.5 \text{ W})(0.310 \text{ °C/W}) = 34.9 \text{ °C}.$$

The inner conductor temperature is therefore,

$$T_i = T_o + \Delta T = 20 + 34.9 = \mathbf{54.9\ °C}.$$

One can now correct for the change in the copper resistance with temperature. The resistivity and hence the resistance of the Cu scales very nearly with the absolute temperature (we are not given the temperature coefficient of resistivity, so this is a reasonable procedure for a nonmagnetic pure metal such as copper), so that

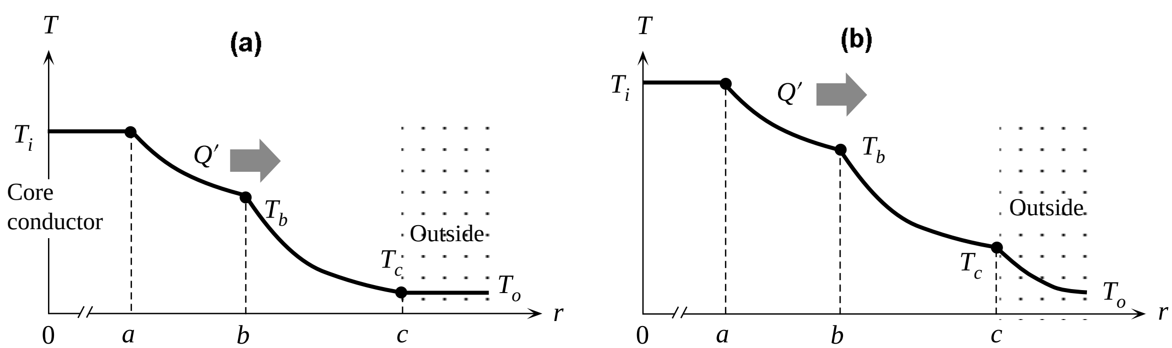
$$\frac{Q'_{54.9}}{Q'_{20}} = \frac{I^2 R_{54.9}}{I^2 R_{20}} = \frac{54.9 + 273 \text{ K}}{20 + 273 \text{ K}} = 1.116$$

where the subscripts relate the quantities to 54.9 °C and 20 °C. The new $\Delta T'$ is

$$\Delta T' = \Delta T \frac{Q'_{54.9}}{Q'_{20}} = (34.9)(1.116) = \mathbf{38.9\ °C}$$

so that the new T_i = **58.9 °C**. (One iteration is sufficient.)

It is left as an exercise to show that the temperature drop ΔT_{ab} from *a* to *b* is 17.5 °C and ΔT_{bc} from *b* to *c* is 21.4 °C using $Q'_{54.9}$. (Use $\Delta T_{ab} = Q'_{54.9}\theta_1$ *etc.*). Thus, T_b = **41.4 °C**.



Heat transfer and temperature profile in a coaxial cable from the inner conductor to the outer conductor through two concentrentic layers of different insulation. The surface convection maintains the outer conductor temperature at T_o. (a) $h = \infty$. (b) h is finite and there is thermal boundary layer around the outer conductor.

Figure 18

To sketch the *T* vs. *r* temperature profile, we note that at a radial distance *r* from the core,

$$Q' = -(2\pi r L)\kappa \frac{dT}{dr}$$

which means that,

$$\frac{dT}{dr} \propto -\frac{\kappa}{r}$$

that is, the temperature gradient decreases with distance r. Further at $r = b$, there is a sudden decrease in the magnitude of the temperature gradient as κ changes from 0.30 to 0.25. W m^{-1} K^{-1}. These features are shown in the temperature profile sketch in Figure 18 (a). The outer conductor temperature T_c is maintained at the ambient temperature T_o due to a very large h.

When h is finite, then there will be a *thermal boundary* layer between the outer conductor and the ambient and the temperature T_c of the outer conductor at $r = c$ will not be To as shown in Figure 18 (b) . The thermal resistance θ_{tb} of this thermal boundary layer is

$$\theta_{tb} = \frac{1}{hS} = \frac{1}{h(2\pi cL)} = \frac{1}{(25)(2\pi)[(5+3.5)\times 10^{-3}](1)} = 0.75 \text{ °C/W}$$

This finite θ_{tb} would change T_c and hence T_i. The temperature drop from the outer conductor to the ambient is,

$$T_c - T_o = Q'\theta_{tb} = (112.5 \text{ W})\ (0.75 \text{ °C/W}) = 64.4 \text{ °C}.$$

Obviously there is a dramatic change in T_c. This change will also appear as an addition on T_i. Our conclusion is that heat removal from the outer conductor is extremely important. We can calculate the required h coefficient for 10% change in T_i as follows. The total thermal resistance from the core conductor to the ambient when $h = \infty$ is 0.310 °C/W. Thus, for 10% increase on this when h is finite, we need, θ_{tb} = 0.031 °C/W. This means we need $h = 1/(\theta_{tb}S)$ = 604 W m^{-2} °C^{-1}. This type of convection requires a fluid such as water around the outer conductor.

Example 5.5: Thermal resistance of heat sinks in electronics

The function of a heat sink is to provide a low resistance thermal path for the heat to flow from the device to the ambient. The low thermal resistance ensures that heat current does not raise the temperature of the device but flows out to the ambient. Figure 19 shows an integrated circuit moulded in a plastic package that is heat sinked by attaching a heat sink to its top flat surface by using a heat sink bond. The electrical power dissipated in the IC flows out from its generation region through the IC package material to the IC surface (case), and then through the interface layer (sink bonding bond) to the heat sink and then the ambient. The heat transfer through the sink is by thermal conduction to the “fin” surfaces of the heat sink and then by air convection flow and radiation from the fin surfaces.

Suppose that T_s is the sink temperature next to the device (the hottest location), the ambient temperature is T_o, and the heat flow from T_s to T_o is Q', then the effective thermal resistance θ_{sink} is defined as

$$\theta_{\text{sink}} = \frac{T_s - T_o}{Q'}$$

This thermal resistance is usually controlled by the rate of heat removal from the sink’s surface by convection and radiation. The heat from the sink escapes by convection and radiation acting simultaneously;

$$Q' = \varepsilon\sigma_S S(T_s^4 - T_o^4) + hS(T_s - T_o)$$

The larger the rate of heat flow Q' from the sink surface to the ambient, the smaller isθ_{sink}. Heat sinks are therefore designed to increase the surface area S as much as possible which is the reason for the fins. Many heat sinks are typically dark oxidized Al surfaces (especially those that are to be used at high T_s) to ensure a reasonable emissivity (ε). The overall thermal resistance is thus,

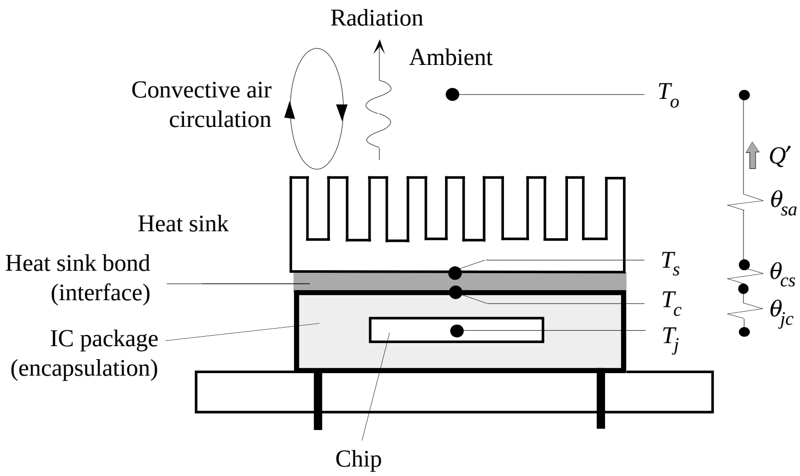
$$\frac{1}{\theta_{\text{sink}}} = \varepsilon\sigma_S S \frac{T_s^4 - T_o^4}{T_s - T_o} + hS = \frac{1}{\theta_{\text{radiation}}} + \frac{1}{\theta_{\text{convection}}}$$

where $\theta_{\text{radiation}}$ and $\theta_{\text{convection}}$ are effective thermal resistances due to radiative and convective heat transfers. The two are combined as if the resistances were parallel to find the overall sink resistance.

As an example, we will take T_o = 25 °C, T_s = 100 °C, Consider a heat sink that has a surface are of ~0.01 m^2, $\varepsilon \approx 0.75$, $h \approx 10$ W m^{-2} °C^{-1}, substituting the values we find $\theta_{\text{radiation}}$ = 15.4 °C/W $\theta_{\text{convection}}$ = 10 °C/W; convective transfer is more important ($\theta_{\text{convection}}$ smaller than $\theta_{\text{radiation}}$). The heat sink has

$$\theta_{\text{sink}} = \theta_{\text{radiation}} // \theta_{\text{convection}} \approx 6.0 \text{ °C/W}$$

The thermal resistance due to thermal conduction to the sink's fins would have to be added to the above value to get the total sink resistance, but this is usually small compared.



A heat sinked IC.

Figure 19

The heat sinked IC in Figure 19 has the heat sink bonded to the top surface of the IC. T_j is the junction temperature. T_c is the IC case temperature. Suppose that the IC has an effective θ_{jc} of 15 °C/W and that its maximum junction temperature T_j is 100 °C. Suppose that we use the heat sink above with a thermal resistance of 6°C/W and we take the ambient temperature T_o = 25 °C. Suppose that the case-to-sink (interface layer between the IC package and the sink) has a thermal resistance of about 1 °C/W. What is the maximum power that can be dissipated?

The maximum power that can be dissipated is the heat current from T_j to T_o,

$$P_d = (T_j - T_o)/(\theta_{jc} + \theta_{cs} + \theta_{sa}) = (100 \text{ °C} - 25 \text{ °C})/ (15 \text{ °C/W} + 1 \text{ °C/W} + 6.0 \text{ °C/W})$$
$$= 5 \text{ W}.$$

ADVANCED TOPICS

6 Heat Equation

Our objective is to derive a general partial differential equation that is based on thermal conduction through the body of the component but also accounts for heat losses from the surface of the component and heat generation within the bulk of the component.

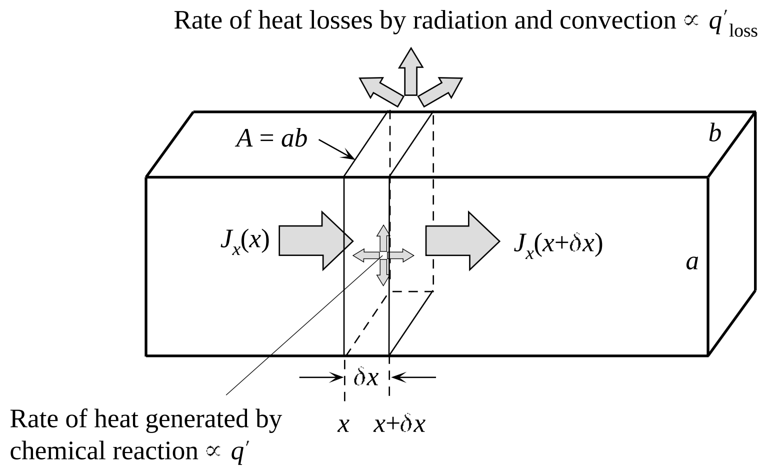


Figure 20

Consider a small volume $A\delta x$ as in Figure 20. The cross sectional area is uniform (does not vary along x). Heat losses from a surface are invariably proportional to the surface area. For example, the rate of heat loss from the surfaces of $A\delta x$ is proportional to the surface area which is circumference × δx, or $(2a + 2b)\delta x$ for the rectangular cross section ($a \times b$) in Figure 20. The following new quantity is defined for the losses:

q'_{loss} is rate of heat loss per unit surface area due to heat loss as a result of thermal radiation (Stefan's law) or convection (Newton's law).

Heat generation processes within the bulk of the component, such as chemical reactions, generate heat that scales with the amount of material, or volume. If J is the electrical current passing through the elemental volume (whatever direction) and σ is the electrical conductivity, then according to Joule's law, the electrical energy converted to heat per unit volume is J^2/σ. It therefore makes sense to define a quantity that accounts for heat generation per unit volume as follows:

$\Delta h_g'$ or q' is **rate of heat generation per unit volume** due to various heat generating processes such as chemical reactions, Joule heating due to an electrical current, phase transformation etc.

We know that if $\partial T/\partial t$ is the rate of increase in the temperature of $A\delta x$, then,

$$\text{Rate of heat accumulation in the elemental volume} = A\delta x \rho c \frac{\partial T}{\partial t}$$

We also know that the *net* rate of heat flow per unit area is $J_x(x) - J_x(x + \delta x)$, which is $-(dJ_x/dx)\delta x$ from Eq. (2-2). Applying the conservation of energy principle,

$$A\delta x \rho c \frac{\partial T}{\partial t} = \textit{net}\text{ rate of heat flow in}$$

$$+ \text{ rate of heat generation by chemical reaction etc.}$$

– rate of heat loss

$$= -A\frac{\partial J_x}{\partial x}\delta x + q'A\delta x - q'_{\text{loss}}C\delta x \qquad \textit{Conservation of energy} \qquad (6\text{-}1)$$

where C is the circumference.

Thus, $$\rho c\frac{\partial T}{\partial t} = -\frac{\partial J_x}{\partial x} + q' - \frac{C}{A}q'_{\text{loss}}$$

Using Fourier's law $J_x = -\kappa\frac{\partial T}{\partial x}$ we get

$$\rho c\frac{\partial T}{\partial t} = -\kappa\frac{\partial^2 T}{\partial x^2} + q' - \frac{C}{A}q'_{\text{loss}}$$

i.e. $$\frac{\partial T}{\partial t} = \left(\frac{\kappa}{\rho c}\right)\frac{\partial^2 T}{\partial x^2} + \frac{1}{\rho c}\left(q' - \frac{C}{A}q'_{\text{loss}}\right) \qquad \textit{Heat equation} \qquad (6\text{-}2)$$

Notice that heat losses scale as *C/A*, *i.e.* as Circumference/Area. If we keep the cross sectional area large, *C/A* will be sufficiently small for the heat loss term to be negligible.

Example 6.1: Temperature gradients in thermally lagged components

Consider a component of length L whose surface is lagged. One end is at a temperature T_A and the other end is at T_B as shown in Figure 21. Suppose that there is no internal heat generation (no chemical or physical processes, no Joule heating etc.). We will consider steady state operation.

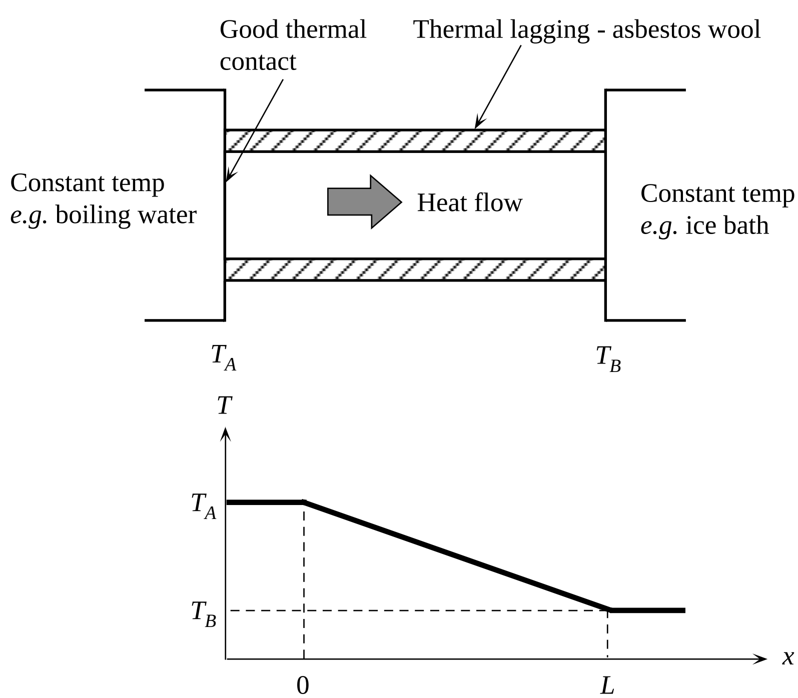


Figure 21

Because of lagging, $q'_{\text{loss}} = 0$, and due to no generation $q' = 0$.

Under steady state conditions $\partial T/\partial t = 0$

Thus, the heat equation gives,

$$\frac{\kappa}{\rho c}\frac{d^2T}{dx^2} = 0\,.$$

Solving this second order equation we find a linear dependence and two arbitrary constants, c_1 and c_2:

$$T = c_1x + c_2$$

Boundary conditions:

$x = 0$, $T = T_A$ gives $c_2 = T_A$

$x = L$, $T = T_B$ gives $c_1 = \dfrac{T_B - T_A}{L}$

Thus, the temperature across the component changes linearly as:

$$T = \left(T_B - T_A\right)\frac{x}{L} + T_A$$

Consider now a finite heat generation in the sample, *i.e.*

$q' =$ finite = Joule heating by a current density $J = J^2/\sigma$.

Then under steady state conditions $\partial T/\partial t = 0$, and the heat equation is

$$\kappa\frac{\partial^2 T}{\partial x^2} + q' = 0$$

Therefore, $$\frac{d^2T}{dx^2} = -\frac{q'}{\kappa}$$

Solving, $$T = -\frac{q'}{2\kappa}x^2 + c_1x + c_2$$

Boundary conditions are: $x = 0$, $T = T_A$ and $x = L$, $T = T_B$

Therefore $T_A = c_2$

and $$T_B = -\frac{q'L^2}{2\kappa} + c_1L + T_A$$

i.e. $$c_1 = \frac{T_B - T_A + \dfrac{q'L^2}{2\kappa}}{L}$$

Thus, the final expression for the *T*–*t* dependence is,

$$T = -\frac{q'L^2}{2\kappa}\left(\frac{x}{L}\right)^2 + \left(T_B - T_A + \frac{q'L^2}{2\kappa}\right)\left(\frac{x}{L}\right) + T_A$$

which is shown in Figure 22

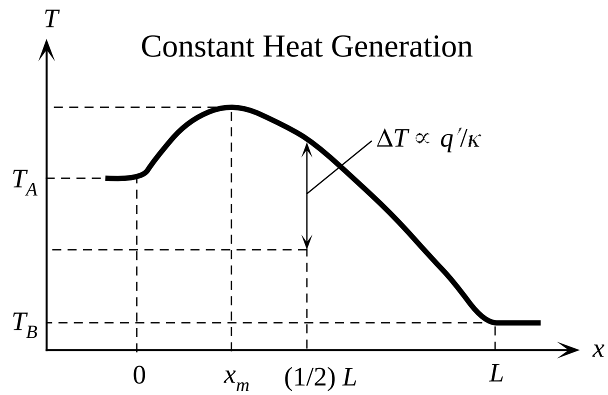


Figure 22

This temperature distribution is a maximum when $dT/dx = 0$, when

$$\frac{dT}{dx} = -\frac{q'L^2}{2\kappa}\frac{x}{L} + \left(T_B - T_A + \frac{q'L^2}{2\kappa}\right) = 0$$

i.e.
$$\left(\frac{x}{L}\right)_m = \frac{T_B - T_A + \dfrac{q'L^2}{2\kappa}}{\dfrac{q'L^2}{\kappa}} = \frac{1}{2} - \frac{(T_A - T_B)}{\dfrac{q'L^2}{\kappa}}$$

i.e.
$$\left(\frac{x}{L}\right)_m = \frac{1}{2} - \frac{(T_A - T_B)\kappa}{q'L^2} < \frac{1}{2}$$

With no internal heat generation, T at $\dfrac{x}{L} = \dfrac{1}{2}$ is

$$T_1 = \frac{T_A + T_B}{2}$$

With heat generation, this temperature is

$$T_2' = -\frac{q'L^2}{8\kappa} + \frac{T_B}{2} - \frac{T_A}{2} + \frac{q'L^2}{4\kappa} + T_A$$

$$T_2' = \frac{T_A + T_B}{2} + \frac{q'L^2}{8\kappa}$$

Thus the temperature rise when heat generation is turned on is

$$\Delta T = \frac{q'L^2}{8\kappa}$$

The change in temperature is thus $\propto q'/\kappa$

NOTATION

$\partial J_x / \partial x$	flux gradient
A	area (m^2)
a	core radius (m)
α_o	temperature coefficient of resistivity (°C^{-1})
C	thermal capacitance of the body ($C = mc$); circumference
C_v	heat capacity per unit volume (J K^{-1} m^{-3})
c	specific heat capacity (heat capacity per unit mass) (J K^{-1} kg^{-1})
D_{th}	thermal diffusivity
Δ	change
ΔH_f	heat of fusion
ΔT	temperature difference
ΔV	voltage difference (V)
d	density (kg m^{-3})
dQ_x / dt	rate of heat flow through an area, A
dT / dx	temperature gradient
δ	small change, an incremental change
$E_{\text{breakdown}}$	dielectric breakdown field (V m^{-1})
EM	electromagnetic
ε	emissivity (no units)
h	coefficient of convective heat transfer (W m^{-2} °C^{-1})
I	current flow (A)
J	electrical current density I / A
J_x	heat flux
κ	thermal conductivity (W m^{-1} K^{-1} or W m^{-1} °C^{-1})
L	length (m)
ℓ	mean free path of the electrons; thickness (m)
m	mass (kg)
P_d	power dissipated (W)
Q'	rate of heat flow ("heat current") (W)
q'	rate of heat generation per unit volume (W m^{-3})
q'_{loss}	rate of heat loss per unit surface area due to thermal radiation or convection (W m^{-2})
θ	thermal resistance (°C/W)
R	resistance (Ω)
r	radius (m)
ρ	density; resistivity (Ω m)
ρ_o	resistivity at the reference temperature (Ω m)
σ	electrical conductivity (Ω^{-1} m^{-1})
σ_S	Stefan's constant (= 5.6×10^{-8} W m^{-2} K^{-4})
T	temperature
T_c	temperature of the outer conductor
T_f	final temperature
T_i	temperature of inner conductor
T_o	ambient temperature; outside temperature; reference temperature
t	time
u	mean speed of the electrons (m s^{-1})
W	width (m)

USEFUL DEFINITIONS AND TERMS

Black body is a hypothetical (ideal) body that absorbs all the electromagnetic radiation falling onto it and therefore appears to be black at all wavelengths. When heated, a black body emits the maximum possible radiation at that temperature. A small hole in the wall of a cavity maintained at a uniform temperature emits radiation that approximately corresponds to that from a black body.

Buoyancy is the reduction in the weight of a body when it is immersed in a fluid such as air, water etc. If the body is less dense than the surrounding fluid, then the object experiences a buyoancy "force" that lifts it "up". *Archimedes principle* states that the buoyant force is equal to the weight of the displaced fluid. The buoyant force has a line of action through the center of mass of the displaced fluid. Hot air rises due to the Archimedes principle because its density is less than cold air. Hot air balon work on Archimedes principle.

Convection is transfer of heat from the surface of a body by motions of fluid atoms adjacent to the surface.

Fourier's Law states that the rate of heat flow through a sample, due to thermal conduction, is proportional to the temperature gradient (dT/dx) and the cross-sectional area (A) *i.e.* if Q' is the rate of heat flow, then $Q' = -\kappa A(dT/dx)$ where κ is the thermal conductivity.

Heat is the amount of energy that is transferred from one system to another (or between the system and its surroundings) as a result of a temperature difference. It is not a new form of energy but rather the transfer of energy from one body to another by virtue of the random motions of their molecules. When a hot body is in contact with a cold body, heat is transferred from the hot to the cold body. What is actually transferred is the excess mean kinetic energy of the molecules in the hot body. Molecules in the hot body have higher kinetic energy and vibrate more violently and, as a result of the collisions between the molecules, there is a net transfer of energy from the hot to the cold body until the molecules in both bodies have the same mean kinetic energy; when their temperatures are the same.

Heat reservoir is a body that can source or sink heat without any change in its temperature. Typically this would be a large body that can easily absorb heat from a hotter body in contact, or provide heat to a colder body in contact. The atmosphere, oceans, rivers, lakes *etc.* are normally considered as practical heat reservoirs though their temperatures may not remain constant if the exchange of heat is not small.

Joule's Law relates the power dissipated per unit volume P_{volume} in a current carrying conductor to the applied field, E, and the current density J so that $P_{\text{volume}} = JE = \sigma E^2 = \rho J^2$, where σ is the conductivity and ρ is the resisitivity.

Laminar flow is a type of flow in which the fluid particles move parallel to each other so that the fluid flow can be viewed as one layer sliding over an adjacent layer. For example, if we were injected a thin red-dye stream into a laminar flow, this stream line would continue to move in a thin line.

Mole of a substance is that amount of the substance which contains N_A number of atoms (or molecules) where N_A is Avogadro's number (6.022×10^{23}). One mole of a substance has a mass as much as its atomic (molecular) mass in grams. For example 1 mole of copper contains 6.022×10^{23} number of copper atoms and has a mass of 63.55 grams.

Phonon is an allowed traveling elastic wave, with a quantized energy, due to the vibrations of the atoms in a crystal, analogous to the photon which is a traveling electromagnetic wave with a quantized energy. More strictly, it is a quantum of energy associated with the vibrations of the atoms in the crystal.) A phonon has an energy of $\hbar\Omega$ where Ω is the frequency of the lattice vibration constituting the phonon, and $\hbar = h/2\pi$, h = Planck's constant.

Steady state flow occurs whenever the local velocity in a flowing fluid, that is the velocity at some point, does not change with time. The velocity may however depend on the position, or distance from the surface of a body. When fluid flow is time dependent, it is said to be unsteady.

Stefan's Law is a phenomenological description of the energy radiated (as electromagnetic waves) per unit second from a surface. When a surface is heated to a temperature T, then it radiates net energy at a rate given by $P_{\text{radiated}} = \varepsilon\sigma_S S(T^4 - T_o^4)$ where σ_S is Stefan's constant ($= 5.67 \times 10^{-8}$ W m^{-2} K^{-4}), ε is the emissivity of the surface, S is the surface area and T_o is the ambient temperature.

Temperature coefficient of resistivity, TCR (α_o) is defined as the fractional change in the electrical resistivity of a material per unit increase in the temperature at some reference temperature T_o. If ρ_o and α_o are the resistivity and TCR at the reference temperature T_o, then the resistivity at temperature T is $\rho = \rho_o[1 + \alpha_o(T - T_o)]$.

Thermal conductivity (κ) is a property of a material that quantifies the ease with which heat flows along a material from higher to lower temperature regions. Since heat flow is due to a temperature gradient, κ is the rate of heat flow across a unit area per unit temperature gradient.

Thermal resistance (θ) is a measure of the difficulty with which heat transfer (Q') takes place across a temperature difference ΔT. It is defined as the temperature drop per unit heat flow, $\theta = \Delta T / Q'$. If heat is transferred easily along unit temperature drop, Q' will be high and θ will be small; little difficulty in transferring heat. For thermal conduction in which heat losses from the surfaces are negligible, then $\theta = L / \kappa A$ where L is the length of the sample (along heat flow), κ is the thermal conductivity, and A is the cross sectional area. Thermal conduction controlled θ depends on both the material and its geometry. For convective heat transfer from a surface, the effective thermal resistance, $\theta = 1 / hS$ where h is the convective heat transfer coefficient and S is the surface area. Thermal resistance is a useful concept in thermal problems in which it can be taken as independent of the temperature to enable Q' and ΔT to be linearly related by $Q' = \Delta T / \theta$.

Turbulent flow in a flowing fluid occurs when the flow velocity is so high that the layers of flowing fluid do not move parallel to each other in contrast to laminar flow in which all fluid particles have parallel velocities. In turbulent flow, the fluid particles move in irregular paths that typically generate local circular flow paths. The fluid flow in a thin layer, called the *boundary layer*, next to a wall is laminar but outside this thin boundary layer away from the wall, the flow becomes trubulent if the flow velocity is high. Onset of turbulent flow depends on the fluid velocity, fluid properties and the geometry of the channel carrying the flowing fluid.

Questions and Problems

1. **Thermal conduction** A brass disk of thermal conductivity 147 W m^{-1} K^{-1} is conducting heat from a heat source to a heat sink at a rate of 10 W. If its diameter is 20 mm and its thickness is 30 mm, what is the temperature drop across the disk neglecting the heat losses from the surface?

[Ans: 6.5 °C]

2 **Wiedemann-Franz-Lorenz law** Given the electrical and thermal conductivities of various metals, plot κ vs σ and hence find an experimental value for the Wiedemann-Franz-Lorenz coefficient C_{WFL}.

Metal	Ag	Ag-3Cu	Ag-20Cu	Al	Au	Be	Brass	Bronze	Cu
σ (nΩ m)	62	52	49	37.5	44	31.3	16.1	10	58
κ (W m^{-1} K^{-1})	427	372	335	237	297	200	120	80	394
Metal	Hg	Mg	Mo	Ni	Pd-40Ag	Pd-40Ag	Steel (1080)	W	
σ (nΩ m)	1.04	22.2	19	14.5	2.4	2.4	5.56	18	
κ (W m^{-1} K^{-1})	8.34	160	142	100	29.3	29.3	46	167	

3. **Thermal resistance** Consider a thin insulating disc made of mica to electrically insulate a semiconductor device from a conducting heat sink. Mica has κ = 0.75 W m^{-1} K^{-1}. The disk thickness is 0.1 mm, and the diameter is 10 mm. What is the thermal resistance of the disk. What is the temperature drop across the disk if the heat current through it is 25 W.

4. **Thermal resistance** An epoxy encapsulated IC (*i.e.* the semiconductor chip is plastic molded) is bonded to a heat sink using a thermal paste (heat sink bond). Suppose the IC area is 2 cm $\times$ 1 cm and the heat sink epoxy thermal conductivity is 1 W m^{-1} K^{-1}. What is the thermal resistance of the heat sink bond if the thickness of the bond is 100 μm?

5. **Thermal conduction through the insulation in a coaxial cable** Consider a coaxial cable that has a copper core conductor and polyethylene (PE) dielectric with the following properties: Core radius, a = 4 mm, dielectric thickness, $b - a$ = 3.5 mm, dielectric thermal conductivity κ = 0.27 W m^{-1} K^{-1}. The resisitivity ρ_o and TCR (α_o) of copper at 0 °C are 16 nΩ m and 4.31 $\times$ 10^{-3} C^{-1}. The outside temperature T_o is 30 °C. The maximum inner temperature is not to exceed 60 °C. What is the maximum dc current that this cable can carry?

6. **Thermal conduction through double insulation in a coaxial cable** Consider a coaxial cable that has an aluminum core conductor with a resistivity ρ_0 = 25 nΩ m and a TCR α_0 = 1/233 °C^{-1} both at 0 °C The core diameter is 7 mm. The dielectric insulation consists of two different concentric layers of polymer insulation. First polymer layer next to the inner core has a thickness of 1.5 mm and the second polymer layer, between the first layer and the outside conductor, has a thickness of 2 mm. The thermal conductivities of the first and the second layers are of 0.24 W m^{-1} K^{-1} and 0.32 W m^{-1} K^{-1} respectively. The coaxial cable is carrying a dc current of 250 A. Assume that the outer conductor is at the ambient temperature which is 25 °C. What is the temperature of the inner conductor?

(Ans: 44.5 °C)

7. **Heat sinks in electronics** A silicon power transistor with maximum power rating of 20 W has a maximum junction temperature of 130 °C. (Maximum rated power refers to the transistor case at 25 °C.)

a What is the junction-to-case thermal resistance and the required thermal resistance between the case and the ambient for operation at maximum rated power if the ambient temperature is 20 °C?

b What is the required thermal resistance from case to ambient if the transistor is to be operated at 15 W with the ambient temperature at 25 °C?

c The transistor is mounted on a heat sink using a mica washer (an electrical insulator) and thermal paste (grease). The heat sink has a thermal resistance of 2.3 °C/W. The thermal resistance of the mica and the thermal paste together is 0.4 °C/W. The power amplifier circuit using this transistor is to be placed in a box (or cabinet) in which the maximum ambient temperature is expected to reach as high as 50 °C. What is the maximum power that can be dissipated by the transistor? Given that we have to use this transistor and the ambient temperature cannot be altered, what is the theoretical maximum power than can be dissipated?

8. Thermal time constant Consider the thermal circuit of a heat sinked transistor that is shown in Figure 9. Assume that C_j is small and C_c and C_s are large so that while the temperature of the junction T_j is rising, T_c remains close to the ambient temperature T_o. By considering the heat flow Q' from the junction into C_s and also into θ_{jc} show that

$$T_j \approx T_o + \theta_{jc}Q'[1 - \exp(-t/\theta_{jc}C_j)]$$

Plot this function taking reasonable values: T_o = 25 °C, Q' = 10 W, and approximately, $\theta_{jc} \approx 10$ °C/W, and $C_j \approx 0.01$ J K^{-1}.

9. Heat transfer by radiation

a Consider 100W, 120V incandescent General Electric bulb (lamp). The tungsten filament is of length 0.579 m and diameter 63.5 μm. Its resistivity at room temperature is 56.5 nΩ m. Given that the resistivity of the filament can at temperature T (K) be represented as

$$\rho = \rho_o\left(\frac{T}{T_o}\right)^n$$

where T is the temperature in K ρ_o is the resistivity at a temperature T_o (K), and n = 1.2, estimate the temperature of the bulb when it is operated at the rated voltage, *i.e.* directly from the mains outlet. Note that the bulb dissipates 100 W at 120 V.

b The emissivity of the tungsten surface is 0.35. Calculate the temperature of the filament using the temperature dependence of the resistivity and also using Stefan's radiation law and the compare the two values.

10. Heat transfer by radiation Consider a 60W, 120V incandescent General Electric light bulb. The tungsten filament is of length 0.533 m and diameter 45.7 μm. Tungsten has a density d = 19300 kg m^{-3}, specific heat capacity c = 130 J kg^{-1} K^{-1}; resistivity at room temperature ρ_o = 56.5 nΩ m, temperature coefficient of resistivity (TCR) α = 0.005 °C^{-1}. What is the time it takes for the filament to reach the operating temperature? Recalculate this time by using the fact that the resistivity is proportional to $T^{1.2}$.

11. Heat transfer by radiation Consider a 60W, 120V incandescent General Electric light bulb. The tungsten filament is of length 0.533 m and diameter 45.7 μm. Tungsten has a density d = 19300 kg m^{-3}, specific heat capacity c = 130 J kg^{-1} K^{-1}; resistivity at room temperature ρ_o = 56.5 nΩ m, and ρ is proportional to $T^{1.2}$ where T is in Kelvins. If the melting temperature of W is 3680

K, what is the voltage that guarantees the blowing of the bulb? How long will it take to blow the bulb if it is accidentally connected to a 350 V supply?

12. **Radiation theory of the electrical fuse:** Consider the principle of operation of an electrical fuse, such as that schematically depicted in Figure 23. As current is passed through the fuse, electrical energy is released in the wire by Joule heating Part of this electrical energy escapes from the surface by radiation (Stefan's Law), and the remaining energy, the net energy released, increases the heat content (internal energy) of the wire, that is, it increases the temperature by an amount determined by the mass m and the specific heat capacity (heat capacity per unit mass) c of the specimen. The final (eventual) temperature T_f of the wire is determined by the steady-state condition:

$$I^2R = \varepsilon\sigma_S S(T_f^4 - T_0^4) \qquad \textit{Steady state} \qquad (1)$$

where, ε is the emissivity of the surface, S is the surface area of the wire, and T_0 is the room temperature (293 K). If T_f is greater than the melting temperature T_m, the fuse metal will melt and break the circuit.

During a small time interval dt, the current I through the fuse wire will release an energy I^2Rdt as heat. Some of this energy will escape from the surface by radiation, and the remainder will raise the temperature of the wire by an amount dT that depends on the mass m and specific heat capacity c of the metal. Thus, energy balance during time dt requires

$$mcdT = \left(I^2R\right)dt - \left[\varepsilon\sigma_S S\left(T^4 - T_0^4\right)\right]dt. \qquad \textit{Energy balance} \qquad (2)$$

a Show that the "minimum current" that will blow the fuse is given by

$$I_{min} = \sqrt{\frac{2\pi^2r^3\varepsilon\sigma_S\left(T_m^4 - T_0^4\right)}{\rho_0\left[1 + \alpha_0\left(T_m - T_0\right)\right]}}$$

where T_m is the melting temperature of the fuse metal, r is the radius of the wire, ρ_0 is the resistivity at T_0 and α_0 is the temperature coefficient of resistivity at T_0.

b If the current I (> I_{min}) is constant, and Δt_m is the time taken to reach the melting temperature T_m of the fuse material, show that

$$\Delta t_m = dc\pi^2r^4 \int_{T_0}^{T_m} \frac{dT}{I^2\rho_0\left[1 + \alpha_0(T - T_0)\right] - 2\pi^2r^3\varepsilon\sigma_S\left[T^4 - T_0^4\right]} \qquad (3)$$

c Based on Eq. (3), sketch schematically the expected dependence of t_m on the current I through the fuse wire.

d At the melting temperature T_m, the metal absorbs an amount of heat per mole called the *heat* (or *enthalpy*) *of fusion*, ΔH_f (J mol^{-1}). During melting, the temperature remains constant at T_m. The net power

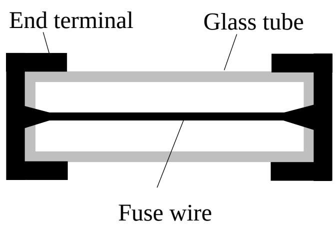
$$I^2R - \varepsilon\sigma_S S(T_m{}^4 - T_o{}^4)$$

goes to supply the heat of fusion. Show that the time t_{melt} it takes to melt the fuse at T_m is

$$t_{melt} = \frac{\left(\pi^2r^4d/M_{at}\right)\Delta H_f}{I^2\rho_o\left[1 + \alpha_o\left(T_m - T_o\right) - 2\pi^2r^3\varepsilon\sigma_S\left(T_m{}^4 - T_o{}^4\right)\right]} \qquad (3)$$

e Consider a Sn fuse wire whose properties are listed below. Suppose that the wire is 1 mm thick. What is the total time t_{fuse} to blow the fuse if complete melting is required.

f What are the assumptions and limitations of the radiation theory of the fuse? What are the limitations of the given fuse theory? When would you expect the theory to fail?



A schematic diagram of a typical fuse for an electronic appliance. The fuse wire is contained in a glass tube and attached to rigid end terminals.

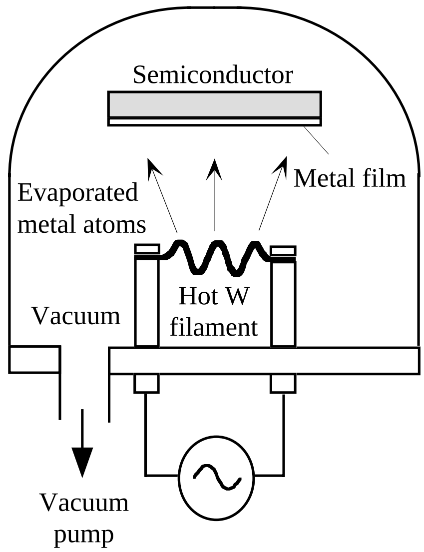
Figure 23

Properties of Sn: Resistivity, ρ = 126 nΩ m; TCR, α_0 = 0.0047 K^{-1}; density d = 7.30 g cm^{-1}; specific heat capacity, c= 230 J K^{-1} kg^{-1}; melting temperature, T_m = 505 K, emissivity, ε = 0.05; ΔH_f = 7.2 kJ mol^{-1}.

13. Vacuum deposition of metals by evaporation in microelectronics Deposition of a metal electrode onto a semiconductor crystal usually involves the evaporation of a metal in a vacuum chamber as shown in Figure 24. A tungsten (W) filament is heated to a very high temperature by passing a current I. Small wires of the electrode metal, such as gold, are wrapped around the filament and melt when the filament temperature is sufficiently high. These metal atoms evaporate from the melt on the hot filament to deposit onto the semiconductor that is held on top of the filament. The evaporated atoms travel in straight lines from the filament to the semiconductor, which requires a vacuum to avoid collisions with air molecules. Consider a particular tungsten filament that has a diameter of 1.5 mm and a length of 0.20 m (20 cm). The resistivity of tungsten at temperature T (K) is given by

$$\rho = \rho_o \left[\frac{T}{T_o} \right]^n \quad ; \quad n = 1.2$$

where ρ_o is the resistivity at the reference temperature T_o (K).



Vacuum deposition of metal electrodes by thermal evaporation.

Figure 24

The resistivity of W at room temperature (25 °C) is 5.7×0^{-8} Ω m. Tungsten has an emissivity ε = 0.33. Calculate the minimum current $I_{\min}$ required to take the filament temperature to the melting temperature of gold, 1064 °C. What is current for melting Pt, which has a melting temperature of 1770 °C? What are your assumptions?

14. **Convective thermal resistance of a vertical plate** Consider a vertical plate that has the dimensions of width = 20 cm, length = 40 cm. Suppose that the ambient temperature is 25 °C. Calculate the effective thermal resistance of this plate when its surface temperature is 100 °C and when it is 150 °C and it is being cooled by convection.

15. **Convective thermal resistance of a vertical surface** The heat transfer coefficient h for convective heat loss from a vertical surface that has a vertical height of d_{vertical} less than 1 m is approximately given by

$$h \approx 1.34\left(\frac{\Delta T}{d_{\text{vertical}}}\right)^{0.25}$$

Consider a vertical thin plate that has the dimensions of width = 30 cm, vertical height = 15 cm. Suppose that the ambient temperature is 25 °C. Calculate the effective thermal resistance of this plate when its surface temperature is 100 °C and when it is 150 °C.

16. **Convection theory of the electrical fuse:** Consider a bare metal wire carrying a constant current I. Let r be the radius and L the length of the wire. The electrical power dissipated in the wire as heat is I^2R. We will assume that, as the wire is bare in air, it will cool by natural convection (Newton's law of cooling) so that the *rate* of heat loss from the surface is given by

$$P_{\text{convection}} = hS(T - T_o) \qquad \textit{Newton's law of cooling} \qquad (1)$$

where h is the convection coefficient, S is the surface area of the wire, $2\pi rL$, T is the temperature of the wire and T_o is the temperature of the ambient.

a Under steady state conditions, the wire will reach an eventual temperature T_f such that the electrical power dissipated in the wire is carried away by convection, *i.e.* $I^2R = P_{\text{convection}}$. Show that the "minimum current", $I_{\min}$, that will just melt the wire (over "infinite time") is given by

$$I_{\min} = \sqrt{\frac{2\pi^2 r^3 h\left(T_m - T_o\right)}{\rho_o\left[1+\alpha\left(T_m - T_o\right)\right]}} \qquad \textit{Minimum current for fusing} \qquad (2)$$

where T_m is the melting temperature of the metal.

b During a small time interval dt, some of the electrical energy released in the metal will be lost by convection and the remainder will increase the heat content of the wire by $mcdT$ where m is the mass and c is the specific heat capacity of the wire. Thus energy balance demands that

$$mcdT = \left(I^2R\right)dt - \left[hS\left(T - T_o\right)\right]dt. \qquad \textit{Energy balance} \qquad (3)$$

Show that the time Δt_m required to reach the melting temperature is,

$$\Delta t_m = \frac{dc\pi^2 r^4 \ln\left[1 + (T_m - T_o)\left(\alpha - \dfrac{2hr^3\pi^2}{I^2\rho_o}\right)\right]}{I^2\alpha\rho_o - 2\pi^2 r^3 h} \qquad (3)$$

Sketch schematically the expected dependence of Δt_m on the current.

c A layman decides to replace a 15 A blown fuse with a thin copper wire across the fuse terminals. He decides to use a 1 mm thick Cu wire based on his intuition. What is the minimum current to fuse this bare wire? How long will it take to melt the wire if an overload of 20 A occurs? Is this sufficiently quick to save an equipment that can only withstand an overload of 20 A for 30 seconds? Assume an average value of h of about 15 W m^{-2} °C^{-1}.

d What are major assumptions and hence the limitations of the convection theory of the electrical fuse?

[Properties of copper at 20 °C, resistivity, $\rho = 17$ nΩ m; TCR, $\alpha = 0.004\text{C}^{-1}$, density, d = 8.9 g cm^{-3}, specific heat capacity c = 380, J kg^{-1} K^{-1}, melting temperature T_m = 1357.6 K].

17. **Coaxial cable cooled by convection** Consider a coaxial cable that has an aluminum core conductor with a resistivity ρ = 23 nΩ m at 0 °C and a thermal coefficient of resistivity 0.0049 K^{-1} at 0 °C. The core diameter is 6 mm. The dielectric insulation consists of a polymer insulation of thickness 3.5 mm. The thermal conductivity of the insulation layer is of 0.25 W m^{-1} K^{-1}. The outside (ambient) temperature T_o is maintained 25 °C. Ambient convection is sufficiently strong (h very large, *i.e.* $h \to \infty$) to maintain T_o at about 25 °C. It is known that the polymeric insulation breaks down when the temperature reaches about 65 °C What is the maximum current that can be carried by this coaxial cable? Sketch the temperature vs. distance profile from the inner conductor to the outside conductor. (Note: this is a sketch not a plot.) How would be the maximum current if the surface of the coaxial cable was convectively cooled with a convective heat transfer coefficient h of 25 W m^{-2} K^{-1}?

(Ans: 275 A with $h = \infty$; 159 A with h = 25)

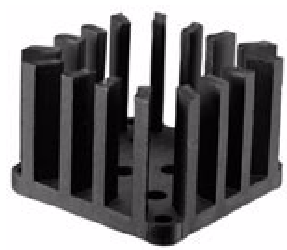
18. **Coaxial cable cooled by convection** Consider an underwater coaxial cable that has a copper core conductor; the resisitivity ρ_o and TCR (α_o) of copper at 0 °C are 16 nΩ m and 4.31 × 10^{-3} C^{-}

[1]. The core radius is 5.5 mm. The dielectric insulation consists of two different concentric layers of polymer insulation. First polymer layer next to the inner core has a thickness of 2.5 mm and the second polymer layer, between the first layer and the outside conductor, has a thickness of 2 mm. The thermal conductivities of the first and the second layers are of 0.3 W m^{-1} K^{-1} and 0.25 W m^{-1} K^{-1} respectively. The outside (ambient) temperature T_o is 25 °C and the fluid convection from the surface of the cable has a convective heat transfer coefficient h of about 400 W m^{-2} K^{-1}. The coaxial cable is carrying a current of 700 A. What is the temperature of the inner conductor?

TO3 Heat Sinks. All black anodized Al.

(Source: http://aavidthermally.com)

3.2 °C/W



4.7 °C/W

6 °C/W

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